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Galileo’s Precursors

Translation of *Studies on Leonardo da Vinci* (vol. 3)*

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*Duhem (1906)

To my wife, who made this translation possible, to the Blessed Virgin Mother, and to the Holy Trinity, Who makes all things possible

Foreword

Note on the Translation

Everything translated from the original French of Duhem (1906). Additions in [brackets] belong to the translator.

Alan Aversa

June 29, 2018

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Preface

To the third series of our *Studies on Leonardo da Vinci*, we give a subtitle: *Galileo's Parisian Precursors*. This subtitle announces the idea of which our previous studies had already discovered a few aspects and which our new researches place in full light. The Science of mechanics inaugurated by Galileo—by his followers, by his disciples Baliani, Torricelli, Descartes, Beeckman, Gassendi—is not a creation; modern intelligence did not produce it from the very start and from all the pieces which reading the art of Archimedes of applying geometry to natural effects had revealed to it. Galileo and his contemporaries used the mathematical skill acquired in the trade of the geometers of Antiquity to clarify and develop a mechanical Science of which the Christian Middle Ages had posed the principles and the most essential propositions. The physicists who taught in the 14th century at the University of Paris had designed this Mechanics by taking observation as their guide; they had substituted it for the Dynamics of Aristotle, convinced of its ineffectualness to "save the phenomena." At the time of the Renaissance, the superstitious archaism, where the wit of the Humanists and the Averroist routine of a retrograde Scholasticism are complacent, rejected this doctrine of the "Moderns." The reaction was strong, particularly in Italy, against the Dynamics of the "Parisians," in favour of the inadmissible dynamics of the Stagirite. But, despite stubborn resistance, the Parisian tradition found, outside schools as well as in Universities, masters and scholars to maintain and develop it. It is of this Parisian tradition that Galileo and his followers were heirs. When we see the science of Galileo triumph over the obstinate Peripateticism of Gremonini, we believe, uninformed in the history of human thought, that we are witnessing the victory of the young modern Science over the medieval Philosophy, obstinate in its psittacism; in truth, we behold the long-prepared triumph of the science that was born in Paris in the 14th century on the doctrines of Aristotle and Averroes, revived by the Italian Renaissance.

No movement can perdure if it is not maintained by the continuous action of a motive power, directly and immediately applied to the mobile. This is the axiom on which the whole Dynamics of Aristotle rests.

According to this principle, the Stagirite wants to apply to the arrow that continues to fly after leaving the bow a motive power which carries it; he believes this power is found in the disturbed air; it is the air, hit by the hand or by the ballistic machine, which supports and guides the projectile.

This assumption, which seems to push its improbability even to ridicule, seems to have been admitted almost unanimously by the physicists of Antiquity; one of them spoke out clearly against it, and he, in the last years of Greek philosophy, is, by his Christian faith, almost separated from this Philosophy; we have named John of Alexandria, nicknamed Philoponus. After showing what was unacceptable in the Peripatetician theory of the motion of projectiles, John Philoponus declares that the arrow continues to move without any mover applied to it, because the rope of the bow has created an *energy* that plays the role of the motive force.

The last thinkers of Greece and even Arab philosophers failed to mention the doctrine of this John the Christian, for whom Simplicius or Averroes had only sarcasm. The Christian Middle Ages, taken by the naïve admiration which inspired the peripatetic Science when it was revealed, shared first, with respect to the assumption of Philoponus, the disdain of the Greek and Arab commentators; St. Thomas Aquinas only mentions it to warn those it could seduce.

But following the condemnations in 1277 by the Bishop of Paris, Étienne Tempier, against a group of theses that supported "Aristotle and those of his suite," a great movement emerges, which will release the Christian thought from the yoke of Peripateticism and Neoplatonism and produce what the archaism of the Renaissance called the Science of the "Moderns."

William of Ockham attacks, with his customary vivacity, the theory of the motion of projectiles proposed by Aristotle; he is content, besides, to simply destroy without building anything; but his critics, with some followers of Duns Scotus, restore the honor of the doctrine of John Philoponus; *energy*, the motive force of which he had spoken, reappears under the name of *impetus*. This hypothesis of the *impetus*, impressed in the projectile by the hand or by the machine which lauched it, is seized upon by a secular master of the Faculty of Arts in Paris, a physicist of genius; Jean Buridan, toward the middle of the 14th century, takes it as the foundation of a Dynamics with which "all the phenemena agree."

The role that the *impetus* plays in the Dynamics of Buridan is very exactly what Galileo will assign to the *impeto* or *momento*; Descartes to the *quantité de mouvement*; and finally Leibniz to the *living force*. So exact is this correspondance that, for explaining in his *Academic Lessons* the Dynamics of Galileo, Torricelli will often take up the reasoning and almost the very words of Buridan.

This *impetus*, which would remain unchanged within the projectile if it were not incessantly destroyed by the resistance of the medium and by the action of gravity contrary to the movement, Buridan takes, at equal speed, as proportional to the *amount of primary matter* that the body contains; he conceived this quantity and described in terms almost identical to those which Newton will use to define mass. At equal mass the *impetus* is as much greater as the speed is greater; prudently, Buridan refrains from further clarifying the relationship between the size of the *impetus* and its speed; more daringly, Galileo and Descartes would agree that this relationship is reduced to a proportionality; they will also obtain an erroneous assessment of the *impeto* and *momentum* which Leibniz will have to rectify.

Like the resistance of the medium, gravity reduces constantly and eventually destroys the *impetus* of a mobile that is launched upward, because such a movement is contrary to the natural tendancy of this gravity; but in a mobile that falls, the movement is in line with the trendancy of gravity; the *impetus* also must be constantly increasing, and the speed, in the course of the movement, must grow constantly. This is, according to Buridan, the explanation of the acceleration observed in the fall of a body, an acceleration that the science of Aristotle already knew, but for which the Hellenic commentators of the Stagirite, Arabs or Christians, had given unacceptable reasons.

This Dynamics expressed by Jean Buridan presents in a purely qualitative but always exact way the truths that the notions of live force and work allow us to formulate in quantitative language.

The philosopher of Béthune is not alone in professing this Dynamics; his most brilliant disciples, Albert of Saxony and Nicole Oresme, adopt it and teach it; the French writings of Oresme make it known even to those who are not clerics.

When no resistant medium and when no natural tendancy analogous to gravity is opposed to movement, the *impetus* maintains an invariable intensity; the mobile to which a movement of translation or rotation is applied continues to move with constant speed indefinitely. It is in this form that the law of inertia presents itself to the mind of Buridan; it is in this same form that Galileo will receive it

From this law of inertia, Buridan draws a corollary, the novelty of which we must now admire.

If the celestial orbs move eternally with a constant speed, it is, according to the axiom of the dynamics of Aristotle, because each of them is subject to an eternal mover of immutable power; the philosophy of the Stagirite requires that such a mover is an intelligence separate from matter. The study of the motive intelligences of the celestial orbs is not only the culmination of Peripatetic Metaphysics; it is the central doctrine around which all the Neoplatonic Metaphysics of the Greeks and of the Arabs revolve, and the Scholastics of the 13th century do not hesitate to receive, into their Christian systems, this legacy of pagan theologies.

Now, Buridan has the audacity to write these lines:

> From the creation world, God has moved the heavens with movements identical to those which currently move them; he impressed on them then an *impetus* by which they continue to be moved uniformly; these *impetus*, indeed, meeting no contrary resistance, are never destroyed nor weakened… According to this conception, it is not necessary to pose the existence of intelligences that move celestial bodies in an appropriate manner.

Buridan stated this thought in various circumstances; Albert of Saxony explains it in turn; and Nicole Oresme, to formulate it, finds this comparison: “Except for violence, it is in no way similar to when a man has made a clock i and lets it go to be moved by itself.”

If one wanted, by a precise line, to separate the reigns of the ancient Science of the reign of modern Science, it would have to be drawn, we believe, at the moment when Jean Buridan developed this theory, at the moment when one stopped looking at the stars as moved by divine beings and when it has been admitted that the celestial and sublunary movements depend on the same mechanics.

This Mechanics—both heavenly and earthly, to which Newton had to give the shape that we admire today—is, besides, that which, from the 14th century, is trying to be built. During this century, the testimonies of Francis of Meyronnes and of Albert of Saxony teach us, one finds physicists upholding that by supposing the earth as moving and the fixed stars as immobile, an astronomical system more satisfactory than that where the earth is deprived of movement would be constructed. Of these physicists, Nicole Oresme developed the reasons with a fullness, clarity, and precision that Copernicus will be far from reaching; to the earth he attributes a natural *impetus* similar to what Buridan attributed to the celestial orbs; to account for the vertical fall of bodies, he admits that one must compose this *impetus* by which the mobile revolves around the Earth with the *impetus* generated by gravity. The principle that he lucidly formulates, Copernicus will simply indicate in a dark way, and Giordano Bruno will repeat it; Galileo will use Geometry to draw the consequences, but without correcting the wrong form of the law of inertia with which he is implicated.

While Dynamics was being established, little by little the laws that govern the fall of bodies are discovered.

In 1368, Albert of Saxony offers these two assumptions:

1. the speed of the fall is proportional to the time elapsed since is departure;
2. the speed of the fall is proportional to the path traversed.

Between these two laws, there is no choice. The theologian Pierre Tataret, who taught in Paris towards the end of the 15th century, reproduced verbatim what Albert of Saxony said. The great reader of Albert of Saxony, Leonardo da Vinci, after admitting the second of these two hypotheses, endorses the first; but he fails to discover the law of the spaces traversed by a falling body; from a reasoning that Baliani will resume, he concludes that the spaces traversed in equal and successive periods of time are as the series of integers, whereas they are, in truth, as the series of odd numbers.

However, the rule which allows the evaluation of traversed space, in a certain time, by a mobile moved by an evenly varying movement, was known for a long time; that this rule was discovered at Paris, in the time of Jean Buridan, or at Oxford, in the time of Swineshead, is clearly formulated in the book where Nicole Oresme poses the essential principles of analytic geometry; in addition, the demonstration he employed to justify it is identical to what Galileo will give.

From the time of Nicole Oresme to that of Leonardo da Vinci, this rule was not forgotten; formulated in the majority of treatises produced by the subtle Dialectic of Oxford, it is discussed in the many commentaries to which these treatises had been subjected, during the 15th century in Italy, then in various books of Physics composed at the beginning of the 16th century by the Parisian Scholastics.

None of the treaties of which we have just spoken contains, however, the idea of applying this rule to falling bodies. We meet this idea for the first time in the *Questions on the Physics of Aristotle*, published in 1545 by Domingo de Soto. A student of Parisian Scholasticism, of which he was the patron and from which he adopts most of his physical theories, the Spanish Dominican Soto admits that the fall of a body is uniformly accelerated, that the vertical ascent of a projectile is uniformly retarded, and to calculate the path taken in each of these two movements, he correctly uses the rule formulated by Oresme. This is to say that he knows the laws of falling bodies whose discovery is attributed to Galileo. Moreover, he not only claims their invention; rather, he seems to give them as commonly received truths; without doubt, they were commonly admitted by the masters whose lessons Soto followed in Paris. Thus, from William of Ockham to Domingo de Soto, we see the physicists of the Parisian school posing all the foundations of the Mechanics that Galileo, his contemporaries, and his followers will develop.

Among those who, before Galileo, received the tradition of Parisian Scholasticism, there is none who deserves more attention than Leonardo da Vinci. At the time when he lived, Italy was opposed with a firm resistance to the penetration of the mechanics of the "*Moderni*," of the "Juniors;" there, among the masters of the Universities, those who looked to the terminist doctrines of Paris have been limited to reproduce, in an abbreviated and sometimes hesitant form, the essential claims of this Mechanics; they were quite far from producing any fruit of which it was the flower.

Leonardo da Vinci, on the contrary, is not content to admit the general principles of the Dynamics of the *impetus*; he ruminated on these principles constantly, turning in all directions, urging them, somehow, to give what they contained. The essential assumption of this Dynamics was like an early form of the law of the live force; Leonardo sees the idea of the conservation of energy, and he finds this idea, to express it, in terms of a prophetic clarity. Between two laws of falling bodies, the one exact and the other inadmissible, Albert of Saxony had left his reader in suspense; after some trial and error that Galileo, too, will know, Leonardo knew how to settle on the exact law; he happily extends it to the fall of a body along an inclined plane. Through the study of the compound *impeto*, he is the first to attempt to explain the curvilinear trajectory of projectiles, an explanation which will receive its completion from Galileo and Torricelli. He sees the correction that should be made to the law of inertia stated by Buridan and prepares the work that Benedetti and Descartes will carry out.

Doubtless, Leonardo did not always acknowledge all the riches of the treasure accumulated by Parisian Scholasticism; he left out some of them whose borrowing would had given his mechanical doctrine the most happy complement; he ignores the role of the *impetus* in the explanation of the accelerated fall of bodies; he ignores the rule that allows the calculation of the path traveled by a body with uniformly accelerated motion. It is no less true that his whole Physics is numbered among those that the Italians of his time called Parisian.

Such a title, moreover, would be justly given to him; indeed, he takes the principles of his physics from his assiduous reading of Albert of Saxony and probably also from his meditation on the writings of Nicolas of Cusa; but Nicolas of Cusa was, too, a follower of the mechanics of Paris. Leonardo is thus in his place among the Parisian precursors of Galileo.

Until recent years, the Science of the Middle Ages was considered non-existent. A philosopher, who admirably knows the history of Science in Antiquity and in modern times, once wrote[1]:

> Suppose that printing press had been discovered two centuries earlier; it would have helped strengthen the orthodoxy and served to propagate, outside of the *Summa* of St. Thomas and a few books of this kind, the bulls of excommunication and the decrees of the Holy Office.

Today, we believe, we are allowed to say:

> If the printing press had been discovered two centuries earlier, it would have published, gradually and when they were composed, the works which, on the ruins of the Physics of Aristotle, have laid the foundations of a Mechanics of which modern times are rightly proud.

This substitution of modern Physics for the Physics of Aristotle was the result of a long-term effort of extraordinary power.

This effort took support on the oldest and the most resplendent of medieval Universities, on the University of Paris. How could a Parisian not be proud of it?

Its most prominent promoters were the Picard, Jean Buridan, and the Norman, Nicole Oresme. How could a Frenchman not feel a legitimate pride?

It resulted from the stubborn fight that the University of Paris, true custodians, at that time, of the Catholic orthodoxy, led against the Peripatetic and Neoplatonist paganism. How could a Christian not give thanks to God for it?

The studies that follow have appeared either in the *Bulletin Italien* or in the *Bulletin Hispanique*; to Mr. G. Radet, Dean of the Faculty of Letters of Bordeaux, to our colleagues, Mr. E. Bouvy and Mr. G. Cirot, we are indebted for the generous hospitality given to our research; may they deign to receive the homage of our gratitude.

Bordeaux, *Pierre*
24 May 1913 *DUHEM.*

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[1] G. Milhaud, *Science grecque et Science moderne* (*Comptes rendus de l'Académie des Sciences morales et politiques*, 1904). — G. Milhaud, *Études sur la pensée scientifique chez les Grecs et les Modernes*, Paris, 1906, pp. 268-269.

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Chapter 1
A date concerning Master Albert of Saxony

The importance of the scientific writings of Albert of Saxony had passed completely unnoticed, in modern times, until the day Thurot, tracing the history of the principle of Archimedes, was brought to report it[1]. In this regard, the learned author stated that, under ms. n° 14723 of the Latin collection, the National Library has a copy of the *Subtilissimæ quæstiones in libros de Cælo et Mundo* composed by Albert; this copy, he said, is from the year 1378. On the basis of Thurot, we had reproduced this information in the study that we called: *Albert of Saxony and Leonardo da Vinci*[2]. However, as we will see, this information was incorrect.

The Administration of the National Library has kindly lent the manuscript cited by Wilson for three months to the Library of the University of Bordeaux; this kind act allowed us to examine very carefully the parts contained in this collection; it is from this review that the present study and the one that will follow it result.

The Latin manuscript 14723 of the National Library is a thick volume; it contains almost three hundred sheets of strong bond paper covered with two columns in a semicursive writing of the 15th century, often very fine, and where ligatures abound; it is bound in green parchment, and on the first plate the arms of the Abbey of Saint-Victor are struck. It is, in fact, from the Saint-Victor collection, where it was under no. 712.

On the front of the second sheet, at the bottom, we find the arms of the Abbey of Saint-Victor with this motto: *Ihs — Maria — S. Victor — S. Augustinus*. Below, reads this indication: *Tabulam hic contentorum reperies folio 270*.

Indeed, the front of folio 270 and the back carries a kind of table of contents which reads:

> *Que secuntur hic habentur, scilicet: Questiones totius libri phisicorum edite a Magistro Ioanne Buridam. 2. — Questione super totum librum de celo et mundo composite a Magistro Alberto de Saxonia. 113. — Questiones super tres primos libros metheororum et super*

[1] Ch. Thurot, *Recherches historiques sur le principe d'Archimède*. 3° article (*Revue archéologique*, nouvelle série, t. XIX; pp. 119-123).

[2] P. Duhem, *Albert de Saxe et Léonard de Vinci*, I (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, I; première série, p. 4).

> *majorem partem quarti a Magistro Jo. Buridam. 164. — X scilicet tercii nec continuit B quia frixata C. 269 et usque 272.*

The manuscript, besides, was mutilated again since the writing of this table of contents, because folios 260 through 269 have disappeared.

Folio 113 col. *a* of this manuscript begins, without any title, with the text mentioned by Wilson; on folio 162 col. *b* the same text ends, and this is the formula that ends it:

> *Et sic cum Dei adjutorio finite sunt questiones super totalem librum de celo et mundo per Magistrum Albertum de Saxonia juxta illa que didicit a Magistris suis. Parisius in facultate arcium anno Domini* $M^{o}C^{e}C^{e}C^{o}$*LXVIIJ.*

It is thus of the year 1368 that this text is dated, and not from the year 1378, as a copy or printing error made it read for Mr. Thurot.

But to what does this date refer? Does it, as Thurot thinks, refer to the work of a copyist? If it were so, the copyist who managed, in 1368, to transcribe the questions of Albert of Saxony cannot be the one whereby the manuscript was preserved at the National Library. The writing of this text clearly indicates the 15th century, and still more convincing evidence compels us to date the collection formerly owned by the Abbey of Saint-Victor in that era; the three pieces that make up this collection are clearly of the same hand, and in a following study we will do on the third of these parts will show us that it reproduces a writing of the 15th century.

Thus, if the date of 1368 is that of a copy, it is that of an old copy of which the manuscript kept in the National Library presents us a replica; the scribe to whom we owe this replica would have religiously preserved the words written by the original scribe.

This completely arbitrary hypothesis is rendered very unlikely by its very mentioning; it, indeed, traces back to the masters of Albert of Saxony the honor of the doctrines that are expressed in the *Quæstiones in libros de Cælo et Mundo*; it would seem very daring if the copyist were irreverent enough to strip the author whose work he reproduced of all personal credit; this case would be very rare, we believe, and perhaps unique in all the Middle Ages.

How this reference seems natural, on the contrary, if we attribute it to Albert of Saxony himself! We see, then, a proof of the modesty of the author and of the gratitude that he had for those whose lessons he took.

Besides, we know that Albert had these sentiments. Let us read the preface in our manuscript which begins the *Quæstiones in libros de Cælo et Mundo*; this preface, which all printed editions have reproduced, concludes:

> Secundum exigentiam istarum materiarum Domino concedente quasdam conscribam questiones super totalem librum Aristotelis antedictum. In quibus si quid minus bene dixero benigne correctioni melius dicentium me subjicio. Pro bene dictis autem non mihi soli sed magistris meis reverendis de nobili facultate arcium parisiensi qui me talia docuerunt peto dari grates et exhibitionem honoris et reverentie.

Is he who put this statement at the beginning of his book obviously not the same who, at the end, wrote the words falsely attributed to the copyist by Mr. Thurot? This statement is even signed by Albert of Saxony himself.

From this signature, it is clear that Albert wrote his *Quæstiones in libros de Cœlo et Mundo* in 1368 and that at that time he belonged to the Faculty of Arts of the University of Paris. A widespread opinion identifies Albert of Helmstedt, known as Albert of Saxony, with Albert of Ricmerstorp, who left Paris in 1365 to become the first rector of the University of Vienna. In another study[3], we showed everything implausible in this opinion; the documents contained in the *Chartularium Universitatis Parisiensis* in the *Liber procuratorum nationis Anglicanæ* allowed us to establish, we believe, that Albert of Helmstedt and Albert of Ricmerstorp were two separate characters. The text we studied leaves no doubt in this regard; in 1368, Albert of Helmstedt still belonged to the Faculty of Arts of the University of Paris, while at this time Albert de Ricmerstorp was, for two years, Bishop of Halberstadt.

[3] P. Duhem, *Albert de Saxe*, II (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*; VIII. Première série, pp. 327-331).

Chapter 2
Jean I Buridan (of Béthune)

At the beginning as in the end of his *Quæstiones in libros de Cælo et Mundo*, Albert of Saxony takes care to proclaim that he owes much to his masters; this commendable modesty is probably not without reasons; indeed, we believe that the teaching of Albert frequently reflects that he had been received "into the noble Faculty of Arts of the University of Paris." Is he, moreover, a master whose lessons are not in large part the echo of those he heard when he was only a student?

The admission of Albert poses a problem: Among the theories that he expresses in his various writings, which ones does he take from predecessors and which are, on the contrary, his own? In particular, when Leonardo da Vinci drew, to fuel his own thoughts, on the *Quæstiones in libros de Cælo*, were the doctrines that he collected even taken from their very source? On the contrary, do they come from elsewhere, and to discover their source must we go further back than Albert of Saxony?

Many times we tried to solve this problem, but the solution always remained very incomplete. To fully and certainly obtain it, the teaching published at the University of Paris when Albert came to sit on the benches of the Rue du Fouarre should be perfectly known. However, regarding this teaching, we are left with only a few records; the few books that conserve it, which have remained manuscripts or have been printed at the time of the Renaissance, are often almost impossible to find; only in the long run, at the price of much research and effort, did we recover the filiation of the main doctrines taught by Albertutius.

The manuscript that we described in the previous paragraph, by reproducing the *Quæstiones totius libri physicorum* of Jean Buridan, provides us with a document that is extremely important for restoring the teaching received by Albert; even a very quick comparison of this writing with the works of the German master is enough to recognize the profound influence that it exerted on the Picard Master. To the question "What does Albert of Saxony owe to his masters?" we will respond at length when we show what Albert owes to Buridan.

Certain data relating to the life of Jean Buridan are sparse; the fame of this philosopher is due, above all, to some dubious legends.

Buridan was born in Béthune; it is the affirmation of a tradition that is nothing but very likely, because many documents prove he was from the diocese of Arras.

His date of birth is unknown; it cannot, however, without large improbability, be placed after the year 1300. In 1327, indeed, Jean Buridan was already Rector of the University of Paris. It is in this capacity that he was called to establish, on 9 February 1328, a statute whose text is preserved[1]; students as well as teachers, for the most trivial reasons, cited before the Curia Conservationis of the University those with whom they were in dispute; to put an end to this abuse, it was decided that a letter of citation would be granted to the complainant after his appearance before the Rector and the University delegates; that statute ends with these words:

> Data fuerant hæc in nostra congregatione generali apud S. Mathurinum facta per venerabilem et discretam virum M. Joannem Buridan rectorem Universitatis supradictæ anno 1327[2] die Martis in octava Purificationis B. Mariæ Virginis.

On 30 August 1329, Jean Buridan, "cleric of the diocese of Arras," is still not equipped with ecclesiastical benefits[3]. But on 2 November 1330, we see[4] that, while continuing to reside in Paris, he is the titular of Illies in his diocese of origin.

Should we, under the pontificate of John XXII, place our philosopher on a journey to Avignon? This conclusion seems to follow from a passage[5] of the *Quæstiones in librum Aristotelis de sensu et sensato* that the Scot Georges Lokert published in Paris in 1516 and 1518 as the work of Jean Buridan. Here is this passage:

> I saw a certain Breton schoolboy who was blind from birth; however, he was talking very well and very clearly about Logic and Physics; I know he went to the Roman Curia, because I was there myself, in the time of Pope John; by the beautiful speech that he argued in front of the Cardinals, he obtained the provisions for his livelihood on the income of an Abbey.

The pontificate of John XXII lasted from 1316 to 1334. It would thus be no implausibility that Buridan had served him, in one of these missions that ensured a constant relationship between the University of Paris and the Pontifical Court. A difficulty arises, however; the passage quoted speaks of the *Curia Romana*, and John XXII was residing in Avignon; certainly, it could be argued that *Curia Romana* simply means the papal court, as it could be named even if it was at Avignon; but such impropriety of words is a bit suprising in the mouth of a master accustomed to the subtle details of Scholasticism; also, we never found the word *Curia Romana* in the many documents regarding the relations of the University with the popes of Avignon, as we read in the *Chartalariam Universitatis Parisiensis*; on the contrary, we

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[1] Bulaeus, *Historia Universitatis Parisiensis*, tomus IV, ab anno 1300 ad annum 1400, p. 212. — Denifle et Chatelain, Chartularium Universitatis Parisiensis, tomus II, sectio I, ab anno MCCLXXXVI ad annum MCCCL, piece n° 870, pp. 306-307.

[2] The year, at this time, began at Easter; this date thus corresponds to 9 February 1328, octave of the Purification.

[3] Reg. Vatican. Comm. Joh. XXII, an. XIII, p. 4, ep. 3169. — Cited by Denifle and Châtelain, *Chartularium Universitatis Parisiensis*, tomus II, sectio I, p. 307, en note.

[4] *Reg. Vatican. Comm. Joh. XXII, an. XIV*, p. 1, ep. 950. — Cited by Denifle and Châtelain, *lbid.*

[5] Joannis Buridani *In librum Aristotelis de sensu et sensato* quaest. III. (Quaestiones et decisiones insignium virorum Alberti de Saxonia, Thimonis, Buridani Parisius, per Jodocum Badium Ascensium et Conrardum Resch, MDXVI et MDXVIII, pars III, fol. XXX, col. *a*. — The description of this edition is found in our *Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, première série, p. 5, en note.)

meet this word frequently in the letters exchanged between the popes of Rome and the University.

We will see that the *Quæstiones in librum Aristotelis de longitudine et brevitate vitæ* that Georges Lokert, in the same editions, attributed to Jean Buridan, were certainly not of the philosopher of Béthune; we will be led, in a forthcoming Study, to assign them to a master who taught in Paris for the first quarter of the 15th century. Also, the questions on the various treaties of Aristotle called *Parva naturalia*, and also questions on the *De anima*, united under the name of Jean Buridan in various editions by Georges Lokert, form a very homogeneous style and doctrine; it is hard not to consider it the work of the same author. The *Quæstiones in librum de sensu et sensato* were thus drafted, no doubt, by the master who, in the 15th century, composed the *Quæstiones in librum de longitudine et brevitate vitæ*; the Pope John mentioned the first of these two writings is not John XXII, who resided in Avignon, but John XXIII, who spent several years in the *Curia Romana* where the University of Paris had with him nuncios in charge of incessant negotiations[6].

To find an authentic document concerning the philosopher of Béthune, we arrive at the year 1340; in that year, according to the *Livre des procureurs de la Nation Anglaise*[7], "Master Jean Brudan (*sic*), of the Picardy Nation," was again named rector of the University of Paris. On 19 June 1342, "while he taught at Paris the books of Physics, Metaphysics, and Morality," he was appointed Canon of Arras[8].

Several times rector, the Canon of Arras, master Jean Buridan, was certainly a very notable character of the University of Paris; an example that we borrow from Boulay[9] shows the esteem in which he was held.

In 1344, to deal with the expenses of the war against the English, Philippe VI of Valois created a tax on salt and the salt marshes. The gabelle was, from the outset, of extreme unpopularity. No one was exempt, not even the University. The University protested against this new charge. "On this occasion, master Jean Buridan, philosopher of great name and reputation, repeatedly named procurator of Picardy, to which he belonged, and twice elected rector of the Academy, was responsible for haranguing the king. But," Boulay adds, "we do not know what the outcome of this harangue was."

Of the great esteem in which master Jean Buridan was held, he would soon receive a new testimony.

In 1308, master Jehan of Thélu, doctor of law, had bequeathed a sum of money so that a chaplain could be at the Saint-André-des-Arcs church.

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[6] Denifle and Châtelain, *Chartularium Universitatis Parisiensis*, ann. 1410 seqq.; tomus IV, ab anno MCCCLXXXXIV ad annum MCCCCLII, pp. 183 seqq.

[7] Denifle and Châtelain, *Auctarium Chartularii Universitatis Parisiensis*; *Liber procuratorum Nationis Anglicanæ*, tomus I, ab anno MCCCXXXIII ad annum MCCCCVI, col. 41.

[8] *Peg. Comm. Clement.* VI, n° 149, fol. 376. — Cited by Denifle and Châtelain, *Chartularium Universitatis Parisiensis, tomus II, sectio I, p. 307*, in note.

[9] Bulaeus, *Historia Universitatis Parisiensis*, tomus IV, ab anno 1300 ad annum 1400, p. 282.

It is only on 22 November 1347 that the testamentary executors of Symon Vayret put the University in possession[10] of the sum bequeathed by Jehan of Thélu; the University immediately made it an obligation to comply with the will of the doctor of law; on 5 August 1348, it presented the "discretum virum Johannem Buridan, in magistrum artibus," to Faucon, Bishop of Paris, so that he would confer on him the title of chaplain of Saint-André-des-Arcs; on 10 October of the same year, Falcon ratified the choice of University[11].

Jean Buridan appears to us, moreover, as a zealous master to the core, always devoted to the interests of the University and, especially, the Picardy Nation. On 22 December 1347, he figures[12] among the masters who settle in a statute a series of measures, practical and financial, relating to the Nation. The roles given to the Pope at Avignon, on 22 May 1349, mention the name[13] of this master, not among the "*nichil actu habentes*" nor among the "*modicum habentes*," but among the "*secundum statum eorum et sufficientiam modicum habentes*;" they were the wealthiest masters.

Time, by prolonging the stay of master Jean Buridan at the University, only increased his reputation and the influence he exerted on his colleagues; in any delicate negotiation, he was the representative of the Picardy Nation.

On 19 February 1807, the English Nation, whose John of Mynda was then procurator, had to judge an embarrassing case[14]; a named John Mast, of the diocese of Liège, after suffering at the Picards the examination of determination (*l'examen de déterminance*), wished to go through the ordeal of the licentiate with the English. Master Themo, the son of a Jew, wanted this request to be rejected; the schoolboy was to stay consistently connected to his nation of birth; to which John Mast responded that Liège was no more Picard than Flemish. During the debate, two Picard masters stood, not as delegates of their nation, but as deprived of title and only as friends of Liégeois; their informal conference with the masters of the English Nation soon appeased the quarrel; John Mast was admitted, according to his request, to take the oath of the two Nations and to split among them the royalties that he had to pay. The two accommodating emissaries who received this transaction were named Johannes Juvenis and Jean Buridan.

The dispute that they had fortunately helped iron out was one of those that can be reproduced; to avoid a relapse, it was important that one decide with rigour the common border of the two nations. Approved by the procurator of the Picardy Nation, Buridan wrote a piece where such a demarcation was proposed; on 29 June 1357,

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[10] Denifle and Châtelain, *Chartularium Universitatis Parisiensis*, tomus II, sectio I, ab anno MCCLXXXI ad annum MCCCL, pièce n° 1155, pp. 619-620.

[11] All the parts related to this presentation, taken from *Livres des procureurs des Nations de Gaule et de Picardie*, are reproduced in: Bulæus, *Historia Universitatis Parisiensis*, tomus IV, ab anno 1300 ad annum 1400, pp. 303-308. — Denifle and Châtelain (*Chartularium Universitatis Parisiensis*, tomus II, sectio I, ab anno MCCLXXXVI ad annum MCCCL) reproducing the presentation of Jean Buridan that the University to Falcon, bishop of Paris Paris (piece n° 1156, pp. 621-622).

[12] Denille and Châtelain, *Chartularium. Universitatis Parisiensis*, tomus II, sectio I, p. 608, piece n° 1146

[13] Denifle and Châtelain, *Ibid.*, p. 645, piece n° 1165.

[14] Denifle and Châtelain, *Auctarium Chartularii Universitatis Parisiensis*; Liber procuratorum Nationis Anglicanæ, t. I, ab anno MCCCXWIII ad annum MCCCCVI

he presented[15] this piece to the English Nation assembled under the presidency of his procurator, the Scot William of Spyny. The proposition of Buridan gave rise, between the two Nations, to active negotiations; these resulted in a concordat where the line of demarcation between the English and Picards was marked with precision. This concordat, whose text is kept in duplicates in the books of the procurators of the two Nations[16], was decreed in the presence of Picardy and English masters belonging to the various faculties; the masters of arts that were among the witnesses were: Jean Buridan, Nicholas of Soissons, Robert son of Godfrey, and Albert of Saxony. According to the book of procurators of the English Nation, this paper was read before the National assembly and sealed with his seal on 12 July 1358.

This document, where the name of the old master of arts Jean Buridan is beside that of Albert of Saxony, his young colleague, is at the same time the last which mentions the presence, at the University of Paris, of the Philosopher of Béthune.

According to tradition, he would have bequeathed to the University, where he had so long taught, a house that he had purchased with his own money and that is shown again in the times of Du Boulay[17].

This tradition seems to prove that Jean Buridan died peacefully in the University where he had lived renowned and honored. A completely contrary tradition shows him driven from Paris by the Realists and taking refuge in Vienna, where he founded a University.

This latter tradition is mentioned for the first time in the first half of the century 16th by the historian John Thurnmaier, more known under the name of Aventine. Aventine gives Buridan[18] a companion in his flight, *Marcilius Balavus*, i.e., Marsilius of Inghen[19], who went to establish the University of Heildelberg; the commentaries of Buridan on the *Almagest* of Ptolemy, Aventine adds, appear even in Vienna.

This story of Aventine sounds implausible. Marsilius of Inghen was still in Paris, where his success was strong, in 1379; the same success was waiting for him at Heidelberg, where he became rector in 1386, and where he died in 1396; there is no evidence that the persecution caused by his Ockhamist doctrines had been the cause of his departure; the extraordinary popularity enjoyed by the teaching of Marsilius in Paris (the classrooms were too small for his audience), the authority which Albert of Saxony and Themo were, few years ago, invested in this same University, proves

[15] Denifle and Châtelain, *Op. cit.*, col. 212

[16] Bulæus, *Historia Universitatis Parisiensis*, tomus IV, p. 346. — Denifle and Châtelain, *Chartularium Universitatis Parisiensis*, tomus 111, ab anno MCCCL usque ad annum MCCCLXXXXIIII, pp. 56-59, piece n° 1240. — Denifle and Châtelain, *Auctarium Chartularii Universitatis Parisiensis; Liber procuratorum Nationis Anglicanæ*, tomus I, ab anno MCCCXXXUI ad annum MCCCCVI, coll. 233-235.

[17] Bulæus, *Historia Universitatis Parisiensis*, t. IV, p. 997.

[18] Aventini *Annalium ducum Boiariæ libri septem*, lib. VII, cap. XXI; ed. Rizler, Bd. Il, p. 474

[19] Du Boulay (Bulæus, *Historia Universitatis Parisiensis*, t. IV, p. 996) thinks that Batavus is mistaken for Patavinus: but Marsilio of Padua had left Paris before 30 May 1329, the time when John XXII wrote to the University to publish parts of the trial where Jean of Jandun and Marsilio of Padua had been convicted (Denille et Châtelain, *Chartularium Universitatis Parisiensis*, t, II, sectio I, p. 326, piece n° 891)

that the Nominalists were not persecuted and Buridan was able to achieve an extreme old age without seeing a decline around him in the favour enjoyed by the doctrines he had professed.

More than one historian has noted with astonishment the constant favor that, at the University of Paris, the principal nominalist masters who have taught there held, from Jean Buridan to Marsilius of Inghen; stangely, this favor has seemed to contradict the repeated prohibitions of which Ockhamism was the object. Perhaps they could conclude *a priori* that the doctrines taught by the Parisian masters differed significantly from those sustained by the *Venerabilis Inceptor*. We have already shown[20] that in the question of Universals, Buridan professed an opinion closer to that of St. Thomas Aquinas than that of William of Ockham. In this study, we will have the opportunity to note other discrepancies between the philosopher of Béthune and the head of the nominalist School; we will see that the former could be treated with honor by those who condemned the excesses of the latter.

In fact, no document has corroborated the account of Aventine; we found nothing that mentions the name of the Philosopher of Béthune among the founders of the University of Vienna.

When in 1365 Rudolph IV, Duke of Austria, established this University, the rector was handed over to a young master of the University of Paris, Albert Ricmerstorp[21], the one who is often confused with Albert of Helmstedt or Saxony.

At the same time that Aventine wrote, in 1514, Georges Tannstatter, professor of Astronomy at the University of Vienna, published the *Tables of Eclipses* of Georges of Peurbach and the *Tables of the First Mobile* of Regiomontanus[22]. He prefaced these tables with a valuable introduction, where he recalled the glorious titles of those who have taught before him in the chair that he occupies. But the astronomer whom he celebrates as the initiator of the Austrian University is not Jean Buridan, which he does not mention; it is Henry Heinbuch of Messe. Here, indeed, he talks in a few words about the founder of the Viennese astronomical school:

> Henry of Messe, German, was a man extremely learned in all science; from the ancient University of Paris[23], he was the first, at the beginning of the founding of our Viennese University, to introduce theology, astronomy, and other most noble studies. He was, with Henry of Oyta, a very famous theologian, the first to teach Theology. The depth and subtlety of his knowledge in astronomy are clearly evidenced by the first book of his *Commentaries on Genesis*. He was the contemporary of the most learned astronomers of Paris, the German

[20] *Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*; seconde série, p. 438.

[21] Heinrich Denifle, *Die Entsehung der Universitaten des Mittelalters bis 1400*, Berlin, 1885; p. 608

[22] *Tabulæ eclypsium Magistri Georgii Purbachii. Tabula primi mobilis Joannis de Monteregio. Indices praeterea monumenlorum quae clarissimi viri Studii Viennensis alurnni in Astronomia et aliis Mathematicia disciplinis scripta reliquerunt…* Viennæ: Austriæ, 1514.

[23] Henry Heinbuch of Hesse underwent the determination in Paris in 1363 (Denifle et Châtelain, *Auctarium Chartularii Universitatis Parisiensis*, t. I, col. 279). An active and renowned master, he was still in Paris on 5 January 1378, the day when the University chose him to go in its name to harangue the Emperor Charles IV, who in the company of Wenceslas, stayed in Paris from 4 through 11 January (Denifle et Châtelain, *Ibid.*, col. 530)

> John of Linières[24] and of John of Saxony. He wrote some theories on the planets and a few other treaties of Astronomy. In Theology, he wrote numerous and famous works that are kept in Vienna, in the Library of Ducal College. He died in 1397, the third day of the ides of February.

What Georges Tannstatter wrote in 1514 was so well-known, at the time, that he was surnamed Henry of Hesse: The Planter of the University of Vienna, *plantator Gymnasii Viennensis*[25].

From where did Aventine take what he said regarding the escape of Buridan and his role in the creation of the University of Vienna? Would he not confuse the Philosopher of Béthune with Henry of Hesse who was, in fact, the contemporary of Marsilius of Inghen, and who left Paris about the same time as the latter?

This is not the only legend about Buridan that Aventine recounted; he mixes it with the wrongdoings, also doubtful, of Jeanne of Navarre, wife of Philippe le Bel; Jeanne of Navarre died in 1305, so this allegation is completely implausible.

Villon makes our philosopher the accomplice of the goings-on in which Jeanne of Burgundy, wife of Philippe le Long, indulged in the tower of Nesles, and the victim of the cruelty of this debauched Queen:

History says that Buridan
Was thrown in a bag into the Seine.

Nowadays, Gaillardet and Alexander Dumas welcomed this fable and made him a character in a long popular melodrama. From the 15th century, however, the historian Robert Gaguin called into doubt[26] these relations of Buridan with a princess who, in 1314, was locked up for adultery.

If the drama of *the Tower of Nesle* formerly popularized the name of Buridan with the public which asks for violent emotions from the theater, this name remained famous among students in philosophy, via a curious argument for or against (you never knew) the freedom of indifference; but the hesitation of the hungry donkey between two completely alike bales of hay seems just as legendary as the love life of the philosopher and Jeanne of Burgundy.

We have searched in vain for the argument of the donkey in the various writings attributed to Buridan; where it could find a place, we encountered completely different examples.

When considering, for example, if there are several separate souls in one man, Buridan wrote this[27]:

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[24] Jean of Linares was neither German nor contemporary of Henry of Hesse. Henry of Oyta and John of Saxony were still in Paris on 11 January 1378 (Denifle et Châtelain, *Auctarium Chartularii Universitatis Parisiensis*, t. I, col. 530). However, on 22 April of the same year, Henry of Oyta was Professor in Prague.

[25] The title of this book testifies/: Henricus de Hassia: plantator Gymnasii Viennensis in Austria: *contra disceptationes et contrarias predicationes fratrum mendicantium super conceptionem Beatissime Marie Virginis et contra maculam sancto Bernhardo mendaciter impositam*. Argentorati, Reinhard Beck, 1516

[26] Cited by BuIæus, *Historia Universitatis Parisiensis*, t. IV, p. 996.

[27] Joannis Buridani *Quœstiones in libros de anima*; in lib. Il quæst. V; edit. cit., fol. VII, col. *b*.

> The will sometimes fights against itself and seems driven by contrary affections, because the voluntary acts are found mixed with involuntary acts. For example, a sailor who sees a sea storm is eager, and in a voluntary way, for the salvation of his body; but, at the same time, there is a strong sadness from the loss of the objects he needs to throw into the sea to be saved; so he wants to throw them in the sea and, in fact, he ends up throwing them; but he resolves with great pain and sadness, and he takes a long time to act; the cause is in the various voluntary actions which fight each other; he wants to escape from the storm and he also wants to save his property.

In the following question, Buridan repeats[28] that "the will sometimes struggles against itself, as happens in a voluntary marriage," then he takes the example we just heard him develop; there is no question regarding the donkey solicited by the attraction of two bales of hay.

This is another instance[29] where this famous example might have been invoked but was not. It involves proving that the sensitive soul of animals plays an active role and not only a passive role in sensation:

> We see, indeed, that the horse or dog, with the help of the senses, composes, divides, and reasons discursively as if it were using a syllogism. If it sees its master on the other side a pond or a ditch, it considers that it cannot reach him by a straight line, but only by a curved path, and it goes around the obstacle. It is not credible that the object suffices produce such a discursive operation; the object has no other virtue than to imprint its *species* into the medium; however, these acts go beyond what such an impression is capable.

Would this not be the case in point that a purely passive sense would let the donkey starve to death between the two equivalent impressions of perfectly equal pecks?

In the *Questions on Nicomachean Ethics*, our philosopher examines especially the problem of free will, which he formulates in these terms[30]:

> The will being placed between two opposed terms, and all other things being perfectly equal, can it sometimes settle to one term and sometimes towards the other?

The author of the *Questions on Ethics* finds, in Philosophy, no compelling reason for or against free will. If he adheres to the opinion that answers in the affirmative to the question posed, it is above all, he says, to submit to the authority of Christian teaching, an authority which one of the condemnations at Paris in 1277 particularly confirmed.

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[28] Joannis Buridani *Quæstiones in libros de anima*; in lib. I quæst. VI; ed. cit., fol. VIII, col. *c*.

[29] Joannis Buridani *Quæstiones in libros de anima*, in lib. II quæst. XIII; ed. cit., fol. XII, col. *a*.

[30] *Proemium*Ioannis Buridani *in questiones super X libros Aris. ad Nicomachum*. Colophon:

> Huc usque producte sunt questiones Buridani morales: robustiori etati precipue perlegende quas Egidius delfus socius Sorbonicus: atque in sacris litteris baccalarius formatus emendatius imprimi curavit. Impressore vuolfgango hopyl. Anno incarnationis domini MCCCCLXXXIX decima quarta die Iulii. In lib. III quæst. I: Utrum sit possibile quod voluntas, cæteris omnibus eodemmodo se habentibus, determinetur aliquando ad unum oppositorum, aliquando ad aliud.

Ed. cit., fol. XLVI, col. *c*.

During his long and interesting discussion, he invokes no argument of the donkey. “I can go from Paris to Avignon either by Lyon or by Dun-le-Roi;” this is the alternative that serves as a concrete example.

Also, he examines this problem[31]:

> Are acts which are done by fear, in the sense that they would not be done without this fear, as the act of throwing goods overboard during a storm, involuntary acts?

> Take,

he said,

> this action that involves throwing goods overboard. In the first place, we can ask generally if the action of throwing goods overboard is a voluntary act; in this case, we should purely and simply answer no… One may ask, secondly, if we accomplish a voluntary act by throwing goods overboard, during a storm, for his own heath and for that of others; we must answer yes.

We already encountered this answer twice, by browsing the *Quæstiones in libros de anima*.

To tell the truth, this discussion does not prove that Buridan had not, in the 14th century, invoked the still famous case of this donkey in the awkward predicament. We will not note any allusion to this argument in the *Quæstiones in libros of anima*; but are these *Quæstiones* of the Philosopher of Béthune? They seem to be intimately tied to the *Quæstiones in parva naturalia* that Georges Lokert published at the same time; one and same author seems to have drafted these and the above questions, too. However, in a forthcoming study, we will postpone until the beginning of the 15th century the composition of the *Quæstiones in parva naturalia*. Should we not act similarly on the subject for the questions on the *De anima*? It is, indeed, the conclusion to which we will be led. We will be led, also, to think that the *Questions on the Nicomachean Ethics* are by the author who wrote the *Quæstiones in libros de anima* and the *Quæstiones in parva naturalia*. What we have just said seems to prove that this author did not think of the argument of the donkey; but we would conclude that the Philosopher of Béthune did not propose this famous comparison. So we come to the review of a book that is undoubtedly of that philosopher; we want to talk about the *Questions on the Metaphysics of Arislote*.

Here is the question that Buridan examines in this book:[32]:

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[31] Joannis Buridani *Quæstiones in X libros Aristotelis ad Nichomachum*; lib. III, quæst. VIII: Utrum operationes quæ propter metum fiunt, scilicet quod alias non fierent, sunt involuntariæ, ut in tempestatibus maris si mercedes ejiciantur. Ed. cit., fol. LVIII, coll. *a* et *b*.

[32] *In Metaphysicen Aristotelis Quæstiones argutissimæ Magistri Ioannis Buridani in ultima prælectione ab ipso recognitæ et emissæ: ac ad archetypon diligenter repositæ: cum duplici indicio: materiarum videlicet in fronte: et quæstionum in operis calce*.

Vænundantur Badio. Colophon:

> Hic terminantur Metaphysicales quæstiones breves et utiles super libros Metaphysice Aristotelis quæ ab excellentissimo magistro Ioanne Buridano diligentissima cura et correctione ac emendatione in formam redactæ fuerunt in ultima prælectione ipsius Recognitæ rursus accuratione et impensis Iodoci Badii Ascensii ad quartum idus Octobris MDXVIII. Deo gratias.

> We distinguish well between the rational and irrational powers when we say: The rational power is equally capable of two opposite acts; it is not the same as the irrational power; it can only produce a single act.

What alternative does Buridan offer to this rational power but that it is our will?

> For the will,

he says,

> to produce the act of volition, it must be that the reason previously judged good and evil. So, let us imagine that the intellect sees a sum of money; it judges that this money would be useful, beneficial, necessary, and it would be good to take this amount; on the other hand, it considers that this money is not his, that it would be dishonest and unfair to seize it. These judgments being posed, and all the other things of the world behaving in a similar manner with respect to the one and the other side, in the absence of any other determining cause, the will can decide to take what it considers useful; it can also decide not to take it, because it was believed that it would be unfair and dishonest to do so; it can remain suspended, producing neither the act of willing or the act of not willing; it can withhold a decision until the intellect will have more extensively considered the two sides and it will be more fully deliberate. The intellect is therefore not enough to determine the will; the will holds its determination of its own freedom.
>
> Consider, on the contrary, the sensitive appetite or any other power that is not free; if this power is indifferent to two acts opposed one to the other, for example acceptance or rejection, it will never be resolved to one or the other of these two effects, unless some other cause determines it. The sensitive appetite of the horse or dog is determined to act by the sole judgment of sense. As soon as the horse or dog judges, by the sense he is endowed, that something is good, that it suits him, the appetite inclines toward this choice. In truth, they sometime compete here as contradictory sense judgements. A dog, for example, is on an empty stomach; it is hungry; it sees food and longs to seize it; but it also sees its master who holds a stick; it considers therefore that it would be wrong to take that meat, and it feared to do it. But one of those two judgments: to take this food, to not take it, which will be the stronger, will determine the more powerful act of appetite, which the external act will in turn will follow.

Is this opposition between the rational and irrational powers supported by irrefutable arguments?

> It seems to me,

says Buridan,

> that to admit such a difference between the freedom of our will and the deprivation of liberty which strikes the sensitive appetite of the dog, it is better to rely on the faith than on natural reason. It would be hard to demonstrate that our will is entirely indifferent to two opposed acts—that it can, which the appetite of a dog cannot, decide to either one or the other side without anything foreign taking it there.

Throughout the course of the debate which ends with this very prudent conclusion, a modern philosopher probably made some allusion to the predicament of the donkey; Buridan does not speak of it.

No text, therefore, allows us to assign this famous comparison either to Jean Buridan of Béthune or to a philosopher who could be in his namesake, who, in the beginning of the 15th century, commentated the *De anima* and the *Nicomachean Ethics*.

The one or the other—or else: the one and the other—were able to use it in the oral exposition of the debates on free will. Did they do it? We can neither affirm nor deny it.

Jean Buridan of Béthune and Albert of Helmstedt, known as Albert of Saxony, taught at the same time in the Faculty of Arts of the University of Paris; the first was much older than the second; the teaching of the one could influence the opinions of the other.

We will meet obvious traces of this influence if we compare the various writings of Albert of Saxony, which has Physics as their subject, to the *Quæstiones totius libri Physicorum* of Buridan.

These questions are kept in the manuscript whose § I contains its description; they occupy 112 pages.

They were printed in Paris, in 1509, by Pierre Ledru, at the expense of the bookseller Denis Roce and under the direction of John Dullaert of Ghent[33]. We were able to consult this edition.

We have already said, and we will show in a forthcoming study, that many writings attributed to Buridan should be dated from the 15th century. We cannot fear that such a fate was in store for the *Quæstiones totius libri Physicorum*; these *Questiones* were probably written in the 14th century; a learned bookseller in Munich, Mr. James Rosenthal, presented to us in his hands a copy on vellum paper of the *Questiones supra libros phisicorum Aristotelis novissime Parisiis disputate*, and this copy is dated the year 1371.

The *Questions on the Physics* of Jean Buridan begin with a proemium[34]; in the proemium, the master teaches us that he wrote his book at the prayers of many of his colleagues and his followers; less modest than Albert of Saxony, he is aware that some inventions are contained in it, and he claims the gratitude of those who would be pleased by these inventions:

> *Bonum, ut habetur primo Ethicorum, quanto est multis communius, tanto est melius et divinius; propter quod multorum de discipulis seu sodalibus meis precibus inclinatus, aliquot scribere præsumpsi de difficultatibus libri Physicorum et hanc illis scripturam communicare, quia non possent, ut debet, multa in scholis audita sine aliquo scripturæ admonitorio memoriæ commandare; super quibus peto et supplico de obmisso et minus bene dicto obtinere veniam; de inventis autem, si quæ faciunt convenientiam, multas habere grates.*

What are these inventions, the subject of which the Philosopher of Béthune demanded his readers recognize? Our object here is not to find them. More limited than that, our object consists in examining if some of the ideas which we attributed

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[33] *Acutissimi philosophi reverendi magistri*Ioannis Buridani *subtilissime questiones super octo physicorum libros diligenter recognite et revise a magistro*Johanne Dullaert de Gandavo *antea nusquam impresse*. Venum exponuntur in edibus Dionisii Roce… Parisius. in vico divi Jacobi, sub divi Martini intersignio. Colophon:

> Hic finem accipiunt questiones reverendi magistri Johannis Buridani super octo phisicorum libros, impresse Parhisiis opera ac industria magistri Pétri Ledru. impensis… Dionisii Roce… anno millesimo quingentesimo nono. octavo calendas novembres.

[34] Ms. cit., fol. 2, col. *b*.

to the discovery of Albert of Saxony have not been suggested by Buridan. So that this study does not exceed its just limits, we will restrict our search to two theories of Albertutius that attracted the attention of of Da Vinci more strongly: the theory of the center of gravity and the theory of *impetus*.

Chapter 3
That the theory of the center of gravity, taught by Albert of Saxony, is not borrowed from Jean Buridan

Albert of Saxony maintained, regarding the center of gravity, a doctrine that is, in his writings, of the greatest importance[1]. We saw this doctrine arise out of the need to resolve certain problems. If we want to appreciate the exact role that Jean Buridan and Albert of Saxony were able to play in the creation of this theory, we need mark precisely where the solution of these problems was at the moment when these two masters began to inquire about it.

The first of these problems can be formulated in the following terms: Is the natural location of the terrestrial element the concave surface of the water or the center of the world? Without reporting here all that was answered to this question since when Aristotle posed it[2], let us see what was said of it at the University of Paris immediately before Buridan and Albert of Helmstedt; Walter Burley will inform us in this regard.

According to Burley[3], the natural place of the terrestrial element is not the inner surface of the element of water; "the Earth is in its natural place if its sphere has its place in the center of the world." "Similarly, water is in its natural place if its sphere has for its center the center of the World, which is the same as that of the Earth." We can say the same for the other elements:

> Nothing is in its natural place if its center is not at the center of the World.
>
> A portion of the Earth, free from obstructions, moves to the center of the World and not to the inner surface of the water.

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[1] Albert de Saxe et Léonard de Vinci ; II. Quelques points de la Physique d'Albert de Saxe (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, 1 ; première série, pp. 8-15).

[2] A summary of these responses is found in our book: *Les origines de la Statique*, t. II, pp. 10-13 [Duhem (1991, 266-269)].

[3] Burleus *Super octo libros physicorum*, Colophon:

> Et in hoc finitur expositio excellentissimi philosophi Gualterii de Burley Anglici in libros octo de physico auditu Aristotelis Stagerite (sic) emendata diligentissime. Impressa arte et diligentia lioneti Locatelli Bergomensis, sumptibus vero et expensis nobilis viri Octaviani Scoti Modoetiensis… Venetiis, anno salutis 1491, quarto nonas decembris.
>
> 93° fol. (unnumbered).

A difficulty, it is true, is:

> When the earth is the center of the World, each of its parts is violated, because, free from any interference, it naturally would move toward the center.
>
> Similarly, if the earth were pierced, from one side to another, with a hole through the center, a clod of earth, thrown into this hole, would move until its middle comes to the middle of the World; one half of this mass would be on one side of the center of the world and the other half on the other side; but this cannot be done unless a part of this clod of earth moves away from the center of the universe to be closer to heaven; however, this latter movement is an upward, thus violent, movement, which is impossible.

To that Burley responds

> that a part of the earth, completely detached from it, is violated when its environment is not the center of the world, because, freed from any obstacle, it is would move to the center of the world; but when it is united with the rest of the earth, it can, without being abused, rest outside the center of the world, because it is at rest not by itself, but in virtue of the rest of the ensemble.

The origin of the second problem must be sought in the writings of Roger Bacon.

Aristotle had conceived nothing, in his Physics, which was similar to our notion of mass; for a body, subject to a certain power, to be able to move with a finite speed, he made that a certain resistance restrain it; in the absence of any resistance, it would come instantly to the end of its movement. A weight, for example, subjected only to its gravity, would reach the ground at the same time that it would freely fall; if its fall lasts a while, it is because it is endowed with a certain resistance that fights against gravity. Aristotle attributed this resistance entirely to the ambient air; this doctrine provides one of his main arguments against the possibility of a vacuum; in a vacuum, a weight would feel no resistance; its fall would be instantaneous.

To face this theory of Aristotle, Roger Bacon undertakes[4] to prove that in a weight that falls, there is not only a natural gravity which plays the role of power, but even internal violence that resists this power even when the medium is removed.

> Physicists believe,

the famous Franciscan said,

> that the descent of weights is entirely natural and is similar to the ascension of a light body, such that these two movements are not violent. But a geometric figure (Figure 3.1) suffices to show the contrary. Indeed, let DBC be a stone or a piece of wood placed in the air, A the center of the World, and GH the diameter of the World. As the three points D, B, C always keep, within the whole, the same mutual distances, they must go down toward the center along parallel lines; D thus descends along the line DE, B by BA, and C by the line CO. D will thus fall away from the center of the world, on the diameter HG, toward the closest point in the Heaven, i.e., point E; C will similarly fall to O. In this descent, D will move away from the center A and will approach from the Heaven according to the distance AE, and C according to the distance AO. But every time that a weight departs from the center to

[4] Fratris Rogeri Bacon, Ordinis Minorum, *Opus majus ad Clementem quartum, Pontificem Bomanum*. Edidit S. Jebb, Londini, typis Gulielmi Bowyer. MDCCXXXIII. Partis quartre dist. IV, cap. XIV: An motus gravium et levium excludat omnem violentiam? Et quomodo motus gignat calorem? Itemque de duplici modo sciendi. pp. 103-104, numbered 99-100 by mistake.

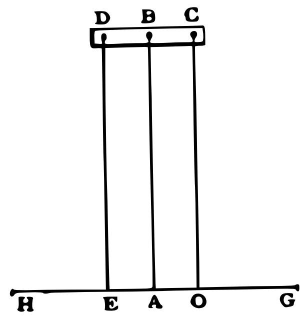

Figure 3.1 [Roger Bacon's illustration that falling weights have natural gravity and internal violence]

> be closer to heaven, there is violence. D and C thus move with a violent movement, and it is the same in all parts of the body DBC, except for part B, which goes only to the center. Thus a great violence occurs here.

Of the two questions of which Walter Burley, on one hand, and Roger Bacon, on the other hand, have given statements, we saw[5] the theory of gravity that Albert of Saxony teaches set out. Specifying what Aristotle and Simplicius had barely indicated, this theory poses the following principles that resolve the difficulties raised:

1. The Earth is in its natural place when its center of gravity coincides with the center of the universe.
2. When a terrestrial fragment is violently separated from the whole of the earth, this fragment and the rest of the terrestrial element move naturally so that their common center of gravity returns back to the center of the world.
3. When he professed the doctrine, was Albert of Saxony simply the disciple of Jean Buridan?

Jean Buridan has also examined the two problems for which this doctrine was created. The solution that he proposed for it has nothing to do with that which Albert adopted. It, through Burley and St. Thomas Aquinas, relates to the tradition of Aristotle and Simplicius; this follows directly from the nominalist principles established by William of Ockham.

William of Ockham affirmed with persistence[6] that in the purely geometric concepts of point, line, surface, there is nothing real, nothing positive; only the volume, the three-dimensional size extended in length, width, and depth, can be realized. A surface is a mere negation, the denial that the volume of a body extends beyond a certain term; similarly, a line is the denial that the extent of a surface crosses a certain boundary; a point, the denial that a line extends beyond a certain point.

[5] *Albert de Saxe et Léonard de Vinci*, II (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, I; première série, pp. 8-19).

[6] Gulielmi de Occam *Tractatus de Sacramento Altaris*, capp. I, II et IV. — *Quodlibeta*, Quodlib. I, quæst. IX. — *Logica*, cap. de Quantitate, etc.

Listen to the famous Nominalist reprimand[7] with his usual enthusiasm the physicists who talk about the stationary poles of the Heaven, of the center of the World, thus realizing points, indivisibles, which are mere abstractions of the geometer:

> What they say of the immobility of the poles and of the center proceeds from a false imagination, i.e., that there exists, in the Heaven, motionless poles and, in the earth, an immobile center. This is impossible. When the object is moving locally, if the attribute remains numerically one, it moves from local movement. But the subject of this accident which are the poles, i.e. the heavenly substance, moves mth local movement; so either the poles will shortly be replaced by other poles numerically distinct from the first or else they will be moving.
>
> Perhaps it can be said that the pole, which is one indivisible point, is not a part of Heaven, because Heaven is a continuity, and the continuous does not consist of indivisibles.
>
> But if the pole exists, and if it is not a part of the Heaven, it is thus a corporeal and incorporeal substance. If it is corporeal, it is divisible and not indivisible. If it is incorporeal, it is of an intellectual nature, and one arrives at the ridiculous conclusion that the pole of the Heaven is an intelligence.

The spirit which guided Ockham when he wrote this passage is also the one that inspired Buridan in the discussion of the two problems we spoke about; the opinion of the Philosopher of Béthune seems to be summed up in these words: The two problems in question are all meaningless, as they attribute the reality and physical properties to the center of the World, while dealing with this center as an indivisible point.

Let us see what the Philosopher of Béthune said regarding the question about the natural place of the earth[8].

According to Buridan[9], the natural place of the terrestrial element is partly the inner surface of the water, partly the inner surface of the air.

> To the opinion which claims that the proper and natural place of the earth is not the water but the center of the world, we will answer[10], in the first place, that the center of the world is the whole earth, and the earth cannot itself be its own place. If by center we mean an indivisible point that the imagination mathematically places at the center of the world, this center cannot be held, because it contains nothing. If it was assumed that the earth was placed elsewhere, under other elements, it would not move toward this point.

One says, it is true, in support of this opinion, that if the earth were pierced from one side to the other, a fragment of earth, thrown into this hole, would descend to the center of the world; but this remark is worthless; "it must be that, according to nature, the hole is somehow filled."

The spirit of Ockham is recognizable in the passage just quoted; it is even more so in the following one, where Buridan examines[11] "if the successive duration that

[7] Gulielmi de Occam *Summulæ in libros Physicorum*, lib. IV, cap. XXII.

[8] Magistri Johannis Buridam *Questiones quarti libri Phisicorum*. Queritur quinto utrum terra sit in aqua sive in superficie aque tanquam in loco proprio et naturali (Bibl. nat., fonds lat., ms. 14723, fol. 63, col. *d*).

[9] Jean Buridan, *loc. cit.*, fol. 64, col. *c*.

[10] Jean Buridan, *loc. cit.*, fol. 65, col. *a*.

[11] Magistri Johannis Buridam *Questiones quarti libri Physicorum*. Queritur nono utrum in motibus gravium et levium ad sua loca naturalia tota successio proveniat a resistentia medii (Bibl. nat., fonds lat., ms. 14723, fol. 66, col. *c*).

affects the movement of the light or heavy body to their natural places is entirely due to the resistance of the medium."

> Notice on this subject, says the Philosopher of Béthune[12], that some physicists readily admit the existence of an intrinsic resistance during the natural fall of a weight.
>
> Suppose that a big man falls; all the parts of this man tend in a straight line to the center. But the extreme lateral parts cannot move in a straight line toward the center since the middle parts prevent them. So, it seems that the parts of this weight experience a certain prevention, a certain resistance against the inclination which brings them to the center. This seems contrary to the conclusion previously posed

which attributes all the resistance in the fall of the weights to the medium.

> Here, it seems to me, is what the answer should be: The center or middle of the World is not something indivisible, similar to the point that one could imagine on a line. The center or middle of the World is something that has a certain magnitude, which is long, wide, and deep; it is, for example, all the earth or a part that has a certain volume (*pars quantitativa*) of that same earth. The lower place, the lowest place, is not the [indivisible] center of the World; rather, this place contains this [indivisible] center of the World. A man who falls has no inclination, nor is directed toward the indivisible center of the World. Much more! If there were no heavy body at the place to which this man falls, if there were only air where currently there is land and water, this man would have the inclination and tendency to become [as a whole] the middle of the World; it is for this, and for this only, that his various parts would all have the inclination and tendency, namely that [the whole body of] this man becomes the middle of the World; in this, the parts do not interfere with each other.
>
> In fact, this man, taken in his entirety, would move much faster than one of his parts taken separately would move; as the various parts prevent and retard each other, so do they render each other livelier and quicker.
>
> Similarly, in one large continuous mass of water, a part does not aspire to descend beneath another part if they both have the same degree of heaviness or lightness. That is why a sailor who goes down to the bottom of the sea does not feel the gravity of the water, although he has on his shoulders a hundred or a thousand tons; this water, in fact, that sits above him, does not tend to go down more. It would have, on the contrary, a similar inclination with respect to the air, if this air were below it.
>
> Although this body of water would not be in its natural place, even when placed in a vase very high on a terrestrial summit, one part of this water would not tend to place itself below another part. Indeed, suppose that in one such place a man lies in a bath and that his leg is at the bottom of this bath, topped with a quantity of water which, in the air, this man could not carry; the man, however, would not feel the weight of this water because this water would have no inclination to stand below the water that surrounds or underlies him.
>
> I say the same about the whole earth, which is the center of the World. Not only is the central part of this earth found naturally at rest, but it is the same regarding its extreme parts; they have no inclination towards this middle point that we imagine to be the center of the earth. The whole earth, and its various parts together, tend, by a continual inclination, to occupy as much space they do currently; that is why they move in a straight line without either the central portions or the end portions mutually preventing or resisting each other.

The principles that the Philosopher of Béthune exhibits in these various passages are again formulated by him in another place[13]. When, in the first book of

[12] Jean Buridan, *loc. cit.*, fol. 67, col. *a*.

[13] Magistri Johannis Buridam *Questiones primi libri Physicorum*. Duodecimo queritur utrum omnia entia naturalia sint determinata ad maximum (Bibl. Nat., fonds latin, ms. 14723, foll. 16, col. *d*, et 17, col. *a*).

the *Physics*, he examines if all beings admit by nature an upper limit, he is taken to formulate and discuss this argument:

If the sustained opinion were accurate,

> an ant falling to the ground would move the whole earth. This consequence is absurd; however, it is logically derived. Indeed, we assume that the earth is exactly balanced at its center. If we imagine, indeed, that one divides the earth with a plane through its center (I understand its center as what the mathematicians conceive), each of the two parts of the earth would have the same weight; each of them would tend to place its center at the centre of the world if the other did not stop it; but neither of these parts can move the other, because they contribute both to the same purpose and are exactly equal in power and resistance. If the weight of a single ant were added to one of them, there would no longer be between the two sides a relation of equality; the part that has the ant would be greater than the other; it would thus move the other half until all is in balance, as previously.

Here is what Buridan responds to this argument:

> This reasoning assumes a false principle, namely that all the parts of the earth tend or have inclination towards a center which we imagine is indivisible. However, this is false. When the whole earth is in its natural place, so that none of its parts is above the water, the air, or the fire, the entire mass of the Earth no longer has an inclination to go further down; it tends to remain at rest where it is, and it is the same for each of its parts. When instead a part of the earth is above some water, air, or fire, then this part has an inclination to come under this water, air, or fire. But the rest of the earth, which is above each part of the water, the air, or fire, is much greater; it has, for resisting, a power that greatly surpasses the motive power of the parts situated above lighter bodies. A small part of the earth thus is not enough to move the whole earth. A very great mass of earth suffices to overcome the resistance of the whole earth, a resistance which comes from the desire to stay at rest in its natural place, because it is in its natural place in its entirety and also by all of its parts which are not above a lighter element.

Here Buridan seems to deny even the Peripatetic theory of gravity, the foundation of the geocentric system; his thought could, it seems, be summed up in these words, which are of Leonardo da Vinci[14]:

> The earth is not in the middle of the circle of the Sun, or in the middle of the world, but in the middle of its elements, which accompany it and are united to it.

And these words, reflections of the doctrines of Nicolas of Cusa[15], prepared for the theory of Copernicus.

The Ockhamist principle according to which a mathematical point cannot have any reality, that the physical center of the world must be not a point but a body, guides Buridan in all discussions analogous to those we have just reported.

For example, in his *Questions on the Metaphysics of Aristotle*[16], it is necessary to define what astronomers mean by the names of homocentric spheres and eccentric spheres; here is the precaution that proceeds this definition:

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[14] *Les Manuscrits* de Léonard de Vinci; ms. F de la Bibliothèque de l'Institut, fol. 41, verso.

[15] *Nicolas de Cues et Léonard de Vinci*, XIV (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, XI; seconde série, pp. 260-268).

[16] *In Metaphysicen Aristotelis. Quæstiones argutissimæ*Magistri Joannis Buridani *in ultima prælectione ab ipso recognitæ et emissæ: ac ad archetypon diligenter repositæ: cum duplice indicio: materiarum videlicet in fronte; et quæstionum in operis calce*. Vænundantur Badio. Colophon:

> It should known that, inside the World, the natural center is the earth itself. We cannot assume there is an indivisible center, if not by imagination. However, we imagine a point in the middle of the earth and regard it as the center of the world. So, all the spheres that will have as their center the center of the earth will be called homocentric…

Buridan does not admit the theory of the center of gravity that Albert of Saxony taught after him; he does not disprove of it, not even formally; it seems that when he composed his *Questions on Physics and on Metaphysics*, this theory was not yet established, nor was it a body of doctrine. In any case, Buridan knew this doctrine in the fullness of its development, which his Ockhamist principles forced him to reject as meaningless.

The theory of gravity that Albert of Saxony supported exercised the greatest influence not only on the mechanical research of Leonardo but also on the whole development of statics until the middle of the 17th century[17]. In addition, this theory spawned the geological system adopted by Da Vinci[18], the system that brought this great artist to the study of fossils where he would lead Cardan[19] and, through Cardan, Bernard Palissy. Thus, few doctrines have played, in the formation of modern Science, a role more important than that theory. Buridan did not participate in the composition of this theory.

After having enjoyed a long vogue that its fertility justified, the theory of the center of gravity that Albert of Saxony taught was eventually driven out of Science; the principle on which it rested, after having been viewed as a "truth of natural light", as a "first principle which nobody doubted", was relegated to the rank of unacceptable errors. The first who dared to doubt that principle is Johannes Kepler[20]. However, some of the attacks that Kepler directed against the proposition of Albert of Saxony seem to be just an echo of the teaching of Ockham and Jean Buridan:

> A mathematical point[21], be it the center of the World or another point, cannot actually move the bodies; neither can it be the object toward which they tend. Thus, physicists prove that such a force can belong to one point, which is not a body, and which is conceived in a completely relative way!

> Hic terminantur Metaphysicales quæstiones breves et utiles super libros Metaphysice Aristotelis quæ ab excellentissimo magistro Ioanne Buridano diligentissima cura et correctione ac emendatione in formam redactæ fuerunt in ultima prælectione ipsius Recognitæ rursus accuratione et impensis Iodoci Badii Ascensii ad quartum idus Octobris MDXVIII. Deo gratias.

Lib. XII, quæst. X: Utrum in corporibus cælestibus ponendi sunt epicycli. fol. LXXIII, col. *b*.

17 P. Duhem, *Les origines de la Statique*, Ch. XV: Les propriétés du centre de gravité, d'Albert de Saxe à Evangelista Torricelli. — Ch. XVI: La doctrine d'Albert de Saxe et les Géostaticiens. T. II, pp. 1-185 [Duhem (1991, 261-379)].

18 *Albert de Saxe et Léonard de Vinci*, IV (*Études sur Léonard de Vinci*, I; première série, p. 33) — *Léonard de Vinci et les origines de la Géologie* (*Études sur Léonard de Vinci*, XII; deuxième série, p. 283).

19 *Léonard de Vinci, Cardan et Bernard Palissy* (*Études sur Léonard de Vinci*, VI; première série, p. 223).

20 P. Duhem, *Les origines de la Statique*, ch. XVI; t. II, pp. 152-156 [Duhem (1991, 357-360)].

21 *Joannis Kepleri De motibus stellæ Martis commentarii, Pragæ, 1609 (Kepleri Opera omnia*, ed. Ch. Frisch, t. III, p. 151).

> It is impossible that the substantial form of the stone, setting in motion the body of this stone, seeks a mathematical point, the center of the World for example, without concern for the body in which the point is located. Thus the physicists show that natural things have sympathy for what does not exist!

Therefore, in the 17th century, discussions that the initiators of modern Science were facing still suffered from the various influences of the teachings that the University of Paris gave in the 14th century.

Chapter 4
The Dynamics of Jean Buridan

Jean Buridan wrote nothing which has directly influenced the development of Statics; the theory of the center of gravity which Albert of Saxony taught was in no way borrowed from him. On the other hand, the system of Dynamics that he adopted, in his *Questions on the Physics*, was called to orient, for two centuries, the thought of the Parisian nominalist School. Welcome, not without resistance, by Italian Geometers in the Renaissance, struggling against Aristotelianism and the routine Averroism of the Universities, it had to develop thanks to their mathematical science, and engender the mechanical doctrine of Galileo and his followers. It suffices to say that the study of the Dynamics of the Philosopher of Béthune is important for the history of Mechanics.

Not that, without doubt, the theory of the *impetus*, which is the foundation of this Dynamics, is due entirely to Buridan. We have seen elsewhere[1] how John Philoponus clearly formulated it; how some Arab thinkers, like the astronomer Al Bitrogi, seemed to adopt it; how Saint Thomas Aquinas and Walter Burley had alluded to rejecting it; and how, finally, William of Ockham gave it a formal and firmly established adherence through a vigorous discussion. Nowhere, however, has this theory been expressed with such magnitude, in conherence and in details than in the twelfth question[2] posed by the Philosopher of Béthune on the subject of the eighth book of the *Physics* of Aristotle.

This question is also formulated: "Is the projectile, after it leaves the hand that launches it, moved by the air?" If not, what moves it?

In the table located at the beginning of the eighth book[3], the matters treated in this question are listed in the following terms:

> Duodecima questio. *Utrum proiectum post exitum a manu proiicientis moveatur ab ære, vel a quo moveatur? Quare longius proiicio lapidem quam plumam vel tantumdem de ligno?*

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[1] *Nicolas de Cues et Léonard de Vinci*; IX. La Dynamique de Nicolas de Cusa et les sources dont elle découle. (*Études sur Léonard de Vinci*, XI, deuxième série, pp. 189-193.)

[2] Magistri Johannis Buridam *Questiones octavi libri physicorum*. Queritur 12° utrum projectum post exitum a manu projicientis moveatur ab ære, vel a quo moveatur. Bibl. nat., fonds lat., ms. 1723, foll. 106, col. *a*, et 107, col. *b*.

[3] Ms. cit., fol. 95, col. *b*.

Quod movetur ab impetu ei impresso a motore. Quare motus naturales gravium sunt velociores in fine quam in principio. An oportet ponere intelligentias ad movendum corpora cælestia? Que res est ille motus? Quare pila de chorda(?) longius reflectitur quam lapis velocius motus?

From the outset, this summary gives an idea of the seriousness of the problems that Buridan addresses in this part of his work. The solutions he proposes to give to these problems make this twelfth question one of the most impressive monuments of medieval Science. We also believe we must give the complete textual translation.

It seems,

says Buridan, that the projectile after leaving the hand that throws it

cannot be moved by the air; the air, indeed, that must be divided by this projectile, seems rather to resist its movement.

In addition, you may say that the one who launches the projectile moves, at the beginning of the movement, not only this projectile, but also the nearby air, and this shaken air then moves the projectile up to a certain distance. But to that we will give this answer: What is it that moves this air once it is no longer driven by the one who launches the projectile? The difficulty is the same for this air as for the cast stone.

Aristotle, in the 8th book of the present work, supports the contrary view in these terms: If projectiles continue to move after having been subjected to the contact of what throws them, it is either by ἀντιπερίστασις, as some claim, or because the air pressed by the projectile pushes, in turn, with a more rapid movement, the air that is in front of it. Aristotle repeats the same thing in the 7th book of the present work, in the 8th book, and in the 3rd book of the *De Cælo*.

This question is, in my opinion, very difficult because, it seems to me, Aristote has not resolved it well.

Aristotle examines two opinions.

The first invokes what he calls ἀντιπερίστασις. The projectile quickly leaves the place where it was. Nature, which does not allow the existence of a vacuum, sends with the same speed some air behind the projectile. This air, animated with a swift movement, meeting the projectile, pushes it forward; the same effect happens again until the body moves a certain distance.

Aristotle does not approve of this theory; he denies it in the 8th book of this work, saying: the ἀντιπερίστασις moves and makes all things move. What one must understand, it seems, is this: If one invokes no other process than the so-called ἀντιπερίστασις, all the bodies that lie behind the projectile, including the Heavens itself, must follow the movement of the projectile; air, in fact, which comes to occupy the place of the projectile, also leaves the place where it was; therefore, it must be replaced another body, and so on, indefinitely. But we can immediately answer what is said in the 4th book of the present work on progressive movement; indeed, it objected that rectilinear movement cannot happen without a vacuum, unless all the bodies placed in front of the mobile begin to move, since bodies cannot compenetrate; we solved this problem by replying that the bodies placed in front of the mobile did not all need to move forward, just some of them need to exert a certain condensation. Similarly, we would say here that there occurs a certain rarefaction of the body placed behind the projectile, so that it is not necessary that all the bodies behind the mobile follow its movement.

But, despite this explanation, it seems to me that the proposed theory was worthless, and this is the result of various experiences.

The *first experience* is that of the spinning top or the wheel of the blacksmith; this body rotates for a very long time; however, this body does not come out of the place it occupies, so the air does not have to follow it to fill the abandoned place. This theory cannot say what moves this top or wheel.

Second experience. You throw a javelin whose posterior end is armed with as sharp a point as the anterior end. It will move as quickly as if it did not have a sharp point in its back end; however, the air that follows the javelin cannot strongly push this tip, because it would be easily divided by its sharpness.

Third experience. A ship that we quickly tow in a river, against the course of the river, cannot stop instantly; it continues to move for a long time after it has ceased being towed. However, the boatman who stands on the deck feels nothing but the air pushing him from behind; he feels only, from the front, the air that resists. Suppose, further, that this boat is loaded with hay or wood, and the boatman is located on the back, against the load; if the air had an impetuosity so great that it was possible to push the ship with so much force, this man would be violently compressed between the load and the air following the boat; experience shows that he is not. If the boat were loaded with hay or straw, the air that follows it would bend, in the direction of the movement, the straws which are located on the back; and this is all false.

Aristotle seems to approve of the second opinion. According to this view, the one who launches the projectile moves, at the same time, the ambient air; and this air, violently shaken, has the power to move, in turn, the projectile; it should not be understood that the same air moves from the point where projectile was to the point where the motion of the projectile ceases, but that the air joined to the projectile is moved by him who launches the mobile, that this air moves another, and so on up to a certain distance; thus, the first air mass moves the projectile until it reaches a second mass air, this second mass up to a third, and so on; also, Aristotle says that there is not only a single mobile but successive mobiles; Aristotle also said that the movement is not a continuous movement, but a series of consecutive or contiguous movements.

But, without a doubt, this opinion and this hypothesis also seems impossible to admit, just like the previous opinion and hypothesis. This explanation does not tell what turns the wheel of the blacksmith or the spinning top when he who put them in motion releases them; indeed, if one were to cover the wheel completely with the help of a cloth that separates it from the ambient air, the wheel however would not cease to turn; it would continue for a long time to move; thus, it is not the air that moves it.

Item, a boat moved quickly remains in motion once the haulers have stopped pulling it; it is not the air that moves the boat; if it were covered with a tarp that one removes and, at the same time, removes the air that is contiguous to it, the boat would not stop for that; in addition, if the boat were loaded with hay or straw and driven by the ambient air, this air would bend the straw which lies on the surface of the load toward the front; on the contrary, the straw is bent toward the back as a result of the resistance of the air that surrounds it.

Item, however much the air moves, it remains easy to divide; it is therefore unclear how it could carry a stone of one thousand pounds launched by a slingshot or machine.

Item, with your hand, without taking anything in that hand, you can move the nearby air as fast and even more quickly than if you had in that same hand a stone you want to launch; so let us assume that this air, thanks to the speed of its movement, has enough impetuosity to quickly move this stone; it seems that if I pushed this air toward you with the same speed, it should make you suffer a very sensible impetuous impulse; however, we do not perceive this.

Item, it would occur that you could throw a feather farther than a stone, and a body weighing less farther than a body of greater weight, their shapes and volumes being identical; however, we experience that this is false; and, the consequence following clearly from the principles, because the shaken air would support, carry, and move a feather more easily than a stone, a light body more easily than a heavy body.

Item, to this explanation the question is raised: By what air is it moved after what lauched the projectile ceased to move it? To this question, the Commentator will answer that this air is driven by its levity, that it is in the nature of air to retain the motive force when it is shaken; thus, it is by this movement that sound, over time, propagates far away; we must, in fact, represent this phenomenon in analogy to what we see in water; one throws a stone in a perfectly tranquil pond of water; the water in which the stone falls moves all around it the water which is nearby, this moves another, and so we see circular waves forming that follow one another until they reach the shore; in the air, therefore, it forms waves of the same kind, and these waves propagate faster than the water in the proportion that the air is more subtle and more easily moveable than water.

To this answer we will object that levity has no property to move but upward, while a mobile can be projected upwards, downwards, or in any direction.

Item, either this lightness is the same as that which the air possessed before the mobile was launched and that it will retain it after the motion of the projectile, or it is something different, a different disposition imprinted on the shaken air by the person who threw the mobile, a disposition that it pleased the Commentator to name lightness. If this lightness is the same as that which the air previously possessed and that it will keep afterwards, the air thus had, before the time when the mobile was launched, the same motive force as at this time; it should therefore, before that moment, move the projectile as it moves it after, because, in nature, any active power, as soon as it is applied to the patient, must act and act in fact. If, on the contrary, this lightness is another thing, if it is a new proper disposition for moving the air, which is impressed by the person who launches the projectile, we can and must also say that such a thing is imprinted on the stone or the thrown mobile, and that this thing is the virtue that moves this body; it is clear that it is better to make that assumption than to resort to air that would move the projectile; rather, indeed, the air seems to resist.

Thus here, it seems to me, is what needs to be said: While the mover moves the mobile, it imprints on it a certain *impetus*, some power able to move this mobile in the same direction that the mover moves it, either upwards or downwards, or sideways, or circularly. The greater the speed that mover moves the mobile, the more powerful is the *impetus* that it imprints in it. It is this *impetus* which moves the stone after the one who throws it ceases to move it; but, by the air resistance, and also by gravity that inclines the stone to move in a direction contrary to that which the *impetus* has power to move, this *impetus* weakens continuously; therefore, the movement of the stone is constantly slowing down; this *impetus* ends up being defeated and destroyed to the point where the gravity prevails over it and now the stone moves to its natural place.

We must, it seems to me, adhere to this explanation because, on the one hand, the other explanations are false, and, on the other hand, because all phenomena are consistent with this explanation.

We will say, for example: I can throw a stone farther than a feather and a piece of iron or lead that fits my hand farther than a piece of wood of the same size. I answer that the cause is the following: All forms and natural dispositions are received in the matter and in proportion to the [quantity of] matter; therefore, the more a body contains matter, the more it can receive this *impetus*, and the intensity is greater with which it can receive it; however, in a dense and heavy body, there is, all things being equal, in fact, more raw material than in a rarefied and light body; a dense and heavy body thus receives more of this *impetus*, and it receives it with more intensity [than a rarefied and light body]; similarly a certain amount of iron can receive more heat than an equal volume of water or wood. A feather receives so weak an *impetus* that it is destroyed immediately by air resistance. Similarly, if the one who launches projectiles moves with equal speed a light piece of wood and a heavy piece of iron, these two pieces also having the same volume and shape, the piece of iron will go farther because the *impetus* which is impressed in it is more intense. It is for the same cause that it is harder to stop a great millstone of a blacksmith, moved rapidly, than a smaller wheel; in the big millstone, indeed, there is, all things being equal, more *impetus* than in the small one. Always in virtue of the same cause, you can throw a one- or half-pound stone farther than the thousandth part of this stone; in this thousandth part, indeed, the *impetus* is so small that it is immediately defeated by the resistance of air.

This also seems to be the cause of why the natural fall of weights constantly accelerates. At the beginning of this fall, in fact, gravity moves only the body; it therefore falls more slowly; but, soon, this gravity impresses a certain *impetus* on the heavy body, an *impetus* which moves the body at the same time as gravity; the movement becomes faster; but the faster it becomes, the more the intense the *impetus* becomes; so, it can be seen that the movement will continually accelerate.

He who wants to jump far retreats and runs with vivacity, in order thereby to gain an *impetus* which, during the jump, brings him a great distance. In fact, while he runs and jumps, he does not feel that the air moves him, but he feels, around him, the air that resists him forcefully.

It is not in the Bible that there are the intelligences responsible for communicating to the celestial orbs their own movement; it is therefore permissible to show that there is no need to assume the existence of such intelligences. You could say, in fact, that God, when he created the world, has moved as it pleased him each of the celestial orbs; he has impressed on each of them an *impetus* that has moved it since then; so that God no longer has to move these orbs, if not in exercizing a general influence, similar to that by which he gives support to all actions that occur; he could also rest on the seventh day from the work he had completed by entrusting mutual actions and passions to created things. These *impetus*, which God has impressed on celestial bodies, are not weakened or destroyed at a later time because there was, in these heavenly bodies, no inclination to other movements, and there was no longer any resistance that could corrupt and suppress these *impetus*. I do not give all of this as assurred; I will only ask the Gentlemen, the Theologians, to teach me how all these things can happen.

But on the occasion of this opinion, some difficulties, which are not small, present themselves.

First difficulty. The stone thrown into the air is driven by an intrinsic principle, namely by the *impetus* that was imprinted on it; it does not appear that this is true, because everyone agrees consider this movement a violent movement, according to the 3rd book of the *Ethics*, which is violent not because of an intrinsic active principle, but because of an extrinsic principle.

Second difficulty. What is this *impetus*? What is the movement itself, or is it something else? If it is something other than the movement, is it a purely successive reality, as the movement itself, or something of permanent nature? Whatever might be, indeed, the affirmation that we adopt, we see arguments to the contrary that are difficult to resolve.

Regarding the first difficulty, one can say that the weight thrown in the air moves by an intrinsic principle which is inherent to it; however, they say that this movement is violent, because this principle, namely the *impetus*, is violent and not natural to the mobile; it is not fitting to the formal nature of this body; it is an extrinsic principle that was violently imprinted in this weight; the nature of the weight inclines to opposing movement and to the destruction of this *impetus*.

Regarding the second doubt, which is very difficult to dispel, it seems to me that one must answer it by posing three conclusions.

The *first conclusion* is the following: This *impetus* is not just the local movement that moves the projectile[4]. This *impetus*, indeed, moves the projectile, and the mover produces the motion; this *impetus* produces the movement, while the movement cannot to produce itself.

Item, all movement comes from a mover that is present at the mobile, which coexists for this mobile; so if this *impetus* were movement, another mover from which this movement

[4] The opinion that Buridan refutes in this conclusion is that which William of Ockham supported. See: *Nicolas de Cues et Léonard de Vinci*, IX: La Dynamique de Nicolas de Cues et les sources dont elle découle (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, XI; seconde série, pp. 192-193).

could come would have to be assigned, and the difficulty from the beginning would remain; it would have served for nothing to pose the existence of a such an *impetus*.

Some quibble on this subject. They claim that the first part of movement, the one that launches the projectile, generates another part of movement, the one that immediately follows the first; and so on until the cessation of any movement. But this view cannot be approved; what produces another thing must exist at the time that something else is made; however, the first part of the movement does not exist when the second part exists, as we have said elsewhere. The consequence that we are establishing can also be made evident by this, as we have said elsewhere: To be moved only consists in the fact of being produced or being destroyed; the movement therefore does not exist when it is done, but when it is made (*Motum esse nihil aliud est quam ipsum fieri et ipsum corrumpi; unde motus non est quando factus est, sed quando fit*).

Here is the *second conclusion*: This *impetus* is not something purely successive; the movement, in fact, is a purely successive reality, as we have said elsewhere, and we just declared that this *impetus* is not the same as the local movement.

Item, all purely successive reality is continually destroyed; it must be constantly produced; however, we cannot assign to this *impetus* something that generates it constantly, because that something would be similar to it.

The *third conclusion* is that this *impetus* is a distinct permanent reality of local movement according to which the projectile moves. This conclusion is the result of the previous two[5] and what has been said previously. It is likely that this *impetus* is a quality whose nature is to move the body to which it was impressed; similarly it is said that a quality imprinted in iron by a magnet moves the iron to this magnet. This is equally likely: While this quality was imprinted in the mobile by the mover at the same time as the movement, similarly is it weakened, destroyed, and prevented by any resistance and any contrary inclination that weakens, prevents, and destroys the movement.

Just as a lucid body that generates light gives the reflected light if an obstacle is opposed to it, likewise, encountering an obstacle, this *impetus* produces a reflected movement. It is true that other causes work together with this *impetus* to produce a reflected movement along its journey. For example, one of these causes is that whereby tennis ball bounces higher than a stone, after hitting the Earth, even though the stone fell down with more speed and impetuosity. Many bodies, indeed, can be bent or compressed on themselves by violence; these bodies have the property to return quickly to their original rectitude or to the disposition that suits them; in return, they can push or pull with impetuosity a body that is joined to them; this is what appears in a bow. So, when the ball hits the hard ground, it is compressed on itself because of the *impetus* of its movement; immediately after, it returns to its sphericity; rising as well, it acquires an *impetus* that moves it into the air at a great height.

Similarly a string of a zither that is highly tense and that one hits remains long agitated with a trembling whereby it emits a sound of a certain duration, and here is how it is done: After it was hit and vibrated, it was curved violently to a certain side, it quickly returns to the first position such that it exceeds this straightness, because of the *impetus*, and it departs in the opposite direction; it then goes back and forth many times. It is a similar case that a bell continues to move sometimes to one side, sometimes on the other, a very long time after its rope stopped being drawn; we cannot stop it easily nor quickly.

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[5] The reasoning of the Philosopher of Béthune essentially supposes that there are only two kinds of realities, permanent realities and successive realities. It is, moreover, what Buridan seems always to admit when discussing, for example, the nature of movement (*Phys.* lib. III, quæst. VII). One can, from this remark, devise an argument to prove that the *Quæstiones in libros de Anima* are not by the Philosopher of Béthune. The author of these questions, in fact, admits that not only purely permanent realities and purely successive realities exist, but also realities which are permanent in a certain way and successive in another way; it is in the latter category that he places light. (Johannis Buridani *Quæstiones in Aristotelis libros de anima*; in lib. II quæst. XIX; éd. Parisiis 1516, fol. xvi, col. *c*.)

> That is what I had to say on this issue; I welcome others to make a more likely answer.

One cannot too much admire the precision with which Buridan has defined this quality to which he gives the name of *impetus*.

For a given mobile, this *impetus* is however as much greater as the speed communicated to the body is greater.

> The bigger the speed with which the body moves the mobile, the more powerful is the *impetus* that it imprints in it.

On the other hand, at equal speed, at the same volume, the *impetus* is greater in a heavy than in a light body:

> If the one who launches projectiles moves with equal speed a light piece of wood and a heavy piece of iron, these two pieces, having, moreover, the same volume and shape, the piece of iron will go farther because the *impetus* imprinted in it is more intense.

Indeed,

> all the forms and natural dispositions are received in the matter and in proportion to the [quantity of] material; therefore, the more material a body contains, the more it can receive this *impetus* and the greater the intensity is with which it can receive it.

The meaning of this sentence is very clear: In different mobiles, launched with the same speed, the intensities of the *impetus* are to one another as the quantities of material that these various mobiles contain.

What is this material? Buridan named it primary matter, *materia prima*. This is not, however, and cannot be the primary matter of Aristotle. Absolutely undetermined, it is not quantifiable. The primary matter of which Buridan speaks is, therefore, this primary matter already provided with dimensions and quantifiable, in which Saint Thomas places the principle of individuation[6].

How can one measure this quantity primary matter contained in a determined body?

> In a dense and heavy body, there is, all things being equal, more primary matter than in a rare and light body. *Modo in denso et gravi, cæteris paribus, est plus de materia prima quam in raro et levi.*

We would also be forcing the thought of Buridan in translating this proposition as: Is the quantity of matter contained in a body proportional to the volume and the density of this body?

If we had some fear in this regard, it would be easy to calm that fear. In one of his questions on the *Metaphysics* of Aristotle, Buridan poses to himself this objection[7]:

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[6] Note the analogy of the thought expressed here by Jean Buridan with that which R. P. Bulliot expressed regarding the identity of primary matter with mass, such as modern mechanics defines it. — Cf.: A. Gardeil, *La Philosophie au Congrès de Bruxelles* (*Revue Thomiste*, 2° année, 1894-1895, pp. 751-758).

[7] *In Metaphysicen Aristotelis Quæstiones argutissimse Magistri Joannis Buridani*. Lib. VIII, quæst. unica: Utrum cælum habeat materiam subjectam formæ substantiali sibi inhærenti. Ed. cit., foll. LV et LVI.

> The density and rarity are due to the amount of material (*ratione materiæ*); a dense body is one that contains much material under a small volume (*sub pauca magnitudine seu quantitate*), and a rare body is one that contains little material under a large volume.

To this objection, the Master responds:

> One may well grant that bodies that have a dense material are those containing more material under a lesser volume.

But how does one measure this density itself? When Jean Buridan composed his questions, a small book, which certainly came from Greek science and was attributed falsely to Archimedes, was commonly studied in the Schools. This *Liber Archimedis de ponderibus*, sometimes named: *Archimedis de incidentibus in humidum*, is reproduced in a large number of manuscripts of the 13th and 14th centuries[8].

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[8] For example, in the following manuscripts of the Latin archive of the National Library: Ms. 8680 A (13th century); Mss. 7215 and 7377 B (14th century). — It was printed twice during the 16th century in the following books:

1. *Sphæra cum commentis in hoc volumine contentis: Cichi Esculani cum textu*, etc. Venetiis, hered. Octaviani Scoti ac soc. 1518.
2. Iordani *opusculum de ponderositate Nicolai Tartaleæ studio correctum*. Venetiis apud Curtium Troianum. MDLXV. Fol. 16, v°, to fol. 19, v°.

In 1565, the Abbot Forcadel, of Béziers, published a French translation whose demonstrations were slightly paraphrased, under the following title:

Le livre d'Archimède *des pois qui aussi est dict des choses tombantes en l'humide, traduict et commenté* by Pierre Forcadel de Bezies *lecteur ordinaire* of the *Roy es Mathématiques* in the University of Paris. *Ensemble ce qui se trouve du Livre d'Euclide intitulé du léger et du pesant* translated and commentated by the same Forcadel. At Paris. In Charles Perier…, 1565.

The title adopted by Forcadel is the exact translation of it, which a hand of the 13th century put in the margins of the text contained in Ms. lat. 8680 A of the National Library (fol. 12, r°): *De ponderibus Archimedis et intitulatur de incidentibus in humidum*. This title is related to a passage where the speed of bodies falling in fluids is treated. This passage is missing all the printed and handwritten texts, including that contained in the Ms. 8680 A of the Latin archive of the National Library. He finished the text contained in the Ms. 7377 B of the same archive.

This title is also the one that Blaise of Parma, in his *Tractatus de ponderibus*, gives to the same writing: "*Nullum elementum in eius propria regione ponderat. Hoc dicit Alaminides in tractatu de incidentibus in liquido.*" (National Library, fonds lat., ms. 10252, fol. 167, v°.)

Everything seems to indicate that this book, like the *De levi et ponderoso* attributed to Euclid, is of ancient origin. It is obviously incomplete and undoubtedly ended with a description of the hydrometer. The full text might have also existed in the 14th and 15th centuries, because Albert of Saxony and Biaise of Parma follow, in a rough description of the hydrometer, the theoretical considerations that they borrow from the so-called treatise of Archimedes.

Thus completed, this treatise would probably represent the source from which the Latin author of the *Carmen de ponderibus*[a] drew.

Maximilian Curtze, who knew nothing of this history, has published[b], giving a unique monument of 14th century Science, the text that concerns us; this text was taken from the Ms. Db. 86 of the library in Dresden, where it bears the title *De insidentibus aquæ*.

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a) *Metrologicorum scriptorum reliquiæ*. Ed. F. Hullsch, Lipsis, 1866; vol. II, pp. 96-200.
b) Maximilian Curtze, *Ein Beitrag zur Geschichte der Physik im 14. Jahrhundert* (*Bibliotheca Mathematica*, 1890, p. 43).

This treatise has been paraphrased, in some quite unfortunate way, by Jean de Murs; under this title: *De ponderibus et metallis*, it forms the fourth part of the *Opus quadripartitum numerorum*[9] whereby the Norman geometer put the final touches, as he himself tells us, on 13 November 1343.

This text has been cited by Albert of Saxony[10] in his questions on the *De Cœlo* of Aristotle.

Finally, at the beginning of the 15th century, Blaise of Parma quoted him in turn[11] and was inspired by the drafting of the third part of his *Tractatus de ponderibus*.

All these treatises defined the notion of specific weight, which they called *gravitas secundum speciem*; they taught to how to compare the specific weight of the various bodies, either by a hydrostatic balance or by using the hydrometer method.

There is no doubt that Jean Buridan has, in his mind—close to the notion of density, at least for solids, liquids, and gases—the notion of specific weight, so well elucidated when he taught; there is no doubt that he acknowledged the equality between the ratio of the densities of two bodies and the ratio of the specific weights of these same two bodies. That is why, in the question of which we have reproduced the translation, we see two adjectives unite as synonyms: *densum* and *grave*, and also the two adjectives: *rarum* and *leve*.

We could thus most certainly translate into modern language what Jean Buridan thought of the *impetus* communicated to a heavy body by saying that the intensity of this *impetus* equals, for him, the product of three factors: an increasing function of the speed, the volume of the body, and a density proportional to the specific weight. If you had asked him to specify the form of the first factor, he would have probably taken it as proportional to the speed, and he had also identified the *impetus* as being what Galileo would one day call *impeto* or *momento*, and Descartes the *quantité de mouvement*.

But all bodies are not heavy; the heavenly substance, in particular, is not; and, however, Buridan does not hesitate to attribute an *impetus* to the orbits of the Heavens. Is the intensity of this *impetus*, for these orbits, determinable by a rule similar to that imposed on heavy bodies?

The solution to this question is made singularly delicate by the opinion that our author professes about the heavenly substance.

We have seen[12] how, in the Middle Ages, the opinions diverged regarding the nature of the fifth essence. We can reduce them to three main heads:

1. The Heavens are not composed of matter and form; it is a simple substance. This is the doctrine of Averroes, taken up by Jean of Jandun in some of his books.

[9] *Quadripartitum numerorum* Magistri Johannis de Muris (Bibliothèque nationale, fonds lat., ms. n° 7190).

[10] *Quæstiones subtilissimæ* Magistri Alberti de Saxonia *in libros De Cœlo et Mundo*; lib. I, quæst. III.

[11] *Tractatus de ponderibus* secundum Magistrum Blasium de Parma. (Bibl. nat., fonds lat., ms. n° 10262.)

[12] *Nicolas de Cues et Léonard de Vinci*, XIV: La nature des astres selon Nicolas de Cusa et Léonard de Vinci (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, XI; seconde série, pp. 255-259).

2. The Heavens are composed of matter and form; but there is no identity of nature between celestial matter and sublunary material; these two materials are only similar. This is the opinion of St. Thomas Aquinas that Jean of Jandun sometimes held.
3. The Heavens are composed of matter and form; the material of the Heavens is of the same nature as the material of bodies subject to the generation and corruption. This is the hypothesis supported, with increasing precision, by St. Bonaventure, Gilles of Rome, and John of Duns Scotus and William of Ockham.

Jean Buridan breaks significantly with this doctrine that seemed to have triumphed at the University of Paris.

> Gilles,

he said[13],

> opposed St. Thomas with very strong arguments; he shows that the material of the Heavens and the material of human beings below may not be substantially different. But one can also prove against Gilles that these two matters cannot be of the same nature.
>
> Gilles, indeed, persuaded himself that this heavenly matter is not affected by any deprivation, that it wants no form other than its own, because it contains virtually all other forms. But there is a difficulty which he cannot escape, and here it is: The matter of inferior beings is deprived of this heavenly form, and, however, it has a natural power to receive it; it does not possess this form, and, however, its intrinsic nature makes it suitable to be subject to this heavenly form or an analogous form, just as the matter that Gilles places in the Heavens is subject to it, because these two matters are similar in nature. Thus the matter of these inferior beings would have an appetite to acquire the substantial form of the celestial bodies; and as it is impossible that it is never subject to this form, its power and natural appetite would be frustrated forever, which one cannot admit.

The solution to such difficulty seems appropriate; it is to return to the doctrine of the Commentator and deny that there is, in the heavenly substance, a matter subject to a form.

Moreover, the only reason why Aristote admitted a matter in the sublunary beings is drawn from the substantial transformations to which these beings are subject; the supposition of a similar matter, free of all generation and corruption, seems superfluous in the heavens, exempt from all generation and corruption.

Thus, the Heavens are not a composite of a matter subject to a form; it is a simple substance that is in actuality of itself.

> It is said to be simple in the sense where this word is opposed to these: composed of matter and form; but it is composed of parts, endowed with magnitude… It is permissible to give it the name of matter, if one intends, by this word "matter", to refer to the subject of the local movement, something that is able to be here at this moment and elsewhere at another moment.

[13] *In Metaphysicen Aristotelis Quæstiones argutissimæ* Magistri Ioannis Buridani. Lib. VIII, quæst. unica: Utrum cælum habeat materiam subiectam formæ substantiali sibi inhærenti. Ed. cit., foll. LV et LVI.

With these definitions we can, for a specific part of the Heavens, consider the speed with which it moves, the quantity of matter that forms it; thus, Buridan does not contradict himself by attributing a certain *impetus* to this part.

While continuing to deny that the heavenly substance is composed of matter and form, he will continue to talk about the density of this substance:

> In Heaven, a part that is denser contains, under a lesser volume, more of this heavenly substance; it is not necessary, for this, to assume the existence of a matter.

Thus the intensity of the *impetus* must be measured, according to the thought of Buridan, by the product of an increasing function of speed, volume of the mobile, and the density of the substance that forms this mobile. For heavy bodies, this density is, without doubt, proportional to the specific gravity. But it is an attribute much more general than specific gravity. There is a density even for celestial bodies that are free of gravity and lightness; these bodies, too, can move under the *impetus* imprinted on them.

This proposition of Buridan is, perhaps, the first clear apperception of a truth that the 17th century will have the glory of making beyond dispute: The same Dynamics is to govern the celestial movements and the movements of sublunary bodies.

We could, among the issues that the Philosopher of Béthune had eximined regarding Physics, well glean passages where some of the thoughts whose presentation[14] we read and analyzed would be repeated, more or less at length. It would take too long to transcribe all these passages here. We will only reproduce one[15] where the philosopher treats, as he did in the previous pages, of the conservation of the movement of the celestial orbs.

> It is an imagination,

said Buridan,

> that I cannot refute in a demonstrative way. According to this imagination, from the creation of the world God has driven the heavens with movements similar to those by which they are currently moved; he then imprinted them with an *impetus* by which they continue to be uniformly moved; these *impetus*, indeed, meet no contrary resistance, are never destroyed or weakened. Similarly we say that a stone launched into the air is moved, after it has left the hand that threw it, by an *impetus* imprinted in it; but the great resistance that comes as much from the medium as from the inclination of the stone toward another place, continually weakens this *impetus* and eventually destroys it. According to this imagination, it is not necessary to pose the existence of intelligences that move celestial bodies in an appropriate manner; much more, it is not necessary that God moves them, if not in the form of a general influence by which we say he cooperates in all that is.

This explanation of the motion of the celestial spheres is very close to the heart of our philosopher, which in another of his writings he gives a third exposure. The

[14] We have already cited elsewhere (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, seconde série, p. 423) a passage where Jean Buridan explains the accelerated fall weights exactly as in the question that will be translated; in the next paragraph, we find this passage.

[15] Magistri Ioannis Buridam *Questiones quarti libri Physicorum*. Queritur nono utrum in motibus gravium et levium ad sua loca naturalia tota successio proveniat ex resistentia medii. Bibl. Nat., fonds latin, ms. 14723, fol. 68, col. *c*.

commentary on the *Metaphysics* of Aristotle led him to discuss the doctrine of the Stagirite whereby each celestial orb is driven by a special intelligence. In this discussion, it is necessary, according to Buridan, to distinguish the suppositions of secular wisdom from the teaching of the Catholic faith. Also, after having examined the opinions of Aristotle and commentators, he continues in these terms[16]:

> One can even imagine another hypothesis, but I do not know whether it is extravagant (*nescio an sit fatua*). Many physicists, you know, assume that the projectile, after leaving the mover that launched it, is driven by an *impetus* that this mover gave it; it moves as long as the *impetus* is stronger than the resistance; this *impetus* would last indefinitely (*in infinitum duraret impetus*) if it were not diminished and destroyed by something contrary which resists it or by something that inclines the mobile to a contrary motion. However, in the celestial movements, there is nothing contrary that resists. In the creation of the world, thus, God moved each sphere with the speed that his will assigned to it, then he stopped moving it; in the course of time, these movements have always persisted under the *impetus* imprinted in the spheres themselves. That is why it is said that God rested, on the seventh day, from all the work he had completed. I do not say, however, that he might cease to act to the point of not continuing this general influence outside of which a man himself, Socrates, for example, could not work; one would make a mistake, in fact, if he claimed that something can move itself, or even only exist, outside of this overall influence.

Buridan concludes this presentation of his bold hypothesis with the following words:

> You see the views of the philosophers, previously reported, differ greatly from the truth of the Catholic faith.

His theory of the motion of the celestial spheres, where our principle of inertia is found powerful, appears in his eyes as the mechanical commentary of the text where Genesis contemplates the divine rest on the seventh day of Creation.

[16] *In Metaphysicen Aristotelis Quæstiones argutissimæ* Magistri Joannis Buridani. Lib. XII, quæst. IX: Utrum quot sint motus cælestes, tot sint intelligentiæ et converso. Edit. cit., fol. LXXIII, col. *a*.

Chapter 5

That the Dynamics of Leonardo da Vinci proceeds, via Albert of Saxony, from that of Jean Buridan. — To what extent it deviates and why. — The various explanations of the accelerated fall of weights that have been proposed before Leonardo.

Jean Buridan certainly attached extreme importance to the hypothesis that the celestial orbs continue to move under the *impetus* that the Creator gave them originally; giving great weight to this view, his judgment did not deceive him. We have seen[1] that this doctrine was reproduced by Albert of Saxony; we have also acknowledged everything that this theory suggested to Nicolas of Fords and, via Nicolas of Cusa, to Johannes Kepler. His influence does not even stop there. The permanence of the *impetus*, rectilinear or circular, in the case where the tendency of this *impetus* is upset neither by the resistance of the medium, nor by the natural gravity of the mobile, is the hypothesis that carries the whole Dynamics of Galileo[2]. Descartes was to achieve a more correct statement of the law of inertia; but by reducing, as has been said, the role of the Creator in the movement of the universe to "a first flick", Jean Buridan could have approved it.

Besides, this theory regarding the movement of the celestial spheres is not the only passage that deserves to be noticed in the question just quoted; there is no part of this question which is not pregnant with discoveries that modern Science will take care to update.

The history of Dynamics would show us the notion of *impetus* through two and a half centuries acquiring nothing that the Philosopher of Béthune had not already given it; it would then show us the stripping of its purely qualitative form for a more precise quantitative form; we would see the *quantité de mouvement* of Descartes evaluated, first incorrectly, and thus becoming the *momento* of Galileo; it would make us finally recognize it under its correct mathematical form in the *live force* of Leibniz.

The same story would tell us that Newton did not have an idea of mass that different from what Buridan defined; open, indeed, his book *Principles* and read the lines which begin:

[1] *Nicolas de Cues et Léonard de Vinci*, IX et X (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, XI; deuxième série, pp. 180-21 1).

[2] Emil Wohlwill, *Die Entdeckung des Beharrungsgesetzes*, Il (*Zeitschrift fur Volkerpsychologie und Sprachwissenschaft*, Bd. XV, pp. 96 sqq.)

Definition I. — *The quantity of matter is the measure of this matter obtained by multiplying the density by the volume.* The quantity of air of twice the density that occupies a space twice as much is quadruple; a space triple as much contains a quantity six times as much. Understand the same thing regarding snow and dust that can condense by liquefaction or compression. It is the same for all the bodies that are likely to condense in various ways by the effect of any causes… This quantity that will follow, I shall sometimes refer to by the names of *body* and mass. It manifests itself in each body by the weight of this body; indeed, using experiments very exactly done on pendula, I found that it was proportional to weight, as it will be taught further on.

Definition II. — *The quantity of movement is the measure of this movement obtained by multiplying the speed by the quantity of matter.*

Certainly the thought of Newton here is very close to that of the Philosopher of Béthune; and, moreover, as the old master of arts said regarding mass carries the seed of the clearest and most natural method that we find today for introducing this concept in our Energetics.

Since the day when Jean Buridan proposed it, this notion of mass, measured by the intensity for *impetus* that corresponds to a given speed, has ceased to be defined[3] in the sammer manner, in France, Germany, and Italy, by all the Nominalists, by Albert of Saxony, Marsilius of Inghen, John Dullaert, Frederick Sunczel, and Cajetan of Tiene, while the Averroists Vernias and Achillini contributed to making it known by fighting it. Kepler adopted it; he clearly formulated it and was in charge of transmiting it to Newton.

Finally, regarding the explanation of Buridan to account for the accelerated fall of bodies, a continuous filiation brought out this great truth of modern Mechanics: A constant force produces a uniformly accelerated motion[4].

The masters of the terminalist school in Paris have, during all the Middle Ages, jealously guarded the deposit of this Mechanics, so rich in fertile thoughts, which Buridan taught on the Rue de Fouarre circa 1350. At the beginning of the Renaissance, it creeps into Italy, where the Averroists of Padua and Bologna gave it, so far, a very bad reception; now, it will find adept followers whom the ancients have trained at the skilled methods of Geometry, who translate it into mathematical language, which will thus explicate the truths contained in potentiality and determine it to produce modern Science. In the writings of Leonardo da Vinci, we grasp this Parisian science at the same time when it passes from the medieval mind to the modern mind. This Mechanics, in fact, about which the great artist constantly thinks, which he tries to apply to all the problems of which his thought is possessed, which he celebrates as "the paradise of mathematical sciences", is the dynamics of Buridan; and the *Question* that we have reproduced is somehow the theme of which the notes of the great painter will develop variations.

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[3] *Nicolas de Cues et Léonard de Vinci*; X. La Dynamique de Nicolas de Cues et la Dynamique de Kepler (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, XII; seconde série, pp. 201-207).

[4] P. Duhem, *De l'accélération produite par une force constante; notes pour servir à l'histoire de la Dynamique* (*Congrès international de Philosophie tenu à Genève en 1904; rapports et comptes rendus*, pp. 850, seqq.),

Hence, we do not conclude that the philosopher of Bethune directly influenced Da Vinci; no indication allows us to assume that Leonardo read the *Questions on the Physics* of master Jean Buridan. But he had read and pondered at length, we know, the *Questiones in libros de Cœlo et Mundo* of Albert of Saxony; in the latter work, he found a concise but precise exposition of the Dynamics that the former had so masterfully made; via Albertutius, it is the teaching of the Philosopher of Béthune that Leonardo received; it is this teaching that developed his own thoughts.

There is one point, however, where the Dynamics of Leonardo remained far behind the dynamics of Jean Buridan; what the one said for explaining the accelerated fall of bodies is not found in the other.

Jean Buridan supports the view that the increasing speed of the weight is due to an *impetus* which adds to the gravity of the mobile and is constantly increasing.

Leonardo da Vinci does not appear to have adopted this theory. If we want to make an exact account of what he thought in this regard, we must measure the power that could incline him toward the explanation that Buridan had proposed, and also the resistances which solicited him to favor other explanations; and for this, we need to trace briefly what the predecessors of Da Vinci had imagined on the subject of the accelerated fall of bodies[5].

When a heavy body falls freely, the speed of its fall grows from one moment to the next. This fact has probably been known from ancient times; Aristotle mentions it several times:

> Always[6], the mobile which tends toward its resting place seems to move with an accelerated movement; on the contrary, the body which moves with a violent movement slows down its course — Ἀλλά τὸ μὲν ἱστάμενον ἀεὶ δοκεῖ φέρεσθαι θᾶττον, τὸ δὲ βία τοὐναντίον[7].

How could this acceleration of the fall of bodies be found? Simplicius cites[8] two specific observations to highlight it:

1. When a trickle of water falls from a high place, from a gutter, for example, it is continuous in the vicinity of its origin; but soon the acceleration of the fall separates the drops of water which on the ground isolated from each other.
2. When a stone falls from a high place, it hits the obstacle more violently if we stop it towards the end of its fall than in the middle or the beginning; the more violent shock signifies a greater speed.

Simplicius takes these comments from a writing entitled: Περί κινήσεως, composed by Straton of Lampsacus, who was a disciple of Theophrastus, the favorite

[5] We have already addressed this issue, in a much less comprehensive way, in the following writing: P. Duhem, *De l'accélération produite par une force constante. Notes pour servir à l'histoire de la Dynamique* (*Congrès international de Philosophie tenu à Genève en septembre 1904; Comptes rendus du Congres*).

[6] Aristote, Φυσικῆς ἀκροάσεως τὸ Ε (book V, ch. VI). — (Édition Didot, vol. II, p. 317.)

[7] Cf.: Aristote, Φυσικῆς ἀκροάσεως τὸ Η, θ (Book VIII, ch. IX) — Περί Οὐρανοῦ τὸ Α, η (book III, ch. VIII); τὸ Γ, β (book III, ch. II). — (Édition Didot, vol. II, pp. 363, 380 et 415.)

[8] Simplicii *in Aristotelis Physicorum libros quattuor posteriores commentaria*. Edidit Hermannus Diels, Berolini, MDCXCV, p. 916 (Comment, in Physicorum lib. V, cap. VI).

pupil of Aristotle. But it is clear that they could be made at any time and it would be puerile to look for the first author.

What explanation does Antiquity give to this acceleration?

Let us go back to the basic principle of Peripeticic Dynamics; based, apparently, on the most frequent and certain observations, this principle can be stated in these terms[9]:

If a force (ἰσχὺς) or power (δύναμις) moves a certain body with a certain speed, it will need twice the strength or power to move the same body with twice the speed.

This principle, which was accepted without question for centuries, required that the increasing speed of a falling body correspond to an increasing value of the force. The problem posed by the accelerated fall of heavy bodies was immediately transformed, for the ancient philosophers, into this: *To what is the continual increase of the force exerted on a body due, while it approaches the ground?*

If you doubted that in the previous lines we had exactly interpreted the doctrine of Greek physicists, the doubt would be dispelled by reading Themistius

Themistius composed a *Paraphrase* of the Περί Οὐρανοῦ of Aristotle; this *Paraphrase* had been translated from the Greek into Syriac, from Syriac into Arabic, and Arabic into Hebrew; in the 16th century, a Jew of Spoleto, Moses Alatino, gave from the Hebrew text a Latin version which, alone, today, is the work we conserve.

Now, in this writing, Themistius deals with the increase of speed in the fall of weights; he maintains[10] that the natural place, the term of the rectilinear movement, must necessarily be a determinate place and located at a finite distance.

> That there can be no undetermined places, because no mobile can move to infinitely (which would happen if indeterminate locations existed), we can still recognize by the following consideration: All earth has an even quicker and faster movement as it approaches the lower place; it is the same for all fire that approaches the higher place; thus, if they moved indefinitely, the speed and rapidity of their movement should grow to infinity. Thus if the places are not located at defined distances, the propensities to these places, i.e. their gravity or levity, will not have limited quantities; they will grow without measure. When a body, indeed, moves downwards with a certain speed, it is from gravity that it holds this speed; also, although the size of the mobile remains invariable as this mobile progresses, it gains a more intense gravity. However, we also taught that no finite body can possess an infinite force; thus, it cannot be that the bodies moving have, for the places to which they tend, an infinite propensity; therefore, they will never gain infinite speed; therefore, it is consistent with reason that the natural attractions are located at limited distances.

It is difficult to express better than Themistius does in this passage the essential principle of all the explanations we will review. The speed with which a determined mobile moves in a given medium is proportional to the force that pulls this mobile; the acceleration of the fall of a weight thus supposes that the weight of this body

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[9] Aristote, Φυσικῆς ἀκροάσεως τὸ Ζ, ε (Book VI, ch. V) — Περί Οὐρανοῦ τὸ Γ, β (book III, ch. 11).

[10] Themistii Peripatetici lucidissimi *Paraphrasis In Libros Quatuor Aristotelis de Cœlo* nunc primum in lucem edita. Moyse Alatino Hebræo Spoletino Medico, ac Philosopho Interprete. Ad Aloysium Estensem Card. amplissimum. Cum Privilegie Venetiis, apud Simonem Galignanum de Karera, MDLXXIIII. Lib. I, circa text. 88, fol. 141 verso.

grows constantly; there is no doubt of the existence of this increase; the whole problem is to discover the cause.

To the question thus formulated, we gave several diverse and numerous answers.

First, here is the opinion that Aristotle seems to have conceived:

Gravity is a quality by which the weight tends to its *natural place*, i.e., to the place where its form reaches its perfection, where its own preservation is best assured. The more the weight approaches this place, the more this quality becomes intense; viz., the closer it approaches the ground, the heavier it becomes.

That such is the opinion of Aristotle is not easy to prove by formal citations; the most one can say is that this opinion is not in disagreement with such a passage of his writings[11]. But his most loyal commentators have interpreted the thought of the Stagirite. Simplicius, notably, formulates it in these terms:

> Ἀριστοτέλης… νομίζει… βάρους γοῦν προσθήκη τὴν θᾶττον φέρεσθαι πρὸς τῷ μέσῳ γινομένην.

Besides, whether opinion is or is not of the Philosopher has been clearly articulated by Themistius:

> Rectilinear movements,

he said[12],

> which are produced by an impulsion and an unnatural violence are certainly not uniform. But it is the same for natural and spontaneous movements; the farther they are from their start, the more they are lively and quick. Indeed, the bodies moving in this way increase their speed more and more; because, the more they approach the term where they tend, the more they are close to being united to the places which are related to them and which must ensure their safety.

This explanation, we will see, was destined to receive a more or less universal favor in the 13th century.

A second explanation of the accelerated fall of bodies has been proposed by Hipparchus in his essay entitled: Περὶ τῶν διὰ βαρύτητα κάτῳ φερομένων; Simplicius has conserved it for us[13].

When a body is thrown into the air, the virtue which drives it upwards first outweighs gravity; but this virtue is constantly weakening; it surpasses therefore the weight less and less, so that the projectile rises up less and less quickly. A moment comes when the lifting force is precisely equal to the weight; the body then ceases to rise and starts to descend. As the lifting force always diminishes, gravity prevails more and more and the weight falls faster and faster.

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11 Cf. Aristotle, Περί Οὐρανοῦ τὸ Α, η (book I, ch. VIII). — (Édition Didot, vol. II, p. 380.)

12 Themistii Peripatetici lucidissimi *Paraphrasis in Aristotelis Posteriora, et Physica*… Hermolao Barbaro Patrick) Veneto Interprete. Venetiis, apud Hieronymum Scotum. 1542. Lib. VIII, circa text. 76; p. 207.

13 Simplicii in Aristotelis de Cælo commentaria edidit J.-L. Heiberg, Berolini, MDCCCXCIV, p. 264. (Comm. in de Cælo lib. I, cap. VIII.).

This text[14] has sometimes been invoked to prove that Hipparchus, rather than assigning the maintenance of the motion of projectiles to the fluid medium, made this maintainence due to a virtue, an *impetus* imprinted in the substance of the mobile.

That Hipparchus admitted the existence of such a virtue is quite possible, but it would be imprudent to be fully sure by what the great astronomer, in the testimony of Simplicius, said regarding the accelerated fall of bodies. The *force projecting upwards*, the ἀναρρίψασα ἰσχὺς of which he speaks, may well be the push that, according to the Peripatetic Physics, the shaken air exerts on the projectile.

The thirty-third of the *Mechanical Questions* attributed to Aristotle asks why large projectiles soon stop.

> It is,

he replies,

> not because the *projecting force* (ἰσχὺς) ends, or because of the rotation, or because the weight of the mobile eventually became more powerful than the *projecting force* (ἰσχὺς ῥίψασά).

The expression used here is the same as what Hipparchus used. However, the thirty-fourth *Mechanical Question* seems to be a summary of the considerations by which Aristotle in his *Physics* explains the maintenance of the motion of projectiles by the successive pushes of the shaken air; and the thirty-fifth *Mechanical Question* formally invokes this theory.

Greek Antiquity has therefore left us a single text where the motion of the projectile was clearly and formally assigned to an *impetus impressus*; this is the comment that Jean Philopon made on the fourth book of the *Physics*.

Let us return to the suppositions of which the accelerated fall of bodies was the object.

Alexander of Aphrodisias[15] does not adhere to the explanation of Aristotle, nor to the explanation of Hipparchus.

Like Hipparchus, he deemed the increase that the weight of a body as it approached the ground would feel as unlikely, but he objected to the opinion of Hipparchus; excellent for explaining the accelerated fall following a violent movement, it is faulty when any violence preceded the downward movement.

In turn, he offers a theory that is not without affinity to that of Hipparchus.

When a weight is kept in a high position, its nature alters and turns it into a contrary nature; as a weight, it tends to become light. Then, removing the obstacle that retained it, it will fall; but during the first moments of its fall it will keep something of this lightness acquired by its stay in the high place, of this virtue which is opposed to the descent; the gravity of the mobile will be reduced by as much and the fall will

[14] Arthur E. Haas, *Über die Originalität der physikalischen Lehren des Johannes Philoponus* (*Bibliotheca Mathematica*, 3 Folge, Bd. VI, p. 337, 1906).

[15] Simplicius conserves the opinion and the text itself of Alexander of Aphrodisias for us. Cf.: Simplicii *in Aristotelis de Cælo commentaria* edidit J.-L. Heibcrg, Berolini MDCCCMCIV, p. 265. (Comm. in de Cælo lib. I, cap. VIII.)

be very slow at first. Then, little by little, the acquired lightness will weaken; it will hinder the gravity less and less, and the fall accelerates.

All that these opinions professed by Aristotle and his commentators—by Hipparchus, by Alexander of Aphrodisias—have in common is that they attribute the constant acceleration in the fall of the bodies to a property of the heavy body itself.

Other interpretations attribute to the medium in which the fall occurs the increase of force which opposes this increase in speed.

Simplicius[16] tells us that, in his time, a number of physicists (τινὲς δὲ χαὶ οὐχ ὀλίγοι) explained the accelerated descent of bodies as follows:

When a body is very far from the ground, a large thickness of air lies below it; this thickness gradually becomes smaller as the weight approaches the ground; therefore, in falling this mobile divides the underlying air easier and thus seems heavier.

In relating these hypotheses to us, Simplicius seems to remain very skeptical about how credible they are. For the theories of Alexander of Aphrodisias and Themistius, he proposes a test; in offering it, he seems to predict well that it will give a result unfavorable to these hypotheses and that proponents of these assumptions will find ways to evade its denial:

> If,

he said[17],

> a weight gradually approaches its natural place, there is an increase in gravity, here is what should happen when one weighs a body in the air: Whether we place ourselves at the top of a tower, or a tree, or on top of a peaked rock, and weigh a body held by a wire that descends from there to the ground, this body must seem heavier than if it were weighed at ground level. This assumption seems fabricated; we could, it is true, object to this experiment that the difference is inperceptible.

However, the opinion of Simplicius seems to tend toward that of Themistius:

> In this world,

he said[18],

> which different property does a body have as it is separated from its natural place by such and such a distance? This one only: It begins to move more weakly to its natural place when it goes from a more distant position, and there is a constant relationship between the weakness of the movement and the greatness of the distance.

Simplicius tells us how several physicists sought to explain the acceleration observed in the fall of of bodies; they admitted that the resistance of the layer of air it traverses through decreases as this layer becomes thinner. Another theory also

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[16] Simplicii *in Aristotelis de Cælo commentaria* edidit J.-L. Heiberg, Berolini, MDCCCXCIV, p. 266. (Comm. in de Cælo lib. I, cap. VIII.).

[17] Simplicii, *loc. cit.*, ed. cit., p. 267.

[18] Simplicius, *loc. cit.*, ed. cit., p. 255. — Cf. *Léonard de Vinci et la pluralité des mondes*: III: Le poids d'un grave varie-t-il avec la distance au centre du monde? — Simplicius, Averroès, Albert le Grand, Saint Thomas d'Aquin (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, X; seconde série, pp. 64-05).

attributed to the medium the increase in speed of weight that falls; but this explanation was probably offered after the time when the famous Athenian commentator wrote, because he made no allusion to it. This theory, which was destined to be very popular, is located in a treatise *De ponderibus* of which we have established its Hellenic origin[19] and whose unknown author we have designated as the Precursor of Leonardo da Vinci.

In the fourth book of the treatise *De ponderibus* with which we deal at the moment, the fifteenth proposition is formulated thus[20]:

> *A liquid flowing continuously forms a jet whose cross-section is even narrower the longer the liquid crossing this section runs.*

The unknown Greek author explains this phenomenon in the following way[21]:

> Let *ab* be the orifice through which it is flowing, and *c* the first flowing part. When this part comes to *df*, the part *o* is at the orifice. Similarly, when part *e* arrives at *df*, part *o* is at the orifice, etc. The more a part descends, the heavier it becomes; part *c* is thus heavier at *df* than it was at *ab*; so it is heavier in *df* than the part *e* is at *ab*; also while *e* reaches *df*, *c* reaches *zl*, such that *fz* is longer than *af*; the jet becomes continually more spindly, because the parts that came out first are the fastest; also, they eventually separate from each other.

This is, we saw, the explanation, according to Simplicius, already given by Straton of Lampsacus. The precision that the Precursor of Leonardo brings in this explanation deserves to be reported. We find there, in fact, this formally reported truth: The cross-section of a given liquid flow is smaller as the fluid flows with more speed. Now, we have seen what role the discovery of this principle played in the evolution of the ideas of Leonardo da Vinci[22]. Would this discovery not be suggested to him in reading the passage we just translated?

The Precursor of Leonardo attributed the accelerated fall of bodies to an increase in their weight; from where does this increase come? He tells us in the fifth question of the same book; here is this question:

> *A heavy thing moves faster the longer it descends.* This is more true in air than in water, because air is suitable for all kinds of movements. So a weight that descends pulls, in its first movement, the fluid that is behind it and starts to move the fluid that is underneath and in immediate contact to it; the parts of the medium thus set in motion move those which follow them, so that the latter, already shaken, put up a lesser obstacle to the weight which descends. Therefore, it becomes heavier and gives a stronger impulsion to the parts of the medium which give way before it, to the point that these are no longer simply driven by it, but that they draw it. So it happens that the gravity of the mobile is aided by their pull and that, conversely, their movement is increased by this gravity, so that this movement continually increases the speed of the weight.

[19] *The Scientia de Ponderibus et Léonard de Vinci*, VIII: Conclusion (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, VIII; première série, pp. 310-316).

[20] *Loc. cit.*, p. 285.

[21] The text, very wrong and almost incomprehensible in the 7378 A manuscript of the Latin archive of the National Library, is much more correct than the manuscript 8680 A of the same archive.

[22] *Themo le fils du Juif et Léonard de Vinci*, VI : L'écoulement uniforme des cours d'eau (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, V ; première série, pp. 195-198).

It well seems that, of the opinions professed by the Greeks affecting the fall of weights, we do not have any more recent text than this one.

Averroes tells us how he was aware of this acceleration, and what he said unfortunately leaves us guessing.

In terms almost as explicit as those of Themistius, he declares[23]

> that the cause for which various things move with different speeds is the diversity that exists in their inclination, i.e. in their gravity or their lightness; as a result, the heaver or lighter a body is, the quicker it moves; moreover, it is clear that this proposition can be reversed and that the faster a body is in its movement, the heavier or lighter it must be; if it is so, when the speed is infinite, the heaviness or lightness is also infinite.

But Averroes follows no more than the opinion of Themistius and Simplicius; he does not admit that the weight of a body varies with its distance from the center of the world.

> Know,

he says[24],

> that the proximity and remoteness have no influence, unless these bodies move under the action of an external cause, because then these bodies could be near or far from their mover.

When a piece of iron is attracted to a magnet, the attraction it feels is greater the closer it is to the stone that moves it. There is nothing analogous when considering the weight of a body, because the body carries in itself the principle of its movement.

One should not, indeed, according to the Commentator[25], confuse the attraction that place exerts on the weight with the attraction that the magnet exerts on iron; although these two actions are both improperly called attractions, they differ greatly from the other:

> Any attraction in which the attracting body remains motionless while the drawn body moves is not, in fact, an attraction. The drawn body moves itself to the attracting body in virtue of its own perfection. So it is for a descending stone or fire going up; and one must understand that it is the same with the movement of iron toward a magnet… But there is a difference between this case and the bodies moving to their natural places. Any of these bodies, in fact, moves similarly to its place, be it near or far away… Iron, on the contrary, moves toward the magnet only when it is endowed with a certain quality that emanates from the magnet; also, if one rubs the magnet with garlic, it loses its virtue, because then the iron does not receive more from the stone thus disposed that quality which makes it able to move toward it.

Despite the opposition from Averroes, it is the hypothesis of Themistius which triumphs among Christian philosophers of the 13th century.

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[23] Aristotelis *De Cœlo… cum* Averrois Cordubensis *variis… commentarius,* lib. I, summa V11I, cap. IV, comm. 88.

[24] Averroes, *loc. cit.*, cap. III, comm. 81. — Cf. *Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu,* seconde série, pp. 66-67.

[25] Aristotelis *De physico auditu libri octo cum* Averrois Cordubensis *variis in eosdem commentariis*; lib. VII, summa III, comm, 10.

Albert the Great shows[26], as did Aristotle and Themistius, that a heavy or light body cannot continue its rectilinear movement to infinity.

> Earth, fire, and, in general, all heavy or light bodies show us that the [natural] movement cannot grow to infinity. All these bodies, indeed, move faster toward the end of their movement, and their speed becomes more intense as they move farther away from their starting point; we have, in Physics, indicated the cause. So if the movement of these bodies continued to infinity, it should be believed that that speed also grows to infinity; as, moreover, any increase in speed can only come from an increase in gravity or lightness, the gravity or lightness would become infinite; and we have previously demonstrated that this is impossible.

The body that moves into its natural place thus becomes continually heavier or lighter; this increase of heaviness or lightness is not an accidental increase due, for example, to any action of the medium; it is a real increase in the natural form which constitutes its heaviness or lightness:

> The natural movement, in effect[27], is a step forward towards the natural form or place (*ubi*); thus the more the mobile moves, the more vigorously it acquires its natural form; therefore, since the movement is the result of the natural form, it must be that the more the mobile acquires this form, the more vigorously and speedily it moves; also, any purely natural movement is faster at the end than at the beginning or in the middle, and faster in the middle than at the beginning. In violent movement, the opposite happens; anything driven by violence loses some of the strength of its form; when the form retakes its vigor, the mobile returns to its natural movement.

Averroes declared that this increase of form, which constitutes gravity or lightness, would prove impossible as a result of its approach to the natural place; Albert does not admit this impossibility; according to him, this increase of the form has the same cause as the form itself, and this cause is that which engendered the heavy or light body:

> One can show in a natural way[28]—using simple bodies and movements of physical bodies like earth, fire, and other similar bodies—that place is a reality. From the movement of these bodies, in fact, one draws the evidence not only that place is a reality, but that it has a certain property in which the form of the bodies that move towards it receives its complement. Any physical body, indeed, as soon as it is not prevented, moves into its proper and natural place as toward that which must give it its perfect form. *As much as this body receives its form from the part of its generating cause, it has received it from place.* One and the same generating cause, although it gives form to this body, gives it a place where this form will be completed and kept.

St. Thomas Aquinas, like all the peripatetics who succeeded him, invokes[29] the acceleration of natural movement in order to prove that this movement cannot con-

[26] Beati Alberti Magni, Ratisponensis episcopi, *De Cœlo et Mundo*; lib. I, tract. III, cap. III: Illorum qui dicunt elementa mundorum non moveri ad invicem eo quod distent in infinitum.

[27] B. Alberti Magni, Ratisponensis episcopi, *Liber physicorum sive physici auditus*; lib. V, tract. III, cap. VIII: De solutione quarumdam dubitationum quæ oriuntur ex præhabitis.

[28] B. Alberti Magni *Op. cit.*, lib. I, tract. I, cap. II: De probatione quod locus sit aliquod in natura.

[29] Sancti Thomæ ab Aquino *Commentaria in libros Aristotelis de Cœlo et Mundo*, lib. I, lect. XVII.

tinue to infinity. He adds the following considerations, where we recognize a summary of the commentaries of Simplicius[30]:

> It is necessary to know that Hipparchus assigned the cause itself that violently moved the body to this accident, to the fact that earth moves faster the more it descends; the more, indeed, the movement continues, the less the virtue of the mover remains, and so the movement slows down. It is for this cause that the violent movement is more powerful at the beginning; towards the end, it weakens more, and a moment comes when the weight can no longer be carried upward; it then begins to descend, because of the smallness of what remains of the virtue provided by the mover, author of the violent movement; the more this virtue weakens, the more the contrary movement quickens.
>
> But this reason is not general; it applies only to the body which, after a violent movement, moves with natural movement; it does not apply to those which move from natural movement because they were engendered outside of their proper places.
>
> Others have sought the cause of this effect in the quantity of the medium—of the air, for example—through which the movement occurs; they admitted that this air would particularly resist more as the natural movement progressed more and, therefore, it placed less and less of an obstacle to this natural movement. But this reason would be as valid for violent movements as for natural movements; and in these violent movements, it is the opposite that happens.
>
> So let us say with Aristotle that the cause of this effect is this: The more the heavy body descends, the more its gravity strengthens, because this body is approaching its own place. It is thus proved that for the speed to increase to infinity, gravity would have to increase to infinity. One can say the same for lightness.

It is likely that St. Thomas rendered this increase in gravity or lightness by the approach of the natural place for the same reason that Albert the Great did. At least this is what Pierre d'Auvergne did, who finished the commentary on the *De Cælo* interrupted by the death of the Angelic Doctor, his master.

> Heavy bodies,

he said[31],

> or light ones are under the power of the natural place, as they are of the form; they are thus moved by the generating cause that gives them their form; insofar as this cause gives them the form, it gives them the place to the same extent.

This proposition reproduces verbatim an affirmation of Albert the Great.

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30 The commentaries on the *De Cælo* composed by Simplicius had been, in 1271, translated from Greek into Latin by William of Moerbeke, who was a friend of St. Thomas Aquinas. He was therefore able to use them, and he used them generously in his own commentary on the *De Cælo*. This commentary was, in fact, the last work of the Angelic Doctor; when he died in 1274, the writing remained unfinished.

31 *Libri de cælo et mundo Aristotelis cum expositione* Sancti Thome de Aquino. *et cum additione* Petri de Alvernia. Colophon:

> Venetiis mandato et sumptibus Nobilis viri domini Octaviani Scoti Civis modoetiensis. Per Bonetum Locatellum Bergomensem. Anno a salutifero partu virginali nonagesimo supra millesimum ac quadringentesimum. Sub Felici ducatu Serenissimi principis Domini Augustini Barbadici. Quinto decimo kalendas Septembres. Lib. IV, comm. 24, fol. 71, col. *c*.

St. Thomas Aquinas made no reference to the explanation of the accelerated fall of weights that the Precursor of Leonardo da Vinci gives in his treatise *De ponderibus*. On the other hand, it is by a similar supposition that he accounts for[32] the so-called initial acceleration of projectiles. We have said elsewhere[33] what vogue this theory of the Angelic Doctor had throughout the history of Dynamics.

Roger Bacon is comprehensive[34] regarding the explanation, accepted by Themistius, of the accelerated fall of weights, and the objections that the Commentator raised against this explanation. The discussion he develops led to adoption of a sort of middle way. First of all, far and near, the body desires to reach its natural place; this place moves it as the final cause, and the motive power that results has an intensity that does not vary with distance. On the other hand, from a certain distance, place moves as the efficient cause, just as a magnet moves the iron; it exercises on the weight an action which reinforces the first power, and all the more so because the heavy body is closer to the term to which it tends.

This complicated assumption is only a simplification of the assumption of Saint Bonaventure.

> To explain the movement of a weight,

the Seraphic Doctor said[35],

> it is not enough to rely on the gravity, the quality proper to the mobile; a virtue eminating from the place that attracts and another virtue issuing from the place that pushes contribute to this movement.

Of the three causes invoked by Saint Bonaventure, Bacon removed one, the repulsive action of the place from which the mobile moves away. But let us listen to the famous Franciscan:

> Does this virtue, by which the mobile naturally goes into its place, exist in this mobile in virtue of an influence emanating from the place? *It seems so*, because, according to what Aristotle said, this power of the place—by which any body, when it is not prevented, is taken into its own place—is remarkable.
>
> *Item*, the movement of bodies toward their place is similar to the movement of iron to the magnet; it is commonly said; Averroes speaks of it in the 8th book of the *Physics* and elsewhere; however, this latter movement is produced by the influence of a certain virtue.
>
> *Item*, natural movement is stronger towards the end; the more a weight descends, the more rapidly it descends, as happens for the iron that is close to a magnet; but the cause of this increased speed is the greater proximity between the mobile and place; so that the place can cause this speed, it must be, it seems, that it has some influence.

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32 Sancti Thomæ Aquinatis *Commentaria in libros de Cælo et Mundo*, lib. II, cap. VI, lect. VIII.

33 Bernardino Baldi, Roberval et Descartes, I: Une opinion de Bernardino Baldi touchant les mouvements accélérés (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, IV; première série, pp. 197-139).

34 *Liber primus communium naturalium* fratris Rogeri Bacon; pars III, dist. II, cap. III: De loco ut est res naturalis conservans locatum. (Bibl. Mazarine, ms. 3576, fol. 58°.

35 Celebratissimi Patris Domini Bonaventuræ, Doctoris Seraphici, *In secundum librum Sententiarum disputata*. Dist. XIV, pars I, art. III, quæst. II: Utrum motus cælorum sit a propria forma vel ab intelligentia.

Item, the force with which the weight moves is continually renewed when the mobile approaches its end; this arises from the fact that a certain disposition is renewed in this body; but it is nothing which can be said to be renewed in this weight except the virtue of the place.

Sed contra: What moves a body by the influence of a certain virtue does not moves it as long as this body is not located within a convenient distance from the source of this influence; this is what occurs for the magnet; the magnet does not move the iron as long as it is not, in relation to the magnet, at a suitable distance so that it can receive the impression of this virtue by which the alteration that obliges it to move is produced in it. The weight, on the contrary, descends down to its place at some distance from where we placed it, and, as Aristotle said in the 4$^{\text{th}}$ book of *On the Heavens and the Earth*, even if one would place it in the concavity of the orb of the Moon. It is therefore clear that place has no influence on the body which moves toward it. This is the opinion of Averroes on the 7$^{\text{th}}$ book of the *Physics*. Although he established a reconciliation between the movement of mobiles toward place, there is, however, between these two movements, a difference; iron, placed at a suitable distance from the magnet, receives a certain impairment, while the mobile receives nothing from the place.

Item, at the end, the matter has, by its form, a more powerful appetite than at the beginning; however, the form does not move the matter as its efficient cause; it could therefore be the case here.

Here is what must be said: Near or far, the virtue of place moves the body as an end loved and desired; but far, this virtue does not move the mobile as its efficient cause; it only moves it as such within a certain distance. As a result of the weight coming to its own place, the weight moves over the whole distance to this place; it naturally tends toward it, moving towards it at some distance that it is placed. But, from the moment when the weight is no longer a fixed distance from the place, it gets from this place a certain virtue which produces in it an alteration by which it moves faster. Iron is not, itself, such an appetite towards the magnet; it is only able to experience this appetite; between its nature and the magnet, there is not such a fittingness that it itself wants to join the magnet and that it moves toward that goal; the fittingness that is between the magnet and the iron only makes the iron apt to receive the virtue emanated from the magnet; it is only when it receives this virtue that it desires the magnet and moves towards it.

The propositions formulated by the various authors who have taken part in this debate could be formulated, in the language of modern Mechanics, roughly as follows:

According to Themistius and his followers, the weight of a body varies with the distance this body is from the center of the world; it decreases when the distance increases; the assertions of Simplicius amount to declaring that the weight is inversely proportional from the distance to the center.

According to Averroes, if a force of attraction increases when the mobile gets close to the attractive center, this force must be annulled when the distance from the mobile to the center exceeds a certain limit; it is, he thinks, what happens to the attraction of iron to a magnet; on the other hand, he admits that a stone remains heavy at any distance from the center of the world; therefore, the weight of this stone remains independent of the distance to the center of the world.

For synthesizing the two opinions, Roger Bacon admits that the weight of a body is the sum of two forces: one of these forces is independent of the distance from the weight to the center of the world; the other is zero as long as this distance exceeds a certain limit; when below this limit, this distance decreases, and the second force becomes increasingly larger.

These discussions have been of great interest in that they have accustomed philosophers to consider attractive forces varying with distance; when Kepler and William Gilbert will try to create a celestial Mechanics using such forces, they will find, carefully preserved by the teaching of the Schools, the ideas that the discussions of the 13th century had analyzed and clarified, and these ideas will provide the primary and essential materials of their theories.

However, the theory of Themistius, inspired by Aristotle and generally adopted in the 13th century, gave the accelerated fall of weights a completely false image. According to this theory, the speed of the weight that falls would depend not on the time elapsed since the beginning of the fall nor on the path traversed during this time, but on the distance from the heavy body to the center of the world. The most common observations sufficed to prove that such a result was grossly erroneous; we do not see, however, any master of Scholasticism making this remark before Richard of Middleton; but he gave this remark extreme precision.

This is, indeed, what the English Franciscan wrote[36], in the last years of the century, commenting on the *Books of Sentences*:

> Some have argued that bodies are moved by a virtue eminating from the place opposed to their natural place, a virtue that would push them.
>
> But we can only say that this is proper cause of the movement of heavy bodies; the more, indeed, these bodies would be far from the center, the more quickly they would move, because they would be more strongly affected by the cause that moves them; however, it is certain that the movement of heavy or light bodies is faster at the end than at the beginning.
>
> Others say that the cause of their movement is a totally attractive virtue emanating from the natural place, so that the movement of the elements to their own place is a pulling motion.
>
> But, contrary this opinion, we can produce the argument here: The Commentator said that an attraction by which the body attracting remains motionless while only the drawn body is moving is not a real and true attraction; in this case, the attracted body moves even to the attracting body, in order to achieve perfection, just as a stone moves downward and fire upward.

Against the theory of Themistius, referred to in the lines that we have just read, Richard of Middleton produced this argument drawn from experience:

> Take two bodies of the same weight and same shape; start the fall of the first in a high place and the fall of the second in a low place, and this in such a way that, when the second (the one that starts from the lowest place) will begin to go down, the first (the one that leaves the highest place) would already reach a distance from the ground equal to that from which the second began to move. The weight that started from the highest place will come to the ground faster than the other weight; and, however, when they were at equal distance from the ground, these two bodies behaved similarly, due to the influence of place.

[36] *Clarissimi theologi Magistri Ricardi de Media Villa Seraphici ord. min. convent. Super quatuor libros Sententiarum Petri Lombardi Quæstiones subtilissimæ*, Nunc demum post alias editiones diligentius, ac laboriosus (quod fieri potuit) recognitæ et ab erroribus innumeris castigatæ, necnon conclusionibus, ac quotationibus ad singulas Quæstiones adauctæ, et illustrat, a R. P. F. Ludovico Silvestrio a S. Angelo in Vado, Doctore Theologo, et eiusdem instituti professore. Cum indice generali, ac locupletissimo totius operis. Ad Illustrissimum et Reverendiss. D. D. Marcum Antonium Gonzagam, Marchionem, Principemq. Rom. Imperii, et Episcopum Casalensem. Brixiæ, de consensu Superiorum, MDXCI. Lib. II, dist. XIV, art. III, quæst. IV; tomus secundus, p. 180.

This objection ruins the explanation that Themistius had proposed for the acceleration in the fall of weights. For this explanation, which is the correct one to substitute according to Richard of Middleton? The one the Precursor of Leonardo da Vinci would give in his treatise *De ponderibus*. Indeed, Richard writes:

> So here is, in my view, what must be said: Although the various elements have been determined by that which has engendered the movements that are natural to them, however it is by their own virtue and [not] by the participation of some influence in their natural places, that they perform the movements to which the generating cause has determined them… But the effectiveness of this movement is helped by the shaking, produced by the light or heavy body that moves, of the medium itself.

The hypothesis of Hipparchus was certainly well-known in the Schools when Richard of Middleton wrote; the translation William of Moerbeke gave on the commentary of the *De Cælo* that Simplicius wrote and the commentary on the *De Cælo* that St. Thomas undertook, could not fail to draw attention to the considerations of the great astronomer. No doubt, these considerations led Richard to write[37]i, regarding a bean thrown in the air, the following lines:

> Note that the ascent of the bean is a violent movement; thus, I say that after the movement of the bean becomes somewhat remote from its starting point, the virtue whereby the bean ascends becomes weaker; so the violent movement is slower near the end than it was in the beginning; that virtue ends up so weakened that it is no longer sufficient to move the bean upward; it is still, however, enough to prevent its descent; and then the bean remains, on its own, immobile; later, this virtue weakens to the point that it can no longer prevent the descent; the natural virtue of the bean then wins over that one, and the bean falls.

In the theory of Hipparchus, Richard of Middleton has introduced something new; he was the first to consider the period of rest which would separate the movement of ascent, which is violent, from the movement of descent, which is natural; we have said elsewhere[38] what fortune this *quies media* doctrine had and how, through the theory of the *impeto* composed by Leonardo da Vinci, it prepared the explanation of the motion of the projectiles that Galileo would one day give.

The theory of Themistius seems to have been struck to death by the objections of Richard of Middleton; the authors who write around the year 1300 do not invoke it to account for the acceleration observed in the fall of weights.

Gilles of Rome teaches[39] that natural movement is faster towards the end, while violent movement is faster at the beginning.

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[37] *Quodlibeta Doctioris eximii* Ricardi de Media Villa, *ordinis minorum, quæstiones octuaginta continentia*. Brixæ, apud Vincentium Sabium, MDXCI. Quodlibitum II, art. Il, quæst. XVI: Utrum faba ascendens obvians lapidi molari quiescat; pp. 54-56. — Cf.: *Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*. seconde série, note II, pi. 442-443

[38] *Nicolas de Cues et Léonard de Vinci*, XI: La Dynamique de Nicolas de Cues et la Dynamique de Léonard de Vinci. Theory of *impeto* composé (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, XI; seconde série, pp. 211-212).

[39] Egidii Romani *in libros de physico auditu Aristotelis commentaria accuratissime emendata: et in marginibus ornata quotationibus textuum et comentorum. ac aliis quamplurimis annotationibus: Cum tabula questionum in fine. Ejusdem questio de gradibus formarum. Cum privilegio*. Colophon: Preclarissimi summique philosophi Egidii Romani De gradibus formarum tractatus Venetiis impressus mandato et expensis Heredum Nobilis viri domini Octaviani Scoti civis Modoetiensis. per Bonetum Locatellum presbyterum. 12° kal. Octobr. 1502. Lib. VIII, comm. 76, fol. 189, col. *c*.

> It should be noted,

he adds,

> that the natural movement starts from a violent rest, while the violent movement departs from a natural rest. So the further the natural movement is from the rest from which it started, the closer it approaches the center; that is why this movement is constantly strengthened by its remoteness from the state of rest from which it left. In violent movement the opposite takes place.

Perhaps you might see in the lines that we just quoted a vague reference to the theory of Themistius; however, we tend to interpret them in an entirely different way when we compare them to each other[40], where Gilles of Rome examines “what a violent rest is and how such a rest can be engendered”:

> It must be said that this violent rest is caused by the violent movement. But it is acknowledged in general that everything generated by such a movement has rather a negative (*privativa*) cause than a positive cause. If, for example, a stone is thrown into the air, it will rest at the top of its course; but the rest comes from a negative principle, namely the lack of impulsion, rather than an effective and positive principle. We must imagine, in effect, that when a stone is thrown into the air, it needs, for it to move quickly, a stronger impulsion than for it to move slowly, so that a stronger impulsion is needed to make it progress upward than for it only to maintain the place that it already reached. However, initially, the impulse is great and strong; then it weakens continually; the stone, or any other object you throw violently upward, thus moves first with force; then, as the impulse is lacking, the projectile moves slower; so it happens that this impulse becomes so weak that the pushed air no longer suffices to raise the stone, although it is sufficient to keep it in the high place that it has reached; finally, in a last period, the thrust of the air weakens so that it can no longer support the heavy body that was launched upward; this body must, therefore, fall. It is clear that such rest is caused by a privation and a defect rather than proceeding from a positive and efficient cause… Thereby, one can resolve the objections made previously. When one says that the movement is always stronger when it approaches its term, it must be understood that this term or this final rest is generated by a movement whose cause is positive and not negative, which is not true of violent rest.

This is true, on the contrary, of the natural repose which a body reaches when it reaches its proper place;

> in this case, indeed, the rest generated by natural movement is the term where the mobile tends, because this term fits the nature of this mobile; this rest has a positive cause and is not caused by privation.

This passage of Gilles of Rome is remarkable in many ways.

We have, in the first place, as we found it in a *Quodlibet* of Richard of Middleton, the idea that a period of rest separates the period during which a projectile rises from the period during which it falls. We also have a recognizable statement of the theory of Hipparchus; but, in this presentation, the continuation of the movement of the projectile upwards is formally assigned to the impulse of the shaken air; so, it is true that the adoption of the theory of Hipparchus in no way implies that an *impetus*, imprinted on the projectile by the hand that threw it, continues to move it after it has left that hand.

[40] Ægidii Romani *Op. cit.*, lib. VI, comm. 64, dubium primum; ed. cit., fol. 117, col. *c*i.

We do not find, however, in these lines written by Gilles Colonna, the explicit definition of the cause that accelerates the fall of a weight. Is this definition very difficult to guess? In the course of the two passages which we have quoted, Gilles does not cease to compare, as Hipparchus did, the accelerated fall of a weight with the slowed ascent of the projectile; what is *positive* in one of these movements is *privative* in the other; today, we would say that our author passes from one movement to another by a simple change of sign; however, he attributes the slowdown observed in the rise of the projectile formally to the decrease of the push that the air exerts on the body; is it not clear that in his mind the acceleration that occurs in the fall of a weight has the growing impulse of an increasingly shaken air for its cause? Like Richard of Middleton, Gilles supported the theory that the Precursor of Leonardo had proposed; now, it is difficult to doubt it; it will be impossible when we read the remarks of Walter Burley on the thought of Gilles of Rome.

It is to the medium traversed by the weight that John of Jandun attributed the acceleration experienced by the fall of this body; but, in his various writings, it must play some different roles.

First, read the commentary on the *De Cælo*[41]; to the theory of Themistius, Jandun objects with various reasons; he reproached him, in particular, to destroy one of the arguments that Aristotle directed against the plurality of worlds; it ends with these words:

> We grant that the natural movement is faster at the end than at the beginning and that a weight, free of any impediment, moves more quickly the closer it is to its natural place. But it is claimed that this cannot be if the fall of this weight takes its principle from a power of the place; we deny this proposition; this occurs not because the weight is moved effectively by virtue of the place, but because the stone which approaches the center is followed by a greater amount of air than it would be in another place, and that air gives the stone a stronger impulsion; that is why this stone moves more quickly.

John of Jandun, in this passage, seems to attribute the accelerated fall of the weight to the amount of air that overcomes the mobile and not to the agitation of this air. We want to clarify his opinion on this and reconcile it with that of the Precursor of Leonardo.

When, in his questions on the *De Cælo* of Aristotle, John of Jandun quotes St. Thomas Aquinas, he named him[42] *Frater Thomas*; when he quotes the same author, in his questions on the *Physics*, he named him[43] *Sanctus Thomas*; the canonization of St. Thomas Aquinas was promulgated in 1323; so we are led to think that John of Jandun had completed his questions on the *De Cælo* before 1323 and that he wrote after that time his questions on the *Physics*.

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[41] Joannis de Janduno *In libros Aristotelis de Cælo et Manda Quæstiones subtilissimæ*. Lib. IV, quæst. XIX: An grave inanimatum quoquomodo moveatur virtute existente in loco.

[42] Joannis de Janduno *In libros Aristotelis de Cælo cl Mundo*; in lib. I quæst. XXIV: An sit possibile esse plures mundos.

[43] Ioannis de Janduno Super octo libros Aristotelis de physico auditu acutissimæ quæstiones; sup. lib. I quæst. II: An eus mobile, vel corpus mobile, sit scientiæ naturalis subjectum; sup. lib. IV quæst. VI: An locus sit immobilis.

Since these questions are later than those, one cannot be surprised when the author criticized, rejected, or fixed some doctrines that he had professed. Hence we will see further precision in his explanation of the accelerated fall of weights.

On the subject of the eighth book of the *Physics*, John of Jandun examines this question[44]: Does an inanimate weight move by itself? The discussion to which he submits this question is one of the most developed that we find in the works of our author; it was also one of the most noticed on the part of the masters of Scholasticism, one whom the name of the Parisian Averroist was most often cited.

Jandun, however, did not deserve the honor of this important question, because this is the confession, full of good faith, by which he ends it:

> That in our posterity those, who from the bottom of the soul will be the friends of the truth rather than of fame, will know one thing well: The proofs given here of the doctrine which I support are not entirely my own; I take them from a theologian that I believe to be, among my contemporaries, one of those who explain Aristotle and the Commentator with the utmost subtlety. However, I have added various things which serve to put order in the explanation and confirmation of this thesis.

Therefore, to this anonymous theologian, and not to Jandun himself, we need to assign the following passage, where the theory of Themistius is, first of all, refuted roughly as it had been by Richard of Middleton:

> They say that the speed of fall of the weight, greater when this weight is near the center than when it is farther away, has no cause other than a certain virtue emanating from the natural place closer to the mobile in the first case than in the second. This proposition can be denied; in effect, the following consequence would result: If we took two bodies of the same weight, where one would begin to descend from the sphere of fire while the starting point of the other would be close to the earth, at the end of the movement these two bodies would traverse equal spaces with equal speeds; obviously, it is the contrary that is true.
>
> If one then says that the greater speed is due to the greater amount of air that follows the mobile falling from a higher place,

which is precisely what Jandun taught in its questions on the *De Cælo*,

> it is not the virtue of place nor the vicinity to this place that is causing this speed; we therefore depart from the first statement.
>
> But if the approach of the natural place is not cause of this greater speed, we will ask what this cause is. Perhaps it must be said, as some do, that it results from the fact that the parts of the air which the weight has divided and which follow it are more numerous at the end of the movement than at the beginning. As a result, they argue, the weight acquires a greater accidental speed from one moment to the next.

The words "*propter scissuram plurium partium æris insequentium*" seem to indicate that the cause is not the thickness of the air mass that overcomes the weight, but the agitation of the air layer it went through.

The theory of Hipparchus has attracted the attention of our Averroist; like Gilles of Rome, he presents it[45] by formally admitting that the motion of a projectile is maintained by the agitation of the ambient air; but he is very reluctant to place the

[44] Ioannis de Janduno *Op. cit.*, sup. lib. VIII quæst. XI: An grave inanimatum moveat seipsum.

[45] Ioannis de Janduno *Op. cit.*, sup. lib. VIIJ quæst. XVIF: An motus reflexus continuus esse valeat.

period of rest in between the two opposed movements, which Richard of Middleton and Gilles of Rome claimed to demonstrate:

> You might say that this part of the air which, with the stone, is moved up to the high place where the ascending movement ends, supports this weight in the air for a while. We will ask by what cause this air so holds the mobile; whereas, indeed, this air is very easily divisible and it yields very easily, it did not seem reasonable that it prevent the fall of the weight… Perhaps we should say this: The part of the air which, by violence, rose at the same time as the weight preserves for a certain time the virtue of moving other parts of air, although in this part, the virtue capable of moving the weight directly ceased to be; throughout, it holds the weight in its elevated position; when in this part of the air, the first of these two virtues ends, in turn the weight moves itself and moves that air. But what is this virtue? Why does it last so long, neither more nor less? What destroyed it? This is what remains to be clarified.

When, in his commentary on the *Books of Sentences*, Durand of St. Pourçain quoted Thomas Aquinas, he named him[46]: *Sanctus Thomas*; the book is thus after 1323. Besides, in closing this writing, Durand teaches us[47] that he started it in his youth and finished it in his old age: "*Scripturam super quatuor Sententiarum libros juvenis inchoavi, sed senex complevi.*" Now Durand died in 1332. Therefore it is after the writings of John of Jandun that we must place the Commentary on the *Sentences* composed by the Dominican Doctor.

Touching the accelerated fall of weights, the opinion of Durand of St. Pourçain is very similar to what Simplicius attributed to many physicists whose names, incidentally, he withheld.

> It is false that the distance to the natural place diminishes the inclination of the mobile to this place,

Durand said[48].

> The inclination that the heavy or light body has toward its own place is the result of the form of this body; as long as this form remains the same, the inclination does not change; the greater or lesser distance to the natural place does nothing by itself. If the natural movement is more intense at the end than at the beginning, the cause is that the resistance of the medium becomes less, while the inclination of the mobile is supposed to be constant. Indeed, the closer the air is to the earth, the less lightness it has and the less it fights against the movement of the weight. One must say the same about the movement of the light body.

Durand of St. Pourçain also does not see that his explanation is as faulty as the explanation of Themistius; like him, he attributed to the weight that falls a speed that depends only on the distance to the ground.

With Walter Burley, we find the thoughts of Gilles of Rome; but we find them accompanied by precisions that clearly reveal their meaning, and this meaning is what we have given them.

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[46] D. Durandi a Sancto Portiano super sententias theologicas Petri Lombardi commentariorum Libri quatuor, per fratem Iacobum Albertum Castrensem ad fidem veterum exemplarium diligenter recogniti. Venundantur Parisiis apud Ioannem Roigny sub basilisco, et quatuor elementis, via ad divum Iacobum. 1539). Lib. I, dist. XVII, quæst. VII, fol. 45, col. *a*.

[47] Durandi a Sancto Portiano *Op. cit.*, conclusio Operis; ed. cit., fol. 324, verso.

[48] Durandi a Sancto Portiano *Op. cit.*, lib. II, dist. XIV, quæst. I: Utrum aliqua: aquæ sint super cælos.

Here is, first of all, a passage[49] concerning the theory of Hipparchus and the *violent rest* separating, according to Gilles of Rome, the two opposed movements of the projectile thrown in the air:

> The generation of violent rest does not happen in the same way as the generation of natural rest. What causes natural rest is the nature of mobile itself; it is also what causes natural movement; the same nature causes the natural rest and the natural movement. Violent rest, on the other hand, is caused by a violent virtue, when it is faulty. The violent virtue is very strong at the beginning of the movement; it is powerful enough to prevent the mobile from moving to its natural place and to move it in the opposite direction. Later, at the end of the [ascending] movement, the violent virtue is so weakened that it is not sufficient to move the mobile in the same direction; it is sufficient only to maintain it at the place that it occupies; it then gives it a violent rest. Indeed, to prevent the mobile from taking the natural movement, it requires a lesser virtue than to move it with a contrary motion; so when the virtue which violates the mobile is so debilitated that it can no longer advance it, it still prevents the movement in the opposite direction and compels the mobile to remain at rest. When the virtue that violates the mobile becomes so weak that it can no longer force the body to move in the original direction, nor prevent the natural movement, then the mobile starts to move with its natural movement. That is why the stone, thrown in the air, rests at the turning point, unless it is prevented. The launching force is the cause of this movement in that it no longer sufficies to lift the mobile, but only to prevent it from leaving the place it occupies and to move to its natural place. This is undoubtedly what certain philosophers mean when they say that violent movement is generated by *defect*, whereas in natural movement, the generation of the rest is *effective*.

All these considerations on violent rest are, very deeply ingrained, the mark of Gilles of Rome.

We come to the passage[50] where Walter Burley explains the accelerated fall of weights. This passage starts with a sentence borrowed verbatim from Gilles of Rome:

> It should be noted that the natural movement starts from a violent rest, while the violent movement starts from a natural rest. So, the more the natural movement departs from the rest from which it started[51], the faster this movement becomes, as a result of the distance to the resting state from where it comes. In violent movement the opposite occurs.

Burley comments on this text of Gilles Colonna in these terms:

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[49] Burleus *super octo libros physicorum*. Colophon:

> Et in hoc finitur expositio excellentissimi philosophi Gualterii de burley anglici in libros octo de physico auditu. Aristo. stagerite. emendata diligentissime. Impressa arte et diligentia Boneti locatelli bergomensis, sumptibus vero et expensis Nobilis viri Octaviani scoti modoetiensis. Et humato Jesu ejusque genitrici virgini Marie sint gratie infinite. Venetiis. Anno salutis nonagesimoprimo supra millesimum et quadringentesimum. Quarto nonas decembris. Tractatus tertius quinti libri in quo agitur de contrarietate motuum et quietum. Caput 2^{m} tractatus tertii: et est de contrarietate motus ad quietem et quietum ad invicem; fol. sign. *v* 2, col. *a*.

[50] Gualterii Burlæi Op, cit., lib. VIII, tract. III, cap. III, in quo ostenditur quod motus localis est primus motuum; ed. cit., fol. sign. DD, coll. *c* et *d*.

[51] The text of Gilles of Rome intercalated here these words: “The more it approaches the center,” which could be an allusion to the theory of Themistius. Burley erased those misleading words.

Therefore, the proposition “all mobile things move away even faster the farther they are from rest” should be understood of natural movement; indeed, any body which moves with natural movement moves more quickly the farther it is from rest, i.e., from the place where it remained immobile by violence. We can also apply it to violent movement as well as to natural movement; it must then be understood thus: Any body moved with natural movement moves much faster the more distant it is from the violent rest from which it began to move; and any body moved by violent movement moves more quickly the more distant it is from the violent rest to which its movement tends.

It is commonly said that the natural movement is accelerating towards the end as a result of the proximity of the term to which it tends; it must be understood that this is not true; it is not only because it approaches the center that a weight moves more quickly. Take, in effect, two bodies of the same weight, and assume all things being equal; we say that these two bodies are of the same shape, same size, and that they have the same degree of all the characteristics that are related to movement; let A and B be these two bodies; place the body A high in the air, in a place whose distance to earth is ten stadiums, and let C be this place; as for B, put it in a place whose distance to the earth is only one stadium, and let D be this place. The body A falls and, at the moment when this body will come to a place one stadium from the ground, the body B begins to descend; let E be the instant when bodies A and B are separated from the ground by the distance of one stadium. It is clear that after the instant E, the body A will descend more quickly than the body B; and yet, at the instant E, these two bodies are also close to the Earth. Thus, it is not the closest approach to the natural place that causes the maximum speed of the natural movement, but the greatest distance to the violent rest from which the movement began. At the instant E, in fact, and for the duration of the movement after this moment, the body A is farthest from the violent rest from which it began to move than is the body B from the violent rest where its fall began; also, after the moment E, the body A moves faster than the body B, although these two bodies are equidistant from the earth and equidistant from their natural place. Thus, this distance to the violent rest from which the body is set in motion is the cause of the continuous acceleration of natural movement.

But this, it seems, is the remote cause; so, it is necessary to assign a more explicit proximate cause.

This is why some argue that the weight, in its fall, continuously acquires a new accidental gravity; it becomes continually heavier and heavier; thus, its movement constantly accelerates. It is the same for a light body; in its upward motion, it constantly acquires a new accidental lightness. Therefore, the farther these bodies are from the state of violent rest from which they began to move, the more quickly they move.

It seems to me that the air is heavy with heavy bodies and light with light bodies. When a heavy body falls, the air mass which is behind it and pushes it down is always greater and greater, whereas the air mass following its movement is also growing, continually; the movement accelerates because the medium that lies ahead of the mobile that gives way to it is heavier and heavier, and the medium following the weight also becomes heavier and gives this body a stronger impulse. Thus, the mobile moves faster the farther away it comes, because its movement is increasingly assisted by the medium, both forward and backward.

The explanation that Burley has just developed is a kind of synthesis where the thoughts of many an author of antiquity compete.

We acknowledge, first of all, the Peripatetic theory which attributes to the medium the continuation of the movement of projectiles.

We find, then, the analogy between the acceleration of natural movement and the slowdown of the violent movement, such as Hipparchus had reported it, according to Simplicius.

The decreasing resistance of the medium preceding the mobile is invoked as it would be by some physicists prior to Simplicius and, more recently, by Durand of St. Pourçain.

Finally, the growing impulsion of the fluid following the weight is admitted as it was by the Precursor of Leonardo da Vinci.

This synthesis is the result of continuous efforts which the work of, firstly, Richard of Middleton, the writings of Gilles of Rome, of John of Jandun, and then of Durand of St. Pourçain testify for us.

These efforts fill a whole period of slow development which the theory of the accelerated fall of weights had undergone.

In a previous period, illustrated by the great Scholastic doctors of the 13th century, the explanation of Themistius was generally admitted.

From Richard of Middleton to Walter Burley, the masters whose attempts to characterize the second period ridded the science of the unacceptable doctrine of Themistius; they clearly highlight this truth: the speed of a falling body depends not on the distance of this weight to the center of the World, but on the distance of the weight to its initial position; they are less fortunate when it comes to explain the increase in speed; they all look for the reason in the influence of the environment.

But the actual text of Burley tells us of the opening of a third period of the history that we retrace here.

Burley alluded to some philosophers who attribute the acceleration of natural movement to the continuous increase of accidental gravity. However, in the Middle Ages, the name of accidental gravity was certainly taken as synonymous with *impetus*. "Some," said Cajetan of Tiene[52], "give the name of accidental gravity or lightness to the virtue that the mover provides to the mobile, but it is more commonly called *impetus*." Cajetan was, moreover, an assiduous reader of Burley, which his writings constantly cite. Thus, in the time of Burley, physicists were demanding an increasing *impetus* to accelerate the fall of weights.

Who were these physicists?

Named Canon of Evreux in 1342[53], Walter Burley was certainly still living in 1343; he ended his career as Jean Buridan began his; the allusion that the commentaries on the *Physics* composed by the English Master cointain could, in the strict sense, be aimed at the teaching of the Picard Master; it is more likely that it relates to the opinion of older physicists, contemporaries of Burley, of whom Buridan was the disciple and whose doctrines adopted and developed.

We have already quoted in the previous paragraph a passage where Buridan explains, using a growing *impetus*, the speed of a falling body; he gives this explanation

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52 *Recollectæ* Gaietani *super octo libros Physicorum cum annotationibus textuum*, fol. 51. Colophon:

> Impressum est hoc opus per Bonetum Locatellum, iussu et expensis nobilis viri Domini Octaviani Scoti civis Modoetiensis. Anno Salutis 1496.

53 Denifle and Châtelain, *Chartularium Universitatis Parisiensis*, tomus II, pars prior, p. 154.

also in another place[54], while the problem of the origin of gravity leads him to assert that a weight does not become heavier when it approaches its natural place.

> You will say,

Master Jean Buridan writes,

> that this reasoning should be retorted in the opposite sense; it is clear, in fact, that a weight, in its fall, moves even more quickly the closer it is to its place; it does not seem that this can be explained, except because place exercises a virtue of attraction nearby more than than it does far away.
>
> To this I say that, all things equal, a weight does not fall faster when it is close to the lower place, when it is, for example, three feet or ten feet distant, than when it is far away and separated by one hundred or one thousand feet. Indeed, suppose that a man is at the top of one of the towers of Notre Dame, and that a stone, located ten feet above him, falls on him; this stone would not hurt this man, neither more nor less, than if he were, instead, in the lowest place of a well deep and this same stone him fell on him from a height of ten feet. It is clear from this that the stone does not move more rapidly in this very low place than in that very high place.
>
> Therefore, it is obvious that if a weight moves faster or slower, it is not because it is closer or farther from its place; but, as we will say later, it is because the heavy body acquires a certain *impetus* which is joined to its gravity to move it; the movement thus becomes faster than when the heavy body was driven by its gravity alone; the faster the movement becomes, the more vigorous the *impetus*; as the weight keeps descending, its movement becomes faster and faster, because by continuing to go down, it moves more and more from the point from which it began to fall; incidentally, it does not matter whether this fall happens in a higher place or in a lower place.

What will be, in the vicissitudes through which the teaching of Scholasticism will pass, the fate of this theory Buridan proposed?

Albert of Saxony adopted, as a whole, the Dynamics of the *impetus* such as Jean Buridan formated it[55]. He even completes it, at one point; he takes[56], using this notion of *impetus*, the analysis of the various phases that the movement of a projectile thrown upward presents, and he was try to clarify the demonstration of that *intermediate rest* which his predecessors had introduced using Peripatetic Mechanics.

Like all successive physicists from Richard of Middleton to Buridan, Albert of Saxony does not want the weight of a body varying with the distance of this body to the center of the Earth. He wrote, on this subject, a remarkable sentence, in which the intensity of gravity is given not as determining the speed with which a weight *moves*, but only as determining the speed with which it *begins to move*. From the assumption that weight is greater the closer the body is to the center of the world,

[54] Magistri Johannis Buridam *questiones totius libri Physicorum*; lib. VIII, quæst. IV: Utrum actu grave existens sursum moveatur per se post remotionem prohibentis, vel a quo moveatur. Bibl. nat., Latin archive, ms. 14723, fol. 92, col. *d* — Cf.: *Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, seconde série, pp. 420-421.

[55] Nicolas de Cues et Léonard de Vinci, IX: La Dynamique de Nicolas de Cusa et les sources dont elle découle (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, XI; seconde série, pp. 194-200).

[56] *Nicolas de Cues et Léonard de Vinci*, XI: La Dynamique de Nicolas de Cues et la Dynamique de Léonard de Vinci. Théorie de l'*impeto* composé (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, XI; séconde serie, pp. 212-213).

> one would draw,

he said[57],

> this conclusion: all things being equal, a weight would not begin to move with the same speed when it departs from points located at different distances from its natural place. This result is contrary to experience and, yet, it is logically deduced; the attractive virtue would be stronger closer than from afar; thus, if a body started moving close to its natural place, the beginning of its movement would be faster than if it began to move far from this same place.

What difference is there between these words of Albert of Saxony and our modern proposition: "The various forces acting on the same mobile are as accelerations that they impart to this mobile"? Obviously, the thought is the same; but to formulate it and refine it, we have the wonderful language that the infinitesimal calculus has created.

In three of his writings, Albertutius, more or less at length, deals with the accelerated fall of weights; we have previously quoted[58] what he said in his *Questions on the Physics* and in his *Questions on the Treatise on the Heavens and the Earth*; without repeating it here, we reproduce what the *Tractatus proportionum* contains on this subject:

> A weight that descends in a uniform medium descends faster at the end than at the beginning; this does not arise, however, from a greater ratio of power to resistance, since the resistance has been assumed to be uniform… To this argument, I respond: When the weight has for a certain time exercised its motion descending in the uniform medium, the ratio of the total motive power to the resistance no longer has the same value at the end than at the beginning; whereas, in fact, the resistance remains uniform, the power becomes more intense thanks to the *impetus* which is acquired by this weight as it descends; this impetus, joined to the principal motive power of the stone, moves it more rapidly at the end than at the beginning.

In our study of *Albert of Saxony and Leonardo da Vinci*, we saw that Leonardo had in his hands and studied with great care the *Quæstiones in libros de Cœlo et Mundo* of Albert of Saxony. We have also seen that in a list of books included in the notebook F, Da Vinci included the *De Calculatione d'Albertucco* next to that of Marliano; in this *De Calculatione*, we have not hesitated to recognize the *Tractatus proportionum* of Albert of Saxony.

Leonardo did not only have the *Tractatus proportionum* in his hands; he studied it and discussed its doctrines; witness this passage[59]:

> On movemenet. Albert of Saxony, in his *On proportions*, said that if a power moves a mobile with a certain speed, it will move half of this mobile with twice the speed; which thing does not seem [accurate] to me…

[57] Alberti de Saxonia *Subtilissimæ quæstiones in libros de Cœlo et Mundo*, lib. II, quæst. XIV (apud edd. Venetiis, 1492 et 1520. This important question is omitted in the editions published in Paris in 1516 and 1518). — Cf. *Léonard de Vinci et la pluralité des mondes*, VI: Le poids d'un grave résulte-t-il d'une attraction exercée à distance? Jean de Jandun, Guillaume d'Ockam, Albert de Saxe (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, X; seconde série, p. 88).

[58] *Bernardino Baldi, Boberval et Descartes*, § I. (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, première série, pp. 130-131.)

[59] *Les manuscrits* de Léonard de Vinci, ms. I of the Bibliothèque de l'Institut, fol. 120, recto.

The conclusion of Albert of Saxony to which this passage alluded is two pages after the text just quoted.

Of the three texts that we have borrowed from Albertutius, two at least were under the eyes of Da Vinci. But should we admit it? If these texts bear the well-known impression of the accelerated fall of weights, this impression is too much effaced to attract much attention; in reading the various writings of Albert of Saxony, Leonardo may have attached but a slight importance to what he presented regarding the accelerated fall of the weight.

It seems, moreover, that the Terminists, while admitting the explanation of the motion of projectiles by the theory of the *impetus*, are hardly concerned with the application that one could make of that same theory to the movement of heavy bodies; Marsilius of Inghen does not speak about it in his *Questions* on the *Physics* of Aristotle; besides, in these questions, it is difficult to discover a few vague and scarce allusions to the Dynamics of *impetus*.

This Dynamics finds a rather extended presentation, and obviously inspired by Buridan and Albert of Saxony, in the *Abbreviationes libri Physicorum*[60] of the same Marsilius of Inghen. One also encounters, in this book, an allusion to the accelerated fall of weights and the explanation that the theory of the *impetus* gives for it. Marsilius of Inghen says that gravity was not an attraction of the natural place; he adds:

> One might ask if it is not because it is attracted by place that the weight moves faster towards the end of its course. We will respond that this effect comes from the *impetus* acquired as a result of the movement.

But how brief and not very explicit this allusion is[61]!

If Marsilius of Inghen quickly glossed over the accelerated fall of weights, on the other hand he strives[62] to explain an entirely imaginary phenomenon, the alleged acceleration that a projectile would experience right after it leaves the hand or the instrument that launched it[63]. Jean Buridan and Albert of Saxony had not spoken of this acceleration, the existence of which, probably, seemed dubious or deniable. Marsilius of Inghen did not imitate their prudent reserve; here is the passage that ends his *Abbreviationes*:

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[60] *Incipiunt subtiles doctrinaque plene abbreviationes libri physicorum edite a prestantissimo philosopho* Marsilio Inguen *doctore parisiensi*. (This book, printed before the year 1500, has no indication related to the name of the publisher, the date, or place of the edition. The sheets are not paginated.) The theory of the *impetus* occupies the last two pages. Cf.: *Nicolas de Cues et Léonard de Vinci*, IX : La Dynamique de Nicolas de Cues et les sources dont elle découle ; X : La Dynamique de Nicolas de Cues et la Dynamique de Kepler; XI : La Dynamique de Nicolas de Cues et la Dynamique de Léonard de Vinci. Théorie de l'*impeto* composé (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, XI ; seconde série, pp. 195-197, 195-197, 203-204, 213-214)

[61] Marsilius of Inghen, *Op. cit.*, col *a*. of the fol. following the folio labeled K. 3.

[62] Marsilius of Inghe, *Op. cit.*, last folio, col *a*.

[63] Regarding this alleged acceleration, see: *Bernardino Baldi, Roberval et Descartes*, I: Une opinion de Bernardino Baldi touchant les mouvements accélérés (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, IV; première série, pp. 127-139).

But, you say, the *impetus* has its largest power nearby what lauches it; the arrow should therefore strike, close to the bow, stronger than at a certain distance; however, this is contrary to experience.

This question is difficult; therefore, we will only give it an evasive and probable answer.

In the first place, we can answer that one who launches a projectile impresses it with an *impetus* starting from the zeroth degree; that, while he launches it, it impresses a certain power to the air; this air moves with the projectile, and, up to a certain distance, it increases the intensity and strength of the *impetus* provided to mobile by the one who projected this body.

Secondly, we can answer that the *impetus* has, indeed, its greatest power when he who launches it ceases to touch this body, but that it is only applied to it later; this method of application constantly improves until the mobile has traveled a certain distance; now, a better application of the force greatly helps the speed of the movement. It would therefore seem that it is the very nature of the *impetus* which determines, at a certain distance, this better application.

Thirdly, you could say this: at the beginning of the movement, a very strong *impetus* is impressed on the part of the mobile that touches him who launched it; but, in the farthest parts, the *impetus* is weak and not very intense. Similarly, if one pushed Socrates, and Plato by the intermediacy of Socrates, the *impetus* transfered would be, in the beginning, confined in Socrates, then, by his intermediacy, it would pass to Plato. Thus, at the beginning of the movement, the parts of the projectile located farthest from the mover would move, it is true, as soon as the parts closest to the mover; but it would be so because the posterior parts would bear, so to speak, and push forward, by their own virtue, the anterior parts. Subsequently, the posterior parts would impress on the anterior parts an *impetus* as strong as or not as significantly differing from what they themselves have; then the projectile would move with more speed and impetuosity. This effect will therefore arise from the fact that the *impetus* was not everywhere equally strong, but that it was weak in the remote parts of the mover; then it became stronger by spreading uniformly in the whole mobile. This is, I believe, the likeliest and easiest sustainable explanation.

The effect that Marsilius of Inghen proposed to explain is devoid of all reality; it is therefore otiose to research if the invoked cause could account for it; but it is not without interest to stop for a moment at the considerations that we just read.

Marsilius, like Buridan, sees in the *impetus* a separate permanent reality of local movement; he can, without alogism, examine how this *form* is distributed at each moment in the mass of the mobile, independently of the distribution that the local speeds affect it.

He found, moreover, in the treatise *De ponderibus* of the Precursor of Leonardo, some considerations of the same kind[64] on the distribution of the impulse within the projectile; now, as the manuscripts attest, the knowledge of this treatise was common in the 14th century.

A great effort should not be made to establish a parallel between the considerations expressed by Marsilius of Inghen and those we develop today when we want to explain how the disturbance produced by a sudden shock propagates in a fluid or elastic medium; also, some similar considerations, in which we easily find the influence the Precursor of Leonardo and Jean Buridan, fortunately serve the future

[64] *La* Scientia de Ponderibus *et Léonard de Vinci*, IV: Les réflexions de Léonard sur le quatrième livre du *Tractatus de ponderibus* composé par son Précurseur (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, VII, p. 281 et p. 286).

rector of Heidelberg when he proposes to analyze[65] the bouncing of a ball that hits an obstacle.

What were the professed opinions concerning the accelerated fall of weights, by the University of Paris and the Universities subject to its influence, during the time that has elapsed since Marsilius of Inghen until the second half of the 15th century? We lack documents that inform us in this regard. Those which we possess refer to the end of the 15th century. They present us with mechanical theories singularly deprived of the degree to which Jean Buridan and Albert of Saxony had brought them.

Nevertheless, among some scholastics of this period, we perceive as a reflection some doctrines which had sprung up in Paris in the middle of the fourteenth century; such a reflection illuminates, for example, the work of Pierre Tataret.

Towards the end of the 14th century, the Parisian Pierre Tataret composed his commentaries on the various writings of Aristotle; all of these commentaries formed a sort of manual where the whole Philosophy was treated, and whose popularity was extreme[66]. In this writing, Pierre Tataret is given to Scotism; but, quite often, his preferences move away from the doctrines of the Subtle Doctor and go to the theories taught by the Parisians Nominalists.

Thus, toward the end of his commentary on the eighth book of the *Physics*, Pierre Tataret explains, by the *impetus*, the continuation of the movement of projectiles. In a very summary but exact way, he indicates how this hypothesis allows one account for various phenomena: the bouncing of a ball that struck the earth, the rotation of a wheel that the craftsman has stopped turning, the movement of the top that the child launched; "if a bean, he said, cannot be launched as far as a lead bullet, it is because of a lack of *impetus*, because one cannot, in this bean, impress an *impetus* as large as in a ball of lead."

This faithful summary of the Parisian Dynamics continues in these terms, where neither Buridan nor Albert of Saxony would have agreed to recognize the expression of their thinking:

> One might ask why the body thus moved by the *impetus* sometimes moves towards the end or the middle of its course faster than at the beginning; we will answer that here is the reason: initially, this *impetus* is not impressed on all parts of the mobile, but only to those which adjoin the mover; it is through the intermediacy of these parts that it is communicated to the distant parts, until finally the *impetus* is distributed through the whole mobile; then it moves with a more rapid movement.

If Tataret abandoned, regarding the accelerated fall of weights, the tradition of Buridan and Albertutius, it is easy to say what influence has led him; it is that of Marsilius of Inghen; he merely extended to the acceleration of the movement of

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[65] Marsilius of Inghen, *Op. cit.*, fol. sign. 1, col. *a*, and the preceding fol., coll. *c* and *d*.

[66] *Commentarii Magistri* Petri Tatareti *in libros Philosophie naturalis et Melaphysice Aristotelis* — or else: Petri Tatareti *Clarissima singularisque totius Philosophie necnon Metaphysice Aristotelis expositio* — or even: *Commentationes Petri Tatareli in libros Aristotelis secundum Subtilissimi Doctoris Scoti sententiam*. According to the *Repertorium bibliographicum* of Hain, seven editions of this manual existed before the year 1500; they continued to increase during the first quarter of the 16th century; it was even published in the 17th century.

weights what Marsilius imagined for explaining the alleged initial acceleration of the motion of projectiles.

In his commentary on the second book of the *De Cælo*, Pierre Tataret returned to the study of the accelerated fall of weights; he seeks to set out the quantitative law which this acceleration obeys and, in this regard, he reproduces a remarkable passage due to Albert of Saxony; but on the subject of the cause which determines the increase in speed, he limits himself to this statement:

> We have seen elsewhere how the *impetus* or motive quality constantly increases in intensity in the mobile.

If Pierre Tataret, despite the Scotism that he affirms, takes anything from the teaching of the Nominalists, others affect indifference and contempt for this teaching they deem too immature; leaving aside what the *moderniores* were able to say, the *juniores* only want to authorize St. Thomas Aquinas or Duns Scotus.

Jean Versor of Paris, who died around 1480, is a convinced Thomist; also, by the example of his master, the Angelic Doctor, he fully admits the theory of Themistius. When he says, for example, that gravity is not due to an attraction exercised by the center of the world on the heavy body, he wrote these lines[67], whose logical coherence leaves much to be desired:

> It would result that a mass of earth which falls would not descend faster at the end of its fall than at the beginning; indeed, the bodies which move by traction move even more slowly the farther away they are from what drives them; however, it is manifest to the senses that earth moves more slowly at first, and its movement accelerates the more it descends. Also, according to St. Thomas, natural movement is faster at the end than at the beginning because the more the mobile approaches the natural place or where the virtue that generates it and conserves it is found, the stronger its motive power becomes; that is why, at the end, it moves faster.

What Versor said here after St. Thomas, he takes for his account in another passage[68] where, more so with him, he attributes to place an attractive virtue similar to that of the magnet:

> Natural rectilinear movement,

he writes,

> when it occurs in a uniform medium, is faster to the end than at the beginning… We say: when it occurs in a uniform medium; in this case, indeed, the resistance remains constant while the power increases constantly. If the medium were not uniform, if it provided at the end a resistance greater than at the beginning, it could be that this movement was also slow or even slower at the end than at the beginning. If one asks what the cause of this acceleration is, we answer that it comes from an attractive virtue of the place; naturally, this place attracts even more powerfully the body it can lodge so that this body is closer; similarly, a magnet attracts a piece of iron with more speed the closer the iron is.

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[67] *Questiones magistri* Johannis versoris *super libros de celo et mundo cum textu Arestotelis*. Colophon: Et sic terminantur questiones versoris super duos libros de generatione et corruptione Aristotelis secundum processum ejusdem versoris diligentissime correcte. Anno incarnationis dominice MCCCCLXXXIX penultimo die Maii. Lib. I, quæst. XII, fol. XIII, col. *d*. — This same work was printed in 1485, 1488 and 1493.

[68] 1. Johannis Versoris *Op. cit.*, lib. II, quæst. VIII, fol. XXVIII, col. *a*.

The Franciscan Nicolas Dorbellus or De Orbellis, who died in 1455 after having professed at Poitiers, was a convinced Scotist; he gave all the books of Aristotle and the *Summulæ* of Petrus Hispanus a brief commentary, drafted according to the spirit of the Subtle Doctor; this commentary, printed several times[69], long served as a Philosophy manual in the Franciscan schools.

In this dry and routine manual, there is no longer a question attributing to *impetus* either the accelerated fall of weights, to which he did not make any allusion, or even the movement of projectiles.

> Although the stone,

he says there[70],

> does not always remain contiguous to the hand that throws it, it remains incessantly in contact with a certain part of air which is for it the proximate mover. In effect, he who throws the stone, as he communicates an impulsion to this stone, also communicates one to the air, and the air which received this impulsion continues to push the stone…

Thus, in French schools one forgets everything that the meditations of the Nominalists had discovered. Let them listen to the teachings of the German-language Universities.

The teaching given by Marsilius of Inghen had greatly contributed to the spread in Germany of the Nominalist doctrines; Frederick Sunczel is one of the masters who claim most willingly the theories professed by the Rector of Heidelberg.

To the study of the motion of projectiles, Sunczel dedicated an important question[71] where we recognize the summary of what Buridan and Albert of Saxony have written; we find, in this question, a short reference to the hypothesis that these authors have proposed regarding the motion of the celestial spheres:

> A wheel of a blacksmith,

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[69] The edition we consulted is the following: *Cursus librorum philosophiæ naturalis venerabilis magistri Nicolai de Orbellis ordinis minorum secundum viam doctoris subtilis Scoti*. — Colophon:

> Eximii ac peritissimi artium ac sacre theologie magistri Nicolai Dorbelli ordinis minorum preclarissima logice expositio: parva quidem volumine: maxima vero doctrine copiositate. Quod opus sicut ceteris logice voluminibus est emendatius: ita profecto omnibus logice libris volentibus in dialectica: et precipue secundum doctrinam doctoris subtilis erudiri est utilius: Imprcssum Basilee: Anno domini millesimo quingentesimotertio.

— The same book was previously published under the title: *Philosophiæ peripateticæ ad mentem Scoti compendium*; Bononiæ, per Magistrum Henricum de Harlem et Matheum Grescentinum, 1485.

[70] Nicolai de Orbellis *Op. cit.*, Physicorum lib. Vil, cap. 11.

[71] *Collecta et exercitata* Friderici Sunczel *Mosellani liberalium studiorum magistri in octo libros Phisicorum Aristotelis: in almo studio Ingolstadiensi*. Cum adjectione textus nove translationis Johannis Argiropoli bizatii (sic) circa questiones. Colophon:

> …Impressa sub hemisperio veneto Impensis Leonardi Alantse Bibliopole viennensis Arte vero et ingenio Petri Lichtenstein Coloniensis anno MDVI Die XXVIII Mensis. Lib. VIII, quæst. XI.

said Sunczel,

> that one has moved, then stopped moving, turns for a while; however, this is not the air pushing it, because it cannot move such a mass; the wheel would still be moving even though the one who turned it would have long since ceased to do so. Similarly, some ancient philosophers said that initially the First Mover produced such an *impetus* in the heavens.

On the subject of the accelerated fall of weights, the same Sunczel expresses himself in an extremely vague manner. In his concise and obscure remarks, we find a pale reflection of the idea put forward by Buridan and Albert, and a slightly clearer reflection of the doctrine that Marsilius of Inghen made known.

> We may ask,

said the professor of Ingolstadt,

> if the *impetus* is stronger at the beginning of the movement or in the middle of that same movement. We answer that it is stronger in the beginning; indeed, it is supposed that this *impetus* is in violent movement; however, in Books II and IV of the *De Cælo*, we see the violent movement is stronger at the beginning. From that moment, the *impetus* starts to weaken gradually, because of the gravity of the mobile and the medium resisting it; in the end, it is so weakened that it does not move anything. And it is obvious the existence of the *impetus* in any violent movement is supposed, in the case, for example, where a heavy body is thrown upward or a light body downward, and even in the cases where a heavy body is thrown down more quickly that it would move itself; it is also assumed in natural movement, because a weight, near the end of its movement, acquires some *impetus*; it is not assumed in voluntary motion nor in the motion of animals, nor even in the movement of extra-natural origin, like the motion of the celestial spheres. Secondly, you could say: Experience shows, however, that a body moved by *impetus* shoots less strongly at the beginning of its movement or at short distance than in the middle of its course, that is, at longer distance. We answer that this is the cause: Initially, the *impetus* did not have enough extension; it is thus, at the beginning, stronger *intensively*, but a little later, it becomes stronger *extensively*.

Within German Universities the fight was fierce; the *Moderni*, like Sunczel, followed the influence of Marsilius of Inghen and professed Philosophy following the principles of the Nominalists of Paris; the *Veteres*, on the contrary, were focusing exclusively on the teachings of St. Thomas and Duns Scotus; some, even more loving of archaism than others, found Thomism too recent and made themselves disciples of Albert the Great.

Thus did Conrad Summenhard.

In 1477, Summenhard had contributed, under Eberhard V the Bearded, count of Württemberg, to the creation of the University of Tübingen; he was twice, in 1483 and in 1487, Rector of this University. He died in 1501 at the convent of Schuttern. After the death of this theologian, a Philosophy course he composed was published[72], which he gave as a commentary on Albert the Great.

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[72] Conradi Summenhard *Commentaria in Summam physice Alberti magni*. Colophon:

> Vuolfgan. fa hage. ad lectorem. Habes nunc Candidissime lector Conradi Summenhard Theologi eruditas commentationes in Albertum recognitas quam plenissime ex corrupto exemplari recognosci potuere. Que miro ingenio literis sunt excuse a solerli Henrico gran Calcographo Hagenaw… Vale ex Hage. cursim Anno 1607 septimo kal. maias.

Despite his pretensions to archaism, Summenhard cannot guard himself from all the influences after Master Albert; he constantly cites St. Thomas and Duns Scotus, and, although anonymous, the Parisian doctrines sometimes seep into his commentaries; also, on the subject of the accelerated fall of weights, he faithfully reproduces, much more so than Frederick Sunczel, the explanation given by Jean Buridan and Albert of Saxony.

Whence,

said Summenhard[73],

> is the natural movement faster at the end than at the beginning? There are three opinions on this subject.
>
> The *first opinion* is that of the ancient philosophers. They placed in the natural place a virtue by which it would attract to it the natural body. The closer the natural body is to its natural place, the better this attractive virtue can act and attract the body; therefore, the body moves faster at the end than at the beginning.
>
> This opinion is false. Then, indeed, a less heavy body would descend, towards the end of its movement, faster than a body of greater weight; the attractive force, in fact, would exert more dominance over a body of lesser gravity than over a heaver body…
>
> According to the *second opinion*, this effect arises from a being tending more strongly to its end to which it is closer. Thus, the more a virtuous man improves, the more powerful is the effort by which he tends to felicity. However, the natural place is the end to which the body which it must lodge tends.
>
> This opinion is refuted thus: If the weight, at the end of its movement, is directed faster toward the center due to the appetite that it feels—as, on the other hand, appetite occurs because of privation—the weight should experience the appetite of its natural place all the more powerfully the more it is deprived of it; so, it should move all the more quickly the farther it is from its natural place; therefore, the natural movement would be faster at the beginning than at the end.
>
> The *third opinion* is this: By natural movement, a certain *impetus* is acquired in the body which moves naturally; this *impetus*, weak at the beginning of the movement, grows at the end; it is because of this *impetus* that the natural movement is faster towards the end, while this *impetus* is acquired, than it is at the beginning. When a rock falls from on high, the larger the height from which it falls and the duration of its fall, the greater is the *impetus* acquired by it. This *impetus* is a certain quality which is added to the natural gravity and which helps move the stone downward. Towards the end of its movement, this *impetus* increases as a result of the speed of the previous movement; that is why, in the end, this movement is faster than at the beginning.

Summenhard continues with these words[74]:

> Why is the violent movement faster initially and slower at the end?… This is because the violent movement is caused by a certain *impetus* that the mover has imprinted in the projectile and that moves this projectile. As the projectile has a natural resistance against this *impetus*, it continually weakens.

In this *Vetus*, Parisian Dynamics found a more faithful interpreter than in the *Modernus* Sunczel.

[73] Conradi Summenhard *Op. cit.*, tract. I, cap. VIII, vicesima difficultas, fol. sign. *f* 4, coll. *a* et *b*.

[74] Conradi Summenhard *Op. cit.*, tract. I, cap. VIII, difficultas vicesimaprima, fol. sign. *f* 4, col. 6.

The explanation, using *impetus*, of the acceleration observed in the fall of weights has been often unknown or little-known by the Nominalists of France or Germany; it could hardly expect a greater favor in Italian universities which Averroism infested.

Paul of Venice has constantly vacillated between the doctrines of the Parisians and the doctrines of the Commentator; we find here a striking example of his hesitation.

In his *Summa totius philosophiæ*, Paul of Venice is a partisan of the Parisian theories.

> The stone,

he said[75],

> after it left the mover that launched it, is moved by a virtue that this extrinsic mover has imprinted on it.

In his summary of the sixth and seventh book of the *Physics*, there is an almost textual reproduction of the *Tractatus proportionum* of Albert of Saxony; in particular, it reads the following[76]:

> In the descent of a weight, as the speed grows, the ratio of the power to the resistance grows; in effect, besides the essential gravity, there is a continual acquisition of accidental gravity, which is called *impetus*, and which makes this ratio constantly increase.

In his large *Exposition* of the *Physics*, Paul of Venice is Averroist. He admits[77] that the "modern opinion", that the movement of the projectile is maintained by a certain "virtue", is "commonly held"; in support of this view, he mentions the main arguments given by Buridan and Albert of Saxony; but, he added, "although this opinion is widely held, it is not true," and he takes up the theory of Aristotle and the Commentator; on the employment of the *impetus* in the explanation of the accelerated fall of weights, he does not say a word.

The *Expositio* of Paul of Venice is dated; in the colophon of this work, the author tells us that he finished it on 30 June 1409, the day of the commemoration of the Apostle St. Paul. We do not know the date of the *Summa totius philosophiæ*; so we do not know if the famous Augustine passed from the Averroist Dynamics to the Parisian Dynamics or if he underwent a conversion in the opposite direction. In any case, whether he affirmed or whether he combatted the Mechanics of the Parisians, he disclosed the principles to his students at Padua.

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[75] Pauli Veneti *Summa totius Philosophiæ*, Pars I, *Physica*, penultimate chapter.

[76] Paul of Venice, *Ibid.*, cap. XXXl (Proæmium non compris).

[77] Expositio Pauli Veneti *super octo libros physicorum Aristotelis nernon super comento Averrois cum dubiis ejusdem*. Colophon:

> Explicit liber Phisicorum Aristotelis: expositus per me fratrem Paulum de Venetiis: artium liberalium et sacre theologie doctorem: ordinis fratrum heremitarum beatissimi Augustini. Anno domini MCCCCIX, die ultima mensis Junii: qua festum celebratur commemorationis doctoris gentium et christianorum apostoli Pauli. Impressum Venetiis per providum virum dominum Gregorium de Gregoriis. Anno nativitatis domini MCCCCXCIX die XXIII mensis Aprilis.

Fol. signed y V.

> Paul of Venice,

Pomponazzi tells us[78],

> was the Preceptor of Cajetan of Tiene.

Among the masters who taught in the 15th century in the Italian Universities, none more than Cajetan of Tiene was subjected to the Parisian trends. In his commentary on the physics of Aristotle, Cajetan gave[79], of the movement of projectiles, an explanation that is very consistent with the principles developed by Jean Buridan. But when it comes to explain the accelerated fall of weights, the famous professor from Padua hesitates between the hypothesis proposed by Buridan and those which won the support of Richard of Middleton, Durand of Saint Pourçain, and Walter Burley. Here, indeed, is what we read in the part of his commentary[80] where he strives to prove that gravity is not due to the attraction exerted on the heavy body by the natural place:

> This assumption is faulty when it proposes to assign the cause for which the natural movement ends up accelerating; this acceleration, indeed, does not occur for the given reason, but because, in the continuation of its natural movement, a heavy or light body acquires by its own nature an accidental gravity or lightness; it is added to gravity or the natural lightness that pre-existed, and it renders the movement faster; or even because at the end of the movement, the mobile has behind it a larger quantity of the medium than at the beginning, and that this medium pushes the mobile and helps the movement.

The most Parisian of the Italian masters frankly did not dare to adhere to the theory of accelerated fall which Buridan and Albertutius proposed.

Regarding this theory, the Averroists of Bologna and Padua were, in general, silent.

In his *Question on Heavy and Light Bodies* [81], Nicolò Vernias of Chieti declares

[78] Petri Pomponatii Mantuani. *Tractatus acutissimi, utilissimi, et mere peripatetici. De intensione et remissione formarum ac de parvitate et magnitudine. De reactione. De modo agendi primarum qualitatum. De immortalitate anime. Apologie libri tres. Contradictoris tractatus doctissimus. Defensorium autoris. Approbationes rationum defensorii, per Fratrem* Chrysostomum *Theologum ordinis predieatorii divinum. De nutritione et augmentatione*. Colophon:

> Venetiis impressum arte et sumptibus heredum quondam domini Octaviani Scoti, civis ac patricii Modoetiensis: ac sociorum. Anno ab incarnatione dominica MDXXV calendis Martii. *Tractatus de reactione*, fol. 27, col. *has*

[79] *Recollecte Gaietani Super octo libros Physicorum cum annotationibus textuum.* Colophon:

> Impressum est hoc opus Venetiis per Bonetum Locatellum jussu et expensis nobilis viri domini Octaviani Scoti Modoetiensis. Anno salutis 1496. Nonis sextilibus. Augustino Barbadico Serenissimo Venetiarum Duce. Lib. VIII, foll. 50, col. *d*, et 51, col. *a*.

[80] Cajetan of Tiene, *Op. cit.*, lib. VIII, fol. 46, col. *d*

[81] Nicoleti Theatini *in celeberrimo studio Patavino ordinarii philosophie legentis Questio de gravibus et levibus ad integerrimum Philosophum et Medicorum principem Gerardurn Holderium Veronensem*. This question spans fol. 91, verso, to fol. 93, verso, in the following book: *Acutissime*

> that Albertutius and the other Terminists deviate both from Aristotle and the truth when they claim that the movement of projectiles is due to an *impetus* conferred by him who launched them to the projectiles themselves, and not only to the air or water that surrounds them.

Solids, indeed, cannot receive such an *impetus*; only fluids—as Averroes, Walter Burley, and John of Jandun wanted—are suitable for this purpose, because they can compress then relax back to their natural state, to communicate the impulse they have received to another body. Vernias admits the alleged acceleration that a projectile would have at the beginning of its journey; he admits that the shaft launched by a ballista strikes at a distance more strongly than with a machine; he explained this alleged observation, which Cajetan of Tiene had the good sense of declaring false, by assigning a property very similar to *impetus* communicated to the medium. But in this question devoted to the movement of heavy and light bodies, he made no mention of the acceleration that manifests itself in the fall of a heavy body.

Alessandro Achillini[82], like Vernias, knows

> the opinion of the Parisians; such is this opinion: the *impetus* is a quality impressed on the projectile; it moves this projectile; but as it is in it by violence, it constantly weakens.

He was aware of the reasons that the Nominalists argue in favour of this opinion; but he refutes these reasons one after the other, in order to preserve the theory of Aristotle and the Commentator.

Achillini believes that a thrown stone begins by accelerating its movement, and he explains this alleged fact more or less as St. Thomas Aquinas explained it[83].

> It must be known,

he said,

> that the stone starts to move more slowly than it will move subsequently; after a while, indeed, the stone is helped by the air; but at the beginning, it is not; before being moved, in fact, or before moving the projectile, the air awaits being driven by another body, because it is in its own sphere; but once the stone has given a boost to the air, it starts to move and to carry the stone.

Not content with explaining this imaginary fact, Achillini described another no less fanciful fact, in order to have the pleasure of taking it into account:

Questiones super libros de Physica auscultatione ab Alberto de Saxonia edite: jam diu in tenebris torpentes: nuperrime vero quam diligentissime a vitiis puryate: ac summo studio emendate: et quantum aniti ars potuit fideliter impresse. — Nicoleti Verniatis Theatini *philosophi perspicacissimi contra perversam Averrois opinionem de unitate intellectus: et de anime felicitate Questiones divine: nuper castigatissime in lucem prodeuntes*. — Ejusdem *etiam de gravibus et levibus questio subtilissima*. Colophon:

> Venetiis sumptibus heredum q. D. Octaviani Scoti Modoetiensis: ac Sociorum. 21 Augusti. 1516.

82 Alexandri Achillini Bononiensis *De dementis liber tertius*, cap. II (Alexandri Achillini Opera, Venetiis, apud Hieronymum Scotum, MDXLV; foll. 135-136).

83 Bernardino Baldi, Roberval et Descartes, § I (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, première série, p. 129)

> One can wonder,

he says

> how it is that a wheel driven by a rotation about its axis moves, after it is launched and left to itself, more quickly than it moved before. This cannot be, it seems, except in virtue of the *impetus* acquired, an *impetus* which is not governed, whereas before it was governed by the mover; it does not seem, indeed, that the air in this case is moved circularly, especially when one could place an obstacle to the circular motion of the air using a canvas or wood frame very close to the wheel… To this I say that the movement of the wheel is comprised of a natural movement and a violent movement; the violent movement is the ascent of the heavy parts, and the natural movement is their descent; so there is some movement that occurs on its own, and it will keep the movement, although the air does not help; here, there is another help, that of the heavy parts which, descending, push the other parts and make them climb… However, how is the movement accelerating? The parts that must be pushed upward have as much power to resist the movement as the parts that will come down have to make them rise. … Here is the answer: The hand applied to the wheel assisted the movement in the time it was pulling downwards, but it put a certain obstacle to the speed in the time that it was removed to raise it.

To a silly question, a silly answer; it is the only reflection that the ramblings of Achillini deserve.

Moreover, our Averroist, who so tediously explained some purely imaginary accelerations, did not say a word on the very real acceleration that is observed in the fall of weights.

Thus, from his Dynamics, Jean Buridan drew an ingenious theory of the accelerated motion of heavy bodies; this theory was called to exert on the development of Mechanics a happy influence, but its fecundity was not initially manifest; expressed three times, but with insufficient precision and development, by Albert of Saxony, it has been forgotten, unknown, or cast into doubt by most of the Nominalists of the Parisian School; as for the Italian Averroists, they buried it in a profound silence.

When, therefore, we see Leonardo da Vinci giving splendid developments to many of Buridan's Dynamics, which the reading of Albert of Saxony had made known to him, and at the same time neglecting the theory of the fall of weights that the two Parisian masters professed, we cease to be astonished at this disparity.

Leonardo, in effect, has continued to deepen this concept of *impetus* using the theory of the motion of projectiles that the School of Paris constructed; combining in his mind the Dynamics of Albert of Saxony and the Metaphysics of Nicholas of Cusa, he constructed[84] a Philosophy of movement and of force where, latent yet but already fruitful, the idea of energy conservation circulates. Inspired by by what the Parisians said about the *intermediate rest* between two contrary movements of a projectile, he conceived[85] the notion of composed *impeto*; by that he introduced into Dynamics a principle from which Galileo drew marvelous consequences; he did understand that the progression of a projectile was subjected to the continuous

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[84] *Nicholas of Cusa and Leonardo da Vinci*, XII: La Dynamique de Nicolas de Cusa et la Dynamique de Léonard de Vinci (*suite*). La théorie métaphysique du mouvement (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, XI; seconde série, pp. 232-238).

[85] *Nicholas of Cusa and Leonardo da Vinci*, XI: La Dynamique de Nicolas de Cues et la Dynamique de Léonard de Vinci. Théorie de l'*impeto* composé (*Ibid.*, pp. 211-222).

dependence of two causes, the initial *impetus* communicated by the mover to the mobile and the natural gravity of this mobile.

He who gave to the principles of the Parisian Dynamics some magnificent developments refused to seek the explanation of the phenomena of acceleration from them.

These phenomena, indeed, constantly attracted the attention of Da Vinci; he not only sought to explain the acceleration that weights undergo in their fall; he also admitted that projectile motion still grew in speed for a while after the separation of the mobile and mover. However, as a host of texts evidences[86], Leonardo constantly sought the explanation of these real or imaginary accelerations in the shaking of the medium; he has, on many occasions, expressed the doctrine of his Precursor, which won the support of St. Thomas Aquinas, Walter Burley, and John of Jandun.

It is with this view that Leonardo sided so constantly that he almost seems to have ignored somehow, on this point, the teaching of Buridan; only once, and in a very short note, do we hear him refer to this teaching; yet it does not involve the fall of weights, but the accelerated movement that the rope of a bow takes at the moment it leaves the fingers of the archer; here is this note[87]:

> *On the movement of the arrow.* Although the strength of the crossbow is great initially and null finally, the movement of the rope, however, by the momentum acquired, is faster towards the end than at the beginning of this movement.

The little importance that the Nominalists themselves seem to have given to the explanation of the accelerated fall of weights by the continuous acquisition of an *impetus*, the complete abandonment of the doctrine by the Italians of the *Quattrocento*, no doubt, diverted Leonardo from adopting this theory. However, the power and originality of his genius were such that he did not hesitate to follow a thought unknown to his contemporaries and his compatriots, as long as he found it justified and fruitful. Now, he could not ignore the hypothesis that makes the *impetus* of a falling weight constantly grow; Albert of Saxony had expressed it in three books, and we know that Leonardo had assiduously studied two of these works, the *De Cælo et Mundo* and the *Tractatus proportionum*. If he completely abandoned the theory of the fall of weights that these writings outlined, it is because another opinion would impose itself so strongly on his thinking that he felt the need to dwell on some explanation different from what seduced him.

The predominant opinion is, we have seen, what the Precursor of Leonardo supported in his *Tractatus de ponderibus*. How could have it won the support of the great artist to the point of abolishing, on the subject of an important problem, the such attentive curiosity of this genius?

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[86] *Bernardino Baldi, Roberval et Descartes*, 1: Une opinion de Bernardino Baldi touchant les mouvements accélérés (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, IV; première série, pp. 132-134). — La Scientia de Ponderibus et Léonard de Vinci, IV: Les réflexions de Léonard sur le quatrième livre du Tractatus de ponderibus composé par son Précurseur (*Ibid.*, VII; première série, pp. 376-277).

[87] *Les manuscrits de Léonard de Vinci*, ms. M. de la Bibliothèque de l'Institut, fol. 74, verso.

Maybe it should be attributed to his particular characteristic of only invoking the action of the medium on the heavy body. Leonardo never ceased to meditate with extreme attention on the subject of the influence exerted on the movement of a project by the air that this movement shakes; he correctly saw in that influence the key to the problem which he still pondered, the problem of the flight of birds; continually haunted by the contemplation of the condensed wave that propagates at the front of a projectile, eddies which rush to the rear, he easily succumbed to the temptation to give them more importance than they have in reality, to see there the agents that precipitate the fall of a weight; and yet his same meditations, which frequently provided him a very exact analysis of the action of the fluid on the projectile, should warn against such an error; they should have him proclaim the truth of which Buridan and Albert of Saxony were aware: The medium retards the movement of the mobile; it does not accelerate it.

This truth would probably would have escaped Da Vinci if a mistake, accepted as unquestionable truth, had not encouraged him to receive the explanation of the accelerated fall of weights that his Precursor had proposed.

Aristotle did not believe only in the acceleration of natural movement; he also believed that the speed of violent movement begins by growing[88] Leonardo admits this false idea unquestioningly. However, nothing in the Dynamics of Buridan and Albert of Saxony allows one to explain this initial acceleration of the motion of projectiles; these two authors did not say a single word on this acceleration, and Cajetan of Tiene, their disciple, resolutely denied it. St. Thomas Aquinas, on the contrary, admitted it and explained it by the agitation of the air that the mobile shakes; Vernias and Achillini followed the thought of the Angelic Doctor. Was Leonardo, who admitted this explanation, not naturally inclined to receive an explanation analogous to the fall of accelerated weights, to rally to the opinion of his Precursor, of Richard of Middleton, Gilles of Rome, Jean de Jandun, and Walter Rurley, to the opinion that Cajetan of Tiene dared not formally condemn?

It seems, therefore, that we can pose this affirmation: If Leonardo da Vinci has not admitted, in its fullness, the Dynamics of Buridan and Albert of Saxony, if he has especially abandoned the fruitful explanation of the fall accelerated weights that this Dynamics proposed, it is because his intelligence remained captive to a serious mechanical heresy. We will see this heresy, according to which a projectile begins by accelerating in its course, pose a solid obstacle to the progress of Dynamics in Italy throughout the 16th century.

[88] *Bernardino Baldi, Roberval et Descartes,* I: Une opinion de Bernardino Baldi touchant les mouvements accélérés (*Études sur Léonard de Vinci ceux qu'il a lus et ceux nui l'ont lu,* IV; première série, pp. 117-139).

Part II
The Tradition of Buridan and Italian Science in the 16th Century

Chapter 6
The Dynamics of the Italians at the time of Leonardo da Vinci, the Averroists, Alexandrists, and Humanists

The explanation by *impetus* of the accelerated fall of weights found so little favor with the same disciples of Buridan and Albert of Saxony that it was met, among the Italian Averroists of the 15th century, with such complete contempt that one is hardly surprised to see Leonardo da Vinci ignoring or misjudging it.

This is not surprising, but we admit it. What the great painter wrote on the subject of Dynamics forms a large ensemble where deep and fertile thoughts abound; we would like to see this ensemble completed with justifiable ideas on the cause that accelerates the fall of weights; the erroneous opinions that Leonardo professes there mar the harmony of his work.

It will be easy to find, for this work, all the admiration that it deserves, to understand all its boldness and originality; we just have to compare it to what one thought and wrote in Italy on the subject of Dynamics when Leonardo jotted his his deep thoughts on paper; Agostino Nifo will inform us in this regard.

He will inform us through two books that he wrote at the beginning of the 16th century.

The first of these two books, the *Exposition on the Books of the Physics*[1], includes, in fact, two separate writings: a detailed commentary, and some *Recognitiones* after this *Commentary*. At the end of the book, we read:

> *Completum in Aviano rure nostro*,XV *Maij*MDVI, *fœlicibus astris.*

This date seems to relate to the *Commentary*; it leaves us ignorant of the time when the *Recognitiones* were written.

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[1] Augustini Niphi Philosophi Suessani *Expositio super octo Aristotelis Stagiritæ libros de Physico Audita: Cum duplici textus translatione, Antiqua videlicet, et Nova eius, ad Græcorum exemplarium veritatem ab eodem Augustino quam fidissime Castigatis: Averrois etiam Cordubensis in eosdem libros Proœmium, Commentaria, cum ipsius Augustini Suessani refertissima Expositione, Annotationibus, ac Postremis in omnes libros Recognitionibus, Castigatissima conspiciuntur*. Venetiis. Apud Hieronymum Scotum. MDXLIX.

The second of the works of Nifo that we cite is the *Exposition on the books De Cælo et Mundo*[2]. This exposition is dated 15 October 1514.

The two books of Nifo were written when Leonardo da Vinci read Albert of Saxony and gave the teaching of this author a magnificent development.

To combat the doctrines of the Parisians, the *Moderniores*, and the *Juniores*, the Italian Averroists not only opposed them with arguments; they gladly ridiculed them with sarcasm. We heard Vernias designate the Nominalists with the epithet of Terminists (*Terministæ*), which seemed ridiculous to those who favor it; Albert of Saxony, whom the Italians were taking as the personification of the Parisian school, received from the Paduan professor the nicknames of *Albertutius* and *Albertus parvus*, which can hardly pass for marks of veneration.

With respect to the same men, Nifo uses some nicknames where the tint of mockery is accentuated.

During their discussions of a subtle logic in the 14th century and a hairsplitting and quibbling logic in the 15th century, the Parisian Nominalists multiplied the hypothetical examples; the supposed character serving in these examples almost invariably received the names of Socrates which an ancient custom abbreviated as: *Sortes*. To speak incorrectly of the name of the Athenian sage would provoke the laughter of the Humanists; and certainly Nifo counts on the echo of those ridicules when he calls his Parisian adversaries *Sorticoles*. Sometimes, too, he turns to their usage of the name of *Calculatores* given to anyone who cared to discuss the rules of the Dynamics, which were collected in the *Liber calculationum* by one of them, the *Calculator* Richard Suiseth; Nifo calls them *Captiunculatores*, and he gladly added this epithet to that of *Sorticolæ*[3].

As for Albert of Saxony, it is no longer for him *Albertutius*, but *Albertilla*; this nickname has, it seems, removed all weight from the arguments of the old German master.

In his *Exposition on the Books of the Physics*, he deals with the movement of projectiles[4]; he comments on the text where Aristotle attributes the continuation of this movement to the condensed wave that precedes the mobile; in this commentary, the names of Themistius and Averroes come up frequently; but the theory of *impetus* does not even have the honor of an allusion.

At the end of this presentation of the peripatetic doctrine, our author merely adds these lines:

> Averroes said with good reason that this text is difficult because modern (*recentiores*) commentators did not understand it. The difficulty of this subject is the reason why Albertilla recklessly took up Aristotle, whose own words he certainly did not know; and all the *Sorticoles* of his time fell into the same error.

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[2] Aristotelis Stagiritæ *de Cælo et Mundo libri quatuor, e Græco in Latinum ab* Augustino Nipho philosopho Suessano *conversi, et ab eodem etiam præclara, neque non longe omnibus aliis in hac scientia resolutiore aucti expositione*…. Venetiis apud Hieronymum Scotum. MDXLIX

[3] See, for example, in the *Expositio librorum de physico auditu*, at the end of book VII. Augustini Niphi *Expositio super octo libros de physico auditu*, lib. VIII; ed. cit., p. 645

[4] Augustini Niphi *Expositio super octo libros de physico auditu*, lib. VIII; ed. cit., p. 645

To the *Recognitiones* which follow that exposition, Nifo tells us that "the *Juniores* make objections to the opinion of Aristotle on the subject of the motion of projectiles". He deigns to mention some of these objections, among others: a feather should, according to this view, allow itself to be thrown farther than a stone. But our author does not even take the trouble to solve these problems; "as these things have been treated exactly in our commentaries, let us move on," he said.

In explaining the Physics, Nifo did not speak of the accelerated fall of weights; he treats[5] this topic in his exposition of the *De Cælo*.

At first he reproduces, according to Simplicius and St. Thomas Aquinas, what the ancients thought about this acceleration; he even adds some information; he designates, for example, "Jamblichus and other Platonists" as those physicists whose names Simplicius gave us and who attributed the accelerated fall of weights to the decrease in the thickness of the resistant medium. Our Averroist shows that this assumption is unacceptable using the argument that, since Richard of Middleton, the School of Paris had ceased to argue:

> Suppose,

he said,

> that the mobile M moves into its natural place C by moving along the line ABC. When M arrives at B, suppose that a mobile R, of the same type as and similar to mobile M, begins to move; it is clear that M will arrive at C faster than R, although the thickness of air to traverse, BC, is the same for both; thus, it is not the thickness of the medium that causes the greater or lesser speed of the weight.

Nifo then presents the explanation of St. Thomas Aquinas, with which he identifies, though wrongly, that of Alexander of Aphrodisias; the reason which made him reject the former supposition is equally valid against the latter; our author, however, no longer seems to regard it as peremptory, as he expresses himself in these terms:

> I think with Alexander and St. Thomas that a weight moves faster when it is near its own place than when it is far away, because the gravity of this body is greater then, or, in other words, because it is fortified, increased, and augmented. But I do not believe, like them, that the only cause of this strengthening is the proximity of the natural place; from a similar position, indeed, a mobile that was not previously moved moves more slowly than another body already in motion, although these two mobiles are also close to the natural place.
>
> There are noted on this subject two kinds of gravities. One is natural gravity; it is given to the body, through the form, in the generation of this body and by the natural agent which produced it… The other is accidental or adventitious gravity; it is accidentally produced in the weight by extrinsic causes; some call it *impetus*, and with good reason.

We could, on reading this passage, believe that Nifo, always so quick to change feelings according to his skepticism, is converted to the Parisian doctrine and that he now adheres to the hypothesis of *impetus*. A unique adherence, in any case, and which is allied with a very imperfect knowledge of the adopted explanation! This is, indeed, how Nifo presents it:

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[5] Augustini Niphi *Expositio in libros de Cælo et Mundo*, liber I, ed. cit., fol. 50, coll. *a* and *b*.

> The fact that concerns us does not have for its sole cause the proximity of place, as Alexander and St. Thomas seem to believe; it seems to me that he admits three causes:
>
> The first and main cause is the mobile itself whose form makes it able to move in this way.
>
> The second cause is a dispositive cause; it is the proximity of the place; the proximity of place disposes, indeed, the mobile for the generation of such a gravity.
>
> The third cause is an instrumental and indespensible (*sine qua non*) cause; it is the natural movement whereby the mobile moves and approaches the place; without this movement this accidental gravity cannot exist; the proof is that the mobile, once at rest, is not heavier than before.

And the author, with such verbiage, has read the clear and concise explanations given by Albert of Saxony in his *Quæstiones in libros De Cælo*! A few lines further down, he cites the book of "Albertillus" and, it is true, immediately exclaims: "This man is wrong, *errat hic vir!*".

The Averroists were not, at the beginning of the 16th century, the undisputed masters of the opinion in Italian universities. Before them a new party arose. The Alexandrists held Averroes as a very unfaithful interpreter of the thought of Aristotle, particularly in the question of the immortality of the soul; the custodian of the real thought of the Philosopher was not, for them, the Commentator; it was Alexander of Aphrodisias.

The Alexandrists recognized the successor of Vernias at the University of Padua, Peter Pomponazzi of Mantua, as chief. Transferred to the chair of Philosophy in Bologna, Pomponazzi supported him against Nifo in debates that remained famous.

The writings of Pomponazzi show us that he knew well some of the theories in vogue at the University of Paris, especially those affecting the intensity of forms, action and reaction, and the conservation of forms in mixtures. Cajetan of Tiene appears to have been, in Italian schools, the most active introducer of these discussions; it seems that they have especially obtained credence among physicians; Cajetan was himself a physician; his main successors or opponents such as Jacques of Forlì or John Marliano were, too.

Pomponazzi studied Parisian theories deeply; but, in most cases, it is to refute them better and more surely uphold the doctrines of Aristotle and his Greek commentators. The judgments that he passes on the masters of the Terminist school are often very severe; at least are they free of the sarcasm and nicknames that Nifo substitutes so willingly for arguments.

The treatise *De intensione et remissione formarum* that Pomponazzi wrote and had printed in Bologna in 1514[6] is devoted in full to combat some conclusions of one of the most read and commentated authors by the Parisian logicians, Richard

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6 Petri Pomponatii Mantuani *Tractatus, in quo disputatur penes quid intensio et remissio formarum intendantur, nec minus parvitas et magnitudo*. Bononiæ, apud II. Platonidem, 1415. — Petri Pomponatii Mantuani. *Tractatus acutissimi, utilissimi, et mere peripatetici. De intensione et remissione formarum ac de parvitale et magnitudine. De reactione. De modo agendi primarum qualitatum. De immortalitate anime. Apologie libri tres. Contradictoris tractatus doctissimus. Defensorium antoris. Approbationes rationum defensorii per Fratrem Chrysostomum Theologum ordinis predicatorum divinum. De nutritione et augmentatione.* Colophon:

Suiseth the Calculator. Pomponazzi acknowledges this author [7] for "a man with a very sharp mind", and it is with courtesy that he discusses the opinions which he prefers to those of the philosophers of antiquity. The disciples of Pomponazzi kept, besides, less reserve than the master; in a letter addressed to the author John Virgil of Urbin[8], he spoke of people "so well twisted by the bends and turns of this Suiseth that it is impossible to see the truth."

In the treatise *De reactione* that Pomponazzi printed in 1515, the tone of the discussion becomes more acerbic. The theory of Aristotle on this subject was, the Professor of Bologna said[9], accepted without question by all the Greek commentators and the ancient Latin commentators.

> But those who came after and, in particular, the English, raised, against the universally accepted proposition, such subtle doubts and difficult arguments that the most famous men toiled to resolve, and they have not, in my opinion, succeeded fully satisfactorily.

Without doubt, in the treatise *De reactione*, we sometimes find[10], cited with praise, the names of the masters who are regarded as leaders of the Parisian sect; these names are Albert of Saxony, Marsilius of Inghen, Paul of Venice, Jacques of Forlì, and Cajetan of Tiene, whom Pomponazzi constantly designates Cajetan of Vicenza. But it is not always praise that accompanies the names of the Nominalists too attached, according to Pomponazzi, to their own doctrines and too dismissive of those of Aristotle.

Suiseth the Calculator receives the largest share of the ridicules that Pomponazzi issues:

> If the Calculator will allow me[11], I will tell him: this is a man who ignores the first rudiments of the Philosophy… It is clear and obvious that this conclusion is of a man very little practiced in the words of Aristotle… Let this learned man read Aristotle!

Sometimes, Japoco da Forlì shares with Suiseth the wicked compliments of Pomponazzi[12]:

> It is strange that these very scholarly characters adhere to the findings of reasoning rather than the testimony of the senses. Aristotle, however, in the IIIrd book of the *Generation of Animals*, at the end of the 9th chapter, says that it is better to rely on the senses than on reasoning…; in the VIIIth book of the *Physics*, he states that the research of reasoning and

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> Venetiis impressum arte et sumptibus heredum quondam domini Octaviani Scoti, civis ac patritii Modoetiensis: et sociorum. Anno ab incarnatione dominica MDXXV calendas martii.

Our quotations and references refer to this edition.

[7] Petri Pomponatii *Tractatus de intensione et remissione formarum*; prohemium; ed. cit., fol. 2, col a.

[8] Petri Pomponatii *Tractatus utilissimi…*; ed. cit., fol. I, verso.

[9] Petri Pomponatii *Tractatus de reactione*; proemium; ed. cit., fol. 21, col. *a*.

[10] Petri Pomponatii *Tractatus de reactione, sect. I, cap. VI*; ed. cit., fol. 23, col. *c*, sect. I, cap. XII; ed. cit., fol. 26, coll. *a* and *b*

[11] Petri Pomponatii *Op. cit.*, sect. I, cap. III; ed. cit., fol. 22, col. *b*.

[12] Petri Pomponatii *Op. cit.*, sect. I, cap. III; ed. cit., fol. 22, coll. *b* et *c*

> the neglect of the senses is proof of intellectual weakness… Nothing can shake these men, neither the testimony of the senses nor arguments nor authority, whatever it might be; they rely only on themselves and remain firmly committed to their fantastic imaginations. They are not only at odds with Aristotle, but also with Galen and Avicenna; finally, they destroy all of medicine.

Suiseth and Jacopo da Forlì are not alone in being treated as such. William of Heytesbury (Hentisberus) is called[13] "the greatest of the sophists". As for Cajetan of Tiene, the writing he composed against John Marliano is judged with the utmost severity[14]:

> One thing amazes me in this so learned and famous man; he abandons, rejects, and denies truths manifest to the senses, which demonstrate the most obvious reasonings and which the clear and great voice of Aristotle proclaims. He pursues opinions that are scarcely imaginable. If one were allowed to speak thus of a man whose reputation is so extensive, I would say: Doing so is the height of ridiculousness… What seems most to blame in this man is that he has not proven his conclusions; his proofs undo each other; they are based on falsehoods and on the void; they are very far from any reasonable Physics.

The treatise *De nutritione et augmentatione* that Peter Pomponazzi wrote in 1521 provides us with new harshness against the masters of Parisian Scholasticism. An opinion that Jean Buridan had issued in his questions on the IV$^{\text{th}}$ book of the *Physics* is refuted[15] with some courtesy. But Gregory of Rimini sees his doctrines treated with utmost brutality[16]:

> His entire discourse is corrupt and monstrous… From one end to the other, it is pure madness… What he says is unintelligible…, reaching the highest degree of the unintelligibility. It is, I think, the need to contradict—or else the desire to preserve his view, according to which nothing can last but an isolated moment—that led this man to such great monstrosities.

As for Paul of Venice, if he contradicts Walter Burley, it is "by ambition"[17].

In Dynamics, what were the views of Pomponazzi? The texts that we had in our hands give us no indication on this subject. But the attachment of this author to the sense of Aristotle and his Greek commentators and the severity with which he treats most of the representatives of the Parisian school makes us believe that the leader of the Alexandrist school did not profess the same mechanical doctrines as Buridan and Albert of Saxony.

Opposing the Averroists and the Alexandrists who populate the universities, particularly those of Bologna and Padua, around the year 1500; Italy is proud to produce the brilliant host of its Humanists.

Love of poetry and eloquence, fine admirers of Roman or Attic elegance, the Humanists had no desire to take part in the discussions which were stirring at the

13 Petri Pomponatii *Op. cit.*, sect. I, cap. VIII; ed. cit., fol. 23, col. *d*.

14 Petri Pomponatii *Op. cit.*, sect. I, cap. XI; ed. cit., fol. 24, col. *d* et fol. 25, col. *a*.

15 Petri Pomponatii Mantuani *De nutritione et augmentatione libellus*, lib. II, cap. IX; ed. cit., fol. 130, col. *d*.

16 Petri Pomponatii *Op. cit.*, lib. II, cap. XI; ed. cit., fol 137, col. *c* et d.

17 Petri Pomponatii *Op. cit.*, lib. I, cap. XIII; ed. cit., fol. 123, col. *b*.

Sorbonne, in the noisy streets of the Fouarre or College of Montaigu; the subjects of these discussions seemed too abstract; the methods by which they were carried out were too subtle; and especially their refined Latinism could not suffer the "style of Paris", the rough technical language in which they did not how to conclude their arguments. A Hermolao Barbaro, for example[18],

> pursues insulting those barbaric philosophers; they are commonly held,

he said,

> for the sordid, coarse, and uneducated; during their lives, they were not alive and, after their death, they live no more; or if they live, it is in pain and shame.

The humility in which these monks and masters of arts had buried their laborious existence repelled and disgusted the Italians of the Renaissance, who were thirsty for fame.

The Parisians, however, had merit in the eyes of the Humanists attached to the Catholic faith; even in Italy there were such Humanists, and they were more numerous than they say. In face of the Alexandrists and Averroists of Bologna and Padua and the Alexandrists who denied the immortality of the soul and Averroists who supported the unity of the human intellect and rejected personal survival, the Sorbonne appeared as the guardian of Christian orthodoxy. The Catholic Italians greeted her as the mistress of sound Theology:

> I spoke according to the thesis of St. Thomas,

wrote Mirandole in 1493[19],

> and in accordance with the common way. I call the common way of the theologians what is commonly held in Paris; it is there, indeed, that the study of theology especially flourishes. Now, on the subject of the presence of the soul in a place, almost all in Paris agree with the Scotists and Nominalists; it is for this reason alone, thus out of respect for the University of Paris, that I did wanted to state my conclusion only as probable.

These lines show us what, according to the Italian Catholics, the authority of the University of Paris was; the condemnations that Étienne Tempier brought forth in 1277 made the citadel which defended Christian thought against the onslaughts of Peripateticism and Hellenic or Muslim Neoplatonism; to penetrate the doctrines of the theologians of Paris, the Humanists were willing to learn the language that they used.

So did John Pic de la Mirandola:

> He had,

[18] According to a letter from John Pic de la Mirandole to Hermolao Barbaro, dated: Florentiæ, III nonas Junias MCCCCLXXXV (Joannis Pici Mirandulæ *Omnia opera*. Colophon, at the end of the Opuscula: Opuscula hæc Ioannis Pici Mirandulæ Concordia? (Comitis Diligenter impressit Bernardinus Venetus, adhibita pro Airibus solertia et diligentia ne ab archetypo aberraret: Venetiis Anno Salulis MCCCCLXXXXVIII, die IX Octobris).

[19] *Apologia* Joannis Pici Mirandulæ Concordiæ Comitis. Qæstio prima. De descensu Christi ad inferos. (Joannis Pici Mirandulæ Concordia Comitis *Omnia opera*.)

his nephew John Francis Galeotti Pic tells us[20],

> a thorough knowledge of the modern theologians, of those who use the style commonly called Parisian. Such was this knowledge that if, unexpectedly, one asked him for an explanation of an abstruse and little-explicit question formulated by one of these theologians,

he immediately gave the most perfect exposition.

John Pic de la Mirandola went further; against the Humanists that the language of the School of Paris repulsed, he dared to stand up for this technical terminology.

> When we consider, for example, the production of a man by the Sun, our authors will say: *hominem causari*. Immediately,

wrote Hermolao Barbaro[21],

> John Pic will cry out to you: This is not Latin. Until then, you are right: This is not Roman. But you add: Thus it is incorrect. Your argument errs; an Arab, an Egyptian will say the same thing; they will not say it in Latin, but they say will say it correctly… Who prevents these philosophers whom you call barbarians from having established by common agreement a certain rule of language and to hold it as sacred, as the Roman language is for you? Why would you say that their language is not correct and that yours is? There is no reason for this, since this imposition of names is quite arbitrary. If you do not want this language to merit the name of Roman, call it the French, English, Spanish, or even Parisian, since this is what the vulgar call it. When those who employ it use it with you, they will be mocked and remain misunderstood many times; but the same thing would happen to you if you were talking in the midst of them; Ἀνάχαρσις παρ' Ἀθηναίοισ σολοιβίζει, Ἀθηναίοι δὲ παρὰ Σχύθαις, Anacharchis made solecisms in Athens, the Athenians among the Scythians.

The orthodoxy of the Parisians saved, among the Christian humanists, the barbarity of their language and their dialectic subtlety; the Paduans would have vainly relied on a similar indulgence, whose efforts were to support "the ungodly dogmas[22] of Alexander, Averroes, and several other ancient philosophers".

Therefore, it is the Averroists, otherwise more numerous and influential than the Nominalists on the chairs of Italian universities, who especially attacked the Humanists. The language of the Averroists, enamelled in Arabic words, surpassed in harshness the Parisian style and, even more than this one, upset the delicate ear; the narrow and intolerant cult that they professed to Aristotle and his commentators appaled the Platonists. The name of Averroes became as the symbol of all that shocked Humanism.

Behold, for example, Giorgio Valla of Piacenza; he is a scholar who taught rhetoric in Milan, in Pavia in 1470, and in Venice in 1481; he is a hellenist who translated many of the works of Aristotle, Cleomedes, Ptolemy, Plutarch, and Proclus; he is a refined Latin scholar who annotated and edited the *Tusculanes*; he is an orthodox Christian, faithful to the teachings of the great Catholic doctors Albert, St. Thomas Aquinas, Duns Scotus, and Gilles of Rome, whom he cites with reverence;

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20 *Joannis Pici Mirandulæ, viri omni disciplinaruni genere consummatissimi, vita per*Joannem Franciscum Illustris Principis Galeotti Pici filium *edita* (Joannis Pici Mirandulæ *Omnia opera*).

21 The letter (already cited) of John Pic de la Mirandole to Hermolao Barbaro, Florentiæ, III nonas Junias MCCCCLXXXV.

22 *Apologia* Joannis Pici Mirandulæ, in fine.

all are willing be fierce opponents to the Averroist School; he is, too; let us hear how[23] he speaks of Aristotle and his Commentator:

> Those who consider things from a penetrating gaze should hardly be surprised that Aristotle, hallucinated in this circumstance, has professed similar errors; he gave many doctrines very inferior to this one; and, in this regard, the Platonists accuse his ignorance and lack of rectitude in judgment. This is why it was left aside, lying under rust; only Plato and the Platonic doctrine were celebrated. But soon we saw Averroes emerge from the mud, a barbarian, an absolutely stupid glutton with a stinking brain (*Aliquanto post Barbarus quidam ineptissimus lurcho, putidique cerebro e lato effossus Averroes*); reveling in captious discussions, he succeeded, using sophistical arguments, to present a Platonic Aristotle to the point that there is no known philosopher who was the same.

This fiery hatred of Aristotle and his Commentator could predispose George Valla to welcome the new antiperipatetics of the nominalist School; also, we find in his writings a kind of reflection of Parisian dynamics; but this reflection is pale and vague!

We hear this reflection in the teaching of our humanist[24] on the subject of the time of rest by which a falling projectile would be separated from the ascent of this body:

> If a movement headed in a straight line is reflected, it produces, it is true, two contiguous movements, but not two movements which continue each other. Between these two movements, in fact, a rest occurs which interrupts the continuity. The first movement ends, then the second is accomplished as starting from another origin; between the ultimate limit of the first and the beginning of the second is an intermediate rest… Thus, the term of the ascent of the stone tossed into the air differs from the beginning of the descent of this body, which falls with speed; this distinction corresponds to the flow of a certain duration; some rest is therefore observed between the two opposite movements of the stone.

If Valla admits the existence of the intermediate rest from which Leonardo da Vinci, at the same time, would draw the fruitful notion of composed *impeto*, he says nothing about the reasoning by which all Parisian masters, from Richard of Middleton to Marsilius of Inghen, tried to give the cause.

With the Nominalist school, and against the unanimous sense of the Peripatetics and Averroists, Valla attributes the motion of projectiles to an impressed force (*vis indita*) on the mobile. But he does not attribute the acceleration of natural motion with an increase of *impetus*; he adopts, regarding this motion, the explanation of Aristotle and Themistius. This is what we see in the following passage:

Only circular motion

[23] Georgii Vallæ Placentini viri clariss. *De expetendis et fugiendis rebus opus, in quo hæc continentur…* In fine tomi secundi: Venetiis in ædibus Aldi Romani impensa ac studio Joannis Petri Vallæ filii pientiss. Mense Decembri MDI. — Totius operis liber XXIII et Physiologiæ quartus ac ultimus, de Cœlo, quodque Mundus non sit æternus, et Aristotelis argumentorum confutatio; c. I. — This large compilation, one of the typographic masterpieces out of the Aldine presses, was published by John Peter Valla two years after the death of his father; he, in fact, died in Venice in 1499.

[24] Georgii Vallæ *Placentini Expetendorum ac fugiendorum quem struebat* liber vigesimus secundus, Physiologiæ vero tertius, quartæ hebdomadis liber primus. De naturalibus principiis et causis. Cap. VI: De motu et quiete.

has the uniformity which is related and natural to it. All bodies moving in a straight line, either by nature or against nature, move to their end with a different speed than at the beginning. If a body moves against nature through the effect of a pull, it starts to move more slowly; then it goes faster gradually as it approaches the mover that pulls it because then the power of this mover dominates more. On the contrary, bodies that are thrown move faster initially, then slower when the force impressed on them by what threw them is destroyed… Finally, bodies moving with natural movement will go faster when they are close to their own places; they wish, in fact, to reach their integrity, and they draw this integrity from new forces, as if they were more amply supplied with form. Thus, any body that moves with rectilinear movement, be it by nature or against nature, provides an uneven course.

Pic de la Mirandola "did not ignore," says his biographer John Francis Pic[25],

everything that concerns the tricks, sophistical arguments, and trifles of Suiseth, which he calls calculations (*captiunculæ cavillæque sophistarum et suisseticæ quisquiliæ, quæ calculationes vocantur*); there are some mathematical considerations that are applied to extremely subtle and, I would say, extremely bizarre (*morosiores*) physical theories. He was very erudite in these matters and read many writings of this genere, writings with which, perhaps, Italy was not very familiar… However, he seemed to hate and detest these questions.

George Valla probably did not have, of the *calculationes* of Paris, as in-depth a knowledge as Pic acquired and which was, in the testimony of his biographer, very rare in Italy; but doubtless, like John Pic, he hated them, and his Physics suffered; he carefully kept the errors that Parisians refuted long ago.

Nifo mockingly bypassed the Dynamics of the *captiunculator* Albert of Saxony; George Valla was without a doubt ignored; Leonardo, more inspired, continued to ponder the lessons of this Dynamics; almost alone among the scholars of his country and of his time, he had the great merit of guessing the most fertile ideas contained in that much-disparaged Parisian Physics.

[25] *Joannis Pici Mirandulæ…. vita per* Joannem Franciscum illustris principis Galeotti Pici filium *edita*.

Chapter 7
The spirit of Parisian Scholasticism in the time of Leonardo da Vinci

While most Italians, far from imitating the brilliant artist, adhered, with the routine of a Nifo, to the antiquated theories of the Mechanics of Aristotle and the Commentator, what did the Parisians, *Moderniores*, *Juniores*, *Terministes*, *Captiunculatores*, and *Sorticolæ* do? What was taught in the early years of the 16th century on the banks of the Seine? What was the spirit that animated this teaching at the very time when Leonardo abandoned Italy and went to die in France?

Against the narrow Averroism of Bologna and Padua, Paris presented the widest eclecticism. We find the proof of this eclecticism in the writings of the Sorbonne doctors and teachers of the Faculty of Arts; but it seems interesting to hear one of them define and justify it.

On the chairs of the Sorbonne and the Rue du Fouarre, many Spaniards sat.

One of them, Pedro Sanchez Cirvelo of Daroca (province of Zaragoza), was certainly, around the late 15th century and beginning of the 16th, one of the most active teachers in the Parisian Faculty of Arts. We owe him a Treatise of practical Arithmetic[1] and a commentary on the *speculative Geometry* of Bradwardine[2]. We owe to him, above all, a commentary on the treatise of the *Sphere* John of Sacro-Bosco; attached to the same text of the *Sphere* and to the *Fourteen Questions* that Peter of Ailly composed on the subject of this same writing, this commentary formed a kind of astronomical textbook that was frequently printed[3] at the end of 15th century and early 16th century.

[1] Petri Cirveli Darocensis Hispani *Tractatus Arithmetice practice qui dicitur Algorismus*. Impressus Parisius in Bellovisu, Anno Domini 1505, die 29 aprilis. — Id., Impressus Parisius per Anthonium Ausourt pro Johanne Lamberto. Anno Domini 1513.

[2] Thome Breuardini *Geometria speculativa recoligens omnes conclusiones geometricas*… Colophon:

> Et sic explicit Geometria Thome Breuardini cum tractatu de quadratura circuli bene revisa a Petro Sancliez Girvelo, expensis bonesti viri Johannis Petit, diligentissime impressa Parisiis in campo Gaillardi. Anno Domini 1511, 6 Marcii.

[3] Johannis de Sacro-Bosco *Sphæræ mundi opusculum una cum additionibus peropportune insertis ac familiarissima textus expositione* Petri. Parisiis, per Wolfgangum Hopyl, 1494.

The commentary of Pedro Girvelo de Daroca is followed by a dialogue[4] among the *Darocensis*, who is the author, and Burgensis, who is his friend Gonzalve Gilles of Burgos. *Darocensis* sought to establish the merits of the innovations contained in his treatise; he is brought, by it, to discuss the degree of submission that we owe to the opinions of ancient writers; Burgensis, conversely, dreams of a disciplined science, where all discussion would be excluded. Here are some passages of this dialogue:

> Darocensis. Hear these words: You know in what honor the doctrine of Peter Lombard has been held; the sentences of this master are cited as texts by all theologians; they do not believe, however, that we should rely on Peter Lombard in everything that he advanced; on the contrary, they usually take no account of him. Thomas, the solemn doctor, argues in a host of cases against those who were his teachers. All the teaching of John Scotus relates only to the refutations of the propositions of Thomas and other theologians. The very subtle Nominalists, who came next, have directed their sharp strokes against Thomas as against John Scotus, and it is not obvious that one of them is less famous. How many others who need the laurels of immortality for their mutual discussions could advocate for our cause! The others that are men to us; as such, they were able to err; to interpret or correct them with respect, and keep the truth with all one's strength, should be the role of a loyal spirit. From what the predecessors of extreme skill were, it must not be concluded that the way of discovery is now closed to their successors; the Philosopher said: The sciences are like rivers; they grow by a continuous influx…
>
> Burgensis. You speak very well. But for your trick, remember all the evil that the altercations between supporters of different opinions made in the Republic of Letters. Despite the precept of Horace, you will find very few men who are enslaved to the word of any master and do not swear by these words:
>
> Nullius addictos jurare in verba magistri.
>
> One is Stoic, the other Peripatetic. This one follows Thomas, that one Scotus, and another a third master. The result is that those who participate in truth and keep it are very rare. Is there, I pray thee, nothing more unworthy of a man of study, nothing more shameful to him, than to block the progress of science and truth? Now our great Aristotle asserts: The stubborn attachment to the cult of a master is a big obstacle for those who want to know…
>
> Darocensis. You claim that the altercations of scholars have resulted in hiding the light to all but a very small number of men. Nothing is less like the truth. As the Philosopher says, it is precisely by solving the questions discussed that truth manifests itself. These are the arguments of the successors which shed light on the opinions of their predecessors. Whoever therefore seeks should doubt; he must remain in doubt until he has heard the alleged reasons for each side, until he calms the passions of his mind so that he can, free from all intellectual emotion, engage in the search for truth. This is realized in the highest degree if we care to discuss what the various authors have thought of the debated question; if one first takes notice of one of them as a basis for examining what others have thought of the posed problem. But I would not say we act rightly if we imitated any doctor to the point of

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Johannis de Sacro-Bosco *Uberrimum sphere mundi commentum insertis etiam questionibus* D^{ni} Petri de Aliaco. Parisius, in campo Gallardo, oppera atque impensis magistri Guidonis Mercatoris, anno 1498 (some copies say 1468; instead of the seal of Guy Marchand, they offer that of Jehan Petit, Johannes Parvus).

Johannis de Sacro-Busco *Sphera cum additionibus et commentis Petri Cirveli insertis questionibus* Petri de Aliaco, Parisiis, 1508, 1515, 1526; Compluti, 1526.

[4] Dialogus disputatorius. P. C. D. in additiones immutationesque opusculi de sphera mundi nuper editas disputatorius dyalogus. Interlocutores Darocensis et Burgensis.

> imagining that what he said is free of errors. The fragility of the human mind does not suffer it to be so, unless by a special help of God. Therefore, our Parisian philosophers refrain from acting in this way. No doubt in most cases they walk in the footsteps of Aristotle; but they do not refuse to hear the opinions of other masters who added a large number of very brilliant discoveries to the work of Aristotle; some, perhaps, are exceptions to this behavior; we must look at them as disciples not of philosophy, but of routine…

What was printed here, in 1494, a master of the Faculty of Arts in Paris heard. From the time when Thomas Aquinas taught there, the University of Paris kept the same spirit, respectful of ancient authorities, welcoming new opinions; Parisian traditionalism knew, throughout the Middle Ages and the time of the Renaissance itself, to remain in perfect equilibrium between routine and the excessive taste for novelty.

This eclecticism appears, first of all, to him who peruses the writings composed by Parisian masters; it manifests itself in the variety of names of authors cited. Aristotle, Alexander of Aphrodisias, Themistius, and Averroes do not occupy, in these writings, the dominant and almost exclusive place that the Italian masters granted them; the great doctors of Scholasticism—Albertus Magnus, Thomas Aquinas, Duns Scotus, and Giles of Rome—are heard with respect, but their opinions are freely discussed and very often rejected; many of the doctrines taught are taken from William of Ockham, Gregory of Rimini, and Robert Holkot; it is, above all, these philosophers whose genius, measurement, and common sense have been able to combine and temper, one by one, the Thomistic, Scotist, and Nominalist doctrines, to Walter Rurley, Jean Buridan, Albert of Saxony, and Marsilius of Inghen; Italian Averroists themselves are not neglected, and Paul of Venice is frequently cited, though his inconsistencies and fallacies are sometimes met severely.

It is especially at the College of Montaigu that the tradition of Buridan and Albert of Saxony seems guarded with particular loyalty; teachers who teach in this College are trying to rescue from oblivion the writings of the great Nominalists of 14th century. Regent in Montaigu, the Scot Joannes Majoris, who in 1516 printed in Paris the *Summulæ* of Buridan; regent in Montaigu, Georges Lokert, another Scot, who in 1516 published the *Questions of Albert of Saxony on the Physics*, *De Cœlo*, and *De generatione*, and those of Themo Meteora and Jean Buridan on the *De anima* and *Parva naturalia*; regent in Montaigu, John Dullaert of Ghent who, in 1509, and still in Paris, gives an edition of the *Questions on the Physics* composed by the Philosopher of Béthune. The same Dullaert, moreover, as if to assert his eclecticism, printed in Paris in 1513 by Thomas Rees, the *Summa totius philosophiæ naturalis* and the *De compositione mundi* of Paul of Venice.

The revival of Nominalism of which the College of Montaigu was the scene at the beginning of the 16th century seems to have been led by the theologian Joannes Majoris.

Joannes Majoris was born around 1478[5] in the small town of Glegorn[6] neighboring Haddington, in Scotland, hence the nickname *Haddinglonanus* which is often given him; since 1504 we see the *Summulæ* of Buridan published and, in 1530, he gives another edition of his commentary on the first book of the *Sentences*; his death is dated the year 1540.

In his long and active career as a teacher, John Majoris trained many disciples. Two of them, on the subject of the doctrines of Dynamics that were professed at the College of Montaigu, give us documents of extreme importance.

One is John Dullaert of Ghent (1471?-1513); he left us *Questions on the Physics* and *De Cælo*[7] that Nicolas Desprez printed in Paris in 1506; these questions were an echo of the teaching that Dullaert gave at Montaigu.

The other is the Spaniard Luis Nuñez Coronel of Segovia whose *Physicæ perscrutationes*, after having been professed at the College of Montaigu, were published in Paris in 1511[8].

Besides, if Montaigu kept faithfully the traditions of Buridan and Albert of Saxony, he was not its only custodian; at St. Barbara, in particular, those traditions were held in high esteem; we have as witness the Spaniard Juan de Celaya; by the title itself of his *Exposition* and *Questions* on the *Physics* of Aristotle[9], printed in Paris in

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[5] *Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu,* seconde série, p. 404

[6] Joannis Majoris doctoris theologi *in Quartum sententiarum quæstiones utilissimæ* … vænundantur a suo impressore lodoco Badio. Colophon:

> … in chalcographia Jodoci Badii Ascensii. Anno a virgineo partu Millesimo quingentesimo decimosexto: circiter Calendas Decembris. Deo Gratias.

Letter of Joannes Major (*sic*) printed on the back of the title page.

[7] Johannis Dullaert *Questiones in libros Phisicorum Arislotelis*. Colophon:

> Hic linem accipiunt questiones phisicales Magistri iohannis dullaert de gandavo quas edidit in cursu artium regentando parisius in collegio montisacuti impensis honesti viri Oliverii senant solertia vero ac caracteribus Nicolai depratis viri hujus artis impressorie solertissimi prout caracteres indicant anno domini millesimo quingentesimo sexto vigesima tertia martii.

[8] *Physicæ perscrutationes* magistri Ludovici Coronel Hispani Segoviensis. Prostant in edibus Joannis Barbier librarii jurati Parrhisiensis academie sub signo ensis in via regia ad divum Jacobum. — The book bears no colophon. The folio following the title begins with a letter: *Ludovicus Nunius Coronel illustrissimo viro Inacho de Mandocia*; this undated letter was written in Paris. It is followed by another letter: *Simon Agobertus Bituricus fratri Joanni Agoberto*. In this letter, dated: Parrhisiis, MDXI, Simon Agobert speaks with great praise of his teacher Luiz Coronel, who taught philosophy at the College of Montaigu. — There exists a second edition of this book: *Physice perscrutationes egregii interpretis* Magistri Ludovici Coronel… Lugduni, in edibus J. Giunti. 1530. We could not consult this second edition.

[9] Expositio magistri ioannis de Celaya Valentini *in octo libros phisicorum Aristotelis: cum questionibus eiusdem, secundum triplicem viam beati Thome, realium, et nominalium*. Venundantur Parrhisijs ab Hemundo le Feure in vico sancti Jacobi prope edem sancti Benedicti sub intersignio crescentis lune commorantis. Cum gratia et Privilegio regis amplissimo. Colophon:

> Explicit in libros phisicorum Aristotelis expositio a magistro Joanne de Celaya Hyspano deregno Valentie edita: dum regeret Parisius in famatissimo dive Barbare gymnasio pro cursu secundo anno a virgineo partu decimo septimo supra millesimum et quingentesimum. VII

1517, this author affirms his eclecticism, since he declares to follow "the triple way of Saint Thomas, the Realists, and the Nominalists".

In the book of Juan de Celaya, the text of Aristotle is still reproduced and accompanied by an exposition or literal commentary; it is only after this commentary that the author announces by the title "*sequitur glosa*" the detailed discussion of the most modern views. John Dullaert completely abandons the commentary on the text of Aristotle; by the example of John of Jandun, John le Chanoine, Jean Buridan, Albert of Saxony, and Marsilius of Inghen, he confines himself to examine a series of *Questions* raised by the various chapters of the work of Aristotle. Louis Coronel goes even further; his writing has the form of an original treatise on the *Physics*; the order in which the diverse materials are presented alone reveals the influence of the Φυσιχὴ ἀχρόασις.

Moreover, despite this variety of form, it is the same spirit that animates the works of these three authors. The problems that play a leading role are those that the Parisian Nominalists—William of Ockham, Gregory of Rimini, Buridan, Albert of Saxony—have posed or renewed; in those chapters of the *Logic* that concern Mathematics, the Science of equilibrium and movement, and the principles of general Physics, in the sense that these words have taken today, are the subjects of most of these problems. No doubt, the form in which the proposed solution is made often shocks our habits; sometimes we have some difficulty following the thought of the author through the *videtur quod sic*s, *sed contra*s, *arguitur*s, *confirmatur*s, which baffles the simplest logic of Buridan and Albert of Saxony to which we are accustomed and delights those too clever dialecticians; perhaps we see *Sortes* constantly placed in the hypothetical case that the divine Omnipotence could only realize and whose interest sometimes escapes us; but if we encourage ourselves to penetrate this antiquated form, to expose the idea that it hides or dons, we are often astonished to find this idea so young and alive. In particular, it will be difficult not to feel this astonishment by studying what John Dullaert, Louis Coronel, and Juan de Celaya taught regarding Dynamics.

The Dynamics that is professed at Montaigu or St. Barbara in the early 16th century is that of the leaders of the Nominalist School of the 14th century, William of Ockham, the *Venerabilis Inceptor*, Jean Buridan, and Albert of Saxony. In these colleges the arguments that Italian Averroists advanced against this Dynamics are thoroughly refuted; sometimes, the sarcasm of the Paduans fell sharply at the skill of the revered masters of the University of Paris.

Before ending this question of the *impetus*, we want,

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idus Decembris. diligenter impressa arte Johannis de prato et Jacobi le Messier in vico puretarum prope collegium cluniacense commorantium: Sumptibus vero honesti viri Hemundi le feure in vico sancti Jacobi prope edem sancti benedicti Sub intersignio crescentis lune moram trahentis. Laus deo.

In 1518, Jean du Pré and Jacques le Messier printed and Hémond le Fevre sold the *Expositio magistri* ioannis de Celaya Valentini, in quattuor libros de celo et mundo Aristotelis: cum questionibus eiusdem, and also the Expositio magistri ioannis de Celaya Valentini, *in libros Aristotelis: de generatione et corruptione: cum questionibus eiusdem*.

said Louis Coronel[10],

> to treat here of the opinion Nicholas of Chieti[11]; he occupies the first chair of ordinary Philosophy at the University of Padua and, as he himself tells us, teaches without a competitor. He published on the movement of heavy and light bodies a certain small question that we recently received. It outlines the opinions of many philosophers and, after having refuted them, at least in his opinion,

he contends that a falling weight, as well as a projectile, is moved by the air.

> He says this opinion is that of Aristotle and the Commentator. He treats with contempt the very subtle Albert of Saxony and names him Albertutius; he gives our other doctors the name of Terminists… He was surprised that a certain master Cajetan wanted to support such errors.
>
> We will not change the name of this master out of respect for him; but we will simply show that he contradicts himself… If, to speak like Sallust, he took some pleasure in reprimanding others, he will lose it by agreeing to reprimand himself, provided however that this document reaches him.

When Louis Coronel wrote these lines, Vernias was dead; but Nifo could have profited from the lesson they contained.

[10] Ludovici Coronel *Op. cit.*, lib. III, pars I: De motu locali, fol. LII, col. *b*.

[11] Luiz Coronel said: *Nicoleti de Thienis* (of Nicolò of Thiene) instead of *Nicoleti Theatini* (of Nicolò of Chieti).

Chapter 8
The Parisian Dynamics in the time of Leonardo da Vinci

The Dynamics that John Dullaert[1] and Louis Coronel[2] taught at Montaigu, which Juan de Celaya[3] professed at St. Barbara, the Dynamics of Jean Buridan and Albert of Saxony, is the Dynamics of *impetus*.

How does the projectile move once it leaves the hand or machine that launched it? All reject the explanations which attribute the continuation of this movement to the air, that the action is a thrust of the air that swirls behind the mobile or an attraction of the condensed wave that propagates in front of it.

> Contrary to these meeting of these two ways of speaking,

Dullaert wrote,

> I brought up a single argument: A mobile can move with a rotational movement, by always remaining in the same place; it is certainly not driven by the air that pushes it, nor by the air that he who launched it would have shaken; these two explanations are insufficient. The consequent clearly results from the antecedent, and it is made manifest by the movement of the clog…
>
> Although one of these two views seems to have been that of the Philosopher, a third is commonly held, as follows: After the rest of the mover that launched it, the movement of the projectile is produced by a certain virtue imprinted on this mobile; that is to say that the first mover gives the projectile the virtue of moving in such a direction that it, like the magnet, we said above, gives the iron the virtue of moving

towards it.

Louis Coronel also rejected, by various arguments, the theories which attribute to the air the continuation of the movement of projectiles; one of these arguments is as follows:

[1] Joannis Dullaert de Gandavo *Op. cit.*, Physicorum lib. VIII, quæst. II: Quæritur secundo utrum projectuin, dum reflectitur, in puncto reflexionis quiescat.

[2] Ludovici Coronel *Op. cit.*, lib. III, pars I: *De motu locali*; ed. cit., fol. L, col. *c* seqq. (Running title: *De impetu*.)

[3] Joannis de Celaya *Op. cit.*, lib. VIII, cap. XI, quæst. III: A quo movetur projectum post separationem illius a quo projicitur; fol. CC, col. *d* et fol. CCI. (Running title: *De motu projecti*.)

> This explanation does not account satisfactorily for the rotational movement of the wheel that a person pulls and which, in its movement, still remains in contact with the same air; we cannot say, in fact, in this case, that the surfaces of the air reunites after being separated, since for the duration of the movement the wheel remains in the same place.

Coronel then tells us that

> many scholars agree to imagine an *impetus* separate from the mobile; first, when a heavy body is thrown into the air or horizontally, it might, after being launched, continue to move if one assumed a certain quality that the instrument of projection impressed on it, which is *impetus*; if we could not admit the existence of this quality, physicists would not know what mover to give to this mobile.

Dullaert teaches us not only that this explanation is commonly accepted, but even that one habitually gives to this virtue imprinted in the mobile the name of *accidental gravity* when the projectile is launched downward and *accidental lightness* when it is launched upward. He does not like these names; in a body, in fact, that we launched horizontally, this virtue can be called neither gravity nor levity; so it is better, in all cases, to call it *impetus*. This wish seems to have been heard in Paris, as Coronel and Celaya do not employ, to designate this virtue impressed on the mobile, any other term than that of *impetus*.

What is, according to our authors, the nature of this virtue? We will now review their opinions in this regard. We follow, for now, how they, according to Buridan and Albert of Saxony, explain the various phenomena of Dynamics.

> A stone,

Dullaert said,

> receives more of this virtue than a feather receives; it can thus be launched farther away.

Juan de Celaya, in imitation of Buridan, is more precise:

> You may ask why, according to this opinion, a stone thrown moves longer than a feather. We reply that the reason is this: The stone has more material and is denser than the feather; so it receives a more intense *impetus*, and it retains it longer; therefore, it is not surprising that it moves longer.

To this explanation Celaya provides an objection:

> A large projectile therefore would move faster than a smaller projectile; this result is contrary to experience;… however, one would prove it thus: The *impetus* impressed on a large projectile is greater than the *impetus* impressed on a small one, so the big projectile moves quicker than the small one.
>
> To this reply we will answer that the consequent is falsely deduced. To demonstrate this, we have to distinguish that the *impetus* imprinted on a projectile can be greater either *intensively* (and we deny that is so in the present case) or *extensively*; we grant that this occurs in the present case, but then we deny consequent; there is in fact no problem that an *impetus*, which is extensively less than another *impetus* but is *intensively* much greater, produces a faster movement than the latter.

The distinctions so familiar to Scholasticism that mark the words *extensive* and *intensive* find their proper translation in this entirely modern form of that statement: The total *impetus* of a body results from an *impetus* given to each member of this body; all things being equal, the elementary *impetus* is more intense as the speed of the element is greater.

The reading of the work of Juan de Celaya shows us that one thought, in Paris, of the extensive distribution of *impetus* in the mass of a body; we were, however, led by the opinions of Marsilius of Inghen that we reported in our previous study[4] and now we will see Louis Coronel discuss. We know[5] how this notion of the extensive distribution of the *impetus* led Leonardo and Bernardino Baldi to conceive the existence of a center of accidental gravity, thereby preparing the way for Roberval, Descartes, and Huygens.

> When a body is thrown up in the air,

Dullaert states,

> it moves faster at the beginning than at the end, and faster in the middle of its course than at the end, and this because the virtue imprinted in it constantly weakens more and more.

> Some say,

writes the same author,

> that the *impetus* caused by violence is corrupted due to the absence of its cause… But it would be better, I think, to say that this *impetus*, caused by violence, is corrupted by the form itself of the projectile, which form inclines the body to a movement contrary to what the *impetus* produced.

Louis Coronel said briefly:

> A violently moved body moves with a movement opposed to the natural movement; while it moves, the *impetus* is constantly weakening; it is more intense at the beginning of the movement and more attenuated at the end; such a mobile therefore moves more and more slowly.

It is in the treatise of Juan de Celaya that we find the *law of inertia*, expressed in the clearest manner, under the form that Jean Buridan gave it, and that even Galileo will keep almost verbatim:

> Against this solution,

the regent of St. Barbara said,

[4] *Jean I Buridan (de Béthune) et Léonard de Vinci*, V : Que la Dynamique de Léonard de Vinci procède, par l'intermédiaire d'Albert de Saxe, de celle de Jean Buridan. — To what extent it deviates, and why. — Les diverses explications de la chute accélérée des graves qui ont été proposées avant Léonard, pp. 94-96.

[5] *Léonard de Vinci et Bernardino Baldi*, IV: Les emprunts de Bernardino Baldi à la Mécanique de Léonard de Vinci (*suite*). Le centre de la gravité accidentelle (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, III; première série, p. 108, seqq.) — *Bernardino Baldi, Boberval et Descaries* (ibid., IV; première série, p. 127, seqq.).

> is opposed the following argument: It follows from this theory that a projected body will always move. This result is false and, however, the reasoning is evident; nothing, indeed, would destroy this *impetus*, so it would always move the projectile.
>
> We respond to this reply by refusing to recognize the value of the reasoning, and that because we deny the antecedent. This *impetus*, in fact, is sometimes destroyed by the resisting medium, sometimes by the form or the virtue of the projectile which exerts a resisting action, and finally sometimes by an obstacle.
>
> … When we throw a weight in the air, the shape of the weight does not cooperate in the upward movement; it resists it, on the contrary, and decreases the *impetus* imprinted in this mobile.

Thus, the *impetus* should last indefinitely if it had to struggle against any of the three causes of destruction that were listed; this is what Celaya admits:

> According to this view, it is not necessary to assume as many intelligences as there are celestial orbs; it would suffice to say that there is in each orb an *impetus*, that the First Cause impressed on it this *impetus*, and that it moves this orb; this *impetus* does not corrupt, because such a celestial orb has no inclination to a contrary movement.

When Buridan had issued, on the subject of celestial motions, this bold hypothesis, he humbly sought the judgment of theologians. Through the voice of John Majoris[6], Theology declares that this assumption is admissible.

John Majoris argues that the Heavens are composed of matter and form. Against this opinion he provides the following objection:

> If Heaven were composite, it would have no need of an extrinsic mover, although this is the opinion of all the sages; so it is not composite.
>
> We reply that this argument contradicts those who treated the movement of Heaven. If there were no objection to fear than those which concern movement, I would say that there is no objection to Heaven being moved by its substantial form; or even that it was moved by an accidental form that would be connatural to it, like a weight descending by its gravity. We see that the wheel of the blacksmith turns by the *impetus* which has been imparted to it; so we must not deny that God could produce an accident capable of moving Heaven in a circular motion, naturally and continuously; it may be said of the substantial form.

Thus, from the beginning of 16th century, the Theology of the University of Paris, which the Catholics of all countries welcome at this time as the faithful guardian of orthodoxy, agreed with this thought: The movements of celestial bodies can depend on the same Dynamics as the movements of sublunary bodies. It is only at the time of

[6] *In secundum Sententiarum disputationes Theologiæ* Joannis Majoris Hadyngtonani *denuo recognitæ et repurgatæ*. Vænundantur Iodoco Badio et Ioanni Parvo. Colophon:

> Finis disputationis Joannis Majoris natione scoti et professione Theologi Parrhisiensis penitus recognite et aucte Impresse impensis communibus Joannis Parvi et Jodoci Badii Ascensii. Opera ipsius Ascensii anno domini MDXXVIII circiter XV calendas septembres. Deo gratias.

— The book begins with two letters, one by Joannes Majoris to two other theologians of the College of Montaigu, Noël Bède and Pierre Tempeste; another by Pierre Peralta to Pierre Desjardins (*ab Hortis*); in those two letters, reference is made to a first edition of the same book given, “many years ago,” to the care of Antoine Coronel. — In dist. XII quæst. III: Utrum cælum sit ex materia et forma conflatum; ed. cit., fol. XXXIX. col. *c*.

Kepler and Galileo that astronomers will frankly adopt this opinion[7]. It is interesting in this regard to observe that John Majoris indicates three possible causes of the persistence of a rotational movement: An impetus imprinted by violence; an accidental but connatural form, analogous to gravity; a substantial form, analogous to the soul, the substantial form of the body. These thoughts of John Majoris offer a striking resemblance with those that Nicolas of Cusa issued, and especially with those that Johannes Kepler will adopt[8].

The continued decrease in *impetus* in a violent movement has been invoked by Albert of Saxony and Marsilius of Inghen to demonstrate precisely that there is an intermediate rest between the ascent and descent of a heavy projectile.

All the regents of Montaigu admit this theory.

The existence of this intermediate rest is the very object of the question where John Dullaert deals with *impetus*. We also find, towards the end of this question, the following conclusion:

> Between two opposing movements, the one direct and the other reflected, only one of which comes from an intrinsic cause, falling through an intermediate rest properly so-called; this is obvious: When a stone is thrown in the air, and is independent of any other mover, a very strong virtue is imprinted in it, according to the opinion which admits the *impetus*; the stone then moves upwards. As this virtue continually weakens, it comes to such a degree of relaxation that it can no longer push the mobile upwards; it however resists the gravity that pulls this body down. Finally, it reaches such a weakness that it no longer suffices to resist. I take the moment when this virtue [ceases moving up but when it] is sufficient to resist, and the moment it no longer suffices to resist; during the intermediate period, the body remains at rest.

Louis Coronel explicitly reproduces the calculation made by Marsilius of Inghen. He has so much confidence in this calculation that he does not hesitate to draw the following conclusion, whose naivety lends to a smile:

> It clearly follows that one can imagine cases where a stone thrown into the air will remain at rest for one, two, or three hours. But, you say, maybe we do not perceive that rest of the stone in the air. This objection does not conclude; the great distance can prevent us from perceiving this rest; or alternatively, it may be that only the stone remains motionless for an imperceptible time.

This theory certainly held an important place in the teaching of Physics at the College of Montaigu; also, the very people who were leaving the College, deeply disgusted from lessons they received there, remained convinced that the *quies intermedia* kept the projectile suspended. Let us hear what Luiz Juan Vives says[9], in

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[7] See P. Duhem, Σώζειν τά φαινόμενα. Essai sur la notion de théorie physique de Platon à Galilée (Annales de Philosophie chrétienne, 79° année, 1908, et Paris, 1908) [Duhem (1969)]. See, in particular, the conclusion of this work.

[8] *Nicolas de Cues et Léonard de Vinci*, X: La Dynamique de Kepler (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, XI; seconde série, pp. 207-211).

[9] Jo. Ludovici Vivis Valentini *De prima philosophia, sive de intimo naturæ opificio liber secundus* (Jo. Lodovici Vivis Valentini *Opera, in duos distincta tomos: quibus omnes ipsius lucubrationes, quotquot unquam in lucem editas volait, complectuntur: præler Commentarios in Augustinum De civitate Dei, quorum desiderio si quis afficiatur, apud Frobenium inveniet*. Basileæ, anno MDLV.

1531, being the student of John Dullaert of Ghent whose imprecations against the Parisian Philosophy will attract our attention:

> Curved or circular movement is one; broken movement is multiple; the breaking of movement corresponds to a stop or gap…
>
> That an interruption is placed between the two parts of such a trajectory, not only because reason teaches it, but also the senses frequently perceive it. Everything, in effect, naturally moves by violence. If it moves naturally, it will remain at rest when it reaches its end. If it moves by violence, between the end of violence and the beginning of the natural inclination, there will be a certain interval during which the violence declines while nature takes over; in this way the stone is thrown into the air. From a violent movement and a natural movement, in fact, a movement that is unique and in one piece can form. Whenever a new force starts and reverses the direction of the movement, there is a certain interval, although too brief to be perceived, during which the first force, fatigued, gives way to the new force that enters vigorously; during this interval a fight occurs, a fight that cannot happen in a single indivisible moment, which requires some time; for this very rapid action, a very short yet divisible time suffices. There are cases where we can, using our senses, see this rest; it is so for the arrow shot into the air; at the time of dropping, it stops a bit, then its second movement begins.

Vives, in this passage, speaks almost like Georges Valla; what he said of the struggle between violence and nature also recalls the considerations by which Leonardo was led to the notion of compound *impeto*[10].

The thought of this fight imposes itself on the mind of the Spanish humanist, because a little further comes this[11]:

> In every action, there is effort to achieve the goal; so there is a distance between the beginning and the end of this action; it is in this interval that effort is exerted, and without this interval the effort would be useless. When the action is contrary to the nature of the patient, the fight is continual; it takes place at the beginning, middle, and end; the violence of the agent and the nature of the patient are never without a fight. When, however, the action is based on the nature of the patient, there is no struggle at the beginning of the movement; this movement, in fact, is excited by the very nature of mobile, and that nature is not fighting against itself. But even when the force is natural, as soon as it kicks in, the medium in which it acts comes into conflict with the agent; the agent or mover, in fact, wants to penetrate the medium to reach its end; and the medium, as soft as it is, resists the penetration; all penetration, in fact, is a kind of division, and the division is the beginning of corruption, while union aids its conservation. The harder the medium is, the closer its parts are united, and the larger its forces and resistance are; this is why movement is more difficult in water than in air, and more difficult in mud than in pure water.

This passage does not only carry the trace of what Vives heard taught at the College de Montaigu regarding violent movement; when the Spanish Humanist shows “nature exciting natural movement”, he certainly remembers what his teachers told him about the accelerated fall of weights. But before researching what they thought about it, we need to examine what they said about the nature of the *impetus*.

In fine: Basileæ per Nic. Episcopimii juniorem, anno MDLV. Tomus I, pp. 504-505). — The *In prima philosophia* is dated: Brugis, anno MDXXXI

[10] *Nicolas de Cues et Léonard de Vinci*, XI: La Dynamique de Nicolas de Cues et la Dynamique de Léonard de Vinci. Theory of *impeto* compound (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, XI; seconde série, pp. 215-222).

[11] Luiz Vives, *loc. cit.*, p. 568.

In this regard, the masters of the University of Paris had a choice between several doctrines.

The first was that of William of Ockham[12].

For the *Venerabilis Inceptor* there is within the projectile no entity, no virtue really existing that one can regard as the mover of this projectile. Moreover, neither is the movement an entity separate from the mobile. For the leader of the nominalist School, the mover, movement, and mobile are the same thing; there is not an *impetus* generating a movement in a body; there is only a body driven impetuously.

Buridan, we have seen, resolutely rejected this theory of William of Ockham. For him, in the projectile in motion, there are three coexistent things that are really distinct from each other: first, the mobile; secondly, a purely sequential reality, a *forma fluens*, which is the local movement; third, a permanent reality, the *impetus*, which produces local movement in the mobile and thus plays the role of mover.

What is the nature of this entity? Buridan does not try to guess it. Albert of Saxony, who admits in its fullness the theory of local movement and *impetus* given by the Philosopher of Béthune, strongly hesitates[13] to resolve this difficult question which rather falls, he said, under Metaphysics than Physics; however, he decided to declare that "the *impetus* is a quality of the second species, consisting in a certain aptitude and facility for movement."

This is in accordance with this opinion, explicitly professed in his *Quaestiones in libros de cœlo et Mundo*, about which Albert, in his Physics, regarding the accelerated fall of weights, spoke in the following terms[14]:

> The mobile animated from natural movement acquired a certain ability in this movement, and this acquired ability, by uniting with gravity, moves the mobile faster.

Marsilius of Inghen found[15] that the *impetus* should be stored both among the qualities of the first species (*habitus vel dispositio*) that are acquired either by the production itself of the subject or by its disposition for the better or for the worse, and among the qualities of the third species (*actio vel passio*).

The comparison of the *impetus* to an acquired aptitude, a habit, had probably attracted the attention of Leonardo da Vinci when he was reading the writings of

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[12] *Nicolas de Cues et Léonard de Vinci*, IX: La Dynamique de Nicolas de Cusa et les sources dont elle découle (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, XI; seconde série, pp. 192-196).

[13] *Nicolas de Cues et Léonard de Vinci*, IX: La Dynamique de Nicolas de Cusa et les sources dont elle découle (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, XI; seconde série, p. 196).

[14] Alberti de Saxonia *Quæstiones in libros de physica auscultatione*; in librum VII quæst XIII. — Cf. Bernadino Baldi, Roberval et Descartes, I: Une opinion de Baldi touchant les mouvements accélérés (*Études sur Léonæ6d de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, IV; première série, p. 130).

[15] *Nicolas de Cues et Léonard de Vinci*, IX: La Dynamique de Nicolas de Cues et les sources dont elle découle (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, XI; seconde série, pp. 196-197).

Albert of Saxony; we find in this comparison the explanation of the last words of this thought[16]:

> If a wheel whose movement became more and more violent gives of itself, after its mover leaves it, many turns, it seems clear that if this mover continues to rotate in addition to its set speed, this perseverance may take place with little force. And I conclude for maintaining this movement, the mover will always have a bit of fatigue and all the more since by nature it will be fixed.

This assimilation of the *impetus* to an acquired aptitude, a habit, was certainly well-known in the time of Leonardo, in the schools of Paris where the works of Albert of Saxony and Marsilius of Inghen were in great vogue.

John Dullaert of Ghent tells us that

> in the opinion of some physicists, the *impetus* generated by violence corrupts gradually due to the absence of its cause, as intuitive knowledge is corrupted by the absence of its object.

Jean de Celaya thinks that the *impetus* is a second quality in the broad sense; he compares it "to the knowledge and dispositions of the soul."

But it is to Louis Coronel that we must go to see the arguments of those who claimed, by this assimilation, to justify the assumption of an *impetus* separate from the mobile and local movement:

> When certain objects are moved repeatedly with local motion, they become more capable of this movement; some ability remains in them, some disposition they gained while they were moving; therefore, during the movement, some actual entity was produced in these bodies; it is this entity that spawned said ability, and this entity was separate from the local movement…
>
> The antecedent of that proposition is made manifest by a large number of experiments. First, when fingers are habituated to writing, they perform the movement of writing much better than before.

And Coronel developes other examples, including that of knowledge acquired by the repetition of the same perception.

> But,

he added,

> he who understands this argument will say that we concluded both the true and the false. If the repetition of actual movements produced an aptitude for movement, a stone tossed repeatedly into the air would acquire a certain aptitude to moving upwards; therefore, all things being equal, it would be easier to toss into the air than previously; experience teaches us otherwise…
>
> This remark does not eliminate the force of the argument. In a man who has bad habits of intemperance, repeated acts of temperance are not sufficient to generate the habit of temperance. Similarly, in a stone where the substantial form and gravity resist upward movement, repetition of several throws produces no aptitude to moving upwards. The argument seems to keep its strength.

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[16] *Les manuscrits de Léonard de Vinci*; ms. B de la Bibliothèque de l'Institut, fol. 26, verso.

We have heard *impetus* compared to the physiological disposition by which the fingers, habituated to writing, write more easily. We are not astonished when Kepler will teach[17] that the *impetus* imprinted by the Creator on the Earth begot within the Earth an anatomical organization and produced a circular arrangement of fibers that ensure the permanence of the rotational movement; he will only follow a well-known opinion in Paris at the beginning of 16th century.

We now know what divergent opinions concerning the nature of the *impetus* sought, at that time, the acceptance of the Parisian masters. Among these parties some suspended their judgment; others were either on one side or the other.

Of the two opinions, Juan de Celaya mentions only one, which likens *impetus* to an aptitude, a disposition which makes it a quality and therefore a permanent entity distinct from the mobile; this is certainly his opinion.

Juahn Dullaert knows

> the other view, according to which the *impetus* is not a quality really distinct from the thing or body that is moved… When an arrow is thrown violently by a ballista, …it is driven by this violent and impetuous movement and not by a quality named *impetus*, and we must say so in other cases.

After setting out the arguments for or against this opinion, the Ghent philosopher appears to remain undecided.

Coronel, who apparently attaches to this discussion more importance than Dullaert does and especially more than Celaya does, takes an intermediate position between that of Ockham and that of Buridan. With Ockham he admits that the *impetus* is identical to local movement, but with Buridan he thinks that the local movement is an entity separate from the mobile. Let us quote his own words, whose keenness is perfect:

> Notice that between *impetus* and local movement I assign no other difference than a difference of more to less, so that all *impetus* would be a local movement, but the converse would not be true; *impetus* is a very intense movement. Moreover, that the movement be strong or weak, we could say that all movement is *impetus*; it does not follow that everything that moves moves impetuously (*impetuose*); but we would see no disadvantage in it; it is not necessary that all that moves with *impetus* is moved impetuously… Willingly, we would have designated *impetus* as the driving quality when it is produced by an extrinsic cause, whereas we would have called it movement (*motus*) when it is produced by an intrinsic cause, if the *impetus* could also be produced by the substantial form and the gravity of a falling weight. We do not worry about it being expressed in one way or another because the difficulty is entirely verbal.
>
> Secondly, note that the mover produces in the mobile a certain entity without which it could not move, and that it is a kind of necessary instrument required by the nature; this entity is the local movement. The weight that moves upward does not have in it any other movement than the *impetus*; in a weight that falls, the substantial form and the gravity produce a movement that can be named *impetus* when sufficiently intense. In short, we can say that in all circumstances where an *impetus* is produced, a local movement is generated; …and all that must be said of the *impetus* as to its instantaneous or successive production must also be said of the movement.

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17 *Nicolas de Cues et Léonard de Vinci*, X: La Dynamique de Nicolas de Cues et la Dynamique de Kepler (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, XI seconde série, pp. 208-211).

Coronel could have accurately translated his thought by calling it *impetus* instead of a *quantité de mouvement*, which Descartes will one day attribute it.

The *impetus* being identical to local movement, the reasons that lead to distinguishing the *impetus* from the body that it moves also establish the distinction between the local movement and the mobile.

> One can make the following argument: The *impetus* is distinct from the thing that moves impetuously; thus local movement is distinct from the mobile. We can justify this result as follows: Any inconvenience resulting from the assumption of an *impetus* distinct from the thing that moves impetuously (if any resulted) follows from the assumption that the movement is distinct from the mobile and *vice versa*; and the one consequence is explained just as well as the other.

Among the arguments that establish that the *impetus* is really distinct from the mobile, Coronel places the explanation of the accelerated fall of weights. It is time to examine what the masters of Montaigu or St. Barbara taught about this explanation.

John Dullaert wrote:

> A certain *impetus* is caused by violence; some other *impetus* is generated naturally. It should be noted in this connection that if a weight is held in the air and if one removes what prevented it from falling, that weight falls faster at the end of the movement than at the beginning, given that the resistance is uniform. The reason is that, in the movement of this weight, the *impetus* of the mobile departs with an intensity of zero degrees (*a non gradu intensionis*), begins to grow in intensity, and constantly and continuously grows until the end of the movement.

The Ghent philosopher adds this sentence worthy of note:

> In weights of different sizes, does the *impetus* increase in proportion to the size of the weight or not? It would be a serious difficulty to examine, but I do not speak of it.

He does not insist anymore on the cause which increases the *impetus* during natural movement.

Coronel is more explicit.

He rejects the explanation of the accelerated fall of weights that Aristotle and Themistius gave. The reasons he argues against this explanation is sometimes due to a peculiar naivety; he thinks[18] that if heaviness were a virtue emanating from natural place, it would be enough to cover the earth with a garment to prevent this virtue from passing through; the bodies placed above this garment would cease to weigh toward the center of the world. Coronel made, moreover, that other happy remark that the theory of Themistius does not explain the slowdown in the movement of a projectile thrown in the air.

The hypothesis of *impetus*, however, saved both the acceleration of natural motion and the slowdown of the violent movement:

> A weight, in fact, falling in a uniform medium descends faster at the end of its movement because, for the duration of its course, the gravity, its own substantial form, or both together have produced a certain *impetus*, a quality that moves it down; and as this *impetus* is, while the mobile is nearing completion, more intense than it was at the beginning of the movement, the weight falls faster towards the end of its fall.

[18] Ludovici Coronel *Op. cit.*, lib. IV, pars I: Do loco; ed. cit., fol. LXXXIIII, col. *c*.

A little later, Coronel repeats:

> On the way down, the gravity produces an *impetus*; …for the successive duration of the descent, the gravity produces an *impetus*.

It is therefore exclusively to the gravity or to the substantial form of the heavy body that this generation of an increasingly intense *impetus* devolves.

This principle was not stated with sufficient clarity in the writings of the old masters; some turns of phrase employed by them could imply that the increase experienced by the *impetus* during a certain time was caused by the *impetus* or the local movement that existed immediately before the movement.

Buridan, for example, wrote:

> …The movement becomes faster; but the faster it becomes, the more intense the *impetus* becomes.

And also:

> The faster the movement becomes, the more vigorous the *impetus* becomes.

More explicitly, Summenhard said:

> Towards the end of the movement, the *impetus* increases due to the speed of the preceding movement.

Some authors thus seem to attribute to the *impetus* or local motion (for Coronel it is all one) that exists at a given moment a part in the subsequent increase of the *impetus*; they prepared a doctrine that we saw Bernardino Baldi[19] make and Roberval adopt[20].

Louis Coronel had formally rejected the opinion of these authors; he objected, as discussed earlier, to a theory of Marsilius of Inghen:

> The *impetus* produced after launching the projectile would be generated by another *impetus*, by the person who lauched it; the *impetus* would be, therefore, an active quality capable of producing a different quality of the same species as itself.

At this point, Jean de Celaya is, we shall see, of the same opinion as Louis Coronel.

Celaya deals several times with the acceleration of natural motion.

Here is the first passage[21]:

> If you ask by what the inanimate heavy bodies are driven when they fall and the light bodies when they ascend, we will answer that a heavy body is moved by its substantial form as a principle and by its gravity as an instrument… You may say: We see from experience that a weight moves faster at the end of its movement than at the beginning; but at the end of

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[19] Bernardino Baldi, Boberval et Descartes, I: Une opinion de Bernardino Baldi touchant les mouvements accélérés (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, IV; première série, pp. 138-139).

[20] *Ibid.*, III: Bernardino Baldi et Roberyal (*Op. cit.*, pp. 144-145).

[21] Joannis de Celaya *Op. cit.*, lib. VIII, cap. V, quæst. II: An animal moveatur ex se; fol. CLXXXVIII, col. *b*.

> its fall, it is closer to its natural place; so it seems that this natural place imprints in a heavy body a certain virtue that moves it quickly. We reply that the cause of this increased speed is not a virtue emanating from natural place. The cause of this increasing speed is the *impetus* which is acquired during the descent; united to the gravity, it produces toward the end a movement faster than what gravity alone produced at the beginning.

Here is a second passage[22] here the same subject is treated again:

> When a certain being moves naturally, a certain quality is caused in this being; this quality, which is called *impetus*, contributes to the movement in an active manner; at the beginning of the movement, this capacity did not exist; the more the mobile progresses, the more intense this quality becomes and the stronger does it support this movement. So… the natural motion is faster at the end than at the beginning. This conclusion is evident because, in the end, the mobile has an *impetus* which aided it and which it did not possess at the start.

To this explanation of the accelerated fall of weights, Jean de Celaya, continuing his exposition, adds another, which had already been proposed before Simplicius and which Durand of St. Pourçain had collected:

> In addition, at the end of the movement, the medium is opposed to its own division with a lesser resistance (I mean its accidental resistance) than it was at the beginning; the medium to traverse is, indeed, thinner toward the end of the movement than it was at the beginning; now, it is very certain that a medium eight feet thick resists more than a medium of four feet, at least regarding accidental resistance. It is not surprising that such natural motion is faster at the end than at the beginning.

Finally, we include this sentence worthy of remark[23]:

> We see from experience that the *impetus* that moves a heavy body up is corrupted by the gravity and the substantial form of the body; on the contrary, the form of the heavy body retains and increases the *impetus* that moves the weight down.

For Celaya as for Coronel, it is the substantial form and the heaviness of the weight which, during the fall of the weight, retain the *impetus* already gained and increase its intensity.

It remains for us to take up one last debate, the opinion of the Parisian masters.

We have seen that Aristotle, and many physicists after him, had admitted the truth of this proposition: In the first moments after its departure, a projectile accelerates. We saw also that Jean Buridan and Albert of Saxony made no reference to the alleged initial acceleration, that Marsilius of Inghen and Cajetan of Tiene had resolutely denied it, while admitting, quite wrongly, the reality of the effects that he attributed to it and trying to give these effects another explanation.

Of this question Juan de Celaya does not speak, but the two regents of Montaigu will lend it some attention.

John Dullaert seems to have foreseen the theory that Bernardino Baldi would maintain one day and to have contradicted this theory in advance. The *impetus* that moves a projectile has its greatest intensity, he said, at the moment when the mobile leaves the mover.

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[22] Joannis de Celaya *Op. cit.*, lib. VII, cap. X, quæst. III: An motus naturalis sit velocior in fine quam in principio; fol. CXCVIII, col. 6.

[23] Joannis de Celaya *Op. cit.*, lib. VIII, cap. XI, quæst. III: A quo movetur projectum post separationem illius a quo projicitur; fol. CCI, coll. *a* and *b*.

> I prove that the *impetus* will not, immediately after that time, have a greater intensity; indeed, if, immediately after that moment, it was more intense than in this moment, its intensity would be increasing for some time; it therefore follows that, during this time, the projectile would move continuously faster and faster; now, this is contrary to the experience and opinion of all those who have dealt with this matter; this is clearly contrary to experience because at the beginning of its movement, the arrow cannot be perceived by the eye, due to its very high speed.
>
> …From the fact that the arrow launched by the ballista moves at the beginning of its movement more quickly than when it is at a distance from the machine, it does not follow that when it is somewhat far from the machine, a more violent shock occurs than at the beginning of its flight. Indeed, for a mobile itself, there is no fixed relationship between the force of the blow and the speed of movement. For this, some assign a cause derived from the nature of the object; this would be a consequence of the very nature itself of *impetus*. But, be that as it may, I do not care; it is enough for me that we can take this argument as evidence that the mobile moves, at a greater distance from what launched it, quicker than it moves at a shorter distance.

Coronel admits, like Dullaert, that a mobile moved by violence moves more slowly; he also admits that the force of the blow is greater when the projectile is at some distance from the instrument that launched it; but he is, more than the Ghent philosopher, anxious to reconcile these two statements.

He begins by outlining the explanation that Marsilius of Inghen had imagined; but he refuses to admit this gradual change in the distribution the *impetus*; in a passage we quoted a moment ago, he refuses to call the *impetus* an active quality, capable of generating its likeness and, by it, of propagating within the projectile.

This explanation rejected, he proposes another, as follows:

> The shock is all the stronger as the amount of air divided by the projectile is greater, the vehemence of the impulse assumed the same; near the beginning of the movement, although the *impetus* is more intense, not much of the air is divided; toward the end, on the contrary, the amount of the air shakes is great, but the *impetus* is very low; at a moderate distance, finally, the *impetus* is intense and the air is shaken a good amount; the blow is thus weaker at the beginning and towards the end of the movement; near the middle of the course the shock is the most violent.

Cajetan of Tiene had already hesitated between this theory and that of Marsilius of Inghen.

In a natural motion, the *impetus* constantly grows; it decreases continuously in a violent motion; from this proposition, which summarized all his Dynamics, Buridan made a remarkable application to vibratory movements; the back-and-forth of a string departing from its equilibrium position and the oscillations of a bell shaken from its equilibrium position had served as examples for him.

Albert of Saxony deduced from the same theory another corollary[24]

> Suppose, he wrote, that the earth be perforated through and through and, through the canal thus dug, a weight descends very quickly towards the center of the World; the body will continue to move beyond it and towards the opposite part of the Sky thanks to the *impetus* it has acquired, which will not corrupt; when the body ascends, this *impetus* missing, the

[24] Alberti de Saxonia *Quæstiones in libros de Cælo et Mundo*; in lib. II quæst. XIV, apud edd. Venetiis, 1492 et 1520. This question is not given in the editions of Paris in 1516 and in 1518.

weight will return to descending; it will thus go, oscillating about the center, until there is no more *impetus* in it; then it will stop.

We saw[25] that this passage of Albert of Saxony seemed to have inspired Leonardo da Vinci with an idea.

In 1516, the Scotsman George Lokert, regent at the College of Montaigu, gave an edition of the *Quæstiones in libros de Cælo et Mundo* of Master Albert of Saxony; in this edition, two questions which are of extreme importance in the history of dynamics were omitted, including that which contains the above passage.

Let us not conclude that this consequence of the Mechanics of Buridan and Albert of Saxony was ignored at the College where Master Georges Lokert professed; it was certainly taught and commented on to the point of striking the most rebellious Parisian Scholastic minds; Desiderius Erasmus of Rotterdam who was, in the last years of the century 15th, a student of the College of Montaigu, will provide us with the evidence.

In 1522, Erasmus published at Basle, for his friend Froben, his *Colloquia*, whose success was extraordinary[26]. Now here is what we read in the ninth dialogue entitled *The Questions*[27]:

Alphius:… It is the opposite in violent movement, which, quicker initially, gradually slows down; that which is opposed to the natural movement…

Curio: But tell me: if some God pierced the Earth by half,… by throwing a stone through that hole, where would it go?

Alphius: It would descend to the center of our Globe; then it would be good enough to rest there; because this Center is the Seat of all heavy Bodies…

Curio: I will reason otherwise: you told me that the natural motion, when it finds no obstacle, increases more and more, progressively; if your thesis is sustainable, the Stone or Lead that one would throw through in the hole in the Earth would find itself near the center, in a very fast movement, and would inevitably go farther, and then it would be a violent motion.

Alphius: The Lead would undergo a bad journey; necessarily melting on the way, it would arrive drip by drip; but if the stone, because of the speed of its movement, could not not stop at the Center, it would immediately begin to move slower and would return to the Center in the same way that a Stone thrown into the air falls back on Earth.

Curio: But as it would be by natural movement that the Stone would return to the Centre, it would still pass it by reason of the high speed and so that poor Stone will be condemned to perpetual motion; it will never have rest.

Alphius: In the end it will rest after running back and forth, until it reaches equilibrium.

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[25] *Léonard de Vinci et la pluralité des Mondes*, VIII: Commentaire aux réflexions sur la pluralité des mondes données par Léonard de Vinci (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, X; seconde série, p. 95).

[26] The existence of an edition earlier than 1522 is unlikely (see: Brunet, Brunet, *Guide du libraire et de l'amateur de livres*, 5° édition, 1861, t. II, col. 1041).

[27] *Les Colloques* d'Erasme, *Ouvrage très intéressant, par la diversité des sujets, par l'Enjouement, et pour l'Utilité Morale. Nouvelle Traduction par* Mons[r] Gueudeville, *Avec des Notes, et des Figures très ingénieuses. Tome cinquiesme, Qui contient, Les trois principaux Mobiles de l'Homme; le Culte, la Nature et l'Art*. A Leide, chez Pierre vander Aa et Boudouin Jansson vander Aa Marcbands Libraires. MDCCXX; pp. 179-181.

The popularity of the *Colloquia* of Erasmus was prodigious. The first edition, printed in 21,000 copies, was taken to Paris in a few weeks. The succeeding editions and translations were countless until the end of the 18th century. For them, the problem of Albert of Saxony was prevalent everywhere. It is by the *Symposia* of Erasmus, we shall see, that Fr. Maurolycus, in Messina, knew about this Parisian problem.

Desiderius Erasmus and Luiz Vives may well deride the masters under whom they studied at Montaigu and the doctrine that these teachers gave them; they will not forget the lessons they received there; when they dress in an elegant Latinity a theory of Mechanics, it is not enough to remove the cloak in which they furnished it to recognize some ancient thought of Albert of Saxony, carefully preserved in the Faculty of Arts of the University of Paris.

This is the Dynamics that was taught in Paris in early 16th century. It is the direct inheritor of the dynamics professed by Jean Buridan; since the mid-14th century, a few points have emerged; others are slightly obscured; the ensemble of them remained the same.

If we compare this Dynamics what at the same time Da Vinci wrote down in his notes, we find between these two doctrines numerous and striking similarities. Among the regents of Montaigu or St. Barbara, more so than among the masters of Bologna or Padua, Leonardo met men whose thoughts echoed his.

Between the science of the Parisians and that of Leonardo, if we seek the differences, we find one that is to the advantage of the great painter.

When a mobile is tossed into the air, the gravity and *impetus* fight among themselves for the duration of the movement; it is a proposition that Da Vinci and the successors of Buridan also admitted; but they rely on that proposition to draw a false conclusion, the existence of a time of rest between the upward and downward movements of a projectile; he sees this fruitful idea: the composed *impeto* accounts for the curvature of the trajectory.

However, the Parisians could proudly show that in several of its parts, their doctrine surpasses that of Leonardo; resolutely, they deny that a projectile launched horizontally starts accelerating its course; and above all, they seek in the acquired *impetus* the correct and fruitful explanation of the accelerated fall of weights.

Chapter 9
The decadence of Parisian Scholasticism after the death of Leonardo da Vinci. The attacks of Humanism. Didier Erasmus and Luiz Vives.

On 2 May 1519, Leonardo da Vinci died in Amboise. At the time when one of his penetrating disciples disappeared, and whom the Parisian Scholasticism had not known, it felt the first symptoms of decay; after having so powerfully contributed to the progress of modern science, it would give up promoting itself.

To discuss with clarity and precision the great problems of Physics, Metaphysics, and Theology, the Parisian Scholastics had to make a dialectical tool as sharp and as penetrating as possible; the Logic already refined by Aristotle no longer seemed subtle enough for them; following Petrus Hispanus, they tried to outsmart the finesse and rigor of Aristotle himself; and, certainly, they had given admirable examples of their ability to define and argue; the analysis of the concept of infinity, which we quickly delineated elsewhere[1], remains as a monument to the power and flexibility of their minds.

But what happened to the Parisian Logic is what has always happened to the sciences where Dialectics plays a vital role. This Logic should only be a means adapted for specific ends, and it exceeds that; it was taken for a purpose, and it was studied for its own sake. It was a weapon to save the truth and deliver a mortal wound to error; it soon served as fencing exercises where each of the two opponents cared only to show his dexterity.

Cultivated for itself and not for its intended use, Dialectics soon produced an abundant and tangled vegetation, to be overburdened with fruits as strange as they were useless. Already the writings of John Majoris, and especially those of John Dullaert, appear entirely encumbered with these subtle quibbles where the author seeks to clarify the proposition that he maintains, only to make us admire his argumentative talent.

Tiring for the reason, to which they paid too much attention, without any attraction for the imagination, to which they escaped with their extreme abstraction, these baffles whose usefulness one could hardly guess repulsed the schoolmen. They rejected them even more surely because the intricacies of the Logic provided no lu-

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[1] *Léonard de Vinci et les deux infinis* (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, IX; seconde série, pp. 1-53). — *Sur les deux infinis* (*Ibid.*, seconde série, note E, pp. 368-407).

crative place for those who wielded them skillfully, while jurists and canonists sold their tricks for fine cash.

This disgust of the students for the scholarly theories of scholastic Logic deeply distressed John Majoris. To fight it, he inserted into his various writings some dialogues where he pitted two of his students against each other, one tired of the intricacies of Dialectics, the other penetrated by the beauties of this art and striving to convince his interlocutor of them.

We have already discussed one of these dialogues[2], where two students of Logic, John Forman and John Dullaert, exchange their grievances; the dues that their studies are forcing them to pay seem heavy; and, on the other hand, John Forman complains that it is a waste time to discuss

> the cases which God could achieve but which never happen, to deal with the infinite, the intensity of the forms in the matter, to consider whether the continuous consists of points, etc.

Master John Annand, appearing, encourages our two logician students in extolling the theologians at the expense of the jurists.

Another dialogue of the same type is inserted in the edition that was printed in 1519, the first book of the commentary on the *Sentences*[3] composed by John Majoris.

Master Gauvin Douglas, pastor of the Church of St. Giles, Edinburgh, and David Granston, bachelor of Theology, exchange their thoughts.

Gauvin complains about the discussions on Logic or Physics that are introduced in the first book of the commentary on the *Sentences*; he is tired of theories that dwell on the subject of relation and the intensity of form; he is disgusted with questions such as this: Should we assume that there are points in a continuum? He prefers the study of the fourth book of the *Sentences*, where one only deals with Theology.

David Granston excuses John Majoris of the logical digressions that he introduced in the first book on the *Sentences*; the teacher only complies with an ancient custom. Gauvin would be wrong to believe, however, that these thorny problems are the cause that distracts the students from engaging in theology. Scarcely having finished studying the *Summulæ*, the young Parisians from wealthy families rush to Law, abandoning Logic and Theology, because a legal career is lucrative. Also, at the College of Navarre and Burgundy one finds, to hear the teaching of the *Summulæ*, a crowd of students from good families; but at the end of the year, there are so few candidates for the licentiate that the regents leave with an empty purse.

At that time, already, to study the thorny questions of Logic and Physics which the first book of the *Sentences* covered, the students of the Sorbonne preferred the purely theological, and hence easier, explanation of the fourth book. There were many when

[2] *Léonard de Vinci et les deux infinis*, II: L'infinimcnt petit dans la Scolastique (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, IX; seconde série, p. 33).

[3] Joannes Major *In primum Sententiarum ex recognitione Jo. Badii.* Venundantur apud eundem Badium. — No colophon. On the verso of the title page one finds an epistle of Joannes Major to George of Hepburn; it is given from Montaigu, on the seventh day of the calends of June 1509, and followed by these words: Impressit autem jam Badius anno MDXIX. The dialogue we are talking about comes immediately after this epistle.

it came to hearing this fourth book commented upon, but there were barely a dozen following the teaching of the first book[4].

It was worse, John Majoris teaches us, when the progress of the Protestant Reformation forced Catholic students to focus their attention on new topics. “The new and horrible calamity of Martin Luther, the execrable heresy,” entails the consequence that in the Sorbonne we abandon reviewing old Theological issues to deal almost exclusively with Holy Scripture.

Despite his preferences, the old theologian of Montaigu is forced to sacrifice to this fashion that he deplores; when he republished his commentaries on the first book of the *Sentences*, he abridges what in the first editions he said on the intensity of forms, infinity, and a variety of other subjects related to the liberal arts. But these sacrifices are not enough to save the Parisian Scholasticism from the decadence to which it will rush.

We heard the cries of alarm of an old man at the thought of the approaching ruin that threatens the place where, long ago, he was a respected leader. Those who, against this place, lead more sharply the assault are the defectors; they lived in the city walls which they wanted to overthrow.

The two most ardent opponents of Parisian Scholasticism were educated in Montaigu; later, the ardent cult of the humanities could make them completely forget, we have seen, the Mechanics lessons they received in that house. These two champions of the Anti-Scolasticism are Didier Erasmus of Rotterdam and Luiz Vives in Valencia.

At the College of Navarre and Burgundy, we found (i.e., Gauvin of Douglas, who told us by the mouth of John Majoris) a large number of young people from a good family; but the House that Gilles Aycelin of Montague had founded in 1313 was the asylum of the most beggarly schoolchildren. There was a meager price; “Montaigu, acute mind, sharp tooth,” said the burghers of Paris, was enlivened by the scrawny figure of subtle logicians. Besides, if we are to believe Rabelais, the cover is worth it; hear ye rather what Pinocrate recounted to Grand-Gousier:

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[4] Joannis Majoris Hadingtonani, *scholæ Parisiensis Theologi, in Primum Magistri Scntentiarum disputationes et decisiones nuper repositæ; cum amplissimis materiarum et quæstionum indicibus seu tabellis.* Vænundantur Joanni Parvo et Jodoco Badio, 1530. Colophon:

> Sub prelo Jodoci Badii Ascensii, communibus ejus et Joannis Parvi impensis: ad Galendas Septembres. MDXXX.

— Letter dated from the College of Montaigu, 1530, from Joannes Major (*sic*) to Joannes Major Eckius Suevus.

In secundum Sententiarum disputationes Theologiæ Joannis Majoris Hadyngtonani *denuo recognitæ et repurgatæ*. Vænundantur Iodoco Badio et loanni Parvo. Colophon:

> Finis disputationis Joannis Majoris natione scoti et professione Theologi Parrhisiensis penitus recognite et aucte Impresse impensis communibus Joannis Parvi et Jodoci Badii Ascensii. opéra ipsius Ascensii anno domini MDXXVIII circiter XV calendas septembris. Deo gratias.

— Epistle of Joannes Major (*sic*) Hadyngtonanus to Noël Bède and Pierre Tempeste, dated: *Ex Collegio Montisacuti, Kal. sept. MDXXVIII.*

> Lord, do not think that I have attended the lousy college called Montaigu; I would rather have liked to be amongst the rants of Saint Innocent for the enormous cruelty and villany that I knew; because it is better to bear the force between the Moors and Tartars, the defects of murderers in a criminal prison, and certainly the dogs in your house than these boors at the said College.

It is in this sad asylum that in 1497 Desiderius Erasmus came, as bursor, to finish his studies; there he contracted the germ of infirmities which poisoned his life, a strange dislike of certain foods, such as fish, and a no less insurmountable distaste for Scholasticism.

Poor Parisian theologians, regents of Montaigu, doctors of the Sorbonne, colleagues of Johannes Majoris! Let us hear what the *Folly*[5] by Erasmus said about this:

> Will I speak of the Theologians?… I ordered my Philautie, the Goddess *Love Herself*, to favor them more than other men; and indeed they are her Minions; as if these bodily Angels were established in the third Heaven, they look from the height of their elevation at all Mortals as like creeping beasts; and they have pity; surrounded by a Troop of magisterial definitions, conclusions, corollaries, explicit and implicit propositions, which comprises the Militia of the sacred School, they find so many meanes of escaping that even Vulcan himself could not remember them… There is no knot that these Gentlemen will not cut with the first blow of the knife of *Distinguo*, a knife made of all these monstrous terms that are in the bosom of the Scholastic subtlety …
>
> There are also many other sharper subtleties: the moments of the Divine Generation, concepts, relationships, formalities, quiddities, hæcceities, and many other chimeras of this nature: I defy anyone who sees them, unless he has a piercing sight to distinguish through the thickest darkness objects that are nowhere…
>
> That which subtilizes even these very deep subtleties are all different ways of the School: you would come out of a labyrinth more easily than you would from the entanglements of the Realists, Nominalists, Thomists, Albertists, Ockhamists, Scotists; ah! I lose breath: and, however, these are only the main sects of the School; really, there are many others! How much of science and of thorns do you think is in all these factions?
>
> …These Hairsplitters are so full of the wind and smoke of their void erudition, and all verbiage, that they do give up: occupied day and night to taste the sweetness of their chicanery, they do not even allot time to read the Gospel or the Epistles of Saint Paul. However, playing the Fool in their Schools, they do not cease to imagine that the Church would collapse as soon as they stopped supporting it, believing themselves Atlas supporting it on their shoulders…
>
> We Peelers have brains so full, so excited with all this nonsense, that Jupiter did not have a bigger brain, when wanting to give birth to Pallas he implored the ax of Vulcan. Do not be surprised if, in public Disputes, they care to wear on their head with so many bands; it is to prevent, by these honorable ties, their brains, overloaded with science, from completely breaking. I cannot help but laugh… when I listen to these famous Characters: they stammer rather than speak; they are not quite reputed Theologians until they know perfectly their barbaric and ugly jargon; there are those in the art who can understand them, but they glory in it, arrogantly saying they do not speak for the common layman. It, they add, debases the dignity of the Holy Scripture to subject it to the rules of Grammar and trifles of Purism. Admire the majesty of the Theologians! Only they were allowed to make

[5] The *Praise of Folly, made in the form of declamation by Erasmus of Rotterdam… that piece, representing the natural man disfigured by silliness, pleasantly teaches him to return to good Sense and Reason: newly translated into by French by* Mr. Gueudeville. In Leiden, to Peter van den Aa, 1713. — The preface of Erasmus, addressed to Thomas More, is dated 10 June 1508, pp. 177-195.

> mistakes in language; and there is, at most, only one scoundrel who has the right to dispute this prerogative.

Three feelings inspired this declamation of Erasmus.

The first of these feelings is the deep weariness that an excessively subtle and punctilious dialectic caused.

The second is the desire for Theology to abandon the useless and complicated logic device, maneuvering tirelessly and fruitlessly; the desire to return to the studies which fertilize and give life to faith, the meditation of Scriptures.

Johannes Majoris already showed us these two feelings to his students; in Erasmus, they are perhaps the most powerful inspirations of the anti-Scholastic spirit; a third sense breathes, even more violently, the hatred of the studies to which one wanted to relegate to his youth, and this is the horror of technical style which the School used; it is the taste for fine language and the worship of Grammar; it is Purism.

The concern for elegance which cannot depart the humanist of Rotterdam banned him from putting, in his diatribes, an exaggerated precision; he would not point the finger at those who mocked it; he did not expressly designate his teachers and his fellow students in Paris. The fiery Vives will have no such qualms.

At the end of 15th century and early 16th century, the Spaniards were prominent in the University of Paris. We had occasion to note the activity of Pedro Cirvelo and to comment on the teaching that John Celaya gave at St. Barbara. John Majoris counted several Spaniards among his favorite pupils. In one of his writings[6] he affectionately cites Louis Coronel, whose *Physicæ perscrutationes* caught our attention; the name of Antoine Coronel, brother of Louis, prolific writer, and editor of several books of the Theologian of Hadington; finally, the name of Gaspard Lax, Sarinyena of Aragon, who in 1512 had printed in Paris three books of Logic, on the *Termini*, *Obligationes*, and *Insolubilia*.

Like many of his compatriots, Juan Luiz Vives was born in Valencia in 1492 and was sent to Paris, attracted by the high reputation of the University; he took a place among the students of the College of Montaigu, where he had two masters of the favorite disciples of John Majoris, the Spaniard Gaspar Lax and the Ghent John Dullaert. A brilliant humanist, Vives could not withstand the harsh discipline of those meticulous logicians; in 1519, we find him a professor at Louvain, where he sarcastically condemns the Parisian University, the teachers who teach lessons

[6] Magister Johannes Majoris Scotus. *Omnia opera in artes quas liberales vocant a perspicacissimo ac famatissimo uno sactarum* (sic) *litterarum professore prof andissimo magistro Johanne Majoris, majori accuratione elaborata, atque castigata quam antehac in lucem prodita sint majorique precio comparanda quam quispiam persolvere possit si ea ab equo judice pensiculantur*. Venumdantur vero a Michæle Augier cive Cadomensi ac Religator Universitatis ejusdem juxta pontem Sancti Petri et a Johanne Mace Redonis commorante e vestigio Sancti Salvatoris sub divo Johanne Evangelista degente. Colophon:

> Impressum Cadomi per Laurentium Hostingue impensis virorum industriosorum Michælis Augier prope pontem ejusdem Cadomi commorantis et Johannis Mace e regione Sancti Salvatoris Redonis residentis.

there, and the lessons they give there. In England, where he passes out of Louvain, to Bruges, where he returned to die in 1540, he continues to carry out a violent battle of humanism against the ancient Scholasticism.

Poor logicians of Montaigu! They were not recruited, David Granston told us, among the sons of wealthy families; their neglected exams did not pay, in their empty purse, but a tiny contribution of the dues, and the students, yet finding these dues too expensive, strove to escape; like our rulers they lived needy and ragged. Listen to this dialogue that Vives makes[7] between Nugo and Gracculus:

> Gracculus: I would like a subject worthy of a poet.
>
> Nugo: What then, is it not a subject worthy of a philosopher that you expected? Ask one of these new famous Parisian masters.
>
> Gracculus: For most, it is customary that they are philosophers, not cerebral.
>
> Nugo: Custumary philosophers? One would rather say cooks or muleteers.
>
> Gracculus: It is that they wear filthy, shredded, torn, muddy, dirty, and flea-ridden clothes.
>
> Nugo: These will therefore be Cynical philosophers?
>
> Gracculus: Worse than that! Regarding the tactful philosophers[8]; they feign to pass for Peripatetics, but they are not, for Aristotle, the leader of the sect, was the most cultured. For me, if I can be a philosopher otherwise, I will say adieu to Philosophy, and for long time.

The portrait that Vives traces of the Parisian masters is undoubtedly little flattered; in any case, it is not flattering. The studies over which these masters preside did not give him a better remembrance. In a writing which he composed in Leuven from 1519, he condemns these studies with more violent diatribes of which his compatriots, the Spanish masters, are copiously tarnished.

> This Paris,

he said,[9]

> should radiate the light of the most complete civilization. Now we see men fiercely embracing the most sordid cruelty and, in addition, engaging in studies that are truly monstrous; such are the *sophismata*, as they call themselves call them; there is nothing more vain, nothing more foolish than these studies. If, sometimes, an intelligent man is left with some attention, his intellectual qualities will go to their peril; likewise, fertile fields that one does not cultivate generate a host of useless herbs. These people dream; they imagine nonsense; they invent a new language that only they understand.
>
> In this situation, most educated people blame the Spaniards who are in Paris; invincible men, they bravely guard the citadel of ignorance…
>
> Is there, in the language of men, a more hackneyed proverb than this one: In Paris we form youth to know nothing, but to be delirious in a senseless chatter? In other universities we certainly studied some vain and trivial matters; but we also learn many solid things; in Paris, one only learns the most hollow balderdash.

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[7] Lodovici Vivis *Exercitationes linguæ latinæ*. Garrientes (lo. Lodovici Vivis Valcntini *Opera in duos distincta tomos*… Basileæ, per Nicolaum Episcopium juniorcm. Anno MDLV. Tomus I, p. 21. — On page 56, these *Exercitationes* are dated: Bredæ Brabanticæ, die Visilationis divæ Virginis MDXXXVIII).

[8] There is here, on the adjectives *cynici* and *cimici*, an untranslatable play on words.

[9] Jo. Lodovicus Vives *In pseudodialecticos*; this piece is dated: Lovani, MDXIX (Jo. Lodovici Vivis Opera, tomus I, p. 272).

> These Spaniards and all their followers should either coerce them to do better science or by public edict banish them as corrupt and as corruptors of both manners and of civilization.

Vives puts the teaching of Paris far below what the other Universities give; is it that he would like to see the Parisians addicted to Averroism, like their rivals in Padua and Bologna? Probably not, if one believes the violence of his invective against Averroes[10]:

> Tell me, I pray thee, Averroes, did you have to ravish the mind of men or, rather, take it away from them? Some authors have led many people by the grace of speech and coaxing words; but nothing is more hideous, more lowbrow, more obscene, more childish than you… Those who formed souls are worthy of admiration and universal praise, those who taught to live well. But you, nothing is more wicked, more irreligious than you; whoever indulges too vehemently in your precepts cannot fail to become an infidel and an atheist.

What Vives criticized his former teachers is not their dislike of Averroism; he shares this aversion. What accuses them, in the first place, is what John Forman complained about in his conversation with Dullaert, which provoked the grievances of Douglas Gauvin in the presence of David Cranston, which most certainly utterly tired and disgusted the students in Paris: the subtlety of a logic which, longly and carefully, analyzes purely abstract problems, solves all hypothetical difficulties, and discusses, in the words that John Majoris lends to Forman, “the cases possible for God, but that never happen.” Let us hear the sarcasm with which Vives echoed the complaints of the students in Logic against their regents[11]:

> What these people could draw from books of Aristotle was very little; much discussion had already crushed, stirred, shaken it to excess; also, this sort of combat seemed the best known, even to the conscripts; we therefore sought a new way of making war and a new subject of battles. They are given, then, to quibble about foolish subtleties, which they themselves call calculations (*calculationes*). It is the Englishman Roger Suiseth who gave a great development to these calculations; also, John Pic was he accustomed to call them the ‘Suisethian trifles” (*Quisquiliæ Suicelicæ*); this is a name that fits them very well; these calculations, in fact, do not apply to science nor to any practical use.
>
> I do not see anyone doubting that these subtleties have no practical use, not even the greatest among those who profess them, among those who are esteemed because they have a deep knowledge of these calculations.
>
> As for science, what can there be in such matters so remote, so completely separated from God, on the one hand, and from sense and intellect, on the other? In an domain where, based on the vacuum, we see a vast edifice of assertions and contradictory opinions arising, regarding the growth and decrease of intensity, density and rarity, uniform motion, non-uniform movement, uniformly varying and non-uniformely varying movement? There are innumerable people, without measure, discussing cases that never happen, that cannot even be present in nature, who speak of infinitely rare or infinitely dense bodies that divide an hour into proportional parts of this or that ratio, who imagine that, in each part, a movement, or a rarefaction or depletion, varies in some relationship…

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10 Joannis Ludovici Vivis *De causis corruptarum artium liber V: De philosophiæ naturæ, medicinoe et artium corruptione*; De philosophia naturæ, Piece dated: Brugis, anno MDXXXI (Jo. Lodovici Vivis Opera, tomus I, p. 412).

11 Luiz Vives, *loc. cit.*, pp. 412-413.

Men like Jacopo da Forlì introduced these logical exercises, these calculations, into medical studies, to the despair of Luis Vives[12]:

> The baffles and minutiae of Jacopo da Forlì are no less thorny or less useless than those of Suiseth, from which John Dullaert made, in our exercises of Physics, some frequent citations; they do not yield to the calculations of Suiseth, neither in prolixity nor in annoying trouble.

These exercises will give not give students of Dialectics any positive knowledge; they are not accustomed to observe the concrete facts; they do not develop in themselves the intellectual qualities necessary for their practice:

> Young people and youth who were educated with these captious and thorny discussions know nothing of plants, animals, the elements, nor all of nature; they are not provided with any experience in the things of nature or any knowledge of realities; no prudence sustains them; their judgment and advice are excessively weak, and we admit them access to honors!

The absence of any concrete and practical knowledge in the teaching of the University of Paris, the abstract and ideal of the problems, and all the complexity and subtlety of the methods used to solve them excites the derisive verve of Vives; but what irritates him to no end, what shocks supremely his delicate taste, is the barbarous language with which this teaching takes place; it is for this "Paris style" that he reserves his most frequent and most violent invectives.

> What then, I pray you[13] is this language which your Dialectic uses? The French or Spanish? The Goth or the Vandal? Because it is certainly not Latin…
>
> These men claim to speak the Latin language; however, not only the do the most learned latinists not understand them, but it happens very often that people of the same stock do not understand even the same sound. Many of the words they use are intelligible only to those who forged them…
>
> Almost everything these professors deal with by dint of syllogisms, oppositions, conjunctions, disjunctions, explanations of propositions, is only a guessing game as between children and good women.
>
> That language[14] where barbarisms and solecisms gush profusely is the only one, it seems, that lends itself to magisterial definitions of theological questions…
>
> Is a book written in an uncultivated way? Whatever the subject, these men are so ignorant, so stupid that they do want to call it neither Philosophy nor Theology, neither Law nor Medicine; they call it Grammar. The *Offices*, *Paradoxes*, *Tusculanes*, *Academics* of Cicero are, they say, of Grammar. Only the writings that they compose—these writings that are not subject to the rules of grammar, hence beyond trivialities of all kinds—are not for them Grammar; and I confess it willingly; it is neither Grammar nor anything else. Scotus, Ockham, Paul of Venice, Hentisber, Gregory of Rimini, Suiseth, Adam Goddam, and Buckingham are not grammarians; they are philosophers and theologians; they understand them. But Cicero, Pliny, Saint Jerome, and Saint Ambrose are banned from the School; for the grammarians understand them!
>
> For me[15], I feel toward God extreme gratitude and thanksgiving and I thank him for finally leaving Paris, to be released from the Cimmerian darkness, to be brought to the light for recognizing what was truly worthy for a man to study, which merits the name of Humanities.

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[12] Ludovici Vivis *Op. cit.*, De medicina (Lodovici Vivis *Opera*, tomus I, p. 415).

[13] Jo. Lodovicus Vives *In pseudodialecticos* (Jo. Lodovici Vivis *Opera*, tomus I, p. 273).

[14] Luiz Vives, loc, cit., p. 381.

[15] Luiz Vives, *loc. cit.*, p. 284.

Humanism! This name refers to the set of repulsions and aspirations that the students in the early 16th century University of Paris entertained! To flee abstract disciplines because we do not see their immediate usefulness, because they require careful and painstaking precision, because the precision demanded technical language dismissive of what charms the ear; to indulge in teachings whose utility is at hand; to gather in his memory concrete observations whose acquisition does not bind up and fatigue the resiliency of intelligence; for the language making cheap harmony, provided it defines thought with rigorous clarity, to prefer the speech that rounds in oratorical periods or veils in poetic images the contours of truth; in short, to abandon reason to embrace imagination that seemed to them more beautiful; such was the dream of many graduates in the noisy Fouarre street in the austere Sorbonne; and to run to the realization of this beautiful dream, they threw away their notebooks, they ripped up their commentaries on the *Summulæ* of Petrus Hispanus, on the *Calculationes* of Suiseth, and on the *Sentences* of Peter Lombard.

So powerfully was the attraction of Humanism that the teachers themselves, those who lived off teaching Dialectics, experienced the seductions of the new studies and despaired of being too old to engage in them:

> You could hear them[16] give to the devil the madness that led their intelligence, to deplore the time they had spent unnecessarily treating these vain trifles. Often,

Luiz Vives continues,

> I heard my old masters, Dullaert and Gaspard Lax, complain with deep pain of having wasted so many years in such futile and hollow studies.

The Parisian masters do not at all waste time, as Dullaert or Lax do, crying about the time and trouble they gave to the thorny problems of logic and physics; resolutely, they turned aside these old methods to run ardently in the new ways; disdainful of knowledge painfully acquired and carefully analyzed by the doctors of the Middle Ages, their predecessors, they regarded as impure everything that was not taken from the source and refused to drink from it; dismissing the crowd of commentators, they wanted Plato and Aristotle directly to teach their Metaphysics; making a clean sweep of all the Scholastic Theology, they intended to enlighten their faith in the sole study of Sacred Scriptures; in any order of things, they wanted to capture the imagination and move the heart rather than convince reason.

For a long time such a movement had begun to occur, some masters of the University of Paris turning away from the nominalistic Scholasticism; from the early 15th century, we find at the head of this movement the two most important people of this University, Cardinal Peter of Ailly and Chancellor Jean Gerson.

They both resent seeing theologians abandon the study of Scripture as the foundation of their science, never looking for more in them than a pretext for purely profane discussions.

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[16] Luiz Vives, *loc. cit.*, p. 284.

Peter of Ailly not only criticizes those "pseudo pastors"[17] their little taste for the study of sacred science, but also their habits of intemperance; and the official *Books of the procurators* of various nations seem to prove that at this point, the reproaches of the Bishop of Cambrai, which were so brutal in form, were just:

> Turn these Pseudo-pastors,

he said,

> more to the study of Scripture, more to the maintenance of divine wisdom; they care only of the wisdom of this world, which is foolishness in the sight of God. And indeed, if they happened by chance in Paris to whisper a few words concerning Holy Scripture, they were only side dishes between the pots in dinners and banquets; they were no more thinkers of a fasting mind, but eructations of a stuffed belly. … O what vile arguments on all sorts of questions! what unnecessary conflict of arguments! There, more often than just, the question reeked of wine and the solution was swollen with venom. They blasphemed, condemned the most proven sentences.

In a manner more precise, Jean Gerson blames the invasion of Theology by the infinite subtleties of the Logic of the Moderns, and his reproaches are exactly those that, in their grievances, the students of John Majoris will resume a century later:

> Why,

he said in his lessons on St. Mark[18], are the Chancellor of the University of Paris

> and the theologians of our time treated as verbose sophists with an unruly imagination? Only for the reason here: What would be useful and intelligible, given the quality of their listeners, is that they leave it aside to devote themselves to indulge in pure Logic or pure Metaphysics, or even Mathematics; then, in a time and place where it does not matter, sometimes they deal with the intensity of forms, sometimes with the division of the continuous; today they express sophisms that only veil the theological terms; tomorrow they will distinguish, in divine things, priorities, measures, durations, instants, the signs of nature, and similar notions. Even if all this is true and solid, which it is not, it would serve mostly to upset the minds of the listeners or excite them to laugh, and not to edify their faith with rectitude.

Peter of Ailly and Jean Gerson accuse the nominalist Logic of harming the study of Holy Scriptures; this reproach came to join another, that of distorting the meaning of the ancient philosophers, and Christian Humanism has formulated its whole program.

At the end of the 15th century, the Christian Humanists comprised, at the University of Paris, a powerful party, of which Jacques Lefèvre d'Étaples can be regarded as the chief[19].

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[17] Domini Petri de Alliaco *Invectiva contra Pseudo-pastores*, unedited writing cited by Launoy (Joannis Launoii Constantiensis, Paris. Theologi, *De varia Aristotelis in Academia Parisiensi fortuna*, tertia editio, Lutetiæ Parisiorum, apud Edmundum Martinum, MDCLXII, pp. 97-98).

[18] Cited by Launoy (Launoii, *Op. laud.*, ed. cit., pp. 98-99).

[19] On Lefèvre d'Étaples, Christian humanist, see P. Imbart de la Tour, *Les Origines de la Réforme*, t. II, ch. I.

Among the writings of Lefèvre d'Etaples, there are few who have been so enjoyed as his *Paraphrases* of the philosophical writings of Aristotle[20]. Habituated to only knowing the mind of the Philosopher through commentaries, glosses, and questions that the Greeks, Arabs, and the masters of the Latin school multiplied profusely, the readers of Lefèvre imagined that the doctrine of Aristotle had been discovered and was revealed to them for the first time.

Ecquæ Stagirites cœcis occlusa latebris
Abdiderat, sunt habitura diem clarum

wrote Josse Clichtove de Newport, doctor of the Sorbonne, in the piece of verse which accompanied the *Paraphrases* of his master. In a letter written in Paris and dated 1504, which accompanies certain editions of this book, Marius Acquicolus d'Oliveto said to Cardinal Francis Soderino, bishop of Volterra:

> Henceforth, keep him who will want his Themistiuses, Alexanders, Simpliciuses; Marius will be content with his dear Lefèvre.

These statements are not the fawning of flatterers; they faithfully paint the enthusiastic loyalty that the writing of the humanist of Étaples has received.

However, when we peruse the *Paraphrasis libri Physicorum*, we cannot help but find his limpid presentation singularly insipid, but colorless, of the great treatise of Aristotle. Certainly the Commentaries and Questions of Burley, Ockham, Buridan, and Albert of Saxony did not have this simplicity; the thought of Aristotle was often hidden under the lush vegetation to which it gave birth; but it is precisely through this scholastic thrust that the peripatetic philosophy was to be fruitful; these thick branches carried the fruits from which modern science would one day extract the juice. To free the stump and manifest it for all to see, the humanism of Lefèvre d'Étaples brutally ripped the tangled branch which he took for parasitic brambles; on the cleared ground, we see more than a withered trunk.

The favorite disciple of Lefèvre d'Étaples was Josse Clichtove[21]. Born in Newport (West Flanders) in 1472, doctor of Sorbonne, then canon of Chartres, Clichtove

[20] Jacobi Fabri Stapulensis *In Aristotelis octo Physicos libros Paraphrasis*. Colophon:

> Impressum Parisiis Anno domini millesimo quingentesimo nonagesimo secundo (Per Johannem Higman). — *In hoc opere continentur totius phylosophiæ naturalis paraphrases: hoc ordine digestæ. Introductio in libros Physicorum. Octo Physicorum Aristotelis: paraphrasis. Quatuor de Cælo et Mundo completorum: paraphrasis. Duorum de Generatione et corruptione: paraphrasis. Quatuor Meteorum completorum: paraphrasis. Introductio in libros de Anima. Trium de Anima completorum: paraphrasis. Libri de Sensu et Sensato: paraphrasis. Libri de Sommo et Vigilia: paraphrasis. Libri de Longitudine et Brevilate vitæ: paraphrasis. Dialogi insuper ad Physicorum, tum facilium tum difficilium intelligentiam introductorii: duo. Introductio Metaphysica. Dialogi quatuor, ad Metaphysicorum intelligentiam introductorii.* Impressum in alma Parrhisiorum achademia per Henricum Stephanum in vico clausi brunelli e regione Schole decretorum. Anno Christi piissimi Salvatoris, entis entium, summique boni. 1512. Pridie Kalendas Februarii.

[21] J.-Al. Clerval, De Judoci Clichtovei Neoporluensis doctoris theologi Parisiensis et Carnotensis canonici vita et operibus (1472-1543). Thesis from Paris, 1894.

died in 1543. Contemporary of John Majoris, he is often found alongside him in theological discussions; but in general, in such disputes, Clichtove and Majoris did not take the same side; the Scottish theologian defended, we have seen, the ancient methods of Parisian Scholasticism; he only gave in, inch by inch and grudgingly, to the requirements of Humanism; the Flemish theologian, however, had rushed eagerly in the way that Lefèvre d'Étaples had opened for him.

Clichtove enriched the Aristotelian *Paraphrases* of his master from the *Scholia*; thus completed, these *Paraphrases* had an extraordinary popularity[22].

However, at the beginning of the *Paraphrasis libri Physicorum*, Clichtove put a preface; in the preface, the author judged and condemned discussions of such punctilious logic to which hitherto Physics gave rise in the schools in Paris; with regard to these discussions, it was expressed in less violent terms, but as severe as those Luiz Vives used.

> By design,

said Clichtove,

> I showed myself sober when it came to discussing questions like the modern faction, shaking with every wind of quibbles contrary to the proven evidence of Philosophy; these things do not engender real science; they generate rather a trivial gossip, an annoying cackle which the quiet and modest philosophy abhors and from which it distances itself; commenting on all the little reasons that fight against the truth of science does not lead the mind to embrace these sciences in their certainty and sincerity; it is rather diverted from it, dropping into captious and sophistical discussions that do not deal with the true doctrine; imbued with these discussions, the minds of adolescents, so they should be pushed to collect the ripe fruit of science, completely dry out and produce sterile herbs in vain… In these scholia that we have joined [to the *Paraphrase* of Lefèvre d'Étaples], we sometimes solve, indeed, questions that pose the same subject matter and that deserve to be agitated; but we do not solve them in the barbaric, repulsive, and crude way that we see used today when we want to examine these questions in teaching.

Thus, from the beginning of 16th century, there were at the University of Paris some masters that the Nominalist Scholasticism left tired and disgusted; fleeing the thorny and subtle discussions, the *captiunculæ*, *calculationes*, and *Suisetiæ quisquiliæ* engaged in the charms of Philosophy and Theology at last humanized; they had only pity and derision for those who continued to treat these sciences according to the *modus barbarus, insulsus et crassus* hitherto in use; they ranged from Lefèvre d'Étaples and Josse Clichtove to those who would favor the students; they turned away from Majoris, Dullaert, and Coronel, who, in turn, forsook the scholars.

Covered with shredded clothes and an empty purse, the unhappy logicians of the University of Paris reflected sadly in their chair around which the students no longer gathered; they listened to the ridicule to which their science was condemned, the

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22 *Totius philosophiæ naturalis Paraphrases, adjecto ad litteram familiari commentario declarato*. According to Abbe Clerval (*Op. cit.*, p. 15), the complete editions, containing the *Paraphrasis libri Physicorum*, are the following: Parisiis, W. Hopylius, 1502; H. Stephanus, 1510 et 1612; Simon Colinæus, 1521 et 1531; Pet. Vidoue, 1533; Joh. Parvus, 15339. — Parisiis et Gadomi, Fr. Regnault et Pet. Vidove, 1533. — Friburgi Brisgoiæ, Fab. Emmeus, 1540. — Lipsiæ, Jac. Thanner, 1006. — Cracoviæ, J. Haller, 1510; Hier. Victor, 1518; J. Haller, 1522.

science they had acquired with much trouble, the science to which they had devoted their laborious life; they heard singing the praises of other more useful, easier, more attractive, more human studies; with a look of envy, they saw success and popularity favor those of their colleagues who had betrayed and abandoned the old disciplines for new studies; they felt the doubt that, painfully, came to envelop their heart, which included their lost years, which reminded them of the harsh and tedious work done for nothing.

If, at least, their melancholic reverie had the leisure to unwind in silence and peace! But Humanism did not even grant them this alleviation to their sadness. Humanism—the delicate Humanism, so concerned with the elegance of its time, so fearful of the slightest grammatical incorrectness—had indulged, to combat Scholasticism, in putting the coarsest invectives into a very pure Latin. Now the violences of language was not enough for it; against the masters that he hunted, against the dialectical method he was chasing, he unleashed the charivari led by the scholarly gentlemen; this is what Humanism called for to restore "the strong and true theology".

In 1521, a trip brings Luis Vives to Paris; from there, he wrote[23] to his master Erasmus, who remained at Louvain; he told him of the popularity and influence that the writings of the Dutch Humanist had with the Parisians.

The Parisians, says Vives,

> exhort and beg to continue to merit well of the true religion, without letting you be scared off by the barking of ignorant people… For them, they take care that during theological discussions, those who take part in the dispute do not utter nonsense. And this is what has happened. In the Sorbonne, if someone presents an argument woven of the threads of a spider of Suiseth, we see immediately the audience frown; they exclaim, jeer, and drive the author of the argument out of the school. It is so even in the philosophical altercations; there comes some speaker of enigmas, equipped with one of these propositions that overdo the *syncategoremata* and whose explanation would require a diviner of Etruria; such a proposition is, moreover, in extreme favor with the scholastic populace; immediately our man is greeted with shouts, whistles, and boos; in a great tumult, he is put to the door of the room where the debate is held. These facts have been, for me, a wonderful spectacle, and I am sure you rejoice in this because of the love that you bring to good studies.
>
> What cannot we hope for,

responds Erasmus[24],

> since the Sorbonne finally despises punctilious subtleties for embracing the true and solid theology!

Pitiful logicians of Paris, reduced to neglect or delivered from jeers! What was given to them to fathom the future, and what comfort they did not find there! Future centuries would quickly tire of Humanism; the Latin elegances of Erasmus or Vives

[23] *Epistolarum* D. Erasmi Roterodami *Libri* XXXI, *et* P. Melancthonis *Libri* IV. *Quibus adjiciuntur* Th. Mori *et* Ludovici Vivis *Epistolæ*… Londini. Excudebant M. Flesher et R. Young. MDCXLII. Sumptibus Adriani Vlacq. — Erasmi Roterodami Epistolarum liber XVII, epist. 10; fol. 753.

[24] Erasmi Roterodami *Epistolarum* liber XVII, epist. II; ed. cit., fol. 755.

were scarcely able to retain for a long time the favor of men of taste, while modern languages were disposed to produce their finest masterpieces. However, from the field plowed by the philosophers and theologians of Paris, there would arise the most wonderful harvest that Science has ever reaped. The *calculationes* in the style of Suiseth, the discussions on division to infinity, on the intensity of forms, and uniform or uniformly varied movement, were so many seeds that were to rise up in the next century and produce analytic Geometry, infinitesimal Calculus, Kinematics, and Dynamics. Humanists treated with disdain Gregory of Rimini, Jean Buridan, Albert of Saxony, and those such as Nicole Oresme, although they were the precursors of Galileo and Descartes, Cavalieri and Torricelli, Fermat and Pascal.

Chapter 10
How, in the 16th century, the Dynamics of Jean Buridan spread in Italy

From this Dynamics that ceased being taught in Paris, there are Galileo and his friends like Torricelli and Baliani, who would, with Descartes and Gassendi, share the inheritance. How, from the beginning to the end of the 16th century, would this legacy be transmitted to them? How would the Dynamics of Jean Buridan, which Leonardo himself had not accepted in its fullness, infiltrate into Italian Science? This is what we will now endeavor to tell.

The infiltration of Parisian Dynamics into Italian Science occurred, however, with extreme difficulty and very slowly, because it is made by gradually suppressing peripatetic prejudices.

These prejudices were strong and well-entrenched in the middle of the 16th century, and we can cite witnesses.

The first witness is Cardinal Gaspard Contarini (1483-1542).

Contarini had written a small book entitled *De elementis* that was published[1], for the first time, in 1548, six years after the death of the author, and that would later have several editions.[2]

In Book I of his work, Contarini asks why

> all the elements, and all the heavy and light bodies moving in the direction in which nature carries the element that predominates in them, will move increasingly faster until they reach the end to which they tend.

The first explanation that Contarini mentions, but to reject it immediately, is the explanation given by the Parisians:

> Some attribute to the *impetus* the cause of this. They say that all of this happens as a result of a constantly growing *impetus*, and that is why the bodies move more quickly. But when you press them further and ask them what is this *impetus*? What kind of quality is it? Where do the elements hold it? They are either silent or they devise some non-existent commentaries that we cannot understand.

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[1] Gasparis Contarini, cardinalis amplissimi, philosophi sua ætate præstantissimi, *De elementis et eorum mixtionibus libri quinque*. Parisiis, MDXLVIII.

[2] The one we have in front of us says: Parisiis, Apud Andream Wechelum, 1564..

The second theory that the Cardinal condemns is that of Themistius; he opposes the objection that Richard of Middleton and, after him, all the School of Paris raised.

He then discusses the enumeration of the many causes he believed must be attributed to the accelerated fall of weights; he begins this list as follows:

> Aristotle, in the eighth book of the *Physics*, deals with the motion of projectiles; he seeks what moves them after they leave the man or machine that launched them; in this regard the Philosopher wrote:
>
> > It is the nature of water and air, when in their proper and natural sphere, and when they have been pushed in any direction, to move immediately after this impulsion, and by their own effort, a certain distance; their movement is fast for a moment; then it gradually slows; finally these bodies return to rest.
>
> Add to this that nature experiences an extreme horror of the existence of an empty space of any kind, which would destroy the unity of the World; when you make a body move in the air or water, the neighboring parts of the air or water rush behind the mobile; pushed first, they in turn push mobile by their own effort, moving it forward.

Although Contarini did not tell us in a formal way, it is clear that he supports this explanation of projectile motion by the propulsion of the shaken medium.

It is, moreover, to this influence of the medium that he attributes the acceleration experienced by the fall of weights. He invokes two causes for this acceleration: first, the propulsive action of the air that the horror of the vacuum forces to rush to the back of the mobile; on the other hand, the decrease experienced by the resistance of the medium, which is located at the front of the mobile when it chases it.

> Some physicists,

Contarini continues,

> invoke a third reason; the whole nature, they say, is led by intelligence; so there is nothing absurd in what we sometimes perceive in the operations of natural agents, traces of reason… This is why, according to these physicists, the longer a heaver or lighter body has moved in accordance with its nature, the closer, therefore, it is to the place that suits it, the more it exerts an effort and pressure; not that a new quality or weight is joined to its gravity; it is with its natural weight itself that it produces a greater and greater effort, of more and more vehemence, gradually, as it has traveled a longer space and is nearer to its end.
>
> I think I do not have to approve or disapprove this reason. Both cases discussed above seem perfectly satisfactory; they seem to me to give the explanation of all the accidents that occur in these movements without it being necessary to invoke the aid of any intelligence or any reason; I therefore content myself with these two causes.

The Commentaries on the *Physics* of Aristotle composed by Francis Vicomercati of Milan[3] are not dated; dedicated to Henry II, they are preceded by an epistle of the author to this king; in this epistle Vicomercati lists the glorious facts that signaled the reign of the sovereign; the last event he cites is the restitution of Boulogne to France; as this restitution was accomplished in the year 1550, it must be assumed that the Commentaries on the *Physics* of Aristotle followed soon after this year.

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[3] Francisci Vicomercati Mediolanensis *In octo libros Aristotelis de naturali auscultatione commentarii, nunc denuo recogniti: et eorundem librorum e græco in latinum per eundem conversio. Ad Henr. II. Galliarum regem*. Venetiis, Apud Hieronymum Scotum. MDLXIIII.

Vicomercati adopts fully[4], on the movement of projectiles, the opinion of Aristotle; it is the fluid medium, initiated by the initial mover, which continues to move itself and advance the projectile.

> Maybe someone will make this objection: This same force, which is imprinted in the air by the mover, might as well be infused in the stone or the arrow that is launched, so that the previous explanation of the motion of projectiles would not be accurate. But, we have already said, it is the property of air and water to receive an *impetus* by which these bodies continue to move themselves when the initial mover is returned to rest, and by which, at the same time they move, they move other bodies; they move the latter, moreover, not from the movement that the mover that launched them had, but from the movement of which these fluids themselves move.

This, Vicomercati says, is the meaning assigned by Alexander and Simplicius to the doctrine of Aristotle; he adds the views of Averroes; he recalls that, according to the Commentator,

> the essence of the water and the air is intermediate between corporeal essence and spiritual essence;… but,

he continues,

> what we have expressed after Alexander and Simplicius is stronger and more easily supplies the solution to all the doubts that may arise about this.

Vicomercati rejected with a most summary casualness the theory of projectile motion that the Parisians argued. The explanation of the accelerated fall of weights proposed by Jean Buridan is even less favored; Vicomercati does not even speak of it. Contarini had made to this explanation a short allusion followed by a no less brief rebuttal; Vicomercati crosses out this allusion and rebuttal; this done, he reproduces[5], almost literally, what the Cardinal said; he declares admitting two causes

> which were approved, in his book *De elementis*, by Cardinal Contarini, the man the sciences adorned with a host of virtues, this gifted philosopher of great judgment and of a profound science. However,

Vicomercati continues,

> of these two explanations, I support especially the first, although preferably Contarini sustains the second. Doubtless, in my opinion, it is of some cogency, but much less than the first.

It is to the reduction in thickness of the medium that the weight must cross that Vicomercati attributes the main role in the acceleration of falling bodies; the impulsion generated at the rear of the projectile by the air that rushes in eddies there appears to be a more dubious effect; of two inadmissible explanations, he hastens to select the more foolish.

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[4] Vicomercati *Commentarii in libros de naturali auscultatione*, lib. VIII; ed. cit., pp. 373 (marked 365) and 374 (marked 373)

[5] Vicomercati, *loc. cit.*; ed. cit., pp. 367-368.

Gaspard Contarini and Francesco Vicomercati are particularly routine minds; the only lessons that their Dynamics takes into account are those of Alexander, Simplicius, and Averroes. Between these latecomer physicists and those who admit the most modern doctrines of the Parisian School, he follows a middle course; they mimic the very strange and little rational eclecticism of which Leonardo da Vinci gave the example; they attribute to *impetus impressus* the continuation of the movement of projectiles; but they ask the action of the shaken air to explain all accelerations, not only the acceleration that is actually observed in the fall of weights, but above all the imaginary acceleration that a projectile would experience at the start of its course. Between the thinking of these physicists and that of Leonardo, the resemblance is so great that it is possible to see in this an echo of it; this assumption is, however, much more probable because the first one of the geometers who has followed, in this matter, in the footsteps of Da Vinci is Tartaglia, a bandit of Mathematics[6]; the second is Jerome Cardan, whose *De subtilitate* was supported by clandestine loans[7] made to a friend of Fazio Cardano.

In the Dynamics of Nicolo Tartaglia, one can distinguish two phases: one corresponds to the statement that the author has given, in 1537, in the course of his *Nova scientia*; the other to what he teaches, in 1546, in his *Qœsiti et inventioni diverse*; nine years later, the geometer of Brescia contradicts himself on almost all points.

The first Dynamics of Tartaglia, that of the *Nova scientia*[8], is purely peripatetic; it receives no reflection of the doctrines of Leonardo da Vinci.

Regarding the impact of a body being more violent the higher the height from which it falls, Tartaglia concludes this proposition[9]:

> *If a body equally heavy moved with a natural movement, the more it moves away from its principle or approaches its end, the quicker it goes.*

On the subject of this acceleration, Tartaglia did not give any explanation other than this:

> The same thing is true for anyone who goes to a desired location; the farther he goes, approaching this place, the more he presses and strives to walk; he seems like a pilgrim, who comes from a distant place: the closer he is to his country, the more he tries to walk in all his power, and so much more when he comes from a more distant country; so does a heavy body; it even hastens to his own nest, which is the center of the World, and the farther from a remote location to this center it comes, the more quickly it approaches it.

The characteristics of violent movement are exactly opposed to those of natural movement:

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6 P. Duhem, *Les Origines de la Statique*, ch. IX, t. I, pp. 194-202 [Duhem (1991, 138-143)].

7 Léonard de Vinci, Cardan et Bernard Palissy (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, VI; seconde série, pp. 223-245).

8 *Nova scientia inventa da* Nicolo Tartalea. Vinegia, Steph. da Sabio, MDXXXVII.

9 Nicolo Tartaglia, *La nova scientia*, primo libro, propositione prima. — He calls a heavy body that which, because of the gravity of its matter and form, is able to experience, in a sensible way, the opposition of the air to any of its movements (def. 1).

> *The more an equally heavy body*[10] *moves away from its start or approaches its end from violent motion, the more slowly it goes…* From there, it is clear that a heavy body also has its greatest speed at the beginning of the violent movement and its smallest at the end.

Tartaglia draws this proposition from observation; he avoids dealing with the nature of the *impetus* that maintains the violent movement.

Projectile motion is strictly divided into two periods, an initial period in which the movement is purely violent and a second period in which it is completely natural.

> No equally heavy body[11] can, for any period of time or place, go with both violent and natural movement.

While the mobile moves with violent movement, it describes at first[12] a straight line AB (Figure 10.1), then an arc of circle BC; at C this arc connects tangentially to

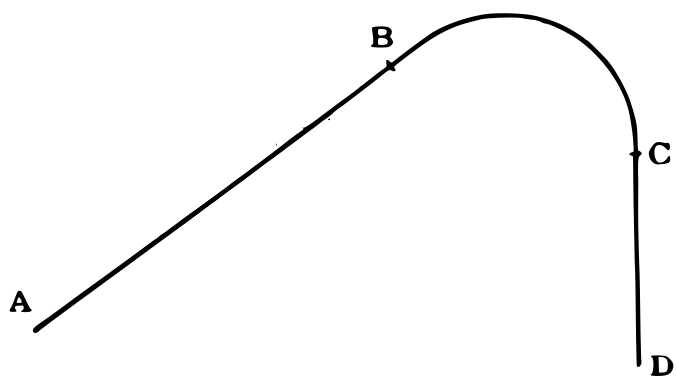

Figure 10.1 [Tartaglia on violent and natural movement]

the described vertical CD with natural movement; at the point C, where the violent movement finishes and the natural movement begins, the speed reaches its lowest value[13].

To this ballistics based purely on peripatetic principles, Tartaglia afterwards made some alterations which closely resembled the opinions supported by Leonardo da Vinci, so closely that he is permitted to believe in an influence exercised on the ideas of the great geometrician[14] by the posthumous notes of the great painter.

In this new ballistics, contrary to what he had supported in the *Nova scientia*, Tartaglia says[15] that, except in the case where the piece would shoot vertically, the

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[10] Nicolo Tartaglia, *La nova scientia*, lib. primo, prop. III.

[11] Nicolo Tartaglia, *La nova scientia*, lib. I, prop. V.

[12] Nicolo Tartaglia, *La nova scientia*, lib. II, suppos. III, propp. IV, V, VI.

[13] Nicolo Tartaglia, *La nova scientia*, lib. I. prop. VI.

[14] *Quesiti et inventioni diverse di Nicolo Tartalea*. Vinegia, Vent. Ruffinelle, ab instantia et requisitione et a propria spese de Nie. Tartalea Brisciano autore; MDXLVI. *Il primo libro delli quesili et inventioni diverse di Nicolo Tartaglia, sopra gli tiri delle artiglierie, et altri suoi varii accidenti.*

[15] Nicolo Tartaglia, *loc. cit.*, quesito terzo.

trajectory of the projectile has no straight portion, not even an inch. What curves the trajectory is the constantly active natural gravity. The high speed is the proper cause of the straightness of the movement; the quicker a heavy body is launched into the air, the less it weighs; therefore, the straighter it travels through the air which even better supports a body that is lighter. The more the speed decreases, the more the gravity increases, and this gravity constantly seeks the body and pulls it to the ground. However, from the moment the cannonball leaves the barrel, the speed of the violent motion is steadily decreasing, and, therefore, the trajectory curves more and more.

We recognize, in this theory, the antagonism and the struggle of *impeto* and gravity, the description of which we read in the notes of Leonardo. In imitation of him, Tartaglia also invokes an accelerating action of the air set in motion. This action serves to answer a question[16] posed by Gabriel Tadino di Martinengo, knight of Rhodes and prior of Barletta:

> The Prior: If one shoots a similar artillery piece twice in quick succession, with a same height, towards the same goal, and with two equal loads, will the two shots be equal?
>
> Tartaglia: Without a doubt they will be unequal; the second shot will go farther than the first.
>
> The prior: For what reason?
>
> Tartaglia: For two reasons. The first is that, at the first shot, the bullet found the air at rest, while, in the second shot, it finds it not only shaken entirely by the bullet launched at the first shot, but still holding strong, runs to the place toward which it was shot. Now it is easier to move and penetrate an already moved and penetrated thing than to move a thing that is at rest and in equilibrium. Therefore, the ball shot the second time, meeting a lesser obstacle to movement than the first, will go farther than this one…

Tartaglia was perhaps borrowing these arguments from any one of the notes left by Leonardo da Vinci; perhaps he conceived them in reading the treatise *De ponderibus* that the mechanist we have called the Precursor of Leonardo da Vinci wrote. We can believe more willingly that from the seventh book of the *Quesiti et inventioni diverse*, Tartaglia plagiarized the statics part of the treatise with an impudence that Ferrari harshly reproached; we also know that this work was published by Curtius Trojanus from a manuscript that Tartaglia left him.

But what Tartaglia could not borrow from the Precursor of Leonardo is the notion compound *impeto*; if formally denied in the *Nova scientia*, it is the assumption that the composition between violent *impetus* and natural gravity is the cause of the curvature of the trajectory, a hypothesis that no one so far, except Da Vinci, appears to have conceived; so complete is the renunciation of Tartaglia of his old ideas that he goes further than his predecessor; he admits that one should consider this composition of *impetus* and gravity, as well as the curvature of the resulting trajectory, during the entire duration of the movement of the projectile. A so sudden and complete change in front supposes some strong impulsion received from without; it is hard not to put the origin of this impulsion in the notes of Leonardo.

If the opinions issued in Dynamics by Tartaglia have, first of all, presented a great conformity to the doctrines of the School, to then parallel the thoughts of Leonardo

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[16] Nicolo Tartaglia, *loc. cit.*, quesito quarto. — Cf.: libro secondo, quesito primo.

da Vinci, it is to these that the theories developed by Jerome Cardan immediately relate[17]. Between the Mechanics of the great painter and that of the famous physician, mathematician, and astrologer from Milan, there are so many parallels, the analogies so intimate, that we are forced very often to regard the second as a plagiarism of the first.

Cardan knows the various opinions that have been expressed on the cause that maintains the violent movement:

> So[18] the first opinion is that the moved thing like the stone is moved by the virtue gained from the one who throws it (*vi acquisita a projiciente*); thus, as the thing heated with fire heats up after other things by its acquired virtue, and the matter remains hot for a long time; so the moved thing receives the force by that which moves, whereby the other is pushed until it rests. This opinion is sensible, which was rejected by the argument of the ancients, allegedly of Aristotle.

Having at length the theories that explain the propulsion of the projectile by the movement of the surrounding air, Cardan added[19]:

> But the first opinion is most necessary, being simply understood and not containing many difficulties. And when it is assumed that everything that is moved is moved by something, that is very true;[20] but what moves is an acquired impetus (*impetus acquisitus*), just as heat in water, which is induced in water by fire beyond nature, and always when the fire is on, the water burns the hand of him who touches it; and so the accident, violently adhering, retains its strength.

Cardan therefore assigns the maintenance of a violent movement to an *impetus acquisitus* similar to the *impeto* invoked by Leonardo da Vinci; and he used this *impetus* to conceive of the nature of the comparison of which Alexander of Aphrodisias used with respect to the χινητιχὴ δύναμις διδομένη that it conferred on the air.

Like Leonardo da Vinci, Cardan distinguishes three periods in the movement of a heavy projectile: a first period in which the projectile moves only under the influence the *impetus acquisitus*; one last period when it is no longer subjected to gravity; finally, an intermediate period where the gravity and the acquired *impetus* violently struggle against each other:

> Thus, the materials[21] that are thrown away consist of three movements: the first violent, the last of all natural, and the middle composed of the other two.

To the two extremal periods in time there correspond two rectilinear portions of the trajectory, the former inclined, the latter vertical; during the interim period, the mobile described a curved arc:

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[17] Hieronymi Cardani medici Mediolancnsis *De sublilitate llbri* XXI. Lugduni, apud Guglielmum Rouillium, sub Scuto Veneto. MDLI. — *Les livres de Hiérome Cardanus, médecin milannois, intitulés de la Subtilité et subtiles inventions, ensemble les causes occultes et raisons d'icelles*, translated from Latin to French by Richard le Blanc. Paris, Charles l'Angelier, MDLVII.

[18] Cardan, *De la Subtilité*, translation by Richard le Blanc, edit. cit., fol. 46, recto.

[19] Cardan, *loc. cit.*, fol. 67, verso.

[20] [This is the Scholastic axiom: *Omne quod movetur ab alio movetur* — Everything that is moved is moved by another.]

[21] Cardan, *loc. cit.*, fol. 49, recto.

> Now, when the ball thrown[22] has straightly reached its extremal place, it neither descends by the figure of a circle, nor also straightly, but almost by a middle line between the two which is almost the surrounding line of a quarter of a circle, as is BC (Figure 10.2); and finally any

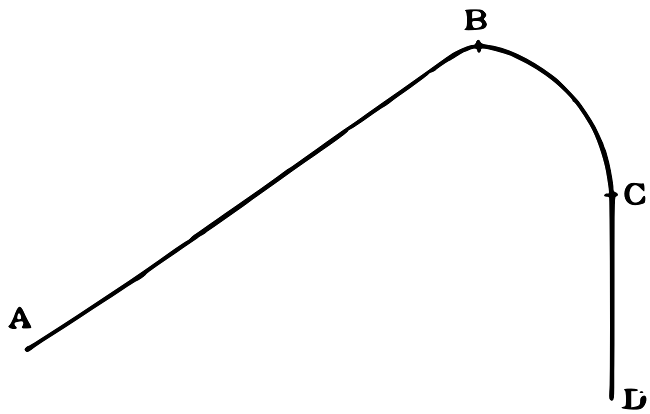

Figure 10.2 [Cardan on violent and natural movement]

> time the ball goes straight from C to D by the movement of the heavy material.

With Aristotle and also with Leonardo, Cardan admits[23] that the greatest velocity of the projectile is reached neither at the beginning nor the end, but at the middle of its course:

> For we see that machines and likewise shafts thrown by the hand move more vehemently by some distance than those that are close, and nearly in artillery.

However, the assistance of the *impetus* and gravity cannot explain this alleged truth of observation: the “natural movement is increased in the end, the violent movement in the beginning”; the transition from violent movement to natural movement should correspond to a minimum speed. The existence of a maximum speed between the start and finish of the projectile cannot be explained[24] as an accelerating action of disturbed air:

> Because the air in the beginning did not help the movement, if not very little; in the course of time, the natural movement of the air, as it is moved, is no longer effective; … because for him it is necessary that the speed of movement increase.

Cardan repeatedly studied this accelerating action of the shaken air; in one of his last works, the *Opus novum proportionibus*[25], he decomposes, as Leonardo had done before him, two other actions: A pull the air expelled at the front of the mobile and a

[22] Cardan, *loc. cit.*, fol. 49, recto.

[23] Cardan, *loc. cit.*, fol. 48, verso.

[24] Cardan, *loc. cit.*, fol. 48, verso.

[25] Hieronymi Cardani Mediolanensis, civisque Bononiensis, philosophi, medici et mathematici clarissimi, *Opus novum de proportionibus numerorum, motuum, ponderum, sonorum, aliarumque re-*

impulsion of fluid that comes, in turbulence, to occupy the place that the projectile leaves void behind itself.

> It evidently results that all movement is either natural or violent, there being some increase in speed after the beginning of the movement up until a certain moment. That is why all kinds of war machines require a certain distance for their shot to attain its greatest violence.

It is thus to the accelerating action of the air that we must attribute[26] the increased speed of natural movement by which a weight falls to the ground:

> *All natural movement, performed in a homogeneous medium, is stronger at the end than at the beginning; this is in contrast to violent movement.*
>
> Indeed, according to the above, natural movement constantly increases by the action of the medium; on the other hand, the cause that moves is perpetual, deriving from an eternal principle; from what we have said, it moves uniformly; this movement will therefore become faster at the end than it is in any other part of its duration. On the contrary, in violent movement, when the mobile approaches the goal, the force that moves the projectile necessarily ends; it is surpassed by the natural force that moves in the opposite direction; so before the movement completely stops, it becomes, in its final part, extremely slow.

What Cardan, in the *Opus novum proportionibus*, clearly explains regarding the acceleration of natural motion allows us to interpret a very obscure passage in the *De subtilitate*; in this passage, one determines[27]

> The reason why a ship is conducted so slightly by sails… Because this ship is hardly moved from the beginning. Yet Aristotle had some doubt, believing that the violent motions are reduced towards the end. It is evident that the movement of the ship is always rendered lighter for equal wind… Is there always movement, even if only up to a certain limit? It is already known that it increases from the beginning. But the cause of it is, when this movement ceases, that the violent movement, as I said, is increased; it will be so even more increased when the motive cause remains.

In his *De rerum natura* of which the first edition was printed in Rome in 1555, Bernardino Telesio professes a Dynamics that is quite similar to that of Cardan, based on that of Leonardo da Vinci.

Telesio presents[28] the explanation that Aristotle gives of the motion of projectiles; he added at once:

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rum mensurandarum, non solum geometrico more stabilitum, sed etiam variis experimentis et observationibus rerum in natura, solerti demonstratione illustratum, ad multiplices usus accommodatum, et in V libros digestum; Basileæ, ex officina Henricpetrina, Anno Salutis MDLXX, Mense Martio. Lib. V, prop. XXX.

[26] Cardani *Opus novum de proportionibus*, lib. V, prop. XXXI.

[27] *The books* by Hierome of Cardanus, Milanese physician, *intitulés of subtlety and subtle inventions*, translated from Latin into French by Richard White; Paris, Charles l'Angelier, 1556, fol. 335. This passage is not in the first edition of *De subtilitate*, published in 1551; it was introduced in the second edition, printed in 1554, upon which the translation of Richard le Blanc was made.

[28] Bernardini Telesii Cosentini De Rerum Natura iuxta propria principia, Liber Primus, et Secundus, denuo editi. Neapoli, apud Iosephum Caccium. Anno MDLXX. Liber primus, cap. 46: Cur gravium ad inferna motus assidue magis concitetur, Peripateticorum nulli satis explicatum est; ed. cit., fol. 32, verso.

> It is a vain reason and based on a completely false basis; bodies which are projected violently, indeed, seem to be not driven, as it pleases Aristotle to maintain, by the air that pushes them forward, but by a *vis impressa*.

If theory of Aristotle were correct,

> any body moved by violence would move forever; a small amount of air is sufficient, according to Aristotle, to raise a stone; *a fortiori* it could do so all the more as when it becomes much larger. It will not be the same if these bodies are moved by a *vis impressa*, a *motus inditus*; the more they move away from the mover that launched them, the more the motion of these projectiles will weaken in a continuous manner; by this moving away, in fact, the *vis impressa*, the *motus inditus*, increasingly weakens and languishes.

If he demands that *impetus* explain the motion of projectiles, Telesio does not assign to it the acceleration of falling bodies; of the theory that he gives this role, he makes no mention. As for the other reasons that have been given for the same phenomenon, he reviewed and found them insufficient; what he proposes as new has many analogies to what Tartaglia gave in his *Nova scientia* and to the subject on which Contarini suspended his judgment:

> The cause for the fall of weights is not uniform[29], for which it is accelerating continuously, all the Peripatetics have sought with great anxiety; but so far it does not seem that they could account for this fact. This reason seems to be manifested very clearly from the principles we have set forth. The true nature of a weight receives its immobility from its proper place, which is the Earth, and from the abstract universality that suits it; but the place that absolutely opposes it, the contact of bodies that are foreign to it and which have hatred for it, confer on this nature a certain strength; it then rushes into its own proper place, toward the bodies that are related to it; it falls more rapidly than these foreign bodies, which hate and repel it, continually accelerating its movement so that it enjoys as soon as possible the immobility within the bodies that are related to it.

Tartaglia and Cardan are really, in Dynamics, the disciples Leonardo da Vinci; Telesio approximates the great painter in that he attributed to an *impetus* imprinted on the projectile the continuation of the movement of the latter, while he does not invoke this *impetus* to explain the acceleration of falling bodies. The physicists that accepted, in this regard, the doctrine of the Parisians, were certainly very rare, in Italy, at the beginning of the 16th century.

It might be temerarious to take as a formal adherence to this doctrine the allusion that Maurolycus made to *impetus* created by the weight. In his *Cosmographia*, which he completed on 21 October 1535 but only published in 1543, the learned abbot of Messina inserts the following dialogue[30]:

> Antimachus: If weights followed a path that allowed them access to the center, from whatever place they were allowed to fall, they would meet at this point.

[29] Bernardino Telesio, *loc. cit.*; ed. cit., fol. 33, recto.

[30] *Cosmographia* Francisci Maurolyci Messanensis Siculi, *In tres dialogos distincta: in quibus de forma, situ, numeroque tam cælorum quam elementorum, aliisque rebus ad astronomica rudimenta spectantibus satis disseritur*. Ad Reverendiss. Cardinalem Bembum. Venetiis MDXXXXIII. In fine: Completum opus Messanæ in freto siculo die Jovis XXI Octobris Vllll indictionis anno salutis MDXXXV. quo die Carolus V Cæsar ab africana expeditione reversus Messanam venit. Venetiis apud hæredes Lucæantonii luntæ Florentini mense lanuario MDXLIII. Dialog. I, pp. 15-16.

> Nicomedes: No doubt, but I will try you with this question: Pierce the earth, as could be done to a wooden ball, with a hole through its center; in this hole, drop a heavy stone; how far do you think it will go?
>
> Antimachus: Will not this point be at the center?
>
> Nicomedes: This is precisely what a man who does not know this subject would say. But be aware that this stone, which is thus abandoned to itself, would stop not first of all at the center. Swept away by the *impetus* of weight, it passes beyond the center a certain length and ascends the opposite hemisphere; then it would fall and, again, go beyond the center, ascending a shorter length than previously; it would go and return, following a path that is due to contract constantly, while the *impetus* would weaken little by little, until the moment when it would be back at the center. Similarly, a lead plumb suspended by a wire that has been drawn from the vertical position does not immediately return to that position; it surpasses it, first, by a certain distance, and then it goes and comes back a number of times; each time, the force that moves it is lower and the difference smaller; it eventually remains at rest in the vertical position.
>
> Antimachus: You have reasoned very penetratingly, and you bolster your speculation with a well-adapted example. I remember now that, in his *Colloquies*, Erasmus of Rotterdam proposes the same question.

Maurolycus remembered, no doubt, having read this in a writing other than that of the *Colloquies* of Erasmus. The dialog where he presents it is full of considerations on the center of gravity of the earth and on the convergence of the verticals that are borrowed from the *De Cælo* of Albert of Saxony. But if an Italian scholar could, without shame, in 1535, allude to the writings of Desiderius Erasmus, had he been able, without blushing, to confess that he asked for his inspirations in a treatise composed, in the 14th century, by a scholastic of Paris?

The year the *Cosmographia* of Maurolycus was printed was also the year of the immortal treatise of Copernicus. It is intriguing to note that the treatise also contained a brief reference to the *impetus* generated by the weight:

> Bodies that are moved upwards or downwards,

wrote the canon of Thorn[31],

> do not perform a simple, uniform, and equal movement. In them, in effect, one can adjust the lightness or *impetus* caused by their own weight. All bodies that fall experience, at the beginning, a very slow movement; then, falling, they increase their speed.

The allusions to the *impetus ponderis* that we found in the *Cosmographia* of Maurolycus, without however involving a formal and complete adherence to the Parisian doctrine of the accelerated fall of weights, show us that this doctrine was not unknown to the Abbot of Messina.

Alessandro Piccolomini, in his *Paraphrase* on the *Mechanical Problems of Aristotle*, whose first edition is from 1547[32], clearly admits this theory of Buridan and Albert of Saxony. Aristotle or the author, whoever he may be, of the Μηχανιχὰ προβλήματα had compared[33], in a falling body, gravity (βάρος) and movement (φορὰ

[31] Nicolai Copernici *De revolutionibus orbium cælestium* libri VI; lib. I, cap. VIII.

[32] Alexandri Piccolominei *In mechanicas quæstiones Aristotelis paraphrasis paulo quidem plenior*, ad Nicholaum Ardinghellum Cardinalem amplissimum. Excussum Romæ, apud Antonium Bladum Asulanum, MDXLVII.

[33] Aristote, Μηχανιχὰ προβλήματα, XVIII and XX (ed. Didot, t. IV, pp. 64 and 65)

or χίνησις). Besides, very vaguely he seemed to indicate that movement can add weight and increase it; these are loose and indecisive thoughts that Piccolomini, in his *Paraphrase*, interprets with help of the Parisian doctrine; moreover, he is careful not to name the authors of this doctrine; in the way he presents it, one believes it is the outcome of Greek Science.

He presents this doctrine, along with his whole theory of violent movement, in his 37th Chapter, devoted to the consideration of the thirty-second question of Aristotle.

> It should be noted,

wrote Piccolomini

> that there are two kinds of gravities: one that is rooted in the very nature of the body; the other, superficial, which the Greeks call ἐπιπόλαιαν. This is none other than a certain temporary *impetus* that can either be acquired in the same body moved by its own tendency (*qui vel acquiritur in re ipsa ex suo nutu mota*), or be imprinted by a mover moving violently.
>
> In effect, when a stone tends downwards, it becomes ever more rapid because, as a result of the movement, it constantly acquires a greater gravity (I intend to speak of the superficial gravity)…
>
> Similarly, when a stone is thrown violently, it receives a certain gravity or some superficial lightness imprinted by what projects it. This is none other than an accidentally acquired *impetus* which moves the stone violently and makes it as movable by itself, until this *impetus* becomes languid and vanishes…

No more for Piccolomini than for Leonardo da Vinci, the *impetus* is not of itself perpetual:

> The superficial heaviness or lightness cannot become sustainable or perfect, because the substantial form of the body which suffers, namely, gravity or lightness that is natural to this body, is opposed to what is perfectly and deeply imprinted in it.

That which weakens the *impetus* and eventually kills it is not only the resistance of the external obstacles; it is the natural gravity:

> The impulsive virtue, an effort resulting from its own nature and which becomes more powerful than the superficial gravity or lightness, ends, which can happen either by the resistance of some object that pushes the mobile or by the tendency of the mobile itself.
>
> As soon as the true gravity surpasses, by the power of its effort, the *impetus* that the mover imprinted in the stone, it ceases to move violently and, by its own motion, tends downward[34].

The Dynamics of the Parisians, almost universally ignored by the Italians, will be called to their attention in a form that will be exempt neither of violence nor of difficulty. An Italian who emigrated to France, Julius Caesar Scaliger, will be its spokesman; through the voice of Scaliger, it will oppose his clear and coherent theories with the indecisions and contradictions of Cardan.

In 1557, Julius Caesar Scaliger published[35], from the *De Subtilitate* of Cardan, which had in France an extreme vogue and which Richard White translated into

[34] Piccolomini, *loc. cit.*; cf.: cap. XXXVIII, quæst. trigesimatertia.

[35] Julii Cæsaris Scaligeri *Exotericarum exercitationum liber XV. De Subtilitate ad Hieronymum Cardanum*. Lutetiæ, apud Vascosanum, MDLVII.

French, a more vivid critique; this criticism, which Scaliger gives as forming the 15th book of his *Exotericæ exercitationes*, is entitled: *De Subtititate ad Hieronymum Cardanum*. As the work which he gave the most malicious critique, the writing of Julies Caesar Scaliger was extremely read[36].

Scaliger is a fanatic admirer of the masters of the Parisian School; a quote will give us the measure of his extraordinary admiration.

In 16th book of the *On the Subtlety*, Cardan had the rather naive idea of ordering the geniuses in descending order of greatness. He awarded the first place to Archimedes, citing, as the reason for this preference, the mechanical inventions of the Syracusan Geometer. The second place was reserved for Aristotle. Euclid came in third; Jean Duns Scotus was the fourth; the fifth was granted to Suiseth the Calculator, whom Cardan made a Scotsman with the first name of John; our Milanese physician regarded, moreover, these three men—Euclid, Duns Scotus, Suiseth—as having possessed equal genius; the greater or lesser length of time they lived alone determined an order of precedence between them. Cardan placed Apollonius of Perga, Archytas of Tarentum, and a host of other geniuses lowest on the scale of human intelligence.

The preeminence given to Archimedes appalls the reason of Julius Caesar Scaliger[37]:

> You gave a simple artisan precedence over Aristotle who, moreover, was no less learned than he in these same mechanical arts; over John Duns Scotus, who was like the file of truth; over John Suiseth the Calculator, who has almost exceeded the measure imposed on human intelligence! You ignored Ockham, whose genius overthrew all past geniuses, who countered the follies we could not defeat until him, because of their elusive subtlety, with new arguments that he manufactured and shaped! You placed Euclid after Archimedes, the torch after the lantern! It seems that you may be swept away by the whirlwind and storm of thy evil genius; it is not you who keep him in check; it is he who gives you the spur!

Whoever esteems William of Ockham and Suiseth the Calculator so highly will profess the Dynamics of the Parisians, and we will not at all be surprised.

We find, in fact, in the book of Julius Caesar Scaliger[38] a very wide explanation and refutation of the various theories that attribute the persistence of projectile motion to air.

> Such a reason is worthless,

says our author,

> and here is a demonstration that will sufficiently demonstrate it:
>
> Let there be a small light board in which a disk was cut using a lathe or a compass edge; suppose that this disk can turn in the circular cavity without rubbing against the edges. The small board being fixed vertically somewhere, pierce the disk with an axle equipped with a crank; lay down the ends of this axle on two forks. After launching this circular disk, you will see clearly that this disk, once the mover is removed, continues to rotate in the circular

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36 In addition to the first edition: Lutotiæ, apud Vascosanum, 1557, we had in our hands the following editions: Francofurti, apud A. Wechelum, 1601; Francofurti, apud A. Wechelum, 1612; Lugduni, apud A. de Harsy, 1615.

37 Julii Cæsaris Scaligeri *Op. cit.*, exercitatio CCCXXIV: Sapientum census.

38 Julii Cæsaris Scaligeri *Op. cit.*, exercitatio XXVIII: De motu projectorum, Motus violentus quis.

> cavity, although no air is pushing it. In this movement of rotation, in effect, the mobile does not leave behind it any place that the air can come fill in. Moreover, the air which is located between the disk and the board is of such a small quantity that it is unable to exert any of its own force to maintain the movement considered. The contour of the disk, perfectly smooth and polished, can feel no impulsion by the effect of the agitation of the surrounding air.

In this experimental refutation of peripatetic theories, we find traces of the discussions so clearly and firmly conducted by Jean Buridan.

It is not the shaken air that keeps the projectile in motion. What is it, then?

Scaliger does not call the cause which maintains the movement *impetus*; he calls it motion, *motio*; but this change of name does not affect the content itself of the idea; he considers the *motio* identical to the *impetus* of Jean Buridan and Albert of Saxony:

> The *motio* is a form that is impressed in the mobile and that it can retain even when the primary mover is removed. I say: the primary mover, the one that caused this form to penetrate into the mobile, because it is not necessary that the efficient cause continue to coexist in its effect.

This form tires[39] and perishes with time, because it is outside of the nature of the elements in which it is impressed. These doctrines are common to many physicists of the 16th century. But here is a passage[40] where Scaliger marks clearly, on the subject of the accelerated movement that a constant mover engenders, the idea that Piccolomini only hinted at for his reader:

> Heavy bodies, a stone for example, have nothing that favors their being put in motion; they are, on the contrary, quite opposed to it. The stone that is put in motion on a horizontal plane does not move with natural movement… Thus, why does the stone move easier after the movement started? Because, according to what we said above about the movement of projectiles, the stone has already received the impression of movement. A second part of the movement succeeds a first part; and, however, the first remains. So that, although a single mover exerts its action, the movements it impresses in this continuous succession are multiple. Because the first impulsion is kept by the second, and the second by the third.

Although Scaliger has very clearly stated the Parisian theory of the accelerated fall of weights, he has scarcely succeeded in making it commonly received in Italy; he could not even convince Cardan.

When in 1560 Cardan published the third edition of his *De Subtilitate*[41], he attached to it an *Apology Against a Calumniator*[42], an apology intended to respond to the criticisms of Scaliger.

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[39] Julii Cæsaris Scaligeri *Op. cit.*, exercitatio LXXVI: Quare sidera motu non frapguntur. Quare non fatigant motores suos.

[40] Julii Cæsaris Scaligeri *Op. cit.*, exercitatio LXXVII: Quamobrem mota rota facilius moveatur postea.

[41] Hieronymi Cardani Mediolanensis medici *de Subtilitate libri XXI. Ab authore plusquam mille locis illustrati, nonnullis etiam cum additionibus. Addita insuper Apologia advenus calumniatorem, qua vis horum librorum aperitur*. Basileæ. In fine: Basileæ, ex ofiicina Petrina, anno MDLX. Mense Martio.

[42] Hieronymi Gardani Mediolanensis medici *In calumniatorem librorum de Subtilitate actio prima ad Franciscum Abundium, S. Abundii Commendatarium perpetuum*. Ed. cit., pp. 1265 seq.

The response is no less vivid than the attack. To dress Scaliger up in a costume that is particularly disgraced in the eyes of Italian Humanists, Cardan dresses his opponent not as a Parisian but as an Averroist[43].

> What will you say of his judgment?

he exclaimed.

> Whenever he wants to compete in natural philosophy, he uses the principles and authority of Aristotle and Averroes; however, when these prove the eternity of the World, a supposition that removes from Christ his divinity and, above all, the hope of a fair remuneration for good and bad deeds. And after that he dares to accuse me of impiety!

If Cardan accuses Scaliger of a too obstinate attachment to the opinion of Aristotle and Averroes, he refuses to share, for the masters of the Nominalist School, the fervent admiration of his opponent[44]:

> What concern could a donkey have for a lyre, and why praise marjoram for swine? He admires the extreme subtlety of Ockham and Hentisber[45]; he places them higher than the pinnacle of humanity. No doubt they have written on everything in an ingenious and clear manner; but in them, invention is void; deny them a single proposition, and fifteen pages will crush you. But as these authors are well accommodated to the disputes of the schools, he smiles at this and heaps it with praise. It is clear that he does not understand them, but he praises it to give the appearance of comprehension.

Although he does not share the opinion of Aristotle about the movement of projectiles, Cardan does not spare his sarcasm for the experiment by which Scaliger claimed to disprove this theory[46]:

> If this wheel has been executed carefully, he does not see, as he is so stupid, that the crank is driven by the air in a rotational movement and with the crank, the wheel itself… He would have done better to rotate it without using a crank, with a finger he suddenly withdraws.

Here is what Cardan thinks about the explanation of the accelerated motion that a millstone subjected to a constant action undergoes, an explanation in which Scaliger has only followed the teaching of Paris[47]:

> He is completely wrong; it is not only the wheel, but the whole mobile, that moves with more ease and speed when it has already taken a certain speed, and this, as we have taught in the second book, because the air of the first movement comes to the aid of the following movement.

Also, in 1570, in his *Opus novum de proportionibus*, Cardan has persisted, as we have seen, in explaining the acceleration of falling bodies by the impulsion of the shaken air.

If Scaliger did not convert Cardan, he has not convinced Bento Pereira more to embrace the Parisian Dynamics.

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[43] Hieronymi Cardani *Apologia*; ed. cit., p. 1268.

[44] Hieronymi Cardani *Apologia*, art. 324; ed. cit., p. 1412.

[45] I.e., William of Heytesbury, of whom Scaliger has not spoken.

[46] Hieronymi Cardani *Apologia*Art. 29; edit. cit., p. 1304.

[47] Hieronymi Cardani *Apologia*, art. 77; ed. cit., p. 1020.

Born in Valencia in 1535, Bento Pereira[48] entered the Society of Jesus early on; he then went to Rome where he passed his life and where he died, 6 March 1610. It was in Rome that Bento Pereira published, in 1562, the first edition of his fifteen books on Physics[49]. This book was very popular; many editions spread it everywhere[50]; Galileo, who had studied it in his youth, cites in his early writings[51].

Bento Pereira devotes an entire chapter[52] of his work to presenting the various explanations of the violent motion of projectiles; among these explanations, he did not forget the one that the Parisian School supported.

> Some philosophers,

he said,

> who are neither few, but noble, nor numerous support this: When a stone is thrown by the force and momentum that launches it, he who sets it in motion impresses in it a certain motive virtue that remains inherent in the stone and continues to move it after it is separated from the one who threw it.

Our author knew the main arguments that this opinion relies on and, on this occasion, he cites the *Exercitationes* of Scaliger. But, immediately, a new and long chapter comes to refute[53] this theory and save the peripatetic opinion.

The Parisian explanation of the accelerated fall of weights is less pleasing than the theory of *impetus*; Bento Pereira did not honor it even a mention.

On the subject of this accelerated fall, our author describes with great care[54] the various ancient hypotheses that Simplicius has preserved for us; he joined to it the assumption that attributes the acceleration to the impulsion of shaken air at the rear of the projectile, a supposition regarding which he quotes Walter Burley and Contarini.

> This latter opinion,

he added,

> seems to me the most likely. In the first place, the other opinions are refuted by reasons that are manifest and necessary, while in the latter, we cannot even imagine any likely argument. Secondly, this explanation assumes nothing that does not perfectly accord with reason and experience, nothing that is not derived from the nature of things. In this opinion, more than any other, my mind delights; in that one only does it delight in a profound rest.

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48 *Nouvelle Biographie générale*publiée par Firmia Didot frères, t. XXXIX, p. 571, 1862.

49 Benedicti Pererii, societatis Jesu, *De communibus omnium rerum naturalium principiis et affectionibus libri quindecim, qui plurimum conferunt, ad eos octo libros Aristotelis, qui de Physico auditu inscribuntur, intelligendos*; Romæ, impensis Venturini Tramezini, apud Franciscum Zanettum et Bartholomæum Tosium, MDLXII.

50 In addition to the first edition, we noted the following: Romæ, 1076; Parisiis, 1579; Romæ, 1585; Venetiis, 1609.

51 *Le opere di* Galileo Galilei *ristampate fedelmente sopra la edizione nationale*, vol. I, *Juvenilia*; Firenze, 1890 [Galilei et al (1890)]; pp. 24, 35, 145, 318, 411.

52 Benedicti Pererii *Op. cit.*, lib. XIV, cap. IV: De caussa motus violenti eorum qui projiciuntur.

53 Benedicti Pererii *Op. cit.*, lib. XIV, cap. V: Refellitur opinio faciens caussam motus projectorum, virtutem quandam impressam projectis.

54 Benedicti Pererii *Op. cit.*, lib. XIV, cap. III: Tractatur secunda divisio motus in naturalem et violentum.

Bento Pereira is of the school of Contarini and Vicomercati; in this School, the Parisian Dynamics is considered null and void; or, if one takes any account of it, it is to refute its assertions.

Cesalpino and Borro are also of this school.

En his *Quæstiones peripateticæ*, which first appeared in Florence in 1569, Andrea Cesalpino says only a few words[55] on the motion of projectiles; but these words are a formal adherence to the theory of Aristotle[56].

Girolamo Borro was of Arezzo, like Cesalpino. In 1576, he published a fairly voluminous treatise dedicated in its entirely to the motion of weights[57]. At the beginning of this treatise, Borro gives the list of the "names of the ancient philosophers whose views are, in this book, either admitted or refuted." Fifty names of the wise Greeks or Latins, among whom they found even those of Homer and Orpheus, ac-

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[55] Andreæ Cæsalpini Aretini medici clarissimi, atque philosophi subtilissimi peritissimique *Peripateticarum Quæstionum libri quinque. Ad Potentissimum et fælicissimum Franciscum Medicen Florentiæ Et Senarum Principem*. Cum Privilegiis. Venetiis, Apud Iuntas. MDLXXI. Lib. IV, quæst. I, fol. 70, recto et verso. — We have not been able to consult the first edition of this book.

[56] We saw Buridan admit that the *impetus* of a body, moved with a given speed, was proportional to the amount of primary matter in this body; he deduced this proposition from this principle: *Receptio omnium formarum et dispositionum naturalium est in materia et ratione materiæ*. We have sought to show that the *quantity of primary matter* to which Buridan here refers is, at least in the case of heavy bodies, the product of the volume by an amount proportional to specific gravity, thus being identical to the *amount of matter* or *mass* that Newton defined.

That this is the idea the Scholastics attached to these words: *quantity of matter*, we find singularly clear evidence in a question that Cesalpino considered (Lib. IV, quæst. II; ed. cit., fol. 71, verso, to fol. 74, verso), a question whose title is precisely: Omnem virtutis intensionem remissionemque ex materiæ quantitate provenire.

A virtue,

Cesalpino said (fol. 72. recto),

> is not measured by the volume or extent of the body, but about an amount of material; this, indeed, is in itself indefinite, can sometimes tighten down to narrower limits, and sometimes extend to a more ample volume… Every body which is carried simply toward the center (fol. 74, verso), that is to say, all the bodies that are simply heavy [those which are not formed by mixing one or more heavy elements with a light element], all these bodies, I say, are heavier than the others because of the amount of matter they contain; the lead plumb is heavier than a stone because in this plumb there is more heavy matter than a stone of similar volume; it is, in fact, more dense. One can also compare between them the weights of different species [of solids, liquids, gases], of water and earth, for example, but in a place, such as the air, where both the weights are; it is still true that the heavier is that which has more matter.

This amount of matter remains, however, invariable in all transformations that the heavy body may experience:

> If a handful of water turns into ten handfuls of air, there will be the same virtue in ten volumes of air as in one volume of water, because in both cases there will be an equal portion of matter

(fol. 72, recto).

[57] Hieronymus Borrius Arretinus *De Motu Gravium, et Levium. Ad Franciscum Medicem Magnum Etrurias Ducem II*. Florentiæ, In Officina Georgii Marescotti. MDLXXVI.

companied by the names of four Arabic philosophers: Algazel (Al Gazali), Avempace (Ibn Badja), Averroes, and Avicenna; but a Christian philosopher does not get even the honor of a quote.

This contempt, pushing for the absolute oblivion of Western Christian Science, of this colossal intellectual movement which received the name of Scholasticism, is the proper mark of Italian Averroism. Borro is an avid Averroist, as he affirms in every page of his writing. The name of Averroes is crowned therein in the most flattering epithets. "*Averroes, omni genere laudis abundans philosophus…*[58]." "*Philosophus nunquam satis laudatus Averroes…*[59]." "*Averroes divinissime probavit…*[60]." The whole doctrine of our author can be summarized as follows: Aristotle is infallible; Averroes is his jealous defender, authorized with infallibility. Moreover, Borro himself provides us with this summary of his thought[61]:

> *Averroes, qui in Aristotelem erroris notam, nec levissimam illam quidem, ab alio quovis inuri non patitur, sed eundem ab omni injuria nunquam non vindicat, ne in hac parte indefensus relinquatur…, ait…*

This is not such a writing, assuredly, that we shall see dominate the Dynamics doctrines of the Parisians; in fact, Borro did not even give the slightest hint to these doctrines; all that the Nominalists could say about the movement of projectiles or the fall of weights does not exist for him; evidently, he is convinced that between Averroes and him, humanity has stopped thinking.

What keeps the projectile in motion is, of course, for Borro[62] as for Aristotle, the air whose shock propagates in front of the mobile. The physicist of Arezzo does not even seem to doubt that this absurd explanation has been refuted a hundredfold.

The shock of the medium also plays a role in the acceleration of natural motion[63]. Borro presents[64] the various explanations that have been proposed to account for this acceleration; in this explanation, it goes without saying, there is no allusion to the Parisian theory; our author summarizes the opinion that he adopts in the following terms[65]:

> The gravity or levity of the elements is increased by the greater number of parts of the medium that rush to back of the mobile; by the slightest resistance of the medium at the end of the movement; by the strongest pulse of air following the mobile; by the perfection that heavy or light bodies acquire, the more completely they are closer to their natural places. The increase that the gravity or lightness received towards the end of the movement increases this movement and makes it faster.

[58] Girolamo Borro, *Op. cit.*, p. 51.

[59] Girolamo Borro, *Op. cit.*, p. 184.

[60] Girolamo Borro, *Op. cit.*, Index, indication of the question discussed on page 185.

[61] Girolamo Borro, *Op. cit.*, pars III, cap. XXV: Demonstratio, quam Aristoteles libro septimo Physicorum literis consignavit, ad veritatis trutinam examinatur; p. 371.

[62] Girolamo Borro, *Op. cit.*, pars III, cap. XIII: Quomodo elementorum motus a medio pendeat; pp. 234-235.

[63] Girolamo Borro, *Op. cit.*, pars III, cap. XIII: Quomodo elementorum motus a medio pendeat.

[64] Girolamo Borro, *Op. cit.*, pars III, capp. XIV, XV, XVI.

[65] Girolamo Borro, *Op. cit.*, pars III, cap. XVI: Quæ sint veræ Peripateticorum causæ, propter quas ea, quæ natura moventur, velocius in fine, quam in principio moveantur; p. 244.

That we have, more than two centuries after Jean Buridan and Albert of Saxony, written a Rome and Florence, some books like those of Bento Pereira, Andrea Cesalpino, and Girolamo Borro; that the absurd theory of the motion of projectiles, proposed by Aristotle, could be regarded as safe from all the objections that had been made against it; much more, that it has been treated as an undisputed and indisputable doctrine is a fact well worthy of capturing one's attention; it gives the measure of the stubborn resistance with which Italian Peripateticism knew how to oppose the penetration of any new idea. That same strength, we see, however, in men of very different situations: a Jesuit whose religious doctrine is more orthodox; a physician, a university professor, who slips into Averroist Pantheism; a philosopher, no less a great admirer of Averroes, but foreign to the Universities; earlier, we found it both in a Venetian prince of the Church, as Gaspard Contarini, and in a Milanese humanist, as Vicomercati. The state of mind that it characterizes is surely very general in 16th century Italy.

Despite this resistance, the principles that Parisians gave to the study of Dynamics sometimes managed to creep into the Italian Science; toward the middle of the 16th century, we have seen them slip into the writings of Alessandro Piccolomini; during the last quarter of that same century, we find them in the work of Bernardino Baldi and in that of Gianbattista Benedetti.

It was in 1582 that Bernardino Baldi had written his *Exercises on the Mechanical Questions of Aristotle*. This paper was printed only in 1621, twenty-eight years after the death of its author.

We have previously studied the *Exercitationes* that the Abbot of Guastalla composed; we reported there[66] the particularly recognizable mark of Da Vinci; we also said[67] how certain ideas that Baldi held from Leonardo drew the attention of Mersenne and caused Boberval and Descartes to make important discoveries.

If Baldi conceals the influence he has proven on the part of Da Vinci, he admits that which Alessandro Piccolomini exerted on him. It is in a question[68] where one finds the *Paraphrase* of Piccolomini cited with praise that we read this passage:

> Projectiles cease to move because the impression that the *impetus* and virtue give them is not a natural projection; it is purely accidental and violent; however, nothing of what is accidental and violent, nothing of what is not natural, can be perpetual. This accidental impression thus ends. While it ceases gradually, the movement of the projectile languishes and the body finally comes to rest.

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[66] *Léonard de Vinci et Bernardino Baldi* (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, III; première série, pp. 89, seqq.).

[67] *Bernardino Baldi, Boberval et Descartes* (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, IV; première série, pp. 127, seqq.).

[68] Bernardini Baldi Urbinatis Guastallæ Abbatis *In Mechanica Aristotelis problemata exercitationes: adjecta succincta narratione de autoris vita et scriptis*. Moguntiæ, Typis et Sumptibus Viduæ Joannis Albini. MDCXXI. Quæst. XXXII: Quæritur hic, cur ea quæ projiciuntur, cessent a latione? P. 279.

Baldi does not only attribute to the *impetus* the continuation of the movement of projectiles; with the Parisians and Piccolomini, he attributes[69] the acceleration of falling bodies to a continual increase in this *impetus*.

> By that, this question, which the physicists considered very difficult, is resolved: Why, in natural movement, does the speed constantly increase? Here, indeed, it is the nature that moves; as it is inseparable from the mobile, it presses it continuously, first slowly, then, by the said cause, more and more quickly. The movement is thus produced in the movement itself; and as this movement is still increased by both the mover and the movement, it progresses to infinity. Nobody, I think, will deny that the cause of this acceleration is that the moving power moves the mobile while it is already in motion. Indeed, the body moved accidentally acquires a certain gravity; and as this weight is increased by movement, it makes this movement easier and faster.

We have mentioned elsewhere[70] how Baldi extended this explanation to the alleged acceleration that a projectile would experience early its course. We will not return to this theory.

It seems that, in the passage of which we just given the translation, Baldi identifies accidental gravity with movement itself; the movement is treated there as a driving power; and this opinion, which is what Ockham sustained, seems consistent with the thinking of even the author of the *Mechanical Questions*.

Stating that thought, Bernardino Baldi does not hesitate to look at movement as a motive power, but also at rest as a resisting power. A few lines before the passage just quoted, he writes[71]:

> The resistance of the object that passed from the resting state to the state of motion is similar to a certain movement in the opposite direction. The opposite happens to the one who moves a mobile that is already in motion; in this case, it is greatly assisted by the very movement of the mobile; the movement cooperates with the action that the mover exerts on the mobile. The mobile increases to some extent the power of the mover; what this mobile would feel on the part of the mover, it does on its own.

These lines carry the mark of an influence other than that of Piccolomini; they recall very exactly, in fact, a passage that, on the subject of the same question, Cardan wrote in his *Opus novum de proportionibus*[72]:

> Imagine,

Cardan said,

> a body in equilibrium, resting, for example, on the floor; if we want to lift it, it will oppose the violent movement with a certain resistance. Why is that? Because it moves with some occult natural movement; the power of this movement measures the force with which the body resists with contrary movement.

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[69] Bernardino Baldi, *Op. cit.*, quæst. XXXI: Cur facilius moveatur commotum quam manens, veluti currus commotos citius agitant, quam moveri incipientes? Hoc quæritur. Pp. 278-279.

[70] *Bernardino Baldi, Roberval et Descartes*: I. Une opinion de Bernardino Baldi touchant les mouvements accélérés (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, IV; première série, pp. 138-139).

[71] Bernardino Baldi, *loc. cit.*, pp. 177-178.

[72] Hieronymi Cardani *Opus novum de proportionibus*, prop. XXXVIIII, p. 41.

> We understand, therefore, why ships and tanks are moved, first slowly and with difficulty; when then they began to move, their movement becomes faster; they in effect resist by the occult natural movement, and it had its greatest intensity when they were at rest, as taught by Aristotle in his Mechanics; this occult movement is, indeed, a natural movement and contrary to the violent movement. When the body began to experience violent movement, it has a lower natural movement and it resists less.

Galileo was one day to accommodate these thoughts of Cardan and Bernardino Baldi on the setting in motion of a body that is at rest[73].

The theory of the accelerated fall of weights given by the Abbot of Guastalla presents a reflection of the Parisian theory; but this reflection is singularly distorted and obscured. It is under an otherwise clear and sharp form that we recognize, in the writings of Giambattista Benedetti, the principles of the Dynamics that Jean Buridan and Albert of Saxony taught.

These writings, probably composed at different times unknown to us, were gathered by the author, in 1585, under the title *Various Speculations of Mathematics and Physics*[74]; it is in this collection that we find frequent borrowings from the Mechanics of the Parisians.

The movement of projectiles that leave the mover that launched them is always attributed to an *impressio impetus*[75], to a *natural impression* or to an *impetuosity received* by the mobile.

This *impetus* initially moves the body in a straight line; then, when it is sufficiently weakened, the force of gravity begins to exert its action and to divert the mobile to the straight path.

> This *impetus impressus*[76] decreases little by little and continually; then the inclination of gravity of the body creeps into it, gradually mixing with the acquired impression; it does not permit the path to remain straight for long; it forces it to curve; the body is driven simultaneously by two virtues: the impressed violence, on the one hand; and nature, on the other; and this is against the opinion of Tartalea, who denied that a body could be animated at the same time with violent and natural movements.

The opinion that Benedetti supports here contradicts, in fact, the one that Tartaglia sets out in his *Nova scientia*, but it is consistent with what the same geometer professed in his *Quesiti et inventioni diverse*, which is that of Leonardo da Vinci, Piccolomini, and Cardan.

Benedetti very clearly said that a constant mover must generate an accelerated motion:

> In natural and rectilinear motion,

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73 Galilei *De motu* (*Le opere di* Galileo Galilei, *ristampate fedelmente sopra la Edizione nazionale*. Vol. I. *Juvenilia*. *Firenze, successori* Le Monnier, 1890 [Galilei et al (1890)], p. 318).

74 Io. Baptistæ Benedicti Patritii Veneti Philosophi. *Diversarum Speculationum Mathematicarum, et Physicarum Liber. Quarum series sequens pagina indicabit. Ad Serenissimum Carolum Emmanuelem Allobrogum, et Subalpinorum Ducem invictissimum*. Taurini, Apud Hæredem Nicolai Bevilaquæ, MDLXXXV.

75 Benedetti, *Op. cit.*, *De Mechanicis*, cap. XVII, p. 160. — *Disputationes de quibusdam placitis Aristotelis*, cap. XXIV, p. 184. — *Responsa physica et mathematica*, p. 287.

76 Benedetti, *Op. cit.*, *De Mechanicis*, cap. XVII, p. 160.

he said[77],

> the *impressio*, *impetuositas recepta*, grows continuously because the mobile has in itself the motive cause, that is to say, the propensity to travel to the place assigned to it; Aristotle would not have had to declare that a body is more rapid the closer it is to its goal (*terminus ad quem*), but rather that the body is speedier the farther away it is from its starting point (*terminus a quo*). Because the *impressio* grows gradually as the natural movement prolongs, the body continually receives a new *impetus*; in fact, it contains in itself the cause of movement, which is the tendency to regain its natural place out of which violence has placed it.

Elsewhere[78], treating the movement of the wheel used to hoist a bucket out of a well, Benedetti wrote:

> Any heavy body that moves naturally or violently receives in itself an *impetus*, an impression of movement, so that, separated from the moving virtue, it continues to move by itself for a certain period of time. When, therefore, this body moves with natural motion, its speed will increase constantly; indeed, the *impetus* and *impressio* that exist in it will grow constantly because it is constantly united to the moving virtue. Thence also it follows that if, after having set the wheel in motion with the hand, one removes his hand, the wheel does not stop immediately; it continues to rotate for a certain time.

The most knowledgeable writers in the history of mechanics generally attributed to John Baptist Benedetti[79] this explanation of accelerated motion produced by a persistent mover. We know how far away from the truth this opinion is. This explanation was known to Walter Burley in the first half of 14th century; in the middle of that century, Jean Buridan and Albert of Saxony taught it; it was widely accepted at the University of Paris in the early 16th century; Scaliger, in the middle of 16th century, strongly criticized Cardan for not supporting it; Benedetti had absolutely no part in the creation of this theory; but he is the first who, in Italy, frankly and completely accepted this doctrine; Alessandro Piccolomini and Bernardino Baldi paraphrased it rather than clearly formulating it.

Did Benedetti know the theory that Bernardino Baldi proposed to account for the alleged acceleration that a projectile would experience early in its trajectory? It is difficult to answer this question conclusively. But it deserves to be remarked: Benedetti offered the same explanation as Baldi, while indicating that he did not take for granted the phenomenon to which he claims it applies. It is in a letter where our author corrects various errors of Tartaglia that the following passage is found[80]:

> The reason that Tartaglia invokes… is absolutely vain; the air which was originally trapped in the cannon is immediately cast out; it yields to the bullet; it is divided by this body… The

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[77] Benedetti, *Op. cit.*, *Disputationes de quibusdam placitis Aristotelis*, cap. XXIV, p. 184.

[78] Benedetti, *Op. cit.*, *Physica et mathematica responsa*, p. 287.

[79] Emil Wohlwill, *Die Entdeckung der Beharrungsgesetzes (Zeitschrift für Völkerpsychologie und Sprachwissenschaft, XVIter Band, p. 394).*

Giovanni Vailati, *Le speculazioni di Giovanni Benedetti sul moto dei gravi* (*Rendiconti dell'Accademia Beale delle Scienze di Torino*, 1897-1898).

Ernst Mach, *La Mécanique, exposé historique et critique de son développement*; Paris, 1904, p. 120.

[80] Io. Baptistæ *Benedicti Diversarum speculationum liber; Physica et mathematica responsa*, p. 209.

> ball moving faster a certain distance than at the beginning of its course, if that were true, would depend on another cause; this cause would be partly similar to that which, in natural movements, makes bodies especially faster the farther they are from the term from which they started to move naturally; along a certain distance, this body would move in the same way as if it were carried by its natural movement.

Like Bernardino Baldi, Benedetti believes he can give to the Parisian theory an illegitimate extension, against which John Dullaert protested in advance; he will be better inspired by some other propositions which he will relate to this same theory.

Chapter 11
On the early progress accomplished in Parisian Dynamics by the Italians

Giovanni Battista Benedetti

The day an Italian mathematician, repudiating the routine of the Peripatetics and Averroists, dared to receive the principles of the Parisian Dynamics in their fullness, his genius, exercised with precision from studying Euclid and Archimedes, made them produce fruits they had not yet born. Benedetti first brought an important complement to the doctrines of Buridan and Albert of Saxony.

Let us recall the passage[1] where Albert of Saxony presents an idea particularly dear to the Philosopher of Béthune:

> Suppose one quickly rotates a very large and heavy blacksmith's wheel, then ceases to move it. It continues to turn a very long time, which cannot be done, it seems, by some intrinsic *impetus* that was acquired, which was imprinted on it by the one who set in motion. If one stops turning the wheel, its movement decreases continuously and finally stops, because the natural form of the wheel has a tendency opposed to this movement… And, perhaps, if this wheel thus set in motion could last forever, without experiencing any decrease or change, if there were no longer any resistance capable of corrupting the *impetus* that was generated, perhaps, I say, that wheel would be perpetually moved by this *impetus*. If this assumption were granted, it would not be necessary to assume that intelligences move the celestial orbs. One might say, indeed, that God, when he created the heavenly spheres, began to move each of them as it pleased him, and they still move through the *impetus* that God then gave them; in this body, this *impetus* undergoes neither corruption nor diminution because the mobile has no inclination opposing the movement that carries it.

Albert of Saxony, Jean Buridan knew, recognizes only two causes that can destroy the *impetus*: the natural form, which would incline the mobile to an opposing movement, and external resistance, such as air resistance and friction of the medium. In an exactly centered grindstone, the weight would not oppose rotational movement; without the resistance of the air, without friction of the axle on the bearings, this movement would last indefinitely.

[1] Magistri Alberti de Saxonia *Subtilissimæ quæstionis in libros de Cælo et Mundo*, lib. II, quæst. XIV.

To this proposition, which is very justified, Benedetti does not want to subscribe; but to support his denial, which is an error, one must draw up an essential truth that nobody, it seems, had yet clearly seen[2].

Benedetti does not want the movement of the grinding wheel to be perpetual, not even in the ideal conditions that Albert of Saxony imagined; it is thus necessary to discover the substance of this stone, which is an intrinsic cause of resistance to rotational movement, a cause capable of corrupting the *impetus*; and here, according to him, is the cause:

> This is not a movement of rotation; the *impetus* of each of the small parts of the wheel would drive them in a rectilinear motion, if it were free; during the rotational movement, each of these partial *impetus* is violated and therefore it corrupts.
>
> Imagine,

said Benedetti[3],

> a horizontal wheel, as perfectly equal as possible and resting on a single point; imprint on it a rotational movement with all the force we can muster, then abandon it; how is it that its rotational movement will not be perpetual?
>
> This occurs for four causes.
>
> The first is that such a movement is not natural to the wheel.
>
> The second consists in the fact that the wheel, even when it rests on a mathematical point, necessarily would require, above it, a second pole capable of holding it horizontal, and this pole should be realized by some physical mechanism; the result would be a certain friction, from where the resistance would come.
>
> The third cause is due to the air adjacent to the wheel that brakes it continually and, by this means, resists the movement.
>
> Now comes the fourth cause: Consider each of the corporeal parts which moves with the *impetus* that has been impressed on it by an extrinsic motive virtue. This part has a natural inclination to rectilinear movement, and not to curvilinear movement; if a particle on the circumference of said wheel were severed from the body, it is not unlikely that, for some time, this detached part would move in a straight line through the air; we can recognize this in an example from slingshots, with the help of which one throws stones; in these slingshots, the *impetus* of the movement, which was imprinted in the projectile, describes, by a kind of natural propensity, a straight path; *the stone launched begins a straight path following the line that is tangent to the circle that it described, first of all, and which touches it at the point where the stone was when it was abandoned*, as it is reasonable to admit.
>
> The same reason make it that the larger a wheel, the greater is the *impetus* or the impression that the various parts of the circumference of this wheel receive; it also often occurs that when we want to stop it, we cannot without effort or difficulty; in fact, the greater the diameter of a circle is, the less curved the circumference of this circle is… The movement of the parts which lie on the said circumference thus approach all the more the movement in conformity with the inclination that nature has assigned to them, an inclination which consists in moving it along the straight line.

These thoughts certainly pleased Benedetti; he returns to them twice; moreover, he completes and specifies them in these two circumstances, together with the affirmation of an important truth; this tendency of the mobile, moved in a circular

[2] Giovanni Vailati is, we believe, the first who reported these discoveries of Benedetti (Giovanni Vailati, *Le speculazioni di Giovanni Benedetti sul moto dei gravi. Accademia Reale delle Scienze di Torino*, anno 1897-1898).

[3] Jo. Baptistæ Benedicti *Diversarum speculationum liber*; *De mechanicis*. cap. XIV, p. 159.

motion, to escape following the tangent to the curved trajectory is the cause of the rope of a stretched sling pulling the hand that holds the rope.

Benedetti formulates the latter proposition in the same letter[4] where he explained, according to the Parisian Dynamics, how the movement of a wheel that a constant power turns accelerates:

> Any heavy body moving either by nature or by violence naturally wants to move in a straight line; we can clearly recognize it when we use our arm to shoot stones with a slingshot; the strings acquire an even greater weight and pull even harder on the hand the faster the hand shoots the slingshot and the faster the movement is; it comes from the natural appetite which has its seat in the stone and that pushes it to go in a straight line.

The same truth is expressed again, and almost in the same terms, in the following passage[5], which also relates to the operation of the sling:

> The hand turns, as much as possible, in a circle; this circular movement of the hand forces the projectile also to take a circular motion, while, by its natural inclination, this body, from where it received a slight *impetus*, would like to continue its path in a straight line… Let us not pass over in silence a fact worthy of remark, which occurs in this circumstance. The more the speed of the gyrating movement makes the *impetus* of the projectile increase, the more the hand is felt pulled by this body, and this by means of the cord; the greater, in fact, the *impetus* of movement which is imprinted on the body, the more powerful is the inclination of the body to move in a straight line, and the greater also is the force with which he pulls it in order to accomplish this movement.

Buridan and his followers had accepted that an *impetus* imprinted in a body may, according to the manner in which it was generated, tend to move the body in a straight line or in a circle; Benedetti, meditating upon the teaching of these philosophers, rectifies what he held to be erroneous; when very little body is free, the *impetus* always tends to move it in a straight line; in a large body, the connections of the various parts may impose curved movements on them, but it is the result of pressures or tractions which testify to the effort exerted by each element following a straight trajectory. By attributing to these *linking actions* the power to detract from the perpetuity of a rotating motion, Benedetti wrongly contradicted one of the most beautiful and important propositions of Buridan and Albert of Saxony; they discovered one of the facets of truth; Benedetti clearly saw another; the future of Mechanics had to show the exact position that the two partial truths occupy in the whole truth.

How did Benedetti manage to make these such important and precise discoveries? An interesting passage from one of his letters will inform us with the steps of his thought.

Here is what Benedetti wrote to Paul Capra de Novara[6]:

> You ask in your letters if the circular motion of a millstone, which was once launched, could last forever, if this wheel rests, so to speak, on a mathematical point and where it is supposed perfectly round and perfectly polished.

4 Jo. Baptistæ Benedicti *Diversarum speculationum liber; Physica et mathematica responsa*, p. 287.

5 Benedetti, *Op. cit.*, *De mechanicis*, cap. XVII, pp. 160-161

6 Io. Baptistæ Benedicti *Diversarum speculationum liber; Physica et mathematica responsa*, pp. 285-286.

> I say that such a movement cannot be perpetual and even that it cannot last long; firstly, the air, which exerts a certain resistance on the periphery of the grinding wheel, restrains it; but, in addition, it is restrained by the resistance of the parts of the mobile themselves. Once these parts are put in motion, they have an *impetus* which naturally leads them to move in a straight line; but as they are joined together, that they carry one another, they suffer violence when moving in a circle; it is by force that in such a movement they remain united among themselves; the more their movement becomes faster, the more this natural inclination to move in a straight line is increased and the more the obligation to turn in a circle is contrary to their proper nature. So that they remain in their natural rest, as their own tendency is to move in a straight line when launched, it is necessary that however much each one of them resists the other, that each of them pulls back, so to speak, more strongly the one before it, the faster the rotational movement is.
>
> Due to this inclination that the various parts of a round body have to straight movement, sometimes the sabot that turns itself with great violence remains for a certain period of time perfectly straight and resting on the point of iron with which it is armed; no more on one side than on the other, it inclines towards the center of the World; in this movement, indeed, none of its parts tilts toward the center of the World; each of them inclines rather to move in a transverse line, perpendicular both to the direction or vertical line and to the horizontal axis; necessarily, thus, such a body must remain straight.
>
> When I say that these parts do not incline towards the center of the World, I only say it in this respect; never, in fact, are they absolutely deprived of this inclination, and this is why the body makes effort in its point of support. It is true, however, that the more the sabot rotates with speed, the less it presses the fulcrum and the lighter the body becomes.
>
> This is clearly seen if we take the example of a ball launched by a crossbow or some other ballistic instrument or machine. Moreover, in its violent motion, the ball is fast the greater its tendency to go straight, the lower is its inclination to go to the center of the World; by this cause it is made lighter.
>
> If you want to see this truth more clearly, imagine that this body, the sabot, moving with a very fast rotation, is cut or split into many parts. You will find that these various parts do not immediately go down to the center of the World, but that they move, so to speak, straight along a horizontal line. No one, to my knowledge, has yet made this observation about the movement of a sabot.
>
> This movement of the sabot or another similar body shows us how the Peripatetics are wrong about violent movement; in effect, they think that the body is pushed by the air which rushes in to occupy the place left by the mobile; it is rather the contrary effect that is born from this movement of the air.

We remember having read[7], in the *Exercitationes* of Bernardino Baldi, of considerations similar to some of those we have just transcribed, and when we met them in the book of the Abbot of Guastalla, we have not hesitated to indicate their origin; they are, we have said, thoughts of Leonardo; we must repeat this judgment here and make it more formal, because the seal of Da Vinci is even more clearly printed in what Benedetti just outlined.

The fundamental idea from which all the reasonings of Benedetti derive is this: The *impetus* caused by violence is analogous to the natural gravity; the *impetus*, when it acts alone, like natural gravity when it acts alone, moves the mobile in a straight line:

> Any heavy body moving either by nature or by violence naturally wants to move in a straight line.

[7] *Léonard de Vinci et Bernardino Baldi*, IV (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, III; première série, pp. 100-115.

But this thought is, in the Dynamics of Da Vinci, an essential principle.

At the end of its movement, a projectile describes a straight path, because it is so moved by nature, without any admixture of violence:

> The arrow will not move in a perpendicular line[8], and if you find it so, it is a sign that the violent movement ended and that it entered into natural movement—i.e., being heavy it freely fell to the center.

At the beginning of the movement, the trajectory is straight, because the *impeto* then destroys the natural gravity; the accidental gravity alone remains, and it weighs in the direction in which the mover has launched the mobile; the ball that the canon, pointed horizontally, fired, moves along a horizontal line because violence has made it lose its natural gravity directed along the vertical:

> Any weight that moves along the position of equality weighs only along the line of its movement[9]. It is proved in the first part what makes the movement of the cannonball, a movement that is in the position of equality.

It is only natural to compare this phrase of Leonardo to that of Benedetti: The

> faster, in its violent movement, the ball is, the greater is its tendency to go straight and the less is its inclination to go to the center of the world; by this cause it is made lighter.

Among the thoughts of the two authors, there is one nuance to report. Leonardo admits that the first part of the trajectory is purely rectilinear, because then, according to him, the violence completely annihilates the natural gravity. The *impetus*, according to Benedetti, attenuates this natural gravity without destroying it completely, if it is violent; also, the trajectory, as close to the straight line as that the movement is faster, does not ever reach this straight line. Here Benedetti corrects the thought of Da Vinci, as did his master Tartaglia.

For Leonardo, then, and for all those who appear to have been influenced by him—by Tartaglia, Cardan, Bernardino Baldi, and Benedetti—the purely violent movement is rectilinear, like the purely natural movement.

> On movement in general,

Leonardo wrote in one of his notebooks[10].

> What movement in itself is. — What that which is put more into action by movement is. — What *impeto* is. — What the cause of *impeto* and the medium where it is created are. — What percussion is. — What the cause of this is. — What the curvature of straight movement and its cause are. Aristotle, 3rd book of *Physics*, and Albert, and Thomas, and others; of reflected movement *from* (*risaltatione*) in the 7th book of the *Physics*.

The principles we have just mentioned confront us, in fact, with this question: What is the cause that determines the curvature of the trajectory described by a projectile, by the various parts of a mobile far from its mover? This cause is that the

[8] *The manuscripts* of Leonardo da Vinci; ms. A of the Bibliothèque de l'Institut, fol. 4, recto.

[9] *The manuscripts* of Leonardo da Vinci, ms. G. de la Bibliothèque de l'Institut, fol. 77, recto.

[10] *The manuscripts* of Leonardo da Vinci, ms. I of the Bibliothèque de l'Institut, fol. 130, verso.

mobile is not influenced by a purely natural gravity or a simple *impeto*; it resides in the fact that the *impeto* is compound.

A preliminary form of the compound *impeto* is evident in what was said. It results from the struggle between the simple *impeto* that launched the projectile and the natural gravity of this same projectile. It is a composed *impeto* of this kind which—according to Leonardo, Cardan, and Bernardino Baldi—curves the middle part of the trajectory of a projectile, which, according to Tartaglia and Benedetti, curves the trajectory in its entirety.

Besides this kind of compound *impeto,* Leonardo has identified a second compound species[11]. In this new *impeto,* whose existence seems to have been revealed by the game of the globe that Nicolas of Cusa described, the form of the mobile intervenes; there is a conflict between the *impeto* impressed by the mover and what Leonardo calls the *impeto* of the mobile.

Da Vinci gives this mobile *impeto* an extreme importance in the theory of the flight of birds; but it does not appear that he ever managed to formulate a clear idea about it. It is that notion, which remained obscure in Leonardo, which Benedetti specified in several passages that we have quoted. Each of the parts of a mobile which move in a gyrating movement is the center of a conflict between two trends, firstly, the simple *impetus*, which tends to cause the particle to follow the straight line; and then a reaction, a result of the link which unites this part to the adjacent parts, a reaction that opposes the continuation of the movement in a straight line.

What indications did Benedetti find on the subject of these two elements of compound *impeto* in the science of his predecessors?

We have seen that Leonardo attributed formally to simple *impeto* the property of moving the mobile in a straight line; he deduced this result, which Benedetti so formally states: Each of the parts of a mobile moving rotationally would escape at once in a straight line if we broke the ties that unite this part to the rest of the body; would this straight line be the last tangent to the curvilinear trajectory that this part described before it was freed? Da Vinci certainly succeeded, but after much trial and error, to recognize at least the first part of this law; reading his manuscripts prove it for us.

Here is a preliminary fragment in which[12], instead of the true law, an erroneous law is stated:

> Everything moved violently in the air will follow the line of movement of its mover. If someone moves the thing in a circle and it is launched in its movement, its movement is curved; and if the movement started in a circle and finished in straightness, its course will be in straightness.

[11] *Nicolas de Cœs et Léonard de Vinci,* XI: La Dynamique de Nicolas de Cues et la Dynamique de Léonard de Vinci. Theory of *impeto* (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, XI; seconde série, pp. 215-223).

[12] *The Manuscripts of Leonardo de Vinci,* published by Ch. Ravaisson-Mollien, Ms n° 2038, Italian, of the Bibliothèque nationale (Acq. 8070 Libri), folio 1, verso. Paris, 1891.

A second fragment[13] testifies of the doubts of Leonardo on the subject of the law that concerns us. The first of the two sentences that comprise this fragment is crossed out in the manuscript.

> So much as the mobile retains in itself some acquired *impeto*, it follows the straightness of the line of the mover.
>
> By the cause a curved route is given to a mover, the thing that separates flees by the line…

A last fragment[14], finally, contains some affirmations very close to the truth:

> *On circular motion*. But the circular motion of uniform speed will drive the mobile as much with one whole revolution as with several.
>
> But it will drive it in creating the first movement all the more that this creation is nearer to its integrity; and the movement of its mobile will not witness such a circular motion, after it is divided from the wheel, but it follows the straight motion.

There is, in this note, a draft of what Benedetti will say with much more precision. It should be noted that this note is found in book E where Leonardo, in studying the game of the globe, is led to the notion of compound *impeto*.

Reading the notes of Leonardo thus led to admitting this truth first formulated by Benedetti: In a body moving rotationally, each part tends, at every moment, to move in a straight line.

To this first truth, the Venetian Geometer joins a second: What opposes the continuation of this movement in a straight line is a force that pulls the particle toward the center of the circle whose circumference it describes. The smaller this circle is, this greater this force is.

This new proposition was virtually dictated by Benedetti in a book he had thoroughly analyzed and discussed, by the *Mechanical Questions* of Aristotle. From a very similar proposition, in fact, Aristotle or the author, whoever he is, derived from these *Questions* the law of the lever, to which he brought back most Mechanics problems[15]: The lever, instead of allowing the weight of to move in a straight line, makes it move in a circle; this constraint is exerted by the force emanating from the center of the circle; it is as much greater as the path opposed to the weight moves further away from straightness, as the circle described by this weight is smaller.

This doctrine had mixed fortunes. Admitted more or less loosely by the Peripatetic commentator of Jordanus of Nemore[16] and Blaise of Parma[17], it was ingeniously refuted by Leonardo da Vinci[18]; but Guidobaldo dal Monte has resumed it[19] in 1677, thus when Benedetti pondered Mechanics.

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[13] *The Manuscripts of Leonardo da Vinci*, published by Ch. Ravaisson-Mollien, ms. I of the Bibliothèque de l'Institut, fol. 98 [50], recto. Paris, 1889.

[14] *The Manuscripts of Leonardo de Vinci*, published by Ch. Ravaisson-Mollien, ms. E of the Bibliothèque de l'Institut, fol. 29, recto. Paris, 1888.

[15] See, on this, our *Origines de la Statique*, chap. VI, t. I, pp. 108-110 [Duhem (1991, 81-83)], and t. II, note A, pp. 298-301 [Duhem (1991, 455-457)].

[16] *Les Origines de la Statique*, t. I, p. 134 [Duhem (1991, p. 98)].

[17] *Ibid.*, t. I, p. 150 [Duhem (1991, p. 109)].

[18] *Ibid.*, t. I, pp. 160-161 [Duhem (1991, 116-117)].

[19] *Ibid.*, t. 1, p. 218 [Duhem (1991, p. 154)].

In truth, the considerations of Aristotle or Guidobaldo were related to a mass which is urged to move rectilinearly by its natural gravity and not by a violently imprinted *impetus*; but the assimilation of natural gravity and accidental gravity, accepted by most of the mechanists and, in particular, by Benedetti, easily led from the former case to the latter.

Of the truth that the Venetian Geometer formulated with a sort of predilection, the elements were thus, for a long time, more than glimpsed and half-clear; there remained, however, to gather and compose a clear and precise proposition; this is what Benedetti did, and the merit of having accomplished such a task need not be put at too high a price.

Benedetti appears as an opponent of the peripatetic Physics.

His treatise *De mechanicis* follows step by step the *Mechanical Questions* of Aristotle in order to criticize, correct, and supplement them.

Another of his works is entitled: *Disputationes de quibusdam placitis Aristotelis*. We know from the testimony of the author[20], that this writing was composed from 1553. Benedetti prefaces it with this short statement[21]:

> The importance and authority of Aristotle are so great that it is dangerous and very difficult to write anything against what he taught; it is especially so for me, for whom the wisdom of this great man was always admirable. Impelled, however, by the study of truth, whose love would arm Aristotle against himself if he were still alive, I have not hesitated to publish certain conclusions contrary to the opinion of the Philosopher; the philosophy of Mathematics, which I still maintain as an immovable base, forced me not to share his thought.

By his doctrines contrary to those of Aristotle, Benedetti found himself numbered among the opponents of the Italian Scholasticism, so firmly attached, even at that time, to peripatetic and Averroist principles. His thoughts were not as strongly antagonistic to the teachings of the Parisian Scholasticism.

He found[22] the doctrine of Aristotle concerning the infinite; he argued, for example, that an infinite body could presently extend out of the sky; the infinitely many parts of a continuum have an actual existence; that the multitude presently infinite is conceivable just as well as the finite number and is, as well, a kind of quantity. All these assertions would seem frightful heresies to Alexandrists or Italian Averroists. But in what way had they offended in the slightest the Parisian Nominalists? Were these propositions not sustained from the beginning of 14th century by John of Bassols, then, in the course of the 14th century, by Gregory of Rimini, the subtle and powerful logician, and by Robert Holkot? In the first half of 16th century, did not John Majoris and his students formally adopt them? At the Sorbonne, on Fouarre Street, and at Montaigu, they would have met with supporters and opponents, but they would not have frightened nor surprised anyone.

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[20] *Resolutio omnium Euclidis problematum aliorumque ad hoc necessario inventorum una tantummodo circini data apertura, per Ioannem Baptistam de Benedictis inventa*. Venetiis MDLIII. In fine: Venetiis apud Bartholomæum Cæsarum. MDLIII. Dedicatory letter to Gabriel de Guzman, sixth unpaged folio, verso.

[21] Io. Baptistæ Benedicti *Diversarum speculationum liber*, p. 168.

[22] Io. Baptistæ Benedicti *Diversarum speculationum liber; Disputationes de quibusdam placitis Aristotelis*) cap. XXI, p. 181.

Benedetti, moreover, shows himself, in many respects, to be a disciple of the physicists of Paris. His Dynamics had, with that of John Buridan and Albert of Saxony, a close kinship. He also admitted the principle of Statics formulated by Albert of Saxony; after recalling the definitions of the center of gravity proposed by Pappus and Gommandin, he added[23]:

> Others say that the center of gravity of each particular body is the point whereby this body would unite to the center of the Universe, if it were not prevented; and all agree in the fact that the Earth is united at the center, properly called, of the Universe itself by the intermediacy of its center of gravity.

It is from Logic and the Parisian Physics that in Italy the originators of modern science borrow some weapons to fight the antiquated teachings of the Philosopher and Commentator; those who strive to shake off the yoke of the tyrannical routine have their eyes set on Paris, whose Nominalist Scholasticism is, for centuries, in possession of intellectual freedom.

Giordano Bruno

When Benedetti printed his *Various Speculations*, the bitterest and most famous opponent of Aristotelian physics is undoubtedly Giordano Bruno. So it is in Paris, at the College of France, that Bruno came to teach his doctrines. In 1585, when the *Diversæ speculationes* appear, he returned to Paris after a trip to London. It is from Paris that he sent to all the universities a kind of cartel where he formulates a long series of propositions contrary to those Aristotle teaches in the eight books *De physico auditu* and two books *De Cælo and Mundo*[24].

Bruno did not, without doubt, expect to see it chivalrously and courteously debated among Averroist Universities of Italy. But from the Parisians, he expects a better reception.

> I would not have offered to you the discussion of these articles,

he writes[25] in his letter to the Rector John Filesac,

> if I could believe that you were ready to approve perpetually the peripatetic discipline as if it were more than true, thinking that your University is more indebted to Aristotle than Aristotle is indebted to the University.

[23] *Consideratione di* Gio. Battista Benedetti. Filosofo del Sereniss. S. Duca di Savoia. *D'intorno al Discorso della grandezza della Terra, et dell' Acqua. Del Eccellent. Sig. Antonio Berga Filosofo nella università di Torino*. In Torino. Presso gli heredi del Bevilacqua, 1579, p. 18.

[24] Jordani Bruni Nolani *Camæracensis Acrotismus seu Rationes articulorum physicorum adveмnus Peripateticos Parisiis propositorum…* Vitebergæ, apud Zachariam Cratonem. Anno 1588. — Reprinted in: Jordani Bruni Nolani *Opera latine conscripta* recensebat F. Fiorentino. Vol. I, pars I. Neapoli, 1879. Our quotations and references relate to this reprint.

[25] Jordani Bruni *Opera latina*, vol. I, pars 1, p. 57.

Certainly, Giordano knows universities where there is this superstitious respect for Peripateticism, but he knows that Paris has been free of it, and that is why he will sustain his articles in Paris and nowhere else.

And indeed, in 1586, at the time of the feast of Pentecost, for three days, Jean Hennequin, *nobilis Parisiensis*, stood at the College of Cambrai, where courses of the Royal College were given, ready to defend against any peripatetic who would face the hundred twenty articles of the philosopher of Nola.

What was the opinion of the Parisian Scholastics of this debate? We do not know.

No doubt, in the clear and simple form up to brutality that Giordano Bruno gave his arguments, they found neither the complicated Dialectic nor the style bristling with technical terms which they had been accustomed to use; the thought of the philosopher of Nola was dressed in a mode different than theirs. But if they parted from this garment, did they not find, in this thought reduced to its essential nakedness, a group of traits related to their own ideas? Very deeply, the philosophy of Bruno was imbued with the doctrines of Nicolas of Cusa, and these, in their turn, were often penetrated with the teachings that the Parisian Nominalists were giving at the time of the German Cardinal. Bruno, however, had not been able to keep for the thought of the Bishop of Brixen all the delicate flexibility with which he penetrated into the heart of metaphysical problems; he had simplified it and, as it were, made it more massive and coarser. However, this transformation, by often distorting the ideas of Nicolas of Cusa, was close to the ones that at the beginning of the 16th century were maintained at Montaigu or St. Barbara. The Parisian Nominalists could meet, in the cartel of Bruno, many of the propositions that they also supported, and for a long time, against the Philosopher and the Commentator.

Let us follow, in some of its features, the comparison that was no doubt on their mind.

Nicolas of Cusa had taught that the world is neither finite nor infinite. His *learned ignorance* had respectfully bowed before this antinomy[26].

Such a prudent reserve does not fit the impetuous dogmatism of Giordano Bruno.

> We say[27] that the Universe is an infinite substance in an infinite space, that is to say, in an infinity both empty and full. The Universe is one; but the worlds are innumerable; each of the bodies of the World, indeed, is of finite size, but taken together they are numerically infinite.

These propositions dominate, we can say, the philosophy of Giordano Bruno.

Nicolas of Cusa would not have confessed them as his own; but the disciples of the Parisian Nominalists were not very far from receiving them; what the Philosopher of Nola, indeed, gave as real, many of them held as possible.

Duns Scotus was the first[28] to challenge certain reasons that denied the possibility of the actual infinitely large; his "audience", John of Bassols[29], had formally

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[26] *Nicolas de Cues et Léonard de Vinci*, III: Esquisse du système philosophique de Nicolas de Cues (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, XI; seconde série, p. 112).

[27] Jordani Bruni Nolani *Camœracensis acrotismus*, art. LX (Jordani Bruni *Opera latina*, tomus I, pars I, p. 173).

[28] *Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*; seconde série, p. 454.

[29] *Ibid.*, pp. 373-378.

affirmed the possibility of such an infinite; Gregory of Rimini[30] and Robert Holkot[31] rigorously developed the teaching of John of Bassols.

This tradition of Bassols and Rimini was also alive in the University of Paris at the beginning of 16th century. John Majoris stated emphatically[32] that the present realization of an infinite magnitude involves no contradiction, and it is in the power of God. His disciple, the Ghent John Dullaert, followed, in this matter, the doctrine of his master.

The eclectic Juan de Celaya shared, on the subject of the actual infinitely large, the opinions of Johannes Majoris.

In his work on the *Physics*, when he treats *de infinito supranaturaliter loquendo*[33], Celaya begins by reporting the opinion of St. Thomas Aquinas, who refuses God the power of actualizing the infinitely large; he prefers “the opinion of Gregory of Rimini and of other moderns that magnify the power of God”; this view implies these propositions:

> God can produce an infinite number of beings that do not constitute a continuous whole… God can produce an infinite magnitude… God can produce a form of infinite intensity…

After having discussed at length the arguments that countered these propositions, Celaya added:

> Some are wont to confront these conclusions with the authority of the Philosopher and with that of the Commentator, but once there is a question of the power of God, these opinions can in no way be received.

The problem of the infinite seems to have long preoccupied Luis Coronel[34] without his meditations firmly connecting it to the one of the solutions proposed by his predecessors. It seems, however, that his preferences are those indicated in this passage[35]:

> When we make propositions about the infinite, considered with regard to the divine power (and it is only in taking into account this power that we treat here of infinity), we recognize the meanings that consist in affirming this: God can produce a syncategorematic infinite; and in denying this: God can produce a categorical infinite. Almost all ancient doctors were of this opinion; he admitted that an infinite could in no way be endowed with current existence.

Among these ancient doctors, there is one whose opinion seems, to Luis Coronel, particularly respectable, and that master is Jean Buridan; we read, in fact, what our Spanish philosopher said about the famous problem of the infinite spiral line; after reporting the words of Hentisbery and Cajetan of Tiene, he goes on to say[36]:

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[30] *Ibid.*, pp. 385-399.

[31] *Ibid.*, pp. 47-48 and pp. 403-407.

[32] *Ibid.*, pp. 48-49.

[33] *Expositio* Magistri Joannis de Celaya Valentini *in octo libros phisicorum Aristotelis*; fol. cxxv, col. *c*, to fol. CXXX, col. 6.

[34] Ludovici Coronel *Physicæ perscrutationes*, lib.VIII, pars II. De infinito: Nullum infinitum magnitudine continetur sub orbe Lune. Ed. cit., fol. CXX, col. *c*.

[35] Ludovici Coronel *Op. cit.*, ed. cit., fol. CXX, col. *d*.

[36] Ludovici Coronel *Op. cit.*, ed. cit., fol. CXXIIII, coll. *b* et *c*.

> All things considered, here is what it seems must be said: Buridan has, as a general rule, demonstrated a very right judgment in the matters he has dealt with in his writings; his intelligence, naturally a friend of the truth, correctly agreed with this proposition: There exists an infinite spiral line in the syncategorematic sense, but there is no infinite spiral line in the categorical sense[37]. But he found it lacking when it came time to prove it.

After treating for the first time the question of the infinitely large with respect to the divine power, Luis Coronel returns to this issue, the question, he tells us, of Master Simon Agobert, his favorite pupil. He then formulated these conclusions, which seem contradictory to each other[38], but are not, because, the author says,

> the word infinite is taken in the categorical sense in the first and in the syncategorematic sense in the second.
>
> Even by supernatural power, no infinite body has actual existence.
>
> Even by supernatural power, it may not presently exist in any infinite multitude that does not constitute a single whole.
>
> Even by supernatural power, there presently exists no corporeal accident of infinite intensity.
>
> To save the infinite force of the first Cause, it is not necessary to produce what will be able to produce a [categorical] infinite effect.
>
> To save the infinite force of the first Cause, he must grant that it can produce a [syncategorematic] infinite effect.
>
> By supernatural power, an infinite size can be produced.
>
> By supernatural power, an infinite multitude can be produced.
>
> By supernatural power, an accident of infinite intensity can be produced.

These findings, which are opposed to those of Juan de Celaya, are very sharp; the discussion, rather diffuse and confused, by which Luis Coronel supports them reveals a lesser firmness in the intimate thoughts of the author.

This incertitude reveals even a sort of repentance of the two sheets that the author adds to his work, after the colophon:

> I come again,

he said[39],

> to the question concerning the infinite force of the first Mover; it is in respect of this infinite force that I have here treated of the infinite; I say that the opinion which says one can produce the infinity implies no contradiction, although its infinite force could be manifested differently.

It is clear that Coronel is not absolutely decided on refusing God the power to create an actual and categorical infinite.

Our philosopher is certainly not willing to acquiesce immediately to the assertion that one thus comes to affirm the existence, beyond the highest heaven seen by astronomers, of an unlimited space occupied by bodies; but even less would he want

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[37] Coronel employs here the manner of speaking introduced by Albert of Saxony; he said: *Infinita est linea girativa et nulla linea girativa est infinita.*

[38] Ludovici Coronel *Op. cit.*, ed. cit., fol. CXXXVI, col. *d*, and fol. CXXXIX, col. *b*.

[39] Ludovici Coronel *Op. cit.*, ed. cit., fol. CL, col. *a*.

to reject it without further consideration; he clearly declares to us, regarding such a proposition, his intentions[40]:

> We must say that beyond the ultimate sphere there is no infinite body, nor even, moreover, any finite body, because no movement or effect on the lower bodies gives us evidence that this body is there; let the one who would assert the existence of such a body beyond the supreme orb present to us the reason for his opinion; we will respond to this reason, or, if it seems efficacious, we will approve the conclusion.
>
> In this circumstance, to hold the negative is a stance that justifies the absence of a natural or revealed reason; the advantage of holding the affirmative would require a reason or revelation.

If Luis Coronel had taken part in the discussion of the thesis proposed by Giordano Bruno in favor of the expansion of the Universe to infinity, he would have impartially weighed the arguments of the philosopher of Nola.

On 7 March 1277, the theologians of Paris, over whom the bishop Étienne Tempier presided, had condemned this article:

> *Quod prima causa non posset plures mundos facere.*

From that day on, many teachers in the School of Paris no longer regarded the plurality of worlds as an absurdity. Henry of Ghent[41] and Richard of Middleton[42] were the first of these masters. William of Ockham[43], Walter Burley[44], Robert Holkot[45], and Cajetan of Tiene[46] maintained, in turn, that God could have created several worlds.

All of these authors, it is true, contented themselves with affirming the possibility of a finite number of worlds; others went further; they admitted that God could have created a presently infinite number of different worlds. Foremost among these we find John Bassols[47].

We see the doctrine of John of Bassols defended in Montaigu in the early 16th century by master John Majoris[48]; John Majoris argues that one cannot find a contradiction in this hypothesis: There are infinite worlds.

Juan de Celaya is content at the College of St. Barbara to examine[49] if there are multiple worlds; he does not examine whether the multitude of these worlds can be infinite. In his teaching, some passages deserve to be reported here.

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40 Ludovici Coronel *Physicæ perscrutationes*, lib. VIII, pars II: De infinito. Nullum cælum est infinite magnum; ed. cit., fol. CXXIX, col. *d*, in fol. CXXX, col. *c*.

41 *Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*; seconde série, pp. 447-448.

42 *Ibid.*, pp. 411-414.

43 *Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*; seconde série, pp. 76-78.

44 *Ibid.*, pp. 414-415.

45 *Ibid.*, pp. 417-419.

46 *Ibid.*, pp. 415-416.

47 *Ibid.*, pp. 416-417.

48 *Ibid.*, pp. 92-94.

49 *Expositio* Magistri ioannis de Celaya Valentini, *in quatuor libros de cælo et mundo Aristotelis: cum questionibus ejusdem*. Venundantur in edibus Hedmundi le Feure in via divi Jacobi prope edem sancti Benedicti sub signo crescentis Lune moram trahentis. Cum Gratia et Privilegio regis amplissimo. Colophon:

> Currently, there is only one World. This conclusion is proved by this: The Catholic faith does not provide any authority from which the existence of several worlds comes, and furthermore, there is no reason that can force us to assume the plurality of worlds; much more, some of the reasons that the Philosopher invokes against this plurality have a certain appearance of truth. Currently, therefore, we do not have to assume the existence of several worlds…
>
> If we speak from the supernatural point of view, there may be several worlds, simultaneous or successive, concentric or eccentric to each other. Let us prove this conclusion: God can do anything that involves no contradiction; now, the existence of several worlds, simultaneous or successive, concentric or eccentric, involves no contradiction. Therefore, by the supernatural power of God, there can be several worlds.
>
> The reasons of the Philosopher have, against this conclusion, no value. Indeed, we deny that this World here below contains all possible matter; this affirmation is heretical, and the Philosopher could never prove it. It is not necessary, in this case, that the earth of a world is brought toward the center of the other world, because it is at the center of its own world that the virtue which preserves it resides. So, the reasons of the Philosopher will no longer have any appearance of truth.

On the subject of the plurality of worlds, the opinion of Luis Coronel is very similar to that of Juan de Celaya.

> We must examine,

he said[50],

> if there exists an infinity of worlds such as the one in which we are… But, you say, it might be; we do not have the right to look at this proposition as doubtful, because we are Catholics, and it is condemned as heretical[51].

Coronel does not reject this conclusion, although he made this remark:

> Gratian no way alleges the council or the decretal letter in which this opinion was condemned.
>
> But,

he added,

> I proceed here in a purely natural way, and according to what we can affirm by the natural light.

What will natural reason therefore dictate to our regent of Montaigu? Here it is:

Explicit expositio Magistri Joannis de Celaya Valentini in quatuor Libros Aristotelis de Celo et Mundo cum questionibus ejusdem, novissime et cum maxima vigilantia in lucem redacta: ac impressa arte ac artificio Joannis du pre et Jacobi le messier. Anno a partu virgineo Millesimo Quingentesimo decimooctavo die vicesimaprima Mensis Junii Sumptibus vero Hedmundi le feure: in vico sancti Jacobi prope edem sancti Benedicti, sub intersignio crescentis Lune moram trahentis. Fol. XV, col. *a*.

50 Ludovici Coronel *Physicæ perscrutationes*, lib. VIII, pars II: De infinito; ed. cit., fol. CXXXII, col. *a*, to fol. CXXXIII, col. 6. 2.

51 Gratiani *Decretum*, quæst. XXIV, cap. III.

> If, besides this world, it is recognized that there are others, we do not see any inconvenience resulting; thus, it is not more inconvenient to admit two or three or four, and so on… In fact, if this assumption presented some inconvenience, it would be above all what Aristotle claimed, that the earth of a world would be brought to a natural place of the earth of another world; but this argument is not convincing… Because the earth in the first world finds, in the place that it occupies, the conditions for a good preservation, there would be no reason to move to the proper place of another earth; likewise, when part of the earth is lodged in a proper and natural way, it does not naturally move to the place that another part of the earth occupies.

Our scholastic therefore thinks that we can, without contradiction, admit of the existence of several concentric or eccentric worlds; he added that this second hypothesis "is less likely than the first".

But do these multiple worlds, whose existence is not contradictory, in fact exist? Leaving aside the dogmatic condemnation contained in the *Decretals* of Gratian,

> and proceeding in a purely natural way, nobody has the right to assert that there are many worlds, because no one has a reason to formulate this statement; if someone has such a reason, he presents it…
>
> … The plurality should never be assumed without necessity; so we must not admit the existence of several worlds, because nothing that experience teaches us requires the reality of another world…
>
> … Because everything that happens in this world can be explained outside the assumption of another world, it seems that one had not wonder if there is another world as there will not be manifested in nature any inconvenience entailed by the absence of this world.

With regard to the plurality of worlds, Coronel observes the same attitude as with respect to the infinite magnitude of the universe; he rejects both of these hypotheses because he has no reason to admit them; but he is ready to accept them the day a similar reason would be provided to him.

How were the scholastics of Paris, guardians of the tradition of John Majoris, Luis Coronel, and Juan de Celaya, scandalized by the teaching of Giordano Bruno regarding the infinite magnitude of the universe and the infinite multitude of worlds?

But, we might say, when Giordano Bruno proposed the *Acrotismus Camæracensis*, the teachings of Johannes Majoris and Juan de Celaya would be forgotten and their books very little read. We are reassured that plagiarists worked to keep and spread the teachings of Parisian Scholasticism.

Of these plagiarists, the most cynical whom we have seen since Nicolò Tartaglia is unquestionably Francesco Giuntini, of Florence; physician, astrologer, a Catholic priest, then a Protestant, then Catholic again, Giuntini appears as the type of beings devoid of all moral sense, that the time of the Renaissance produced with a generous profusion.

In 1577 and 1578, Francesco Giuntini had printed in Lyons a commentary on the *Sphere* of John of Sacrobosco. The composition of this commentary required of him but a very mediocre effort[52].

[52] Fr. Iunctini Florentini, Sacræ Theologiæ Doctoris, *Commentaria in Sphæram Ioannis de Sacro Bosco accuratissima*. Lugduni, apud Philippum Tinghium, MDLXXXVIII. — Fr. Iunctini Florentini, Sacræ Theologies Doctoris, *Commentaria in tertium et quartum capitulum Sphæræ Io. de Sacro Bosco accuratissima*. Lugduni, apud Philippum Tinghium, MDLXXXVII. — Bibliographers

Giuntini, in fact, formed his work by copying long pages borrowed from other authors. When the copied pages are some verses of Dante, our astrologer names the poet; it would have been difficult to take these verses for his work. But he no longer believes in taking a similar probity when the passages that he appropriates were written by some Parisian Scholastic. There is no question, then, of the name of the owner.

Thus, much of the *De Cælo* of Albert of Saxony was spent under the veil of anonymity in the *Commentary* of Giuntini; this is the way that the considerations of our astrologer on the nature of eccentrics and epicycles were borrowed[53] verbatim from the *Commentary* of Pedro Girvelo.

When Giuntini wants to deal with the plurality of worlds, he loots a new author; it is the *Expositio in libros de Cælo et Mundo* of Juan de Celaya which is, this time, put to use; the *Commentary* of Giuntini reproduced[54], of this book, every chapter from which we extracted some passages. This chapter could therefore be forgotten when Jean Hennequin argued the propositions of Giordano Bruno.

If, therefore, the theses of the Philosopher of Nola touching the infinitude of the universe and the plurality of worlds met, at the College of Cambrai, some obstinate opponents, determined to reject them as contradictory or heretical, we could not find them in the ranks of the Nominalists. Rather, they might offend the beliefs of the Humanists faithful to the traditions of Lefèvre d'Étaples. Let us hear what this latter one puts in the mouth of his interlocutors when he wrote, in the form of dialogues, his *Introduction to the Metaphysics of Aristotle*[55]:

> Theoreticus: By this, Eutycherus, you can imagine why some philosophers—who had, regarding the mystery of unity, equality, and their connection, a faulty intelligence, like Anarch of Abdera—have stated that there an infinity of worlds.
>
> Eutycherus: Why is that?
>
> Theoreticus: In order that the infinite plenitude of the divine Goodness should communicate and propagate to infinity, so that they were not forced to declare any part futile or idle. But the world was created as good as it could be; and it was better that it was one, to resemble more its divine Artisan; through its unity, indeed, the world is the rival of the supreme unity of the One who made it, as it is, by its goodness, the rival of the goodness of the Creator. This world exists; it is one, good, true, and full to the extent that its nature could be capable of existence, unity, goodness, truth, and wholeness. So do those who have affirmed the existence of many worlds seem to you to have done wrong?
>
> Eutycherus: They have done wrong.

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generally think, relying on these titles, that the second volume was printed before the first; but this cannot be, because it contains an *Index Rerum* where the materials of the first volume are shown and the pages where these materials are treated.

[53] Fr. Iunctini *Op. cit.*, pars II, pp. 301-304. This plagiarism was a recurrence; Giuntini had already committed it in his *Sphæra Ioannis de Sacro Bosco emendata*, whose the first edition, which was followed by many others, appeared in 1564. The *Sphæra emendata* also contained many verbatim copies of pages of the *De Cælo* of Albert of Saxony.

[54] Fr. Junctini, *Op. cit.*, lib. I, cap. I, pp. 85-87.

[55] Jacobi Fabri Stapulensis *In introductionem metaphysicorum Aristotelis commentarii per dialogos digesti*. Dialogus tertius. (*In hoc opere continentur totius phylosophiæ naturalis paraphrases… Dialogi quatuor ad Metaphysicorum intelligentiam introductorii*, Parrhisiis, Henricus Stephanus, 1512, fol. 327.)

> Theoheticus: And the view of those who have affirmed the existence of an infinity of worlds does not present, truthfully, a much greater disagreement?
>
> Eutycherus: A greater disagreement, certainly, because nothing is more opposed to the sovereign unity…
>
> Theoreticus: Many others have admitted that this corporeal world was unique; but they tried to prove that it was infinite.
>
> Eutycherus: This is, I think, for the same reason.
>
> Theorericus: For the same reason; namely, that the supreme goodness could spread and propagate to infinity. But this opinion is not sensible. Whereas this corporeal mass impedes the fullness of perfection, they do not see that they equate it to the sovereign plenitude; nay, they equate the entire being of this infinite world to the infinite being of God, and the unity of the world to the unity of God. Otherwise the supreme being, supreme unity, and supreme goodness would not spread or communicate itself to infinity, as they wish. But their assumptions, and those of previous philosophers, face the difficulties that Aristotle reported.

Against the theses of Bruno, the disciples of Lefèvre of'Étaples strengthened the Physics of Aristotle with the Metaphysics of Nicolas of Cusa; to support this same Physics, the followers of Melanchthon invoked the texts of Scripture.

To demonstrate that the world is finite, Melanchthon briefly summarizes some of the weakest arguments of Aristotle[56].

> This manifest demonstration,

he said,

> convinces both the eyes and mind of any healthy person and forces him to confess that the world is finite.

Against the plurality of worlds, Melanchthon recalls[57], in a very concise manner, some peripatetic arguments:

> These absurd consequences,

he said,

> follow the affirmation of one who imagines many worlds; it follows, therefore, that there is only a single world…
>
> But for us, who are in the Church, we have an easier and more certain proof to affirm the existence of a single world. The celestial science, in fact, tells us that this world in which God is manifested—in which he delivered his doctrine to men, in which he sent his Son to the human race—was founded by God. Then it expressly adds that God has stopped and that he has not created any other bodies nor other living creatures. Indeed, in the second chapter of the first book of Moses, it says: *Cessavit ab omni opere suo*, which we must understand as: He did not create other worlds, other living creatures, or any new species. Therefore, it is necessary that there be a single world, and that there are not several worlds.

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[56] *Initia doctrinæ physicæ, Dictata ia Academia Vuitebergensi*. Philip. Melanth. Iterum edita, Wittergre, per Iohannem Lufft. 1550. Lib. I. Cap. intitul.: Est ne mundus finitus an infinitus? — The first edition of this book, which had a large number, is from 1549; we were unable to consult it. — Ed. cit. fol. 38.

[57] Philippi Melanchthonis *Op. cit.*, lib. 1, cap. intitul.: Quomodo confirmari potest unum esse mundum, et non plures; ed. cit., foll. 42 and 43.

Melancthon does not have, against the philosophy of Aristotle, the boiling hostility that animates Pierre La Ramée, better known under the name Petrus Ramus. We know that the violent attacks of Ramus against the Stagirite earned him a condemnation by the University before his Huguenot fanaticism caused the same University to expel him. A refugee in Germany, he continued to fight the peripatetic doctrines with tenacity. In 1562, his *Scholæ physicæ* especially undertook to reform what Aristotle taught about the infinite; but the reform proposed by Ramus did not resemble—far from it—what Giordano Bruno proclaimed. Ramus wants the notion of the infinitely large and infinitely small to have no place outside of mathematics. In physical reality, everything is essentially finite:

> Not only in the nature of things is there no infinity in actuality[58]…; but there is not even an infinity in potentiality; nothing physical, nothing sensible is infinite; all that is physical and sensible is finite and only susceptible to finite division… And yet, Aristotle deserves eternal gratitude, for if he conceived the evil, he also showed the remedy for this evil to those who know how to look attentively. This evil, in fact, broke into our schools at the same time as this plague of sophistry, the deadliest that ever was for the Christian religion. But I turn to other parts of the Physics of Aristotle with a soul filled with great joy and bright gaiety; now that we have blunted, or rather drastically crushed this horn of the infinite, it looks like the rest is no more than games and banter at this unparalleled monster produced by impiety.

Catholic Humanists and reformed Humanists were therefore very reluctant to subscribe to the theses of Giordano Bruno on the infinite magnitude of the universe or the plurality of worlds; their feelings with regard to these propositions differed little, no doubt, from the opinion of the most hardened Averroists; only the Scotists and Nominalists would listen to these statements without fear and discuss them without bias.

The hypothesis of the multiplicity of Worlds leads to the rejection of the theory of gravity as proposed by Aristotle; heavy bodies which are in these different worlds cannot all strive toward the same point.

John of Bassols writes[59]:

> It is not necessary that the earth of one of these two worlds is naturally brought toward the earth of the other world, nor even that it could thus move to another earth; the tendency of an earth toward the centre would not exceed, in effect, the bounds of its own world… If you tell me that in this case the earth of the other world would not be of the same species as the earth here, I answer that it is not necessary that it be of the same species. But admitting that this second earth was the same species as ours, the earth of each of these two worlds would not move toward the center of the other world, but only to the center of the world to which it belongs, so that the natural appetite of this earth would not extend beyond the whole to which it belongs.

> A weight, placed in one of the two worlds,

wrote Robert Holkot in turn[60],

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[58] P. Rami *Schollarum physicorum libri octo, in totidem acroamaticos libros Aristotelis*. Recens emendati per Joannem Piscatorem Argent. Francofurti, apud hæredes Andreæ Wecheli, MDLXXXIII. Lib. III in tertium physicum, in cap. VIII; pp. 97-98.

[59] *Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, seconde série, pp. 416-417.

[60] *Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, seconde série, p. 419.

> would naturally move toward the center of this world in which it is located; another weight, placed in the other world, would tend towards the center of that world.

Finally, a moment ago we heard Juan de Celaya:

> It is not necessary that the earth of a world is is carried to the center of the other world, because it is at the center of its own world that the virtue which preserves it resides.

Luis Coronel corrects in a different and, shall I say, most modern way, the peripatetic theory of gravity; according to him, a mass of earth, placed outside of the centers of various worlds, would direct itself toward the nearest center; this is at least the opinion that one easily concludes from what Coronel said[61] on the subject of the movement of fire:

> Assuming that there existed several worlds, one might ask the following question: Would the natural place for fire from one of these worlds equally suit the fire of another world? If the answer is the affirmative, we would also say, it seems, that the fire of one of the worlds should move towards the fire from the other world or toward the place of the fire. This is, moreover, what Aristotle argues in the first book of *On the Heavens*… It seems that we cannot deny this proposition, since in similar places, taken in different worlds, the same qualities of conservation are found… But in this case, it must be said that natural beings seek to acquire what suits them and strive to acquire the means that are for them the easiest in the course of the circumstances where they are found; this is why a weight, in the absence of any impediment, descends by a straight line, not the curved line that is longer; similarly, if there were another world, the concavity of the lunar orb of that other world would be a suitable place for the fire of our world; this fire, however, would not move to that place because it would be easier for it to stay in the lunar orb of our world.

Were the disciples of John of Bassols, Robert Holkot, and Juan de Celaya not quite ready to welcome the following thoughts that Giordano Bruno conceives[62] under the inspiration of Copernicus?

> These words of Aristotle are meaningless and far removed from the contemplation of nature: If someone transported the Earth to where the Moon is now, each part of the Earth would carry itself not toward there but to its own place. Much better! We say that the parts of an earth have no more power to become part of another earth than the parts a certain animal have the power to become parts of another animal.

Bruno continues in these terms:

> Rectilinear motion is not natural to any of the spheres; it is only natural to a part of a sphere when this part is located outside of its proper region; when this part is within its sphere, it moves more rectilinearly

and directed toward the center.

How does the Philosopher of Nola confirm[63] this thought?

> The various parts of the earth,

[61] Ludovici Coronel *Physicæ perscrutationes*, lib. IV, pars prima quæ est de loco; ed. cit., fol. LXXXIV, col. *c*.

[62] Jordani Bruni Nolani *Camœracensis acrotismus*, art. LXXIV (Jordani Bruni *Opera latina*, tomus I, pars I, p. 186).

[63] Giordano Bruno, *loc. cit.*

he said,

> have a circular motion… There are continually, in the earth, a divergent and convergent flux of the various parts, similar to what happens to the particles of the animals. Also, the parts which are found at the center eventually reach the circumference, to return from the circumference back to the center or to some different place. Hence a continual change in the face of the earth; sometimes we see the sea occupy areas where land was, sometimes mountains appear where valleys are hollowed out… In all these changes, I cannot grant that there is anything violent; I recognize only an absolutely natural movement; I called violent, in fact, what is foreign or contrary to the work of nature and the purpose to which it tends. It is against nature that all parts of the earth do not come, in turn, to occupy the center, that they are not all found, at one time or another, on the surface… Nature wants all that is born to move in a centrifugal movement to be also born for carrying itself toward the center. We do not see the particles of the earth remain at rest, nor the parts of an animal.

In this argument that Bruno seems to have developed with a sort of predilection, do we not recognize the theory of Albert of Saxony regarding the incessant movements of the earth, the theory from which Leonardo da Vinci drew all his Geology?

Let us not imagine, moreover, that he concerned himself there with a theory forgotten since the 14th century and which Bruno revived; this theory never ceased to be studied in the schools of Paris; accepted or rejected, it was constantly discussed there.

Juan de Celaya, for example, gives us in his writing on the *De Cælo*, a very clear presentation[64] of the principles underlying this theory; in terms which are almost literally borrowed from Albert of Saxony, he distinguishes, in the earth, the center of gravity from the center of magnitude; he teaches that the center of gravity is in the middle of the World, while the center of magnitude is eccentric to the world; he concluded that one part of the earth, which the heat of the sun and air keeps lighter, emerges from the sphere of water, whose center is at the center of the World; he rejects, indeed, and for the same reasons that Albert of Saxony did, the opinion which places at the center of the World the common center of gravity of the earth and water.

It is from these principles that Albertutius had concluded the incessant movement of the earth; twice, Juan de Celaya borrowed what he said.

In his *Explanation on the Physics*, he writes[65]:

> It may happen that that, continuously, the excess of the weight of one of the parts of the earth on the weight of the other part is so great that the weight of the first half exceeds the weight of the second half augmented by the resistance of the air which covers the latter; if it is so, the earth would move continuously.

To this remark he replied:

> It is not necessary that the earth is in constant motion; much more, it could be that it does not currently move. This conclusion is obvious; indeed, even if half of the earth would be heavier than the other half, it does not necessarily follow that the first pushes up the second, because of the air resistance around the lighter half.

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[64] Joannis de Celaya *Expositio in libros de Cælo et Mundo*, lib. II, cap. XIII, fol. XLI, col. *d* et fol. XLII, col. *a*.

[65] Joannis de Celaya *Expositio in libros phisicorum*, lib. VIII, cap. V, fol. CLXXXVII, coll. *c* et *d*.

This doubt had also made Albert of Saxony hesitate, who, however, came to reject it; Celaya, too, in his writing on the *De Cælo*, gives us a firm conclusion[66]:

> It is likely that the earth, according to some of its parts, moves in linear motion; this conclusion is obvious; indeed, of this elemental earth, in the region that is not covered by water, some parts are continually carried by rivers to the bottom of the sea; the earth thus grows in the part covered by the water while it decreases in the emerging portion; thus, it constantly moves, by its parts, in a rectilinear movement.

The transport of land by rainwater is, in fact, according to Albert of Saxony, what the previous lines borrowed verbatim, the cause which constantly increases one half of the globe at the expense of the other, which forces the lighter part away from the middle of the World, and which eventually brings to the surface the terrestrial parts which were at the center; Juan de Celaya taught at Saint Barbara all the principles of the doctrine that had so powerfully solicited the attention of Leonardo and which Giordano Bruno used as a weapon against the Aristotelian theory of gravity.

According to the doctrine of Albert of Saxony, one should distinguish in the earth two regions: a lighter one, emerging in its larger part; the other, heavier, almost entirely submerged. The great geographical discoveries, by showing that the constitution of the earth and the seas did not have a similar regularity, led the physicists to modify this opinion; they thought that the center of gravity of the earth was a short distance from its center of magnitude; this view was, in particular, that of Copernicus.

In Paris, some opponents of the Philosophy of Albert of Saxony and of the Moderns took advantage of this change to challenge the incessant movement that Nominalists attributed to the terrestrial mass. Among these was Jean Fernel, the first physician of Henry II. In a work published in 1628, Jean Fernel opposed[67] this quasi-identity of the two centers of the earth theory to the theory the *philosophi juniores* favored; according to him, the earth, thus disposed, remains absolutely still; by this is rejected the opinion of our philosophers "according to which, contrary to the doctrine of Aristotle, the earth could move out of the center."

Prepared by the discussion of the theory of Albert of Saxony, the Parisian Scholasticism should not have been unduly surprised that Copernicus attributed various movements to the earth and that Giordano Bruno accepted this hypothesis.

This is not to say that the Copernican system mattered to the well-known adepts in the University of Paris in the 16th century; far from it; the system of Ptolemy prevailed uncontested in the *alma Parisiorum Academia*; it is admitted, therefore, that the Earth is motionless and that the supreme Heaven rotates with a movement of

[66] Joannis de Celaya *Expositio in libros de Celo et Mundo*, lib. II, cap. XIV, fol. XLI, col. 6.

[67] Joannis Fernelii Ambianatis *Cosmotheoria, libros duas compleæ. — Prior, mundi totius et formant et compositionem: ejus subinde partium (quæ elementa et cælestia sunt corpora) situs et magnitudines: orbium tandem motus quosvis solerter referat. — Posterior ex motibus, siderum loco et passiones disquirit: interspersis documentis haud pœnitendum aditum ad astronomicas tabulas suppeditantibus. Hæcque seiunctim tandem expedite præbet Planethodium. — Cuique capiti, perbrevia, demonstrationum loco, adiecta sunt scholia.* Parisiis, in ædibus Simonis Golini, 1528. Cosmotheoriæ liber primus, et elementorum, et cælestium corporum magnitudines, situs, motusque universim aperiens. — De omnimoda terræ et maris dispositione, cap. I.

uniform rotation, which is the diurnal movement; but to these hypotheses, they attributed a value and meaning of the value totally different from what the Peripatetics recognized in them[68].

For Aristotle, the highest heaven is forced, by its own nature, to move with a uniform and eternal rotation; the very possibility of this rotation requires that the point around which it is executed belongs to a fixed body by its essence. Denying the uniform rotational motion of Heaven, denying the immobility of the Earth, was to formulate two propositions subject to metaphysical absurdity and logical contradiction; the first mover itself was unable to stop the Heaven or modify its movement; and, in his writing *On the Movement of Animals*, the Stagirite maintained the statement contained in this in verse of Homer: All the gods and goddesses, uniting their efforts, could not shake the Earth. The Commentator added even the rigor of these teachings of the Philosopher, and the Averroist Peripatetics had said about the dogmatism of absolute masters, to whom they attributed an infallible omniscience.

Catholic orthodoxy could not admit that the peripatetic Physics impose such limitations on the omnipotence of God. In 1277 the bishop of Paris, Étienne Tempier, and theologians of the University added to the number of items they condemned the following two propositions:

God could not give to Heaven a translational movement.

Theologians are wrong when they claim that Heaven can stop.

It is difficult to measure the importance that this decision and change that resulted from it would have on the opinion of the philosophers regarding celestial motions. In Paris, Oxford, and all the universities who took the watchword of these two illustrious academies, one continued to think that the Heaven moved with a uniform rotation, the Earth being stationary; but one stopped looking at these two propositions as necessary truths, of metaphysical or logical necessity; one regarded them as truths of fact, purely contingent; it was admitted that it was possible to deny them without contradiction; it was permitted to discuss them without going mad.

After 1277 the Parisians still believed in the Earth being at rest, but they believed it in virtue of an experiment[69]: A stone thrown vertically into the air falls exactly to the place where it was launched; they did not know how to reconcile the result of this experiment with the hypothesis of the movement of the Earth. They believed mostly in the immobility of the Earth because this immobility was one of the postulates of the Ptolomaic system and that this system was the only one which made it possible

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[68] We merely summarize in a few lines a chapter of the history of Physics that we fully processed elsewhere; the reader, desirous of knowing the texts that support our assertions, will find them contained in the study entitled: *Le mouvement absolu et le mouvement relatif*, a historical essay which the *Journal of Philosophy* was kind enough to publish in its issues bearing the following dates: 1 September 1907, 1 October 1907, 1 December 1907, 1 February 1908, 1 March 1908, 1 April 1908, 1 May 1908, 1 June 1908, 1 August 1908, 1 September 1908, 1 November 1908, 1 December 1908, 1 February 1909, 1 March 1909, 1 April 1909, 1 May 1909.

[69] Regarding this experiment, see: *Nicolas de Cues et Léonard de Vinci*, XIII: La Mécanique de Xicolas de Cues et la Mécanique de Léonard de Vinci. L'hygromètre, le sulcomètre et le mouvement de la terre (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, XI; seconde série, pp. 247-250).

to describe and calculate the movements of stars, the only one that *saved the celestial phenomena*[70].

But in their discussions on the nature of local movement, the Parisian Scotists and Nominalists did not hesitate to investigate hypotheses where God would have imprinted on the Earth or Heaven movements different from those which the Aristotelian Physics attributes to them. Richard of Middleton examines the case of God giving to Heaven a translational movement; John of Duns Scotus treats the hypothesis that the Universe would be reduced to a homogeneous sphere endowed with rotation; William of Ockham, Jean Buridan, and Albert of Saxony admit that the Earth could have been given a rotational movement identical to or different from what they attribute to Heaven.

It is not only as a philosophical hypothesis and for discussing the nature of local movement that 14th century Parisians admitted the motion of the Earth or the rest of Heaven; there were those who would not have shied away from taking this supposition as the basis of an astronomical system.

> One of my teachers,

wrote Albert of Saxony[71],

> seems to support this view: We cannot show that the hypothesis of the movement of the Earth and the rest of Heaven does not accord with the facts; but, except for the respect I owe him, it is the opposite that seems true, and this for the following reason: Assuming that the Heaven is motionless and that the Earth moves, we could not at all save the conjunctions and oppositions of the planets, nor the eclipses of the Moon and Sun. It is true that my master neither poses nor resolves that objection, although he poses and solves several other arguments designed to prove that the Earth is stationary and that the Heavens move.

The master of whom Albert of Saxony speaks without a doubt attributed to the Earth a simple diurnal rotational movement; surely, such a supposition was not enough to *save all the celestial phenomena*; is it not very remarkable, however, that in the Faculty of Arts of the University of Paris in the first half of 14th century, we could look at this assumption as a defensible astronomical hypothesis?

Albert of Saxony, moreover, has felt some inclination to attribute the phenomenon of the precession of the equinoxes no longer to a movement of a special celestial sphere, but to a slow movement of the Earth.

> We can maintain,

he said[72],

> that there are eight orbs… and that, however, the eighth sphere does not move with several movements; if this sphere seems to move with several movements, this comes from the

[70] In this regard, we refer the reader to the study that we published under the title Σώζειν τά φαινόμενα. *Essai sur la notion de théorie physique de Platon à Galilée* (*Annales de Philosophie chrétienne*, mai 1908, juin 1908, juillet 1908, août 1908, septembre 1908, et Paris, A. Hermann, 1908) [Duhem (1969)].

[71] Alberti de Saxonia *Subtilissimæ quæstiones in libros de Cœlo et Mundo*; lib. II, quæst. XXVI.

[72] Alberti de Saxonia *Op. cit.*, lib. II, quæst. VI.

> following combination: While the eighth sphere revolves from east to west about the poles of the World, the Earth itself rotates from west to east around an imaginary line that terminates in the poles of the zodiac; and this movement is such that in a hundred years, the Earth has rotated a degree.
>
> How, it will perhaps be said, do you save the processional and recessional movement of the eighth sphere, a movement that Thabit imagined? I would reply that this phenomenon could also be saved by giving the Earth another movement like the one that Thabit attributed to the eighth sphere. We also declare that, by this double movement of the Earth, the eighth sphere seems animated, in addition to the diurnal motion, with two other movements, namely, a movement by which it appears to rotate from west to east, one degree in a hundred years, and the movement that Thabit called processional and recessional movement; the eighth sphere, however, would move with a single uniform rotation from east to west.
>
> This theory does not seem absolutely sure; indeed, what moves the Earth thus is not apparent at first sight; however, if someone devoted his efforts to defend this opinion, he can perhaps conceive a way to avoid this difficulty and find many reasons able to give this theory a strong hint of truth.

This was written "in the Faculty of Arts of the University of Paris and in the 1368th year of the Lord."

These teachings, however, nor the book in which they were recorded, were forgotten in Paris in the early 16th century; it is from this book, for example, that Juan de Celaya borrowed almost verbatim the passages of which we spoke a moment ago.

Duhamel provides us with valuable evidence of what is commonly thought in Paris, some time before the *Acrotismus Camœracensis*, from the astronomical system of Aristarchus and Copernicus.

Duhamel was "royal mathematician", i.e., mathematics teacher at the Royal College, where Giordano Bruno would teach a few years after him. In 1557 Duhamel gave[73] a commentary on the *Psammites* of Archimedes. It was in this book that the great Syracusan made known to us the astronomical system of Aristarchus of Samos, the first draft of the Copernican system; the calculations of the *Psammites* are conducted as if the reader admitted the exactitude of this system.

Duhamel does not believe this system is admissible:

> That the Earth,

he said[74],

> be deprived of any overall movement, that it is found in the center of the World, that the Sun is endowed with a double movement, that the fixed stars and the sphere that carries them embrace the rest of the universe, one can, by very clear demonstrations, prove it and refute contrary hypotheses, as I have shown in another work. So I think only one task is left for me that fits my present purpose; it is to show how we will deduce the same size for the World, how we will conclude very few different appearances, regardless whether we adopt one or the other assumption; whether, according to that which is, we regard the Earth as immobile and located at the center of the world, or whether we attribute these properties to the Sun and transfer to the Earth the sphere and movements that are of the Sun.

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[73] Paschasii Hamellii *Begii mathematici Commentarius in Archimedis Syracusani præclari Mathematici librum de numero arenæ, multis locis per eundem* Hamellium *emendatum*. Lutetiæ Apud Gulielmum Cavellat, sub pingui Gallina, ex adverse collegii Cameracensis, 1567.

[74] Paschasii Hamellii *loc. cit.*, pp. 10-11.

Those are the words of an opponent of the Copernican system. Duhamel thought he had good evidence to oppose that system; he had no intention of treating it as a metaphysical impossibility or a logical absurdity, to regard as fools those who adopted an opinion contrary to his own.

The feelings that animated Parisians to respect the hypothesis of the movement of the Earth can be guessed, we believe, if we compare the attitude of Peter Ramus to that of Melancthon.

Member of the University of Wittenberg, illustrious for many astronomers, where Erasmus Reinhold teaches, Melanchthon knows neither the work of Copernicus nor the astronomical significance of this work. But he agrees with what he put forth regarding the motion of the Earth, provided that this discussion will be a pure mind game, an exercise of geometers.

> The men of science with nimble mind,
>
> he says on this subject[75], are pleased to discuss a host of questions where their ingenuity is exercised; but the young people know that these scientists do not have the intent to assert such things. Let these young men, therefore, grant their favors, in the first place, to opinions which benefit from the common consent of competent persons, which are by no means absurd, and thereby may they understand that the truth was revealed by God, embrace it with respect, and rest in it.

Melanchthon then tries to prove that the Earth is truly at rest; he not only summarizes in this aim the reasons provided by the Aristotelian physics, but above all he accumulates texts from Holy Scripture; reasons and texts that are exactly those that the Inquisition invoked to declare, against Galileo, that the hypothesis of the movement of the Earth is *falsa in philosophia et formaliter hæretica*.

Ramus, raised in Paris, and whose life is passed in large part in teaching, professes a very different opinion. Shall we say that he regards the Copernican system as an assured truth? This might be forcing his thought. But for sure, he regards it, in 1562, as a physically plausible hypothesis; and he does not hesitate to confront the Physics of Aristotle with the possibility of such a supposition.

Aristotle argued that time was the measure of the motion of Heaven. To which La Ramée answers[76]:

> Copernicus, the greatest astronomer of our time, has taken away from Heaven all movement; and by the movement of the Earth alone, he measures time more accurately than any astronomer had done before him.

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[75] *Initia doctrinæ physicæ dictata in Academia Vuitebergensi* Philip. Melanth. Iterum edita Witebergæ, per Johannem Lufft, 1550. — We were able to consult the first edition of this book, which is from 1549). — Lib. I, cap: Quis est motus mundi?

[76] P. Rami *Scholarum physicarum libri octo, in totidem acroamaticos libros Aristotelis*. Recens emendati per Joannem Piscatorem Argent. Francofurti. Apud hæredes Wecheli, MDLXXXIII. Lib. IV, in cap. XIV; p. 123.

Jean Hennequin, in the speech he held at the College of Cambrai, on the feast of Pentecost in the year 1586, in presenting the questions that Bruno formulated, dared pronounce these words[77]:

> The most foolish of men are those who say only fools can doubt the rest of the Earth.

He could say so without appearing to insult the masters of the University of Paris. Most of them, and perhaps all, believed in the immobility of the Earth and admitted the Ptolemaic system; but, certainly, one could not find among these *stultissimi omnium* those who dealt foolishly with the hypothesis of terrestrial movement.

We will not push these reconciliations further; those we have indicated, we believe, are so numerous and important that our conclusion does not seem temerarious:

The anti-peripatetic theses of Bruno were often far from being admitted, in the University of Paris, as established truths; but much less were they regarded at paradoxes that we could not support without scandal, in which one could not believe without madness. Many of these theses were only exaggerated corollaries of principles that Étienne Tempier opposed to Aristotle and Averroes in 1277 and which the Scotists or Nominalists supported since that time. Some of them were finally defended for a long time by the doctors of the Sorbonne, by the masters of the Faculty of Arts.

There is more. In a certain question, the anti-peripateticism of Bruno remained far behind the Parisian anti-peripateticism; the question we want to talk about is that of the void.

For Giordano Bruno, the void—which he identifies elsewhere, like John Philoponus, with space and place—cannot be realized; it can only be conceived by abstraction:

> We do not assume[78] that the void is a space in which nothing presently exists; we admit that it is a space within which sometimes one body, sometimes another, necessarily lies… Thus, we define the void as: a space or term that contains bodies; [it is] not a space in which there is nothing. When we say that the vacuum is a place without a body, it is not in reality, but only in the reason that we split the place and the body… By these considerations it is evident that place, space, plenum, and void are the same thing.

In the problem of the void, many Parisian doctors dared to adopt a solution much more bold, much more formal in its opposition to the teaching of the Stagirite.

Some physicists of the 13th century had reasoned thus:

> God could not imprint the Heavens with a translational movement because this movement produces a vacuum, whose existence cannot be admitted without absurdity.

The theologians under the chairmanship of Étienne Tempier in 1277 disapproved of this reasoning with one word: *error*.

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[77] *Excubitor seu* Joh. Hennequini *apologetica declamatio habita in auditorio regio Parisiensis Academiæ in fest. Pentec. anno 1586 pro Nolani articulis* (Jordani Bruni *Opera latina*, tomus I, pars I, p. 70).

[78] Jordani Bruni Nolani *Camœracensis acrolismus*, art. XXXIII (Jordani Bruni *Opera latina*, tomus I, pars I, pp. 130-133.)

From that day many masters of the University of Paris supported the following proposition: By the forces of nature, the vacuum can not be achieved; natural actions immediately fill any place from which the body that contained it is removed. But the existence of the vacuum is not an absurdity, and God, who can do anything that does not involve a contradiction, could produce and preserve a free space. This view, so disconcerting for all peripatetics, was made at the end of the 13th century by Henri de Gand[79] and Richard of Middleton[80] in the 14th century. Walter Burley went even further; he thought[81] that a Catholic can, without heresy, deny the actual existence of the void outside of the Heaven enclosing the World.

Rejected by Jean Buridan, Albert of Saxony, and their disciples, the doctrine of Middleton on the possibility of the vacuum appears to have met great favor in the early 16th century in Parisian Scholasticism; in fact, we find it reproduced, with nuances of minimal importance, in the writings of John Dullaert Ghent[82], Luis Coronel[83], and Juan de Celaya[84]. These masters of the Faculty of Arts of the University of Paris might have criticized Giordano Bruno for the timidity of his peripatetic theses.

The discussion of the arguments that Aristotle against the possibility of the vacuum causes Giordano Bruno to reveal to us[85] his feelings regarding the cause that moves projectiles:

> For bodies that are willfully launched and devoid of reason, Aristotle claims that they draw their power from the air or from any other body that comprises the medium; they are rather hindered by this body. The mobile has a certain innate or imprinted virtue able to carry it in the direction it is launched; as long as this *virtus impressa* lasts, it pushes the body. One, for example, who throws a ball in the air impresses on it something that is comparable to lightness.

We know how firmly the Parisians had not ceased, since Buridan, to defend the principles of this theory of *impetus*, so contrary to the teaching of Aristotle. All opponents of Peripateticism borrowed these principles. Before Giordano Bruno, Ramus did not hesitate to be a weapon against the strange explanation that the Stagirite gave of the motion of projectiles.

> Philoponus,

[79] *Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*; seconde série pp. 447-451.

[80] *Ibid.*, p. 412. — *Le mouvement absolu et le mouvement relatif*, appendice, § VII *bis* (*Revue de Philosophie*, 1er février 1909).

[81] *Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*; seconde série, pp. 414-415.

[82] Joannis Dullaert Questiones in libros phisicorum Aristotelis, lib. IV, quæst. III, fol. sign. oiij, col. 6.

[83] Ludovici Coronel *Perscrutationes physicæ*, lib. IV, secunda pars quæ est de vacuo; ed. cit., fol. LXXXIV, col. *c*, et fol. LXXXV, coll. *a* et *b*.

[84] Expositio magistri Joannis de Celaya Valentini in octo libros phisicorum Aristotelis, lib. IV, cap. XII, fol. CXLIII, col. *d*.

[85] Jordani Bruni Nolani *Camœracensis acrotismus*, art. XXXV (Jordani Bruni *Opera latina*, tomus I, pars I, p. 38.)

Ramus said[86],

> is strongly opposed to this explanation of the cause of the movement generated by projection; he discusses it in detail. The cause of the movement is, according to him, the power of the instrument projecting that was imprinted in the projectile and which received some assistance from the interposed void.
>
> …Philiponus thus said that a certain ἐνέργεια is imprinted in the projectile by the launcher, and this ἐνέργεια more easily traverses a vacuum than a plenum.

From this theory of *impetus*, the principle of which the *Acrotismus Camœracensis* expresses very clearly, Giordano Bruno deduced a consequence of the utmost importance and which no one before him had realized, at least to our knowledge. This result was made in an Italian writing, *La cena de le ceneri*[87], printed in London in 1584, thus two years before John Hennequin supported the *Acrotismus Camœracensis*.

Against the hypothesis of the movement of the Earth, Aristotle presented an experiment; a stone thrown vertically into the air always lands where it departed[88]. Ptolemy, Averroes, and the Christian Middle Ages had, again, relied on this observation to prove that the Earth is motionless. Nicolas of Cusa had, from the false premise from where he drew this conclusion, deduced other no less faulty corollaries. Leonardo da Vinci added a few others; he had, in particular, by the erroneous Mechanics the Stagirite, determined the trajectory that would seem to describe a projectile vertically launched if the Earth were rotating. Copernicus, after briefly recalling the objection of Aristotle, the invention of which he also attributed to Ptolemy, said nothing of what was truly capable of lifting it.

Giordano Bruno condemns[89] with perfect clarity the erroneous principle on which the classic argument against the motion of the Earth rests. When an object is launched from the deck of a ship in motion, it does not move as if it were thrown from a stationary place.

> If this were not true, it would be impossible, when ship runs on the sea, to launch something directly from one side to the other; the feet of a passenger who jumps could not fall back to the place from where they were removed. All the things that are on the Earth thus move with the Earth. If from a place outside the Earth something were thrown down, that thing would seem, following the movement of the earth, to lose the verticality of its movement. This is what we see when a ship goes down a river; when someone standing on the bank of

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[86] P. Rami *Scholarum physicarum libri octo, in totidem acroamaticos libros Aristotelis*. Recens emendati per Joannem Piscatorem Argent. Francofurti. Apud hærcdcs Andreæ Wecheli, MDLXXXIII. Lib. IV, in cap. VIII; p. 114.

[87] *La cena de le ceneri. Descritta in cinque dialogi, per quattro interlocutori, Con tre Considerations, Circa doi suggettj.* All' unico refugio de le Muse. l'lllustrissi. Michel de Castelnuovo 1584. — Reprinted in *Le opere italiane di Giordano Bruno* ristampate da Paolo de Lagarde. Volume primo. Gottinga, 1888. Our quotations and references are to this edition.

[88] We recall here in a few lines what we have elsewhere described in some detail: *Nicolas de Cues et Léonard de Vinci*, XIII: La Mécanique de Nicolas de Cues et la Mécanique de Léonard de Vinci. L'hygromètre, le sulcomètre et le mouvement de la Terre (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, X I; seconde série, pp. 241-255).

[89] Giordano Bruno, *La cena de le ceneri*, dialogo terzo; *Opere ilaliane*, pp. 167-169.

> the river throws a stone straight to the vessel, his throw will be found distorted in proportion to the speed of the ship.
>
> But when we place a man on top of the mast ship and this ship runs as fast as we wish, this man will not be deceived in his sight; from a point situated at the masthead or in the mast top to another point located at the foot of the mast or in the hold or in any other place in the body of the vessel, the stone, or other object that this man will cast, will come in a straight line. Similarly, if someone in the ship throws a stone from the base to the masthead, this stone will fall by the same route, regardless of the motion of the ship, provided however that it feels no oscillation.
>
> So, let us suppose that of two men, one is in the running ship and the other outside [on the river bank]; that the one and the other start roughly at the same place; that from the same place, in the same time, each one drops a stone [on the deck] without giving it any jolt; the stone of the first man will come, without losing its verticality or deviating in any way, to hit the point fixed in advance; the stone the second man dropped will be transported rearward.

These truths are the exact opposite of propositions that Averroes and his successors made, of those on which Nicolas de Cusa had relied in order to measure the speed of a moving vessel; it is much, surely, to have stated these truths so often ignored; but it is not enough; it is still necessary to give the reason, and this is what Bruno did:

> This does not come from any other cause than this: The stone which leaves the hand of the man carried by the ship is moved by the movement of the ship itself; thus, it has a certain *virtus impressa* that the other stone, which the man who remained outside of the ship dropped, does not possess—although these stones have the same gravity, pass through the same air, depart (as much as possible) from the same point, and have undergone the same initial shock. We cannot give any reason for this difference except that things which are fixed to the vessel or belong to it move with it and that the first stone carries with it the virtue of its mover that moves with the ship, while the second stone does not participate in this power. It is thus seen that a projectile neither takes on the virtue of going in a straight line from the term of where it departs, nor from the term of where it goes, nor from the medium through which it moves, but from the efficacy of the virtue that was first impressed on it.

Giordano Bruno published *La cena de le ceneri* a year before Benedetti printed his *Diversæ speculationes*; in bringing together what these two works added back to the Dynamics of Jean Buridan, one obtains nearly all of the principles that Gassendi would adopt in his *Epistolæ tres de molu impresso a motore translato*.

The year when *La cena de le ceneri* appeared is also when Galileo was twenty years old. The Pisan came at the right time. For centuries philosophers mulled over in every sense the thoughts that contained the seeds of the Science of movement; now these thoughts were ripe; they expected that a geometer of genius would render in full light the truths that lived in them and would give rise to the Mechanics of modern times. Galileo was that geometer.

Part III
Domingo Soto and Parisian Scholasticism

Chapter 12
Foreword

The Italian Science of the 15th and 16th centuries comprised a number of works which speaks of falling bodies and the motion of projectiles; the careful reading of these works[1] very easily leads to some conclusions that can be formulated as follows:

The intellectual progress which was to produce modern Dynamics was created, before the middle of the 14th century, at the University of Paris; it was born of the idea that the movement of a projectile can only be maintained not by the movement of the ambient air, as Aristotle wanted, but by the effect of an *impetus* imprinted in the mobile itself. The refutation of the theory of Aristotle was conducted by the rigorous and simultaneously violent dialectic of William of Ockham; the explanation of the theory of *impetus* had been presented in a very clear and complete manner by Jean Buridan, and, shortly after him, by Albert of Saxony.

Throughout the end of the Middle Ages and up to the middle of the 16th century, the Dynamics of Buridan and Albertutius was almost exclusively professed at Paris and in the German universities which formed, in a way, from some communities of the Parisian University; carefully preserved during this long series of years, it had, however, not advanced.

The new Mechanics had great difficulty gaining the support of the Italian masters; the Alexandrists, Humanists, and especially the Averroists formed, in and around the universities, strong parties, ardent rivals of each other but willingly agreeing to combat the language and doctrines of Paris .

At the beginning of the 16th century, few Italians shared the foresight of Leonardo da Vinci and were able to recognize, in the Dynamics of Paris, the key to the mechanics, "of this paradise of the mathematical sciences which makes us attain the mathematical fruit". Even Leonardo himself did not accepted in its fullness the mechanical teaching of Buridan and Albert of Saxony; he did not accept the explanation of the accelerated fall of weights that the authors had given; this explanation, however, would one day release one of the propositions underpinning our science of

[1] See the two previous studies: *John Buridan I (of Béthune) and Leonardo da Vinci* [Part I]. — *The tradition of Jean Buridan and Italian Science in 16th century* [Part II].

motion: the claim that the force moving a body is proportional to the acceleration that the movement of this body experiences.

The first three quarters of the 16th century are witnesses of the slow infiltration of the Dynamics of Paris into Italian Science; and by the end of this long period, most of the Italian masters renounced their stubborn resistance. But if the followers of the new doctrines are few, at least they are able to develop and fructify the ideas whose seed they gathered; thanks to Giovanni Battista Benedetti and Giordano Bruno, the Parisian principles, clarified and generalized, begin to be applied to the solution of new problems; they prepare the advent of the science that Baliani, Galileo, and Torricelli will develop in Italy; Descartes and Pierre Gassendi, in France; and Isaac Beckmann, in Holland; thus we see in all these great men the heirs of William of Ockham, Jean Buridan, and Albert of Saxony.

The study of the influence that Parisian Scholasticism exercised, during the 16th century, on Italian Science thus calls for a sort of counter-party; it seems natural to look for what were, in this same time, the relations between the doctrines taught in the Spanish universities with the theories created by the School of Paris.

We can expect that to discover in this new study facts very different from those that the former study revealed to us; as Italy reacted stubbornly to the Parisian doctrines that tried to penetrate into the teaching of its schools, so we must be prepared to find Spain welcoming to the theories that are professed at the Sorbonne, Fouarre Street, or Montaigu.

The conquest of the Spanish and Portuguese universities by the ideas coming from Paris will be the very natural result, and like the reciprocal of the conquest of the chairs of Paris by the masters coming from the Iberian Peninsula.

On the banks of the Seine, in fact, the Spanish and Portuguese masters were numerous and influential in the 15th century and the beginning of the 16th century[2].

Towards the end of 15th century, we noticed the activity, in the Faculty of Arts, employed by Pedro Sanchez Giruelo de Daroca, who had obtained his degree in Salamanca[3]. In the early 16th century, a collection of Spanish masters surrounds, at the College of Montaigu, the Scot Joannes Majoris; there we find Antonio Nuñez Coronel and his brother Luis Nuñez Coronel, both from Segovia, along with Gaspar Lax, of Sarinena, who will be a master of Vives; at the same time, Juan de Celaya professes at the College of St. Barbara. The Spaniards, however, held at that time such an important place in the University of Paris that their compatriot Juan Luis Vives regards them as the main cause of the defects of Parisian education, which he very harshly accuses[4].

Of the many Spanish students who, like Vives, went to the Parisian University to introduce themselves to its very subtle Scholasticism, famous throughout Europe,

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[2] *The tradition of Buridan and Italian Science in 16th century* II: The spirit of Parisian Scholasticism in the time of Leonardo da Vinci [chapter 7]; pp. 130 [89] ff.

[3] Demetrio Espurz Campodarbe, *Discurso leido en la solemne apertura del curso académico de 1909 a 1910 en la Universidad de Oviedo*; Oviedo, 1909.

[4] *The tradition of Buridan and the Italian Science in the 16th century* IV: The decadence of Parisian Scholasticism after the death of Leonardo da Vinci. The attacks of Humanism; Desiderius Erasmus and Luis Vives [chapter 9]; p. 169 [111].

many, like Ciruelo, the two Coronels, Lax, and Celaya, remained in Paris and in turn sat in the chairs from which they had been taught. Many, no doubt, took the road to their homeland, eager to spread there the knowledge they acquired. They went to Salamanca, proud of its University, one of the oldest and most famous of Europe, and to Alcalá of Henares, the ancient *Complutum*, where, in 1499, Ximenes founded a university, soon to rival Salamanca; others headed for Portugal where, since 1308, Coimbra inherited from the University of Lisbon.

What a welcome the young people who studied in Paris received in these universities, Quétif and Échard tell us[5]:

> We repeated everywhere, and with a unanimous voice, that the study of literature was flourishing more at the Academy of Paris than in any other; the very name of Paris was worth an extra honor and consideration not only for the masters who graduated from the University and those who professed there, but also for those who had simply, as auditors or students, studied in this Academy.

The Spanish and Portuguese chairs were therefore often occupied by those who had gone to Paris to obtain knowledge of the doctrines in vogue or who had professed these doctrines; Pedro Ciruelo, for example, returned to teach at Alcalá[6]; and the very history of Domingo Soto will allow us to see the influence of the Parisian Scholasticism on both the ancient University of Salamanca and on the young University of Alcalá.

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[5] Jacobus Quetif et Jacobus Echard, *Scriptores ordinis prædicatorum*, tomus secundus, p. 171 (Art. Dominicus de Soto); Lutetiæ Parisiorum, MDCCXXI.

[6] See the *Prohemium* from the following writing: *Opusculum de sphera mundi* Joannis de Sacrobusto: cum additionibus et familiarissimo commentario Petri Ciruelli *Darocensis*: nunc recenter correctis a suo auctore: intersertis etiam egregiis questionibus domini Petri de Aliaco. Colophon:

> Fuit excussum hoc opusculum in Alma Complutensi Universitate. Anno Domini Millesimo quingentesimo vigesimo sexto. Die vero decimaquinta Decembris. Apud Michælem de Eguia. E regione Divi Eugenii commorantem: ubi venundatur.

Chapter 13
The life of Domingo Soto, Friar Preacher

Francisco Soto, father of the religious scholar whose work will occupy us, was a very modest gardener of Segovia[1]. In 1494, Soto had a son who received, like his father, the first name of Francisco.

The family resources were far too modest for the young Francisco to be educated at Seville; so he was placed as caretaker of the parish church of the village of Ochando, located not far from Segovia; there, he without a doubt received his first literary initiation from the clergy.

Eager to push his studies further, he went to the University, still young, of Alcalá de Henares. He was there acquired with a young nobleman, Pedro Francisco Saavedra, born in Benalcazar in Andalusia. Soto and Saavedra followed all the lessons given by the teachers of the University, among others by Thomas of Villanova who was one day to be canonized. But the voice that boasted of the Parisian Science, which acclaimed the students trained by the University of Paris, buzzed in their ears; they yielded to the temptation that seduced, in large numbers, the Spanish students; abandoning Alcalá, they took the road to France together.

In Paris our two students were welcomed "*humaniter et festive*", the friars Quétif and Échard say, by two famous masters in the University, the Nuñez brothers Coronel, Antoine, and Louis, who, like Soto, were natives of Segovia. By these compatriots of Soto, the two young Spaniards found themselves introduced to one of the liveliest and most interesting circles found at that time in the Parisian University. The two Coronel brothers were among the followers of the most active and most dedicated of the old Scottish master Joannes Majoris; and he was, certainly, the leader of the conservative party; he tried to keep, in the study of Theology, the traditions of the Nominalist Scholasticism; he forcefully resisted the attempts of Lefèvre d'Étaples and Josse Clichtove to substitute for the scholarly dialectical discussions the sole study of Holy Scripture and the Fathers; the resistance of Joannes Majoris, incidentally, was not a blind obstinacy; he knew how to subtract from his lessons the quibbles of a too subtle logic and the embarrassment of a too barbarous language.

[1] We have gathered all of our information concerning the life of Soto in: Jacobus Quetif et Jacobus Echard, *Scriptores ordinis prædicatorum*, tomus secundus, pp. 171-172. Lutetiæ Parisiorum, MDCCXXI.

The disciples of Joannes Majoris were not unworthy of the master; if John Dullaert of Ghent and Luis Coronel of Segovia linger too much, in our option, on the fastidious quibbles which the disputes of the school were willing to use, at least they managed to preserve and explain all the teachings, of the modern Science that the tradition of Jean Buridan, Albert of Saxony, and Nicole Oresme brought to them.

It is in this milieu—where Humanism was unable to exert its influence, where Nominalism is gradually stripped of its dialectic jumble, where positive Science was cultivated with particular favor—that Soto and Saavedra lived for some years, completing their theological studies together. Around 1620 they returned to Alcalá.

At Alcalá, Francis Soto wins, after a brilliant competitive examination, the chair of Arts at Saint Alphonsus College. But soon the monastic vocation is heard within him. He first retired to the Monastery of Monserrat, then at Burgos; there, he takes the habit of a friar preacher; making his profession on 23 July 1525, he changed his name of Francisco to that of Domingo.

Pedro Francisco Saavedra soon follows the example of his friend Soto; he took the Dominican habit at Segovia along with the name Domingo of the Cross; the desire to evangelize the Indians drew him to America; after a missionary life, he died in Mexico around 1540.

The science of Soto was quickly noticed by the order of Saint Dominic, which he had just entered. His superiors sent him to Bruges first, that he should teach philosophy and theology to his brothers. But soon, one of the two chairs of Theology of Salamanca, the *evening chair* (*chaire du soir*), became vacant; Soto took part in the contest for designating its holder; his success was very great; on 22 November 1532, he entered the chair he was to occupy for sixteen years.

The reputation and influence of Soto did not cease to grow in the order of Saint Dominic and the whole Church.

In December 1545 the Council of Trent opened its sessions. For over a year, the Dominican order lost its Superior General, Alberto de las Casas, and he had not been replaced. Among the friars preachers who attended the Council, over fifty were clothed in episcopal dignity; Domingo Soto, a simple monk, however, was instructed to speak in the name of the whole order, as if he were its superior general; he exercised these important functions during the first four sessions of the Council. On 12 June 1546, a new superior general, Francesco Romeo, was elected; but, as he could not make it to Trent, he was represented by Soto in the fifth and sixth sessions of the Council.

In the meantime, Charles V having chosen Domingo Soto as his confessor, our Dominican had to follow the Emperor into Germany. But by 1550 he returned to Salamanca, where he received the title of honorary professor. In 1551 he preached Lenten sermons in the cathedral. In 1552 the famous Melchior Cano, named bishop of the Canary Islands, left vacant one of the chairs of Theology of the University, the *morning chair* (*chaire du matin*); Soto held this chair until his death.

The conquerors of America too often treated the Indians with the utmost barbarity; Ginés de Sepúlveda thought he found in the teachings of the Church the justification of those cruelties; in his dialogue *Democrates Secundus, seu De justis belli* *causis*[2], he dared to argue that th Christians had the right and duty to exterminate the infidels, rebels of the evangelization. This monstrous thesis raised the indignant protests of a pious and heroic Dominican, Bartolomé de Las Casas, Bishop of Ghiapa. In 1552 the old companion of Christopher Columbus published in Seville his *Brevissima relación de la destrucción de las Indias*, an admirable plea for the unfortunate populations of the New World.

The dispute between Sepúlveda and Las Casas raised a theological question where the cause of the Church and of humanity were involved; Soto was responsible to decide; he did not hesitate to rule in favor of the thesis sustained by Las Casas.

Soto died in Salamanca on 15 November 1560 at the age of sixty-six.

[2] Published by Mr. Menéndez Pelayo in the *Boletin de la Real Academia de la Historia*, T. XXI, pp. 257-369, oct. 1892.

Chapter 14
Domingo Soto and Parisian Nominalism

Soto studied in Paris when the most furious attacks were carried out against the Scholasticism of the Nominalists, the people who prided themselves on Humanism, simultaneously condemning futile curiosity, quibbling dialectic, and barbarous language. The masters who welcomed our student did not follow the new fashions introduced into education by the likes of Lefèvre d'Étaples and Josse Clichtove; still less do they echo the taunts and jeers that Desiderius Erasmus unleashed against the Theology professed at the Sorbonne; conservatives, but in moderation, they readily acknowledged that it was necessary to prune the tree that the Nominalism of 14^th^ century planted and to remove its many unnecessary and cumbersome subtleties; they tried their best to introduce into their lessons more simplicity and clarity than their predecessors were accustomed to put.

The students often went, in this reformist direction, further than the masters; from the College of Montaigu, illustrated by the long and active regency of Joannes Majoris, the most loyal followers of the old Scottish theologian, Dullaert and Lax, saw one of their auditors, the Spanish Luis Vivés, heaping mockery and insults on the masters who taught in Paris and the doctrines they professed.

Soto did not go to the extremes that his compatriot did; he does not stoop to involving himself in the impeccable Ciceronian periods of the puns of lackeys and profanities of a goujat; he did not give in to Humanism and remained a Scholastic philosopher; but he arose as a convinced opponent of Nominalism.

Quétif and Échard show us the young professor of Alcalá busy chasing from the teachings of the University "the opinions or, rather, the nebulousness of the Nominalists" who reigned there.

Later, while Soto, for many years, taught theology at Salamanca, the academic body of this city, eager to "eliminate from its colleges the sect of the Nominalists", asked the learned Dominican for help. The latter wrote for this purpose the *Questions on the Physics of Aristotle*, which we propose to study[1].

[1] According to Quétif and Échard (*Scriptores ordinis prædicatorum*, t. II, p. 172), the first edition of the *In octo libros physicorum commentarii et quæstiones* was published in Salamanca in 1545.

We consulted the second of the editions that Quétif and Échard mention; it is entitled:

We have recognized, moreover, what an extraordinary authority Soto had acquired among the Dominicans; thus, we are not astonished to see his philosophical preferences go toward, in the majority of the problems, the Thomistic solutions which the Friars Preachers have always particularly esteemed.

But it is very mistaken to find in him an exclusive and stubborn Thomist, determined to embrace, in any subject and to the extreme limits, the views of the Angel of the School; it is also a mistake to expect to see him mercilessly condemn all doctrines professed by the Paris Nominalists. Often, even in matters of great importance, we shall see him abandon the positions that St. Thomas held and defend those that the likes of Buridan and Albert of Saxony chose.

This way of proceeding, incidentally, was in the spirit of Parisian Scholasticism. Widely eclectic, the Parisians feared the strong stubborn attachment to the opinion of a single teacher[2]; from their eclecticism a Spaniard, Pedro Ciruelo, formulated at the end of 15th century the very decisive statement; and at the same time when Soto was studying in Paris, another Spaniard, Juan de Celaya, pretended to enlighten his teaching of Physics by the triple light that Thomism, Scotism, and Nominalism casted.

During his sojourn on the banks of the Seine, Soto learned from his masters to practice that intellectual justice which guards against settling a debate before hearing and weighing the views of the parties in dispute. Also, this Dominican, whose biographers show us a resolute and persevering adversary of Nominalism, is wonderfully informed of the treatises composed by the masters whom the Nominalists most willingly acclaim; his *Questions on the Physics of Aristotle* reveal a deep knowledge not only of the various books of Walter Burley and Paul of Venice, but also of those that William of Ockham, Gregory of Rimini, Marsilius of Inghen, and Joannes Majoris have written.

The desire to fight on their own ground the philosophers whose excessive doctrines he proposes to suppress led him to follow closely, in writing his book on Physics, the order and method that Nominalists of Paris had adopted. This book provides a very easily recognizable analogy with the *Physicæ perscrutationes* that

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> Reverendi Patris Dominici Soto Segobiensis, Theologi ordinis Prædicatorum in inclyta Salmanticensi Academia professons ac Cæsareæ Maiestati a sacris confessionibus *super octo libros Physicorum Aristotelis Commentaria*. Tertia æditio nuperrime ab Authore recognita, multisque in locis aucta et a mendis quam maxime fieri potuit repurgata. Cum Privilegio. Salmanticæ, In ædibus Dominici a Portonariis, Cath. M. Typographi. MDLXXII.

The second volume is entitled:

> Reverendi Patris Dominici Soto Segobiensis Theologi ordinis prædicatorum super octo libros Physicorum Aristotelis Quæstiones. Salmanticæ. In ædibus Dominici a Portonariis, Cath. M. Typographi. MDLXXII.

Quétif and Échard cite two more editions after this one, namely: Salmanticæ, per Ildephonsum a Terranova et Neyla, 1582. Duaci, *una cum Dialectica*, curis Jacobi Howerii Hoogstratani ordinis Prædicatorum.

[2] *The tradition of Jean Buridan and the Italian Science in 16th century* II: The spirit of Parisian Scholasticism in the time of Leonardo da Vinci [chapter 7]; pp. 130 [89] ff.

Luis Coronel published in 1511; the questions treated and the arguments targeted in these two writings are often the same, yet the solutions adopted are, in many cases, different.

It even happens that, in order to make his fencing against the Nominalists tighter, Soto comes to borrow their game. Desiring to discourse convincingly against very subtle opponents, he is often reduced to compete with them subtly. Hence, his anti-nominalist dialectic sometimes also becomes as twisted pettifogging as that of the Nominalists; in reading his *Questions*, Luis Vivés would have probably recognized the execrated memories of the teaching he received at Montaigu. It is not only by the moderation of a Thomism welcoming to more modern solutions that Soto shows the links that attach him to the school of Joannes Majoris; it is even the form of his argument, very close to what prevailed in the disputes of the Sorbonne.

How much the Thomism of Soto was tinged with Parisian Nominalism, and this in the most essential theses themselves, we shall see by reviewing some of his opinions and, first of all, by reporting what he taught about infinity.

Chapter 15
Potential infinity and actual infinity

On the subject of the infinite, the doctors of Scholasticism are divided into three major camps[1].

The first party held the thesis of Aristotle and his commentator Averroes: Infinite magnitude is impossible because it is contradictory; not only does no infinite magnitude exist in an actual manner, but also one cannot attribute being in potentiality to infinite magnitude; no magnitude can be increased so as to exceed all limits.

St. Thomas Aquinas admitted this peripatetic doctrine; he even denied the omnipotence of God the power to make an actual infinite magnitude, or a potential infinite magnitude, because if God can do everything which does not imply any contradiction, he cannot realize an absurdity.

The refined logic introduced into the School of Paris by the *Summulæ* of Petrus Hispanus was not content to substitute for the concepts of *actual* infinite and *potential* infinite the somewhat different notions of *categorical* infinite and *syncategorematic* infinite; it also gave birth, on the subject of infinity, to two theories very different from the peripatetic theory.

Of the two theories, one is opposed, in the most absolute manner, to the doctrine of Aristotle, Averroes, and St. Thomas Aquinas; it holds as exempt from all contradiction the existence of an infinite magnitude and infinite multitude, whether syncategorematic or even categorical; God can therefore create a categorically infinite volume or multitude; he can actually divide a continuum into an infinite number of infinitely small parts. First proposed, it seems, by John of Bassols, the immediate disciple of Duns Scotus, this opinion was supported with a prodigious logical force by Gregory of Rimini.

Between the peripatetic doctrine and the doctrine of Gregory of Rimini, it is possible to hold an intermediate stance; one could argue that the categorical infinite cannot be realized without contradiction, but that the realization of the syncategorematic infinite is free from absurdity. According to this view, God could not produce a multitude or quantity that was categorically infinite; but the production of a multi-

[1] *Léonard de Vinci et les deux infinis* (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, seconde série, pp. 3-53). — Sur les deux infinis (*Ibid.*, pp. 368-407).

tude or magnitude that grows beyond all limits and the indefinite division of continuous parts whose size eventually falls below all limits are things that are within his omnipotence. Proposed at the end of the 13$^{\text{th}}$ century by Richard of Middleton, this doctrine rallied in the 14$^{\text{th}}$ century the most illustrious among the Parisian doctors; William of Ockham, Waller Burleey, Jean Buridan, and Albert of Saxony have professed and supported it against the opinion of Gregory of Rimini. Less arrested in his opinion, Marsilius of Inghen, following the example of a certain propeller whose pitch decreases in geometric progression, thinks that a categorically infinite length can be achieved, although the existence of a categorically infinite volume implies a contradiction.

Between the proponents of the categorical infinite and the supporters of the syncategorematic infinite alone, the discussion was very ardent when Soto came to sit on the benches of the University of Paris. Joannes Majoris ostentatiously professed the possibility of the categorical infinite, but he had not received a share, in supporting this opinion, of the rigor and logical power of Gregory of Rimini. John Dullaert and Juan Celaya clearly supported, too, the opinion of Gregory of Rimini[2], while Luis Coronel, not without having experienced some temptation to embrace the same position, considered it prudent to support, with Jean Buridan, the possibility of the syncategorematic infinity alone. None of these authors, however, seemed to think that one could keep the opinion of Aristotle, Averroes, and St. Thomas Aquinas and deny God the power to produce a potential infinite magnitude or multitude.

It seems that the education the young Spanish student received in Paris made a deep and lasting impression on him, because in this serious question of infinity, the learned Dominican doctor neglects entirely the doctrine of St. Thomas for attaching himself to that of Jean Buridan and Albert of Saxony, to that which won the support of his host Luis Coronel.

Soto, in fact, argues that the actual infinite magnitude and multitude are not only unattainable by natural means[3], but they are also contradictory[4], such that the omnipotence of God cannot produce them. On the other hand, he grants[5] that the infinite magnitude and multitude, unattainable in actuality, are achievable in potentiality.

In his presentation of this thesis, Soto defends himself as best as he can from using the terminology of the Parisians, whose rules, however, he knew very well:

> Modern philosophers (*Neoterici philosophi*),

he said[6],

> declare that in respect to continuous magnitudes, the term *infinite* can be understood in two ways; firstly, it can be taken categorically…; secondly, it can be taken syncategorematically;

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[2] *The tradition of Buridan and the Italian science of the 16th century*, VII: Of the first progress made in the Parisian Dynamics by the Italians (*suite*). Giordano Bruno. [section 11]

[3] Dominici Soto *Quæstiones in libros Physicorum*; in lib. III quæst. III: Utrum infinitum sit naturaliter possibile; ed. cit., t. II, fol. 53, col. *c*.

[4] Dominici Soto *Op. laud.*; in lib. III quæst. IV: Utrum de potentia Dei absoluta possit fieri supranaturaliter infinitum in actu.

[5] Dominici Soto *Op. laud.*; in lib. III quæst. III; ed. cit., t. II, fol. 53, col. *d*.

[6] Dominici Soto *Op. laud.*; in lib. III quæst. III; ed. cit., t. II, fol. 53, col. *a*.

> the meaning of this adverb can be explained by these words: an amount that is never so great that it cannot become more (*non tantum quin majus*)… In addition, they pose this rule: When the word "infinite" is placed on the side of the predicate of a proposition, it is taken in the literal (*nominaliter*) and categoric sense, as in these sentences: *Deus est infinitus, continuum habet partes infinitas*. When, however, the word "infinite" is put side on the side of the subject, it is taken in the syncategorematic and explanatory sense (*exponibiliter*), as in this proposition: *Infinita parva est pars continui*.

Soto noted that neither Aristotle nor St. Thomas used the phrases categorical infinite or syncategorematic infinite, which correspond to the names actually infinite or potentially infinite, which they did use. Following the example of the great peripatetics, the professor of Salamanca will use the old ways of speaking rather than everyday language among the juniors, although he sometimes appeals to it there.

But if the form of discourse of Soto is guarded, very imperfectly indeed, from the Parisian innovations, its foundation is entirely composed of arguments that were developed at Montaigu, Fouarre Street, and the Sorbonne. How, indeed, could it be otherwise? The thesis that our author undertakes to refute, in combating it inch by inch, it is that of Gregory of Rimini; so it is not surprising that the name and reasons of this great Nominalist are offered on almost every page. Against these reasons of Gregory of Rimini, how can one not use the responses devised by Jean Buridan and Albert of Saxony, since it is their opinion that it is to prevail? We therefore are not surprised to find, in the book by Soto, long discussions on the division of time into proportional parts and on that helical line "*de qua tam se anxie affligunt multi*"[7].

[7] Dominici Soto *Op. laud.*; in lib. III, Quaest. IV; ed. cit., t. II, fol. 55, col. *c*.

Chapter 16
The equilibrium of the Earth and seas

Seeing Domingo Soto abandon the doctrine of Aristotle and St. Thomas Aquinas for attaching himself to one of the opinions received by the Parisians, when the question in dispute is one of the most weighty of Metaphysics, we measure the entire depth of the impression that the Nominalist teaching marked on the reasoning of the future professor of Salamanca. We are not astonished when our author will prove himself a faithful disciple of modern philosophers in certain theories of physics where the authority of the peripatetic discipline had almost no opportunity to exercise itself.

Thus, we can note, in one of the questions Soto addressed[1], a full and entire adherence to the theory of equilibrium of the earth and seas that Albert of Saxony had, if not imagined, at least greatly developed[2].

Soto admits[3] that the earth is in its natural place when the center of gravity of this mass is at the center of the World:

> The words "natural place" do not merely express, like the words "mathematical place", a containing surface; they also express a conserving virtue; this conserving virtue, no doubt, has its seat in any area that is bounded by the concave surface of the water and also by the concave surface of the air in the entire region where the earth is not covered by water; but it resides in the most perfect manner at the center of gravity of the earth; and that is why the earth moves toward the center of the World.

Now here[4] is the reason why a part of the earth rises above the sphere of water:

> Do not be surprised that the sphere of water is lower than our continent; the part of the earth that is above water is much lighter than the part that is covered by water, because it is drier; also, the center of gravity of the earth is not it the same as the center of magnitude; this center of gravity is much nearer to the surface of the earth covered by water than it is on

[1] Dominici Soto *Op. laud.*; in lib. IV, quæst. II: Utrum omne corpus locum sibi vindicat naturalem, atque adeo, omne ens necessario sit in loco uno; Art. 1: De naturalibus locis corporum.

[2] *Albert de Saxe et Léonard de Vinci*, II: Quelques points de la Physique d'Albert de Saxe (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, I; première série, pp. 7 seqq.) — *Léonard de Vinci et les origines de la Géologie*, X: Albert de Saxe (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, XII; deuxième série, pp. 327 seqq.).

[3] Soto, *loc. cit.*; ed. cit., t. II, fol. 62, col. *b*.

[4] Soto, *loc. cit.*; ed, cit., t. Il, fol. 63, col. *a*.

our continent. As, moreover, the center of gravity coincides with the center of the World where the earth descends, that the sphere of water should be everywhere equidistant from the center of the World, this is what happens: If, on the side where the sea is found, the surface of water is, for example, one hundred miles from the center, on our side the natural place for water will also extend up to one hundred miles from the center of gravity; on our side what is left of the earth, [beyond these hundred miles, emerges, and the earth] occupies a large part of the natural sphere of water.

Chapter 17
The Dynamics of Jean Buridan and the Dynamics of Soto

Where the Parisian Physics was did not contradict the teaching of St. Thomas Aquinas, Domingo Soto would eagerly adopt the assertions; he put, in supporting it, a few more ways when it came to this, unexpectedly supporting some formal conclusion of Aristotle and the Angelic Doctor; he knew very well, however, how to reconcile the traditional respect in the Order of Saint Dominic for these masters of peripateticism with the worship of truths that were demonstrated to him in Paris by strong arguments. Of this freedom of spirit that could, if necessary, put the exigencies of science above Thomistic influences, we have a clear testimony in analyzing the doctrines Soto professed on the subject of Dynamics.

Shaken air is the only cause that allows a projectile to continue its movement; this is the teaching of Aristotle and his commentator Averroes; St. Thomas made a formal adherence to this teaching in his commentary *De Cælo*, which is one of his last writings and which death prevented him from completing.

William of Ockham shows with decisive sharpness to what extent this theory is ridiculous. After him the School of Paris admits an explanation that St. Thomas already knew but expressly rejected: The movement of the projectile is maintained by a certain quality or *impetus*, which was imprinted in the mobile at the time it was launched. Jean Buridan and Albert of Saxony developed the hypothesis of *impetus* with such clarity and precision that they can be placed among the first initiators of modern Dynamics.

Now, is it this doctrine of *impetus* that Soto teaches in detail.

On the explanation given by Aristotle, the professor of Salamanca does not hesitate to say[1] "that it is difficult to prove and even more difficult to admit: *ægre probatur et ægrius creditur*." Here, indeed, is how he develops[2] the objections that one can make to this explanation:

> Most physicists cannot be convinced of this opinion of the Philosopher.

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[1] Dominici Soto *Quæstiones in libros Physicorum*; in lib. VIII quæst. III: Utrum omne quod movetur moveatur ab alio; ed. cit., t. II, fol. 99, col. *c*

[2] Soto, *loc. cit.*; ed. cit., t. II, fol. 100, col. *c*.

> *In the first place*, they do not see that it is possible for the one who launches the projectile to communicate to the air a large enough force for it to be able to move an arrow or an even heavier shaft.
>
> *Secondly*, air cannot even sustain an ounce of lead; so how could it not only support but even move a voluminous ball with a great speed over a great distance?
>
> Experience permits us, furthermore, to see that the air is sometimes agitated by a very strong wind; this wind, however, is not in itself strong enough to move a stone that we ourselves can move by throwing it.
>
> The cause that moves the projectile is not the movement of air, but the one who launches the projectile, or better, the *impetus* he impresses on this body.
>
> Moreover, here is what confirms this reasoning: If the movement of the air were in question, it would drive a feather or flake of wool faster than a stone or a piece of iron; however, experience teaches us the contrary.
>
> *In the third place*, this argument is cited: When a strong wind blows in your face and you throw a stone in the direction opposed to the fast course of this wind, it is clear that you can, in this case, push the air the opposite direction of its movement, yet the stone is moved against the air current; so, therefore, the stone is driven not by the air but by whoever throws it.
>
> One can even use the argument of the motion of a millstone, which one gives a strong impulse, making it rotate, then leaves it to itself; it will continue to turn; it does not seem, however, that the air moves circularly; what cause, indeed, should communicate this movement to it? All the more since the impulsion was not given to the circumference of grinding wheel, where the air might have been touched by the man who gave the impulsion, but to the axis through the middle of the wheel.
>
> Many people, convinced by these arguments and other similar evidence, teach that projectile motion is not the effect of air, but the effect of an *impetus* that was imprinted on the mobile, at the moment it was thrown, either by man or by the machine that launched this body.

These arguments won the support of Soto; here are, in fact, the conclusions he endorses[3]:

> *First conclusion*: It cannot be denied that the human or machine, by launching the projectile, shakes the air at the same time, as the experience of the circular ripples around a stone thrown in water shows us. The truth of this conclusion is particularly evident for the cannons from which air is expelled, in the form of a very violent explosion, at the same time as the cannonball…
>
> *Second conclusion*: The air is not the only cause that moves the projectile; what launched the mobile is also the cause, by the intermediacy of the *impetus* that it imprinted on the projectile.

The argument by which Soto refuted the theory of Aristotle is one that was commonly held in Paris since the time of Ockham, Buridan, and Albert of Saxony; the corollaries that he deduced from the theory of *impetus* are also those that the Nominalists were accustomed draw.

> By that,

he said[4],

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[3] Soto, *loc. cit.*; ed. cit., t. II, fol. ioo, coll. *c* and *d*.

[4] Soto, *loc. cit.*; ed. cit., t. II, fol. 101, col. *a*.

> we can discover the cause by which we a launch a shaft, commensurate with our strength, with more violence and farther than we would throw a small stone. The cause of it is, I say, that where there is least resistance, there is also less ability to receive the impression of *impetus*; the forces exerted do not find an object, then, in which they can fully spread. This is also the reason why a feather does not fly with such impetuosity [as a stone]; furthermore, it is not as well suited to slice through the air…
>
> The alternating motion of oscillation by which, before standing still, the wheel turns slightly in one direction, then turns in the opposite direction, must be attributed to the uneven weight unevenly distributed in various parts of the stone; indeed, when the movement ends as a result of the weakening of the *impetus*, the wheel can be fixed in the position it occupies; it is necessary that the raised parts fall by lifting those on the other side; in turn, when they fall, they raise the first, and this is so until the heaviest parts come to stop in the lowest position.
>
> If we conceive such a uniform wheel that does not weigh more on one side than the other, the movement would stop, I think, at the moment when the force of the *impetus* would end. Unless, however, you do not want, according to what others assume, to hold the following language: The parts of the air which lie on the front of the wheel, on the side towards which the motion tends, are condensed; the *impetus* of the wheel extinguished, they are rarefied and push the wheel back; but the air from the other side in turn launches the wheel forward, and this until the rarefaction of the air reaches the desired degree.

This passage shows us, in Soto, the concern not to attribute everything to the *impetus* in the various effects of projectile motion and to keep some account of the movement of the air. This concern is evident, in particular, in what our author says of the alleged initial acceleration of projectiles, the subject of many debates in the Middle Ages and the Renaissance[5]:

> It is another experience,

Soto said[6],

> that testifies that the air, too, is the cause of the motion of projectiles. We experience, in fact, that an arrow does not hit with such violence a very near object as it does a slightly more distant one; that is why Aristotle says, in the second book *On the Heavens*, that natural motion is more intense towards the end, while the greater intensity of the motion of projectiles is not attained at the beginning or end, but at the middle.

Some assume that, for this purpose, the cause is the following: The *impetus* is not, from the first moment, impressed in full on the arrow; it then becomes more intense or it spreads within the extent of the arrow, so that it moves it a more urgent manner. Soto alludes to this explanation given by Marsilius of Inghen. He goes on to say:

> But it is not easy to understand. We do not see, in fact, once the arrow is far from the ballista, what could increase the intensity of the *impetus*, because an accident does not of itself

[5] *Bernardino Baldi, Roberval et Descartes*, I: Une opinion de Bernardino Baldi touchant les mouvements accélérés (*Études sur Léonard de Vinci*, IV; première série, pp. 127 seqq.) — *John Buridan I (Béthune) and Leonardo da Vinci*, V: That the Dynamics Leonardo da Vinci proceeds via Albert of Saxony and Jean Buridan. In how it deviates, and why. The various explanations for the accelerated fall of weights that were proposed before Leonardo. [chapter 5] — *The tradition of Jean Buridan and Italian science in the 16th century*, III: The Parisian dynamics in the time of Leonardo da Vinci; V: How in the 16th century, the Dynamics of Jean Buridan spread in Italy. [chapter 10]

[6] Soto, *loc. cit.*; ed. cit., t. II, fol. 100, col. *d*.

become more intense. On the other hand, as the arrow is a continuous body, the *impetus* is imprinted simultaneously on the entire body; it therefore cannot expand further.

John Dullaert and Luis Coronel had already presented similar objections to the theory of Marsilius of Inghen; very wisely, they concluded that the speed of a projectile has its greatest value at the time when the mobile is launched. The professor of Salamanca was wrong not to side with this justified conclusion. He let himself here be lead by the desire to follow the opinion of Albert the Great and St. Thomas Aquinas.

That is why,

he wrote,

St. Thomas, when he commentated on the same text of the second book of the *De Cælo*, rightly assigns this experience to the amount of shaken air[7]. A portion of this air puts another in movement, this one shaking a third one, and the cause of motion is thereby increased. The thought of Albert the Great tends to the same object when he says in the same place: The impetuosity of the air moves all the more strongly as it is spread in a larger mass.

On this point Soto shows himself inadvertently unfaithful to the teaching of Luis Coronel and his masters of Paris; but can one reproach him severely? Leonardo, too, was, on the same question, clearly separated from the Nominalist doctrine; and several years after the professor of Salamanca had published his *Questions on the Physics of Aristotle*, Tartaglia and Cardan did not think differently from him regarding the movement of projectiles.

Yet Soto does not fully accept the opinion that Leonardo, Tartaglia, and Cardan so strongly embraced; he wonders if it could not explain the acceleration of the projectile, an acceleration he has no intention of calling into question, by putting forth a principle established by St. Thomas[8] for a totally different purpose:

St. Thomas made a fortunate appeal to another cause: As anything wants its own preservation, it happens that its virtue becomes all the more intense as this thing faces more resistance, provided, however, that it can overcome this resistance; so it may be that the *impetus* of the arrow itself grows in intensity thanks to the resistance that opposes it; but as it is, in the arrow, foreign and coming from the outside, it soon begins to weaken.

Concerning the nature of the *impetus*, Soto formulates this conclusion[9]:

The *impetus* is like heaviness and lightness, a distinct quality of the subject where it occurs.

The assimilation of the *impetus* to gravity was, we know, commonplace in the teaching of Paris in the early 16th century; *impetus* frequently received the names of accidental gravity, accidental lightness; Leonardo da Vinci also willingly gave *impeto* or *forza* the name of accidental gravity.

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[7] *Études sur Léonard de Vinci*, première série, p. 129.

[8] Sancti Thomæ Aquinatis *Summa theologica*, pars I, quæst. LXXV, art. 6: Utrum anima humana sit corruptibilis. St. Thomas is content to pose this principle: *Unumquodque naturaliter suo modo esse desiderat*, without making any application of it to projectile motion

[9] Soto, *loc. cit.*; ed. cit., t. II, fol. 101, col. *a*.

Soto pushes this assimilation as far as possible; he does not believe he is able to clarify the nature of gravity and lightness by defining it as a natural *impetus*[10]:

> That which creates a thing, at the same time gives a form to this thing, and gives it all the accidental properties that are unique to this form, which result from this form, which are necessary for the natural perfection of the thing created. However, the perfect state of a weight, a stone for example, is to reside at the center of the World. Therefore, what engenders a stone gives it a certain natural *impetus* in order that it descend down to the center when it is not prevented. This is why the movement of a weight is attributed to what has generated this weight. Similarly, one who throws a stone imprints on it an *impetus* that moves it…
>
> When bodies are outside their natural places, they are always out of state that suits them and their natural perfection; the movement that brings each body to its natural place is attributed to the cause that engendered it and that, somehow, launches what it engendered toward the perfection that suits it.
>
> Perhaps we can make this objection: When a weight falls, sometimes it happens that the cause that engendered it ceased to be.
>
> Here is what we will answer: a thing that no longer exists can continue to move as long as the virtue that it has produced lasts. This is evident in the example that the launched arrow or the ball projected by the cannon provides us. It is the fire that moves this ball, although it moves at a distance, by the *impetus* it imprinted on it.

The explanation of the motion of projectiles by means of an *impetus* impressed on the mobile has satisfied the reason of Soto to the point that it serves to illuminate, by way of comparison, the solution of other problems in Physics and, in particular, to seek the cause of the movement of heavy bodies. Nothing is more likely to manifest the lasting influence of the teaching of the Parisian Nominalists on the spirit of the professor of Salamanca.

[10] Dominici Soto *Op. laud.*; Super lib. II quæst. prima: De natura; utrum definitio naturæ sit bona? Ed. cit., t. II, fol. 32, col. *c*.

Chapter 18
Soto tries to make the views of Aristotle and St. Thomas agree with the hypothesis of *impetus*

Such a complete break from the theory of the movement of projectiles that Aristotle conceived and St. Thomas Aquinas supported is particularly noteworthy from a prominent member of the Order of Saint Dominic; it is well known, in fact, how much, in all circumstances, this order proved faithfully attached to the peripatetic philosophy, converted to Christianity by the Angel of the School. Soto could not ignore this rupture, which the service of truth imposed on him, but he could not recognize it without suffering. He did, however, all that was in his power to attenuate its brutality and restrict its scope. Unable to constrain it to the opinion of his teachers, he tried to persuade himself that his masters held his opinion.

Regarding Aristotle, the enterprise was difficult; so formally, and in so many parts of his work, the Philosopher attributed the conservation of movement of projectiles only to the air! Soto, however, tried. He imagined that Aristotle had implicitly admitted the hypothesis of *impetus*; he had only assigned to the air, in the movement of the projectiles, an auxiliary role, similar to what one day Leonardo da Vinci, Cardan, and Soto himself would assign to it; he had long insisted on the motive action of the air to distinguish better the problem of the movement of projectiles from the problem of the fall of weights.

> We must not believe,

Soto said[1],

> that Aristotle doubted [this hypothesis of *impetus*], but he did not mention it, holding it evident from the analogy with light or heavy bodies. It is, in fact, the primary reason for affirming the reality of an *impetus* of this kind. Just as the generative cause of weight confers on it a natural quality, which is gravity, that pushes it to the center of the World, so the one who launches a projectile imprints an *impetus* on it.

It is hardly necessary to say how this interpretation of the thought of Aristotle is untenable.

Soto is found in conditions a bit less unfavorable when he claims to make St. Thomas Aquinas a supporter of the *impetus impressus*; he believes, in fact, that in two

[1] Dominici Soto *Op. laud.*; in lib. VIII quæst. III; ed. cit., t. II, fol. 100, col. *d*.

texts of the Angelic Doctor, “*mentis Aristotelis sedulus explorator*,” he recognizes a clear allusion to this quality imprinted in the projectile.

Take a look at these two texts; in the one as in the other, he is explaining how the seed retains its generative power that the male has communicated to it.

Here is the first passage[2]:

> An instrument is considered moved by the agent that was its principle of movement as long as it retains the virtue that was imprinted in it by that principal agent; so the arrow is driven by what launched it as long as the strength of the impulsion of the agent which launched it lasts. Similarly, among heavy or light bodies, a created body is moved by the cause that created it as it retains in it the form that was given to it by this cause; so it is with the seed… It is necessary that the thing that moves and the thing moved are joined together at the beginning of the movement, but not for the whole duration of the movement, as seen in the movement of projectiles…

Now comes the second text[3]:

> The virtue which comes from the father and is a permanent virtue of intrinsic origin; it does not come from outside, such as from the virtue coming from the motive cause that is in projectiles… However, it is, in a way, similar to the latter. Similarly, indeed, that the virtue of the projecting cause, because it is a finite virtue, only moves with local movement a certain distance, likewise, the virtue of the one who begets only moves with the movement of generation up until a determined form.

The authenticity of these passages is not doubtful[4]; at a first reading, it is difficult not to recognize this obvious allusion to the theory of *impetus* that Soto saw there. If given such a sense, however, how will one make them agree with this other passage, not of less certain authenticity, which St. Thomas writes[5] in his commentary on the *De Cælo* of Aristotle:

> We must not assume that the mover by which the violence is produced impresses in the violently moved stone some virtue that moves this stone, as the thing that generates produces in the thing generated a form from which its natural movement results. If it were so, indeed, violent movement would come from a principle intrinsic to the mobile, which is contrary to the very notion of violent movement. In addition, it would result that the stone, by the very fact that it moves with local movement, is altered in its substantial form, which is contrary to common sense.

Soto, who in the previous two texts could see a confirmation of the hypothesis of *impetus* of which his reason was convinced, would find in the new text the formal condemnation of the ideas that are dear to him and, in particular, of the assimilation between violent *impetus* and natural gravity.

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[2] Sancti Thomæ Aquinatis *Quæstiones disputatæ. De potentia Dei*, quæst. III: De creatione. Art. XI: Utrum anima sensibilis vel vegetabilis sit per creationem vel traducatur ex semine? [ad 5]

[3] Sancti Thomæ Aquinatis *Op. laud.*, De anima quæst. unica. Art. XI: Utrum in homine anima rationalis, sensibilis et vegetabilis sit una substantia? [ad 2]

[4] On the authenticity of the *Quæstiones disputatæ*, see: J. Quétif and J. Echard, *Scriptores ordinis prædicatorum*, t. 1, pp. 288-289.

[5] Sancti Thomæ Aquinatis *Commentaria in libros Aristotelis de Cælo et Mundo*, in lib. III, lect. VII [n. 6].

This apparent contradiction embarrassed the various authors who, after Soto, wanted to find in the *Quæstiones disputatæ* allusions to the theory of *impetus*, such as John of Saint Thomas[6]. To solve it we believe it is best to ask for some clarification from St. Thomas himself.

Let us continue the lecture of the commentary *De Cælo* which we quoted at the beginning:

> The mover that moves violently therefore imparts to the stone only the movement which takes place while the mover is in contact with the stone. But the air is more likely to receive such an impression, either because it is more subtle or because it is endowed with a kind of lightness; it is thus moved faster than the stone by the impression that the mover which exerts the violence communicates to it; when this violent mover stops acting, the air moved by it pushes the stone and makes it advance; it also pushes the air that is conjoined to it, and it pushes the stone farther; and this takes place so long as the impressing of the first violent mover lasts, as it says in book VIII of the *Physics*. It is the same to say this: Although the mover that produced the violence does not follow the mobile which is carried by this violence—the stone, for example—in such a way that it moves it while staying present, it moves it, however, by the impression imparted on the air (*per impressionem æris*); if there did not exist bodies such as air, there would be no violent movement. It is therefore obvious that air is the necessary instrument of violent movement; it does not only contribute to the perfection (*propter bene esse*) of this movement.

Now it is, we believe, impossible to ignore the thought of St. Thomas. In the launched stone, there is no quality, no *impetus* imparted by the mover. But the mover imports such a quality to the air surrounding the projectile. All the comparisons where the vernacular speaks of virtue imparted to the mobile by what launches it must, for the physicist, be understood of the impression imparted to the air by the mover. These comparisons can then be accepted without being unfaithful to the Mechanics of Aristotle and Averroes.

It is of this Mechanics that St. Thomas Aquinas is proclaimed the very firm believer, while he rejected with all his might the hypothesis of *impetus* on which the Parisians were, the following century, to establish all their Dynamics. By accepting this hypothesis, Soto remains the disciple of the Nominalist teaching; in vain he tries to deceive and convince himself that he does not deviate from the peripatetic doctrine.

We will see these Nominalist theories, which influenced the professor of Salamanca during his stay in Paris, produce in his works one of their most important results. But to understand how the Dominican theologian was led to formulate exactly the laws of falling bodies, sixty years before Galileo, we must go very far back into the past and follow a lengthy digression; we have to show, in fact, how the double tradition of Albert of Saxony and Nicole Oresme led, so to speak, to this great discovery.

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[6] R[mi] P. Joannis a Sancto Thoma, ordinis prædicatorum, *Cursus philosophicus Thomisticus, secundum exactam, veram et genuinam Aristotelis et Doctoris Angelici mentem*. Quæstiones et articuli super octo libros physicorum. Circa librum octavum, de motus æternitate et reductione in primum motorem, quæst. XXIII: De motu naturalium et projectorum. Art. 2: Qua vi moveantur projecta?

Chapter 19
The origins of Kinematics

The treatise *De proportionalitate motuum et magnitudinum*

When a heavy body falls freely, it moves with a uniformly accelerated motion.

It follows that *the space traversed in a certain time by such a weight is the product of the duration of the fall with the average between the initial speed and the final speed.*

Both laws dominate the whole theory of falling bodies. The discovery is ordinarily attributed to Galileo. We will see, however, that Domingo Soto formally admits of its correctness; he admits it, moreover, as a common truth, in the way he would admit a commonly received proposition in his time in the schools. And indeed, these two laws would not be called into question in the Spanish universities at the beginning of the 16^{th} century because they quite naturally resulted from the teaching of the Parisian Nominalists.

But this teaching, from which Domingo Soto and his contemporaries could draw such corollaries, itself consisted in successive advances whose history we will try to trace.

We must first examine how the concept of uniformly accelerated motion was clarified.

Physicists and astronomers of Antiquity and those of the Middle Ages until the middle of the 14^{th} century attentively considered two kinds of movement: uniform movement of translation and uniform rotation. Sometimes, indeed, they happened to meet, in their speculations, a movement that did not belong to either of these two categories; Aristotle was well aware, for example, that a weight moves more quickly the farther it falls, and many others after wrote about this accelerated motion; but those who spoke of it were content with purely qualitative indications; they were not trying to describe with geometric precision this change in speed.

In two uniform translations, the comparison of the speeds is made, so to speak, by itself; the speeds of the two mobiles are among themselves like the lengths described, during the same time, by a point of the first mobile and by a point of the second mobile; it is not necessary to clarify further the time during which the two lengths are described, nor to designate, in each of the two mobiles, the point which measures the path.

One can easily compare two uniform rotations by evaluating the ratio of the two *angular velocities*; the concept of angular velocity in a uniform rotation is presented so simply and so naturally to the mind of astronomer, that we found it, from the beginning of Greek Astronomy, implicitly present in all the writings on the science of celestial movements, without its being given any formal definition.

What is the speed of a body in which the various parts move in a different way or which do not move the same at different times? This question was posed explicitly in the mind of physicists only recently.

It appears to have been, first of all, clothed in this form: What should we call the speed in a body in which all its parts are not animated with the same movement and, specifically, in a body animated with a uniform rotation?

Answering this question is, in fact, the subject of an anonymous piece that, we believe, was never printed and which is in a manuscript of the late 13th century in the Bibliothèque Nationale[1]. This piece seems to be placed at the origin of the entire intellectual movement that we propose to study.

This short treatise begins, in an Euclidean manner, with the statement of seven propositions which we will reproduce in their Latin:

1. *Quæ magis removentur a centro, magis moventur, et quæ minus, minus.*
2. *Quando linea æqualiter, et uniformiter, et æquidistanter movetur, in omnibus partibus suis et in punctis ipsis æqualiter movetur.*
3. *Quando medietates æqualiter et uniformiter moventur a se invicem, totum æqualiter movetur suæ medietati.*
4. *Inter lineas rectus æquales æqualibus temporibus motas, quæ majus spatium transit et ad majores terminos, magis movetur, et quæ minus [spatium] et ad minores terminos, illa minus movetur.*
5. *Quod nec majus spatium nec ad majores terminos, magis non movetur.*
6. *Quod nec minus spatiurn nec ad minores terminos, minus non movetur.*
7. *Proportio motuum punctorum est tanquam linearum in eodem tempore descriptarum.*

The last of these postulates, which obviously implies that movement is uniform in time, calls for a remark: The word *movement* (*motus*) is taken, for a point that progresses uniformly, as meaning what we today give to the word *speed*. This is a synonym that we often invoke to interpret the texts we quote in this history.

The other postulates are intended to clarify the rules that will allow us to compare *movements* of two equal straight lines; the notion that the author thereby seeks to define corresponds to what we would call the *average speed* of the various points of this line.

[1] Bibliothèque Nationale, fonds latin, ms. n° 8680 A. The piece in question begins at the bottom of fol. 6, r°, with these words: *Que magis renoventur* [read: *removentur*] *a centra magis moventur et que minus minus*. It ends at the bottom of fol. 7, r°, with these words: *Residuum igitur quod est.* g. f. *equale est duplo*. c. d. *et linee.* o. b. *In tant uni erit.* h. a.

The author states the fundamental proposition that he intends to demonstrate in these terms:

> If on a radius which describes a circle one takes a portion of arbitrary length which does not terminate at the center, this straight portion has a movement equal (*æquatiter movetur*) to that of its midpoint. It follows that the radius also has a movement equal to that of its midpoint.

We will not analyze here the relatively complicated demonstration that this theorem receives; rather, we seek to identify the exact thoughts of the author. In stating that this portion of the radius has *an equal movement* to that of its midpoint or, in more modern language, an *average speed* equal to the *speed* of its midpoint, here is exactly what he means: By its uniform rotation, this straight line sweeps out, at a given time, an area equal to what it would sweep in the same time by a movement of translation perpendicular to its proper direction and having a speed equal to the speed of its midpoint. Under the artifices of reasoning, this is where the main idea we can discover is.

The little treatise which we have just analyzed seems to have initiated the Middle Ages to the considerations of Kinematics. What date must we attach to this writing, whose author is unknown to us? Are we to believe that it was written by a geometer of the Middle Ages, for example by a disciple of Jordanus of Nemore like that other treatise contained in the same manuscript collection? Should we regard it as a relic of Antiquity? It seems impossible to answer these questions categorically. All we can observe is that the letters by which the various points of the figures are designated do not succeed one another in the order characteristic of the Greek alphabet, as almost always happens in treatises of Hellenic origin; also, no Greek or Arabic word is found in the Latin in which this booklet is written.

In the 14th century, Thomas Bradwardine, in a writing which we will discuss in the next paragraph, quotes the treatise of which we just presented a brief analysis; he gives it this title: *De proportionalitate motuum et magnitudinum*; but he does not know—or, at least, does not make known to us—the name of who composed it; he is confined, in effect, to designate it as follows[2]:

> *Auctor vero de proportionalitate motuum et magnitudinum subtiliorem istis intellectum ponit, quod linearum rectarum æqualium, temporibus æqualibus quibuslibet motarum, quæ pertransit majus spatium et ad majores terminos, moveri velocius; et quæ minus et ad minores terminos, lardius; et quæ æquale et ad æquales terminos æqualiter moveri supponit; et intelligit per terminos majores terminos ad quos a terminis a quibus magis distantes.*

One can notice that Bradwardine, to whom we owe this very recognizable allusion to the anonymous treatise *De proportionalitate motuum et magnitudinum*, also cited, and in the same book, the *De ponderibus* of Jordanus of Nemore; both writings seem, we have said, to present some formal analogies, as if they came from the same school.

The book *De sex inconvenientibus* is an anonymous book which was written in Oxford, probably towards the end of the 14th century; this work, with which we

[2] Bradewardyn *proporciones*; 2ª pars quarti capituli. Bibl. Nat., fonds latin, ms. n° 6559, fol. 56, col. *d*.

will deal at length in a later paragraph[3], is one of those which *Jordanis* (*sic*) and his treatise *De ponderibus* willingly cite. We find there a detailed discussion[4] of this question: Is the speed of the rotational movement of a spherical orb measured by the speed of the point which is midway between the point nearest to the center and the farthest point? The opinion that says yes is given as that which was produced "in his treatise, *in tractatu suo*" by an author that a manuscript[5] calls *Magister Ricardus de Versellys* and that another manuscript[6] calls *Magister Ricardus de Uselis*.

But is this master Richard *de Versellys* or *de Uselis* the author of the little book that Bradwardine cited and that we analyzed? Is he just some newer philosopher who had adopted the doctrine formulated by this writing? It is impossible to say. We are obliged to respect the mystery where the first creator of a theory, the development of which we will study, is hiding.

Thomas Bradwardine. John of Murs. Jean Buridan.

The first author whose research was influenced by the treatise *De proportionalitate motuum et magnitudinum*, the first who attempted to clarify the concept of speed more precisely than this treatise had, is Thomas Bradwardine.

Thomas Bradwardine was born towards the end of the 13th century at Hartfield, near Chichester. In 1325 he was procurator of the University of Oxford. Confessor of Edward III, he accompanied the king in France. He died on 26 August 1349, a few days after his appointment to the archbishopric of Canterbury.

In turn mathematician, philosopher, and theologian, Bradwardine, by his teaching and writings, exercised a profound and lasting influence on the Scholasticism of the Middle Ages; but this influence was particularly strong in the University of Oxford, as we shall later have the opportunity to see.

Among the most read writings, the most frequently cited of Bradwardine, his *Treatise of Proportions* should be placed first; this book was still in great favor at the time of the discovery of the printing press, which gave it multiple editions[7]. However, the historian must use these editions with caution; there are some very

[3] See § XX [section 19].

[4] *Liber sex inconvenientium*. Quarta questio: Utrum in motu locali sit certa assignanda velocitas? Articulus secundus: Utrum velocitas motus spere cujuslibet penes punctum vel speram aliquod (*sic*) attendatur?

[5] Bibl. Nat., fonds latin, ms. n° 6559, fol. 34, col. *a*, et fol. 36, col. *a*.

[6] Bibl. Nat., fonds latin, ms. n° 7368, fol. 102, col. *a*, et fol. 162, col. *a*.

[7] We could not consult the first two; the third, which we had in hand, will be described in the following note:

1. *Tractatus proportionum* Alberti de Saxonia. — *Tractatus proportionum* Thomæ Braduardini. — *Tractatus proportionum* Nicholai horen. — Venales reperiuntur Parisius in vico divi Jacobi juxta templum Sancti Yvonis sub signo Pellicani (no date).
2. Benedicti Victorii Faventini *Commentaria in Tractatum proportionum Alberti de Saxonia*. — Thome Bravardini Anglici *tractatus proportionum perutilis*. Colophon:

incomplete ones[8], which lack certain parts of undoubted authenticity, and which the Middle Ages has constantly brought honor to the Master of Oxford. Also, we will query a manuscript for the text of the *Proportiones* of Bradwardine; this manuscript[9], formed exclusively of pieces written by the Oxford masters, gives us serious guarantees of its integrity and accuracy.

The arithmetic theory of proportions is not the subject of the book written by Thomas Bradwardine; it is especially Mechanics that this author meant to treat, as he tells us in this preamble[10]:

> *Omne motum successivum alteri in velocitate proportionari convenu; quapropter philosophia naturalis, quæ de motu considerat, proportionem motuum et velocitatum in motibus ignorare non debet; et quia cognitio ejus est necessaria et multum difficilis, ideo de proportione velocitatam in motibus fecimus illud opus; et quia, testante Boetio, primo Arismeticæ suæ, quisquis scientias mathematicales prætermisit, constat eam omnem philosophiæ perdidisse doctrinam, ideo matematicalia quibas ad propositam indigemus præmisimus…*

According to the program this preamble has traced, four chapters make up the whole work, and the first of these chapters is only dedicated to the arithmetical study of ratios and proportions.

The second and third chapter are designed to analyze the relationship between the speed of a movement, the size of the motive power, and the magnitude of the resistance; in modern language we would say that they treat Dynamics.

In the second chapter, Bradwardine refutes the opinions that he regarded as erroneous; this is where we see him[11] invoke "the first conclusion of the *De ponderibus*, which says: *Inter quælibet gravia est velocitatis in descendendo et ponderis eodem ordine sumpta proportio*."

The third chapter is devoted to the presentation of the law that the Master of Oxford regards as exact and which he states in these terms[12]:

> Et sic impositus est finis subtilissimis tractatibus de proportionibus, proportionalitatibus et motuum comparationibus in velocitate excellentis Doctoris Alberti de Saxonia una cum clarissimis annotationibus Benedicti Victorii Faventini. Et venerabilis sacre pagine Doctoris Thome Bravardini Anglici. Impressi autem sunt Bononie per Benedictum Hectoris bibliopolam Bononiensem. Anno domini MCCCCCVI. die XX Martii.

[8] This is the case of the *Tractatus brevis proportionum: abbreviatus ex libro de Proportionibus*. D. Thome Braguardini Anglici, which is found in the following book: *Contenta in hoc libello. Aritlunetica communis. Proportiones breves. De latitudinibus formarum. Algorithmus* M. Georgii Peurbachii in integris. Algorithmus Magistri Joannis de Gmunden de minuciis phisicis. Colophon:

> Impressum Vienne per Joannem Singrenium Expensis vero Leonardi et Luce Alantse fratrum Anno domini MCCCCCXV. Decimonono die Maii.

[9] Bibl. Nat., fonds latin, ms. n° 6559. — The *Proporciones Bradewardyn* begins at fol. kg, col. *a*, ending in fol. 58, col. *a*.

[10] Bibl. Nat., fonds latin, ms. n° 6069; fol. 49, col. *a*.

[11] Ms. cit., fol. 53, col. *a*.

[12] Ms. cit., fol. 54, col. *c*.

> In diverse movements, the speed is proportional to the ratio of the power to the resistance; *Proportio velocitatum in motibus sequitur proportionem potentiæ motoris ad potentiam rei motæ*.

Bradwardine confirms this law, among other reasons, by the authority of various passages of Aristotle and Averroes; and indeed it is undeniable that it represents the most widely recognized and most clearly articulated principle by peripatetic Dynamics; the English mathematician therefore did not recognize how this Dynamics is difficult to reconcile with the truths that observation reveals.

He did not even acknowledge how it is incompatible with some other claims of the Dynamics of Aristotle; the Stagirite admits, indeed, and Bradwardine with him, that there is no movement when the power is equal to the resistance; the speed is then zero.

The Oxford mathematician only notices that some specific laws that he has criticized and rejected are simple corollaries of the general law he regards as true. In this discussion of Dynamics, his logical sense was singularly deceived; but the inconsistencies of Bradwardine, in this difficult subject, are often found, barely mitigated, among his successors.

Bradwardine begins with these words[13] the fourth chapter of his *Treatise of Proportions*:

> After determining in general the connection between the speeds of these various movements when the motive power is compared with the resistances, we will, in the following, demonstrate some special propositions regarding the relations between circular movements when the magnitude of the body moved and the magnitude of the space traversed is considered.

It is the Kinematics of uniform rotational motion that will be discussed in this chapter.

The author begins by reviewing and refuting the opinions that seem inadmissible to him. It is among these that he ranks, with some hesitation, the opinion supported in the treatise *De proportionalitate*; according to this opinion, remarks Bradwardine[14], "any radial portion not terminated at the center, and even the whole radius, moves equally fast with their midpoint."

For this doctrine, the Oxford mathematician substitutes another that he formulates in these terms:

> The speed of the local movement [in a body animated by a uniform rotational movement] is measured by the speed of the point that, in this body moved with local movement, moves the fastest. — *Ideo videtur rationaliter magis dici quod velocitas motus localis attenditur penes velocitatem puncti velocissime moti in corpore moto localiter.*

This way of defining the speed in a rotational movement seems very strange, and less satisfactory, certainly, than the one which the *De proportionalitate motuum et magnitudinum* attempted to justify. It was in no less than the greatest vogue, and

[13] Ms. cit., fol. 56, col. *b*. — This chapter is missing in the edition, printed in Vienna in 1515, of which we have previously given the title.

[14] Ms. cit., fol. 56, col. *d*.

Scholasticism did not tire, for two centuries, from teaching it. It remained as a witness to the profound influence of the treatise that Bradwardine concluded in this ingenious invocation[15]:

> *Perfectum est igitur opus de proportione velocitatum in motibus, cum illius Motoris auxilio a quo motus cuncti procedunt; cujus ad summum mobile proportio nulla reperitur; cui sit honor et gloria quamdiu fuerit ullus motus. Amen.*

Besides, we know the date of this *Treatise of Proportions*; it was composed in 1328, as we learn from the words by which it ends in two manuscripts preserved at the National Library[16], which are:

> *Explicit tractatus de proportionibus editus a Magistro Thoma de Breduardin anno domini M° CCC° 28°.*

The influence of the writing of Bradwardine did not remain confined to Oxford; soon it was felt in Paris; but the two chapters on Dynamics seem to have, first of all, attracted attention; it is probably to them that it is fitting to attribute the composition of various writings for fixing the relationship between the speed with which a mobile moves, the power that moves this mobile into movement, and the contrary power that restrains it.

It seems, for example, that the influence of Bradwardine leaves one guessing of what Walter Burley said of this relation[17], when he comments on book VII of the *Physics* of Aristotle; the terms in which Burley affirms that the speed of movement is proportional to the ratio of power to resistance is reminiscent of those employed by the mathematician of whom he was probably, in Oxford, the fellow student or colleague.

Arguably, the dynamical theories of Thomas Bradwardine contributed to suggesting the theories, all similar in their conclusions, that Master John of Murs explained at length in his *Opus quadripartitum numerorum*[18].

The date of this work is precisely known to us because it ends with this statement[19]:

> Laus et honor, motus (?), gloria, potestas sit summo Deo a quo omnis sapientia derivatur, qui me servum suum ad terminum attulit præoptatum. Actum anno Domini Jesu Chrisii 1343,

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[15] Ms. cit., fol. 58, col. *a*.

[16] Bibliothèque Nationale, fonds latin, ms. n° 16621, fol. 212, v° — ms. n° 1/457C, fol. 261, col. *c*. In the latter ms., instead of *Breduardin*, we read: *Bradelbardin*; the scribe had read the letters *Ib* where the text he copied had a *w*.

[17] Burleus *super octo libros physicorum*. Colophon:

> Impressa arte et diligentia Boneti locatelli bergomensis, sumptibus vero et expensis Nobilis viri Octaviani scoti modoetiensis… Venetiis. Anno salutis nonagesimoprimo supra millesimum et quadringentesimum. Quarto nonas decembris.

[18] Bibliothèque Nationale, fonds latin, ms. n° 7190, fol. I, r°, to fol. 100, v°. — Under this title: Johannis de Muris *De mensurandi ratione*, this same Treatise is found in mss. 7380 and 7381 of the same collection; we did not consult these last two manuscripts.

[19] Ms. cit., fol. 100, v°.

> Novembris 13 die, orto jam Sole, initio Serpentarii exeunte, Luna quoque in Libra, in fine primæ faciei, secundum veritatem tabularum illustris principis Alfonsi regis Castellæ quæ compositæ sunt ad meridiem Toletanum. Explicit quadripartitum numerorum Johannis de Muris.

In the fourth book of *Quadripartitum numerorum*, the first treatise, entitled *De moventibus et motis*, is entirely[20] dedicated to presenting this law, the basis of peripatetic Dynamics: Any mobile under constant power and constant resistance moves with a uniform motion whose speed is proportional to the magnitude of the power and inversely proportional to the size of the resistance.

In this analysis of John of Meurs, he explicitly admits that all the movements considered are uniform and, moreover, it is implicitly assumed that all points of the mobile move with the same speed; the discussions of Kinematics thus have no place in the work of the Norman master.

By accepting without restriction or hesitation the rules that Aristotle, in book VII of his *Physics*, imposed on Dynamics, Thomas Bradwardine and Master John of Murs much more easily satisfy what Master Jean Buridan, a few years after, will.

In his great work on the *Physics* of Aristotle, the Philosopher of Béthune devotes two questions[21] to discussing the rules of Dynamics that the Stagirite posed; and the pitiless discussion clearly reveals this truth: There is in nature no movement to which these rules are correctly applied.

Jean Buridan, however, took care to note repeatedly that some of the rules laid down by Aristotle are patently false when the movement does not continue at a constant speed; but of the variable speed that certain movements such as the fall of weights present, he makes no attempt at a specific study; if the problems of Dynamics are of concern, the questions of pure Kinematics do not solicit his attention.

Albert of Saxony

The first author we saw after Bradwardine, anxious to clarify the concept of speed, is Albert of Saxony; the writings by this author clearly manifest to us, moreover, the dual influence that Albert suffered from Thomas Bradwardine and Jean Buridan.

The influence of the master of Oxford is obvious to one who opens the little book of Albert of Saxony so often printed with this title: *Tractatus proportionum*. This book, in fact, which some manuscripts[22] entitled *De proportionibus motuum*, is not

[20] Ms. cit., fol. 72, r°, to fol. 81, r°.

[21] *Questiones totius libri phisicorum edite a Magistro Johanne Buridam*. De motu. Liber VII^us^ phisicorum. Queritur 7° circa ultimum capitulum hujus VII^i^, in quo Aristotiles ponit multas regulas de comparationibus motuum secundum habitudinem ad motores, et est hoc questio de primis duobus regulis, videlicet utrum he due regule sunt vere. — Queritur 8° et ultimo magis generaliter de illis regulis Aristotilis quas ipse ponit in ultimo capitulo hujus VII^i^ phisicorum utrum sint universaliter vere. (Biblio. Nat., fonds lat., ms. n° 14723, fol. 94, col *a*, à fol. 95, col. *a*.)

[22] For example, ms. n° 7368 (Latin collection) of the National Library which, of fol. 14, r°, au fol. 26, v°, reproduced this treatise, and which says, at fol. 26, v°: *Expliciunt proportiones motuum. Deo gratias.*

a treatise on Arithmetic; like the *De proportione velocitatum in motibus*, it intends to discuss Mechanics. Also, the book of Albert of Saxony is composed on exactly the same plan as the book of Bradwardine.

In that book, as in this one, one first of all reads of a purely mathematical theory of ratios and proportions; but this theory is there only as an introduction to considerations of Mechanics that follow.

When the author addresses these latter, he hastens to warn us that they are the main subject of his teaching:

> *His visis, videndum est de principali intento, scilicet penes quid attendatur proportio velocitatum in motibus; et primo, penes quid tanquam penes causam; secundo, penes quid tanquam penes effectum.*

Not only is the subject which Albert intends to discuss the one that occupied Bradwardine, but Albert will also divide his discourse as Bradwardine did.

He will examine, firstly, how the speed of a movement depends on the cause that produces this movement (*penes quid tanquam penes causant*), that is to say that he will seek how this speed depends on the magnitudes of the power and resistance. The first chapter is on Dynamics.

Secondly, the Parisian Master will analyze the mode of variation of the speed with respect to its effect (*penes quid tanquam penes effectum*); he will seek how the magnitude of the velocity is related to the space traversed by the various parts of the mobile and to the time used to describe them. This second chapter will form a small treatise of Kinematics.

The Dynamics of Albert of Saxony, like that of Bradwardine, can be summarized in the great peripatetic law: The speed with which a mobile moves is proportional to the ratio of the power to the resistance. But the Master of Paris displays less confidence in admitting this law than the Master of Oxford; obviously, his confidence was shaken by the discussion of Buridan; in the presentation that the *Tractatus proportionum* gives, various borrowings are made from this discussion; these borrowings are even more numerous and more recognizable in the course of two questions[23] that Albert of Saxony devoted to the discussion of the rules laid down by Aristotle in book VII of the *Physics*. Among these borrowings there is one that we find in these two writings of Albert of Saxony which deserves special mention; it concerns the supposition that explains the acceleration of falling bodies by an *impetus acquisitus*.

But the chapter of the *Tractatus proportionum* devoted to Dynamics should not hold us back any longer here; what should solicit our attention is the chapter devoted to Kinematics, by which the book ends.

This chapter begins with the these words:

> *Nunc restat videre penes quid attendatur velocitas motus tanquam penes effectum; et primo, de motu locali; secundo, de motu augmentationis; tertio, de motu alterationis.*

This program not only marks the divisions of the chapter that we propose to analyze; it simultaneously discloses its entire scope. Formed by peripatetic Philosophy,

[23] *Acutissimæ quæstiones super libros de physica auscultatione ab* Alberto de Saxonia *editæ*; lib. VII, quæst. VII et quæst. VIII.

Albert gives the word “movement” the entire extent that it takes in the Physics of Aristotle. He will not only discuss, like Bradwardine and our modern Kinematics, local movement, but even movement of augmentation and movement of alteration. That way, his *Tractatus proportionum* will become the exemplar of the treatises *De tribus motibus*, *De triplici motu*, and *De tribus prædicamentis in quibus fit motus* that we will see produced until the early years of the 16th century.

What he says of local movement is divided in two paragraphs devoted to the law of local movement, i.e., to the movement of *translation*, and the other to circular local motion, i.e., to the movement of *rotation*.

The speed of rectilinear motion is measured, according to Albert of Saxony, by the length of the line described in terms of time by a point of the mobile.

However, in the formula that states this definition, a complication is introduced; Albert gives this statement:

> *Velocitas motus localis recti attenditur penes spatium lineale verum vel imaginatum descriptum a puncto medio vel æquivalenti corporis moti in tanto vel in tanto tempore.*

Our author, in fact, does not want a definition that would apply only to the translation of a point or of an indeformable body; he wants that the various points of the body animated with rectilinear motion can, together, move relative to each other, that the body can experience condensations and expansions. The various points of the body, in this case, do not all move with the same speed; what is the speed which one must choose as suitable for measuring the speed of the body itself? It is inadmissible, according to Albert, that it be the point whose movement is the fastest. The rectilinear movement taken by the mobile is, in this case, the speed of a certain average point which can be physically realized within the body, but which can also, from one moment to the other, coincide with different material parts of the body, so that it remains the same point only *by equivalence*.

Obviously, these considerations bear traces of the influence of the little treatise *De proportionalitate motuum et magnitudinum* that we analyzed in § VIII [section 19]. This influence is revealed again, and even more clearly, in what Albertutius will say about circular movement.

In a uniform rotation, what should we call the speed of the mobile?

Is the speed measured by the linear space described by the midpoint of the radius of the mobile, “*sicut vult una opinio*”, or by the linear space that the point equidistant from the concavity and convexity of the orb animated with a movement of rotation describes, “*sicut voluit una opinio*”? the opinion to which Albert makes this double allusion is what the little writing that Bradwardine entitled *De proportionalitate motuum et magnitudinum* supported. It fits very well, it seems, with what the Parisian Master, probably inspired by this little treatise, admitted regarding rectilinear motion. He refuses, however, to measure the speed of the rotational movement in this way.

The definition which, quite unintentionally, it is preferred is what we have heard Thomas Bradwardine advocate: The speed of the circular movement is measured by the length of the line that describes the point of the mobile which moves quickest.

If Albert of Saxony seems to have been misguided when he followed, in this question, the footsteps of Thomas Bradwardine, he seems to have received from his own genius a happier suggestion when he defined *velocitas circuitionis* as what we would call today the *angular velocity*:

> The rotational speed (*velocitas circuitionis*),

he said,

> is measured by the angle described about the center or the axis of this rotation, this angle being compared to the time [used to describe it] so that, if two mobile revolve around the same axis and, in equal time, describe equal angles, we say they rotate equally [fast] around this axis; and if the angles described are unequal, they rotate unequally fast. This conclusion evidently results from the manner of speaking commonly used by astrologers. It is known that such speed, strictly speaking, cannot be compared to the speed of rectilinear movement or to the speed of circular motion, as an angle[24] and a line are not comparable with each other.

Surely, as Master Albert of Saxony actually remarks here, the concept of angular velocity was, always, wrapped in language that astronomers were accustomed to use; yet is it fair to attribute some merit to the one who was the first to define it formally.

We will leave, for the moment, what the Parisian Master said about movement of augmentation and alteration; the result of this study will encourage us to come back to it.

The analysis of the *Tractatus proportionum* showed us how Albert of Saxony was attached to the study of the speed in a body whose various parts do not move equally fast. But, in this writing, we have not met anything that treats of a speed varying from one moment to the next. It is not that this new subject was foreign to the meditations of Albertutius because he will tell us about it in one of his questions on the *De Cælo* of Aristotle[25].

This question reads as follows:

> Is the movement of Heaven, from east to west, regular?

In order to answer that, Albert of Saxony poses a distinction the principle[26] and that we will reproduce:

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[24] The text we have before us is the one with this colophon: *Magistri alberti de Saxonia proportionum libellus finit feliciter qui Venexie summa cum diligentia fuit impressus per magistrum Andream catharensem Die XXI Iulii MCCCCXXXLVII* (*sic*). In this place, by an obvious error, it says *arcus* instead of *angulus*.

[25] *Questiones subtilissime* Alberti de Saxonia *in libros de celo et mundo*. Colophon:

> Expliciunt questiones… Impresse autem Venetiis Arte Boneti de locatellis Bergomensis. Impensa vero nobilis viri Octaviani scoti civis modoetiensis. Anno salutis nostre 1492 nono kalendas novembris Ducante inclito principe Augustino barbadico. Lib. Il, quæst. XIII.

This question, and question XIV, which will be discussed in the next paragraph, were omitted in the editions of the *Quæstiones* of Albert of Saxony, Themo, and Buridan that Josse Bade and Conrad Bosch published at Paris in 1516 and 1518. We are ensured that these two questions were on the handwritten text that the Cod. n° 14723 of the Latin collection of the Bibliothèque Nationale contains.

[26] Burleus *super octo libros physicorum* of which Walter Burley had already established. Colophon:

It is necessary to know,

he said,

that there is a difference between *regular* movement and *uniform* movement. The uniformity of movement is related to various parts of the mobile; uniform motion is the movement of a mobile whose parts move as quickly as any other part. If a stone falls, although its movement is, in the end, faster than at the beginning, however, it is said in the proper sense of the word, because one half of the stone descends as fast as the other half.

On the contrary, movement is called *difform* where one part moves more quickly and another more slowly, as the movement of a wheel; in fact, the parts of the wheel which are adjacent to the axis do not move as quickly as those which are adjacent to the circumference, although these various parts have the same rotational speed. It is not contradictory that the motion of a body be a difform motion and that rotation (*circulatio*) of this body is uniform; indeed, the speed of the movement depends on one thing and the speed of rotation on another thing; movements are said to have equal speeds when in equal times they describe equal lengths; and rotations are said to have equal speeds when the bodies moved by these rotations describe, in equal times, equal angles around the centers of their rotations.

On the other hand, the regularity of movement is relative to time; this movement is said to be regular if the mobile moves with equal speed at all moments in time; but this movement is called irregular if the mobile moved faster in one part of the time and slower during another part.

However, it is known that some make a distinction regarding the uniformity of the motion, saying that it can come either from the various parts of the mobile or from the various parts of the time. Uniformity understood in the first sense is exactly the same thing as the uniformity that we have distinguished from regularity; uniformity understood in the second sense is the same thing as regularity. But these authors do not use the term "uniformity" with as much propriety as we can do, with such definitions.

It must be realized, moreover, that there is no contradiction that a certain movement is uniform and not regular. It is so for the fall of a weight in a uniform medium; this weight moves uniformly because one part moves as fast as any other part; and yet it does not move regularly because it moves faster at the end than at the beginning.

Similarly, a movement can, without contradiction, be regular and not uniform; this can be seen clearly by a wheel which, in equal times, describe equal angles; such movement of the wheel would be regular, but it would not be uniform, since the central parts of the wheel do not move as fast as the peripheral parts.

Thirdly, it should be noted that the same movement could, without contradiction, be both uniform and regular; if, for example, a weight fell in a medium whose resistance would be so exactly proportioned that this weight traversed equal spaces in equal times, the movement of the weight would be both uniform and regular.

In this passage of such perfect clarity, the Parisian Master shows how two problems were found close together, in the thinking of philosophers of the School, by their obvious analogy; one of these problems consisted in studying how, in a *difform* movement, the speed varies from one part to another of the mobile; the other consisted in analyzing how, in an *irregular* movement, the speed varies from one

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Et in hoc finit excellentissimi philosophi Gualterii de burley anglici in libros octo de physico auditu. Aristo. stragerite (*sic*), emendata diligentissime. Impressa arte et diligentia Boneti locatelli bergomensis. sumptibus vero et expensis Nobilis viri Octaviani scoti modoetiensis… Venetiis. Anno salutis nonagesimo primo supra millesimum et quadringentesimum. Quarto nonas decembris.

147e fol. (unnumbered), col. 6.

moment to another. The first problem had already solicited the attention of the author of the *De proportionalitate motuum et magnitudinum*, of Thomas Bradwardine, and of Albert of Saxony; the second could no longer remain neglected.

From the time of Albert of Saxony, the similarity between the two problems led several scholastics to state them in similar language; the words *uniformitas, difformitas* were employed in one case as in the other; one merely needs to specify them by mentioning *quoad mobile* or *quoad tempus*. Albert tried, we have just seen, to adapt different terminology to the two questions; but the attempt seems to have been unsuccessful; the words *regular*, *irregular* were left behind, and the words *uniform*, *difform* were used.

Soon there appeared a term whose inventor would be impossible to name; this term served to designate the movement whose speed increases or decreases in proportion to the time, the movement we call *uniformly varied*; the scholastics designated such a movement *uniformly difform* (*uniformiter difformis*). We find this term in the common usage of masters of the School of Oxford who were contemporaries of Albert of Saxony or were even older than he.

Chapter 20
Albert of Saxony and the law according to which the fall of a weight accelerates

Albert of Saxony is not content to define the regular or irregular movement in time; straightway[1], he is concerned to seek the law that governs the movement he had taken as an example of irregular movement, the accelerated fall of a weight; and what he said about it can be rightly regarded as one of the most remarkable passages of his *Quæstiones* one the *De Cælo* of Aristotle.

Albert notes, first, that the proposition "The movement becomes more intense towards the end" may be understood in various ways. According to the first sense, movement (and by the word *motus*, Albert, like all his contemporaries, understood what we mean by *instantaneous velocity*) can grow by becoming double, triple, quadruple, etc. According to the second sense, it can grow in such a way that to its first value is added half of this value, then half of the half, etc. In modern language, it seems that the speed can grow in an arithmetical progression, or that successive increases in this velocity can form a decreasing geometric progression.

These statements appear incomplete us. What is the independent variable of the values of speed mentioned there? The silence of Albert in this respect comes from what he assumes his reader knows of the science of his time, and the knowledge of this science allows us to compensate for this silence. When the scholastics of the 14th century dealt with the intensity of any property (*intensio formae*), they regarded it as a function of the extension (*extensio*) of the property; in the case of movement, they distinguished two types of extensions, the extension according to the path traversed (*extensio secundum distantiam*) and extension according to duration (*extensio secundum tempus*).

The brief statements of Albert should therefore be understood thus:

When arranging an increasing arithmetic progression according to either the paths traversed by the weight or the durations of the fall, it can be assumed that the values of speed increase in an arithmetical progression or that successive increases in these values follow a geometric progression with a ratio less than unity.

[1] Alberti de Saxonia *Quæstiones in libros de Cælo et Mundo*; lib. II, quæst. XIV: Utrum omnis motus naturalis sit velocior in fine quam in principio? — As we have said, this question is missing in the editions published in Paris in 1516 and 1518.

To admit that the law of falling bodies necessarily belongs to one of these four types is to make an assumption that seems particularly narrow; countless other laws appear possible to us. Albert does not ignore that one can conceive other laws of falling bodies, and later he will define what he will discuss. But these four, by their great simplicity, especially seduce his attention and seem to him most likely. And besides, Huygens, in 1646[2], did not consider it certain that the fall of bodies had to follow one of these four laws, and did it did not seem sufficient for him to decide, by the exclusion of three of them, that the fourth was accurate?

Albert of Saxony proposes something analogous to what Christiaan Huygens, one day, strove to achieve.

To fix his choice, he invokes, as an axiom, a proposition that he views as expressing the thought of Aristotle: If a weight was placed infinitely far away from the center of the World and if we let it drop, the speed of this weight will grow beyond any limit, and it would become infinite before the mobile reached the center of the Universe.

With this axiom, our author excludes laws of free-fall of the second form, because according to these laws, however great the duration of the fall or something along of the path traveled by the mobile, the speed would never exceed a some limit assignable in advance.

A similar consideration allows him to exclude certain other laws that could be proposed; one could imagine the speed growing in arithmetic progression whereas the successive increments of time would form a geometric progression of fractional ratio, $\frac{1}{2}$ for example, or else, when the successive increments of the space traversed would follow a similar progression. These hypotheses, indeed, allow the rate of fall to take any value, however great it may be, before the end of the movement, however small the duration of this movement or however small the distance traveled, which is absurd:

> *Nam tunc sequeretur quod quilibet motus naturalis qui per quantumcunque tempus parvum duraret, vel quo quantumcunque parvum spatium pertransiretur, ad quemcunque gradum velocitatis pertingeret ante finem; modo est falsum.*

It is possible to admire the delicacy and precision with which, in the middle of 14th century, a master of arts knew how to highlight the absurdity of certain assumptions concerning the law of the accelerated fall of weights.

Albert gives the following conclusion to the discussion that we have just analyzed:

> It is necessary, therefore, to understand that the intensity of the movement of the weight becomes double, triple, etc, in the following sense: When a certain space has been traversed, this movement has a certain intensity (speed); when a double space has been traveled, the speed is doubled; when the space traversed is triple, it is triple, and so on. *Et ideo teriia conclusio intelligitur, quod intenditur per duplum, triplum etc., ad istum intellectum quod, quando ipso pertransitum est aliquod spatium, est aliquantus; et quando ipso est pertransitum duplum spatium, est in duplo velocior; et quando ipso pertransitum est triplum spatium, est in triplo velocior; et sic ultra.*

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[2] *Huygens et Roberval, Documents nouveaux*, par G. Henry; Leyde, 1880. Letter of Christiaan Huygens to Mersenne dated 38 October 1640.

The law thus formulated by Albert of Saxony as a possible law of falling bodies is not the proportionality of the speed to the duration of the fall; it is the proportionality of the speed to the space traversed by the mobile. We know that this law was to seduce Galileo in his youth and that he was to later prove its absurdity. But it should be noted that in the analysis of Albert, the *extensio secundum tempus* is, constantly, paralleled with the *extensio secundum distantiam*; except the conclusion that we just mentioned, our author is always careful to repeat what he said of the one for the other; only the concision of his presentation was, undoubtedly, diverted from prolonging this repetition until the end, and to report the proportionality of the speed to the duration of the fall as acceptable; between this exact law and the erroneous one, his choice, certainly, remained suspended; the attention of an intelligent reader could relate to the exact law which Albert had not formulated as to the erroneous law of which he had given the explicit statement.

In no contemporary or immediate successors of Albert of Saxony have we found anything that specifies the law he believes the speed of the fall of a weight follows. But the great popularity of the *Quaestiones in libros de caelo* composed by our author is sufficient to ensure that the School of Paris, during the Middle Ages, did not remain ignorant of what he had taught concerning this important issue. The printing press also undertook, at the time of the Renaissance, to give greater circulation to this teaching. In truth, both editions of the *Quaestiones in libros de caelo*, those that were published in Paris in 1516 and 1518, omitted the question where the law of the increased speed in the accelerated fall of a weight is found; but the editions—given at Pavia in 1481 and at Venice in 1492, in 1497, and 1520—were sufficient to correct this omission.

At the end of 15th century until the early 16th century, the *Questions* written by Master Albert of Saxony were attentively read, and the testimonies of it are innumerable; we can cite convincing proof that the passage that we just analyzed attracted, at that time, the attention of some of the scholastics.

Towards the end of 15th century, the Parisian Pierre Tataret wrote a manual of Philosophy entitled: *Clarissima singularisque totius Philosophie necnon Metaphysicæ Aristotelis expositio*, or: *Commentationes in libros Aristotelis secundum Subtilissimi Doctoris Scoti sententiam*. Like many of those who, in the 15th century, taught Theology at the Sorbonne, Pierre Tataret, by his metaphysical doctrines, is connected to the Scotist School, while he borrows his theories of Mechanics from the Parisian Nominalist School and, in particular, from Albert of Saxony and Marsilius of Inghen. Thus his manual, with respect to the law according to which the fall of weights accelerates, merely reproduces verbatim[3] what Albertutius wrote in his *Quæstiones in libros de Cælo et Mundo*.

However, the summary of Philosophy composed by Pierre Tataret was extremely popular; the *Repertorium bibliographicum* of Hain mentions seven incunabula, and other editions, very numerous, were printed during the first third of the 16th century. Hence, the doctrine of Albert of Saxony received a new and very considerable diffusion. No one knew, no doubt, among the Parisian masters when Leonardo went

[3] Petri Tatareti, *Op. laud.*, *De Cælo et Mundo* lib. II^{us}, tract. II, circa finem.

to France to finish his glorious existence, when Soto collected the lessons of the Parisian University. So when we hear Leonardo first, then Domingo Soto, teach that the fall of a weight is a uniformly accelerated motion, we are entitled to think that their claim has been suggested by the suppositions that Albert of Saxony indicated.

We will have, it seems, discovered the source of one of the basic laws of falling bodies. From where does the second law come, which connects the space traversed by the mobile to the duration of the fall? This is what we will now seek; and this research will lead us to recognize the huge role played, in this act of scientific progress, by a contemporary of Albert of Saxony, Master Nicole Oresme.

De intensione et remissione formarum

Quantity and quality were for Aristotle two essentially distinct *categories*. Discontinuous quantity, such as number, is a sum of units; a number grows by the addition of new units to those which already composed it. Continuous quantity—like length, area, or volume—is a juxtaposition of parts; the parts of a quantity are all of the same nature as each other and the same nature as the quantity formed by their union; all parts of a length are lengths, all parts of a surface are surfaces, all parts of a volume are the volumes; a quantity increases by the addition of new parts to the preexisting parts, and the added parts are of the same species as the parts to which they are added.

So whether it is discontinuous or continuous quantity, some propositions remain equally true; quantities of different sizes can be of the same nature, of the same species; both are formed by the union of homogeneous parts to each other; only the larger of two quantities contains more parts than the smaller; it can be generated from this smaller quantity by adding new parts absolutely the same as those which form the smallest quantity; in the largest quantity thus obtained, the smallest quantity remains contained; the operation by which one increases it, simple juxtaposition of new parts, has neither destroyed nor changed it.

The category of quality is essentially distinct from the category of quantity; anything that can be said of one would be recklessly extended to the other.

Sometimes two qualities of the same kind do not even have the same *intensity*; one body is hotter than another; for the first body, this qualitative *form* that is heat is more *intense* (*intenditur*); for the second, it is more *attenuated* (*remittitur*). Let us be careful not to repeat on the subject of *intensio* and *remissio* of heat what we are entitled to say of the *greatness* and *smallness* of a quantity. Neither the intense heat nor attenuated heat is a union of parts of heat which are all of the same species, which are all homogeneous to the more intense heat they would provide by adding to each other; the more intense heat can in no way be generated by taking, without destroying or modifying it, the less intense heat and adding to it new parts of heat; the less intense heat does not exist, presently and actually, in the more intense heat in the same way that the smallest content, presently and actually, exists inside the larger container. Each heat of a given intensity is a heat of a given species, and this species is distinct from the species of heat of another intensity; an attenuated heat cannot be regarded as part of a more intense heat; any given intensity of heat is something essentially indivisible.

Since an attenuated heat does not turn into intense heat by the addition of new parts of heat, in the manner of a magnitude that is growing, how is this transformation produced? This question raises the problem of the intensification of intensity and attenuation of qualitative forms, *de intensione et remissione formarum*, which has so long preoccupied medieval Scholasticism. It is connected by apparently very close and strong ties to some discussions of modern Physics; can we, for example, define what is meant by the word temperature without analyzing again, as the masters of the Middle Ages did, the characteristics that distinguish the category of quality from that of quantity?

As desirous of the precisions of Logic as of the truths discovered by positive Science, the theologians of the Middle Ages willingly sought in the study of dogma the opportunity to show their dialectical subtlety or their knowledge of physics; so also has modern Science, rather than Apologetics, taken advantage of many a conversation of which the doctors in Theology adorned or overloaded their teaching.

Thus was the problem *de intensione et remissione formarum*. In his first book of the *Sentences*, Peter Lombard remarked[4]:

> In humans charity increases or decreases, and at various times it is more or less intense.

This text provided doctors in Theology a pretext that allowed them to expand their view on the intensification and attenuation of qualitative forms; and so, theories intended to enlighten the study of the various properties which the physicist is called upon to consider, have been presented, first of all, with regard to charity.

These theories can be classified into two groups; there are those who, faithful to the principles of Peripatetic logic, establish an extreme difference between the operation by which intensity of a qualitative form is increased and the addition by which quantity increases; there are those, on the contrary, who suppose an analogy between the two operations and who, therefore, tend to obliterate the borderline between the category of quality and category of quantity.

St. Thomas Aquinas clearly ranks among the followers of Peripatetic distinction; let us hear he says in his *Commentary* on the books of *Sentences*[5] on the operation by which charity increases in intensity:

> Those who argue that charity can be increased in its essence profess opinions which can be reduced to two. One of them claims that this virtue increases by the addition of charity to another charity, the other opinion holding that charity grows in intensity as it approaches nearer to its end, that is to say, to the perfection of charity… But I cannot understand the first opinion; in any addition, in fact, two different things must be understood, of which the one is added to the other. Hence, the two charities are different; they are distinct either by specific difference or by numerical difference; but they cannot differ specifically because all

[4] Petri Lombardi Episcopi Parisiensis *Sententiorum libri IV*, Lib. I, Dist. XVII; De missione Spiritus sancti qua invisibiliter mittitur.

[5] Sancti Thomæ Aquinatis *Scriptum super primum librum Sententiarum*, Lib. I, Dist. XVII, pars II, quæst. II: Utrum charitas augeatur per additionem?

> charities are a virtue of a single species; they cannot be numerically distinct because several accidental forms of the same species can not coexist in a subject numerically one, especially when it is a matter of absolute forms and not relative forms. This assumption, therefore, comes from a false imagining; some conceive the increase of charity in the manner of the growth of a body, an operation in which there is addition of a quantity to another quantity. I say that when charity grows, it does not produce, in this change, any addition; similarly, in the fourth book of the *Physics*, the Philosopher says that a body becomes whiter or warmer without any addition of whiteness or heat; but the preexisting quality becomes more intense because it is closer to its end.

The same thoughts are reflected in the *Summa Theologica* by the Angelic Doctor[6].

Thus according to St. Thomas, it is the essence of charity, whiteness, and heat to be more or less adjacent to perfect charity, absolute whiteness, and extreme heat, and this proximity more or less great to the supreme term constitutes the intensity, the more or less strong *intensio*; for a quality, to become more intense is not to increase by addition; is to develop in its own essence.

Giles of Rome no more believed than St. Thomas did that the addition of one charity with another charity gives a third charity more intense than each of the first two; but he differs from the Dominican Doctor in that he placed[7] in the existence (*esse*) the reason for the intensity which St. Thomas placed in the essence (*essentia*). According to Giles of Rome, charity is not in essence more or less intense, nor is whiteness in essence more or less white; there is only one degree of charity, only one degree of whiteness; but this unique charity and this unique whiteness are more or less completely realized in the subject in which they reside, and, therefore, the subject is charitable or white to a greater or lesser degree.

The debate between Giles of Rome and St. Thomas Aquinas also depends on the distinction between essence and existence, a subtle distinction but which plays a role of utmost importance in the Metaphysics of the Angelic Doctor of his successors.

In this debate Henry of Ghent (1217-1293) clearly ranks with the party of St. Thomas Aquinas:

> The *intensio* and *remissio*,

he said[8],

> will have to be produced in their essence and by their nature, because in their essence they have a certain latitude (*latitudo*). It is therefore not in the nature of the subject, but in the nature of the form, considered in itself, where it is necessary to look for the reason and cause of the increase of which this form is susceptible.

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[6] Santi Thomæ Aquinatis *Summa theologica*, II^{a} II^{æ}, quest. XXIV, art. 5.

[7] Ægidii Romani *In quatuor libros Sententiarum quæstiones*; Lib. I, Dist. XVII. Æigidii Romani *Quodlibeta*; Quodlib. V, quæst. XIV.

[8] *Quodlibeta* Magistri Henrici Goethals a Gandavo *doctoris Solemnis: Socii Sorbonici: et archidiaconi Tornacensis. cum. duplici tabella.* Venundantur ab Iodoco Badio Ascensio, sub gratia et privilegio ad finem explicandis. — Colophon:

> In chalcographia lodoci Badii Ascensii… undecimo kalendas Septembres Anno domini MDXVIII. Quodlibetum V, quæst. XIX; fol. CXCV, r° et v°.

In its essence this form is capable of several degrees; each lower degree is in potentiality of the greater degree; the enactment of this higher degree constitutes the growth of the form.

Henry of Ghent did not refrain from saying that each degree is a certain *quantity* of the form, that the lower degree is a *part* of the higher degree; but he certainly understood these words in the metaphorical sense, where one can say that potential existence is a part of existence in actuality, that this existence is greater than that one. He is careful not to think that the growth of a form is done as an augmentation of magnitude, which results from the apposition of new parts to existing parts.

> The augmentation of forms,

he said,

> is not done by an apposition of parts in their substance or essence; this is an increase in force (*in virtute*), through which the increased form becomes more efficacious in its own operation, which cannot produce the addition of a similar to similar; a warmth added to an equal warmth is no more heat.

The example that the Solemn Doctor just used to highlight the distinction between the augmentation of a quantity and the intensification of a quality will be in constant use in the scholastic discussions.

The essence itself of form, according to the Thomistic doctrine, includes varying degrees, each of which, more perfect than the lower grades, has in act something that was only in potentiality in the lower grades; imitating the divine perfection better than the lower grades, the higher degree is a greater perfection of magnitude (*magnitudo perfectionis*) and not a quantity of mass (*magnitudo molis*)[9].

In order to understand the relationships between the more and more perfect degrees of the same qualitative form, Hervé de Nédellec († 1322) uses a comparison[10] which clearly highlights the essential thought of the Thomistic doctrine:

> The attenuated degree,

the Breton Doctor said,

> is contained in the most intense degree as the vegetative soul is involved in the sensitive soul and the latter in the intellectual soul.

From the pen of Henry of Ghent, we met for the first time, this new term: latitude of a form (*latitudo formæ*); this term refers to the essential property by which the form is more or less close to its highest term, more or less perfect, thus more or less intense; we shall see this new word gain a singular popularity in the Scholasticism of the 14th century.

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[9] Henrici a Gandavo *Quodlibeta*; Quodlibetum V, quæst. Il; ed. cit., fol. CLVI, v°.

[10] *Sublilissima* Hervei Natalis Britonis… *quodlibeta undecim cum octo ipsius profundissimis tractatibus… De beatitudine, De verbo, De eternitate mundi, De materia celi, De relatione, De pluralitate formarum, De virtutibus, De motu angeli.* — Venetiis, 1513. Quodlibetum VII, quæst. XVII.

The expression *latitudo formæ* is clearly delineated in a *Summa of Logic* encountered among the *Opuscules* of St. Thomas Aquinas, but that was probably written long after the time the Angelic Doctor lived[11]. Here is what we read in this *Summa*[12]:

> Substance has, in common with certain accidents, two characteristics: It admits nothing that is contrary to it, and it is not susceptible to more or less. To understand these propositions, it must be known that some forms are endowed with latitude and others not; and this is because some forms are subject to the aforesaid latitude, that they admit a contrary, although this is not true of all these forms.
>
> To find out what the latitude is, note that, for spiritual things, we conceive augmentation by extension of what we know about the magnitude of corporeal things; now, when it comes to physical quantity, it is said of a thing that is great as it approaches the perfection that suits its magnitude; that is why such a thing susceptible to quantity is called great in a man which would not be deemed great in an elephant. Similarly, when it comes to forms, one thing is called great insofar as it is perfect.
>
> But the perfection of a form can be considered from two points of view, as we consider the form itself or the participation of the subject in this form. In the former case the form is called small or large; we say, for example, a small whiteness. In the second case, we use the words more or less; we say of a body that it is more or less white. When a form is endowed itself with an indetermination such that it can be performed more or less in the subject—viz., more or less perfectly—it is said to be endowed with latitude and that it reaches such and such a degree of intensity or remission.

Henry of Ghent took the word *latitude* to formulate the Thomistic theory of the intensity of forms; he made latitude a property which resided in the *essence* itself of the form. It is in the sense of Giles that the author of the *Summa of Logic* takes this same expression; it is not by *essence*, but by *existence*, that the form is endowed with latitude; undetermined by itself, it is determined to this or that latitude, to this or that degree of intensity, as it is put into action, in the subject, more or less perfectly.

The *intensio* of the form, which marks its degree of perfection, should be well distinguished from the *extensio*, which marks the magnitude of the subject where this form is realized. It is one thing for a body to offer to the eyes a more or less intense whiteness, and it is another thing to be a white object of greater or lesser extent. It is so natural to make this distinction that we find it more or less clearly marked by all Scholastics and, in particular, by St. Thomas Aquinas. The author of the *Summa of Logic*, in turn, reports it; he is careful to contrast *latitudo* with *extensio*:

> The perfection or the imperfection of the quantity depends on the more or less great extension; it is according to this extension that an object is said to be larger or smaller. But a more or less great extension is not always a sufficient cause for us to say a thing is more or less, because one can only judge of its existence through the extension… Certain forms, one sees, are susceptible to more or less, and certain others are not; those which are capable of more or less are those equipped with what is named latitude.

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[11] Cari Prantl, *Geschichte der Logik im Abendlande*, Leipzig, 1867; Bd. III, pp. 250-257. Duhem, *Le mouvement absolu et le mouvement relatif*. Note : *Sur une Somme de Logique attribuée à Saint Thomas d'Aquin* (*Revue de Philosophie*, 9° année, n° 4, 1er avril 1909 ; p. 436). — P. Mandonnet O.P., *Des écrits authentiques de Saint Thomas d'Aquin*; Fribourg, 1910 (Extrait de la Revue Thomiste, 1909-1910).

[12] Sancti Thomæ Aquinatis *Opuscula*; *Opusc.* XLVIII: *Totius logicæ Aristotelis summa*; tract. II: De prædicamentis; cap. IV.

Followers of Giles, we have noted, use the word *latitudo formæ*, whereas Henry of Ghent used it to formulate the Thomistic theory. We constantly find this word in the writings of Durand of Saint Pourçain who, in his *Commentary on the Sentences* written circa 1330, adopts the Thomist theory of the intensity of forms[13] and strongly combats the theory of Giles. Durand makes statements such as these:

> We must say that the intensity and remission of form depend on the various degrees of the essence of this form. This can be demonstrated as follows: What greater or lesser extension is for quantity, greater or lesser intensity is for quality. But greater or lesser extension depends on the essence itself of the quantity; the latter, indeed, has in its essence a latitude capable of being extended more or less. Greater or lesser intensity thus also depends on the essence itself of quality, as this quality is endowed for this purpose with a latitude susceptible to varying degrees.
>
> Secondly, it can also been seen in the fallowing way: the indivisibility of a form is why this form is not capable of more or less; likewise, divisibility in degrees is the reason that makes the form capable of more or less; yet the indivisibility of a form depends on the essence of this form; it must be the same for divisibility.

Divisibility of the form into degrees does not resemble, in the thought of Durand of Saint Pourçain, the divisibility of a quantity into parts; the successive degrees designate an increasingly great perfection of the form; each of them is virtually contained in the highest degree; but it cannot be detached as a part can from a whole; the division of a form into degrees must be considered as the division of a genus into species that can be ranked according to their greater or lesser degree of perfection.

From this comparison, it is easy to slip into a doctrine that Durand strongly fought[14], but which, before and after him, had many supporters.

All the authors whose opinions we have so far analyzed attributed a certain indeterminacy, a certain latitude, to a qualitative form; by this latitude, the form may, in a subject, remain the same and yet achieve various intensities of varying degrees; whether its essence more or less approaches the perfection of which it is capable, or whether this essence does not become more or less perfect, it is found more or less fully realized in the subject.

Other philosophers want, on the contrary, that a form not be affected by any indeterminacy—no indeterminacy in the essence of this form means this essence can be called more or less perfect, no indeterminacy in existence, by which the subject could participate in the form of a more or less complete manner. Each form is entirely determined in its essence and in its existence; it is only susceptible to a single perfection and can only affect in one way the subject in which it is realized.

Every form, therefore, is incapable of greater or lesser intensity; each of them possesses an absolutely invariable degree. When, by a depraved language, we speak of the varying degrees of the same form, one will, in fact, describe various forms, specifically distinct from one another, and only belonging to the same genus; in this genus it is possible to rank them in such a way that each of them is more perfect

[13] Durandi a Sancto Portiatio *Super sententias Petri Lombardi commentarii*; Lib. I, Dist. XVII, quæst. V: Utrum charitas possit augeri?

[14] Durandi a Sancto Portiano *Op. laud.*, Lib. I, Dist. XVII, quæst. VII: Utrum eadem forma numero possit esse intensa et remissa?

than the previous and less perfect than the one that follows it; but none of them may, by *intensio*, transform into the following one nor, by *remissio*, be reduced to the preceding one.

So how should we conceive of the increase of a quality? What will, for example, a body that warms up be?

Whether one admits the Thomistic doctrine or adopts the theory of Giles, in a body which is heated, the heat is numerically one, always being the same form; only from moment to moment does the essence of this heat become more and more perfect or, better still, its essence is more and more realized in the heated body.

In the body that heats up, the theory that we presented sees in this moment not one and the same heat that it acquires successively by higher and higher degrees, but an infinity of heats numerically and specifically distinct from each other. At any time a heat is destroyed and, in its place, a more perfect heat is generated; in the second heat, nothing remains of the first. Heating is not the movement by which the essence of a unique form tends towards perfection; it is not the movement by which form of a determined essence is actualized better and better in a certain subject; it is a continuous succession of generations and destructions by which a form is produced only to be immediately annihilated.

We do not doubt that this opinion already had supporters in the time of St. Thomas Aquinas; the Angelic Doctor wrote[15] in his *Commentary on the Sentences*:

> Some claim that charity does not suffer, in essence, any increase; that, when charity becomes larger, the lesser charity that existed before is destroyed; thus it is said that the days are getting longer when longer days are followed by shorter days.

This doctrine is most certainly that of an unknown author whose treatise *On the Plurality of Forms* is wrongly numbered[16] among the opuscules of St. Thomas. Here is what this treatise reads[17] on the subject of the increase of quantities and the operation that intensifies the intensity of a form; the clearness of this passage is worthy of note:

> Of two forms that are similar, there is one, the most perfect, which contains virtually the other, the less perfect; if a lesser form of perfection were conjoined with a more perfect form, it would not give a more perfect form; this addition would be futile. But in nature, nothing is done in vain; thus, there cannot, between different species, be such an addition in which a preexisting form remains at the same time as the form that supervenes. Here, then, is how to understand the analogy that we discussed: When a more perfect form occurs, the preexisting form is destroyed, so only one form remains in what is composed; this unique form contains the less perfect form and contains even more; therefore, it adds something to the less perfect form; as the greater number contains in itself the lesser number that also exists outside of it, and to which it adds something; as, for example, the number four contains in itself, in a virtual and quantitative way, the number three which also exists separately, and adds a unit to it; likewise, the most perfect form adds a certain perfection to the less perfect form

[15] Sancti Thomæ Aquinatis *Scriptum in libros Sententiarum*; Lib. I, Dist. XVII, pars II, quæst. I: Utrum charitas augeatur?

[16] On the apocryphal nature of the booklet *De pluralitate formarum*, see: Mandonnet, O.P., *Des écrits authentiques de St. Thomas d'Aquin*, Fribourg, 1910, p. 95 (Extrait de la *Revue Thomiste*, 1909-1910).

[17] Sancti Thomæ Aquinatis *Opuscula*; *Opusc.* XLV: *De pluralitate formarum*, Cap. I.

> that it contains virtually. But in regard to the numbers, we may, to the smaller number—to the number three, for example—add a new unit which constitutes, together with the three previous units, the number four, which is greater; on the subject of forms, a similar operation is no longer possible; a new form cannot supervene and be added to an already existing form in the matter to constitute a more perfect form.
>
> And the reason for this difference is twofold. The addition of a number to a number is done by whole and quantitative parts which represent the magnitude of the excess of one number over the other; and this excess is of such a nature that it amounts to the same thing, for obtaining the greater number, that we take the smaller number and add something, which makes the smaller number a part of the greater, or that we form the greater number independently, in reuniting all the units out of which it is composed; in one way or the other, the larger number surpasses the smaller of the same quantity. But if a form surpasses another form of the same genus, this is in perfection [and not in quantity]; all the perfection that is in the least perfect form is also, of course, in the most perfect form; in this latter, therefore, the perfection would not would grow if the less perfect form were added to it. All forms are simple; none of them is composed of several forms; the more a form is simple, the more it is perfect; however, in regards to numbers, it is quite to the contrary, because a number is more composed the greater it is; thus there cannot be an addition of a form to a preexisting form as there can be of a number to a preexisting number.
>
> Here is the second reason for this difference: Number is not something that is simply one; it is an aggregate of units; it is of its nature to have several parts, each of which exist actually; so that, in any manner that one portion is added to another portion, a larger number is obtained. But a material substance is something that is simply one; it cannot, in itself, be several realities in actuality. This is why when a substantial form supervenes, it is necessary that the preexisting substantial form gives up its place for it… Likewise should it be for any addition or subtraction that is done in the substance of things; when a new form comes, that which existed before should be destroyed.

Godfrey of Fontaine is ordinarily taken as the determined supporter of the opinion that has been presented; however, his conviction in this regard fluctuated. Those of his *Quodlibets* which were published by Mr. De Wulf and Mr. Pelzer contain a question[18] where the author professes an opinion very different from that of St. Thomas, very close to what Giles of Rome has held. The specific essence of charity or of a similar quality is essentially indivisible, essentially incapable of more or less; it cannot approach or depart from perfection except by changing species. Thus, if a quality is capable of showing various degrees, if it is susceptible of more or less, this cannot be essentially, but only accidentally, insofar as the subject participates more or less in this form.

> If whiteness were separate from any subject, and if we assume that there could be several separate whitenesses, all of these whitenesses would also be perfect… So if they can have some virtual degrees, while substantial forms are not considered to be endowed with such degrees as susceptible of more or less, this is what we must certainly understand by this: These qualities have a nature and a virtue such that the subject can participate of them in varying degrees, be they more or less, or even that the subject is able to receive from them a greater or lesser perfection.

These lines formulate the doctrine of Giles.

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[18] Magistri Godefridi de Fontibus *Quodlibeta reportata*; Quodlibetum II, quæst. II: Utrum caritas sive quicumque habitus possit augeri per essentiam? (*Les philosophes belges ; textes et éludes*. Tome II : *Les quatre premiers quodlibets de* Godefroid de Fontaines, par De Wulf et Pelzer; Louvain, 1904 ; pp. 139 seqq.)

In yet another unpublished *Quodlibet*[19], Godfrey of Fontaine understands the increase of charity thus: The less charity that preexisted was destroyed; another charity is generated, which contains virtually the first, but which surpasses it in perfection and that, for this reason, is called more intense than the first.

Gerard d'Odon of Châteauroux, who was, in 1329, elected superior general of the Franciscan order; who became, in 1342, bishop of Gatane and, around 1348, Patriarch of Antioch; who finally died at Catania in 1349, Gerard d'Odon, we say, adopted, concerning the increase of qualitative forms, the theory which we presented. This is, at least, what John the Canon affirms:

> It must be known,

he said[20],

> what the opinion of Gerard d'Odon is: when something that was white becomes more or less white, the previous form is completely destroyed and a new form, which is a new individual, is generated.

But no has scholastic has more firmly adhered to this opinion than Walter Burley; however, like Godfrey of Fontaine, our author first admitted the theory of Giles.

We find, in fact, a first presentation of the ideas of Burley in the *Commentary on the Categories of Aristotle* that this master composed; here is this presentation[21]:

> I say that no form is susceptible to more or less, but that the form is more or less received by the subject, so that the subject is more perfect or less perfect. No whiteness is susceptible to more or less, but the white body is susceptible to more or less because it takes on a more or less perfect whiteness — *quia suscipii albedinem magis perfectam et minus perfectam*.

The last words of this passage already slip from the theory Giles of Rome to the theory that is commonly attributed to Godfrey of Fontaine. If no whiteness is susceptible to change in intensity, they imply the existence of multiple, unequally perfect whitenesses, and they assume that in a body that becomes more or less white, these various whitenesses replace each other.

It is this doctrine that Burley then developed in a special treatise he entitled: *De intensione et remissione formarum*[22]. This treatise, more than any other, helped to publicize among the Scholastics the theory to which we have just alluded.

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[19] Godefridi de Fontibus *Quodlibeta*; Quodlib. VII, quæst. VII. We derive this information from the following work: *Commentariorum in primum librum Sententiarum. Pars prima.* Auctore Petro Aureolo Verberio. Romæ. Ex typographia Vaticana. MDXCVI; p. 435, col. *at*.

[20] Joannis Canonici *Quæstiones super VIII libros Physicorum Aristotelis*; libri V quæst. III; quantum ad 4^{m} articulum.

[21] *Expositio* Burlei *super libro predicamentorum*; coll. *a* and *b* of the fol. following the fol. marked *e* 4 in the edition with the title: *Preclarissimi viri Gualterii Burlei anglici sacre pagine professoris excellentissimi super artem, veterem Porphyrii et Aristotelis expositio sive scriptum feliciter incipit.* The colophon is: Explicit scriptum preclarissimi viri Gualterii Burlei Anglici sacre pagine professons eximii. in artem veterem Porphyrii et Aristotelis. arte et diligentia Boneti de locatellis sumptibus vero D. Octaviani Scoti Impressum Venetiis Anno 1488. Octavo idus. Julii.

[22] Burleus *de intensione et remissione formarum*. — Jacobus de forlivio *de intensione et remissione formarum*. — *Tractatus proportionum* Alberti de Saxonia. — Colophon:

The system of Godfrey of Fontaine, Gerard d'Odon, and Walter Burley marks the highest point of the peripatetic opposition between quality and quantity. While some Scholastics clung to defending such a system, others tried to reconcile as far as possible the categories of quality and quantity.

We have heard St. Thomas Aquinas rise up strongly, in his writing on the *Sentences* of Peter Lombard, against those who see, in the increase of charity, the addition of a new charity to a preexisting charity; so there were in his time some philosophers for whom the intensity of a quality became intensified by the addition of one part to another part, as a quantity grows.

These philosophers will become numerous in the last years of the 13th century, when the anti-peripatetic reaction provoked or signaled the condemnations brought in 1277 by the Bishop of Paris, Étienne Tempier, and by theologians of the Sorbonne.

One of the promoters of the Scholasticism freed from Peripateticism was the Franciscan Richard of Middleton, whose *Commentaries on the Sentences* of Peter Lombard were probably composed soon after the year 1281.

Richard of Middleton did not hesitate to see, in the increase of a qualitative form such as charity, the result of an addition of parts to each other; the analogy that results between the intensity of a quality and size of a quantity does not escape him; far from seeking to dissimulate this analogy, he declares it in the most formal way[23]; next to quantity understood in the Aristotelian sense, and which he names *quantity of mass* (*quantitas molis*), he places the intensity of quality, which he calls *quantity of force* (*quantitas virtutis*).

> Charity may increase,

he said,

> because any quantity that is imperfect can increase. Now there are two kinds of quantities, namely, the quantity of mass (*quantitas molis*) and the quantity of force (*quantitas virtutis*); therefore, there are two kinds of augmentations, the augmentation relative to the quantity of mass and the augmentation relative to the quantity of force. Charity being a quantity, it can increase in force until it has reached its end. And as, in essence, love is force, so that the charity and force of love are not distinct from one another except in reason alone, it must admitted that charity grows essentially. …
>
> The quantity of force is not only measured by the number of objects (subjected to the action of this force), which gives it extensive measure, analogous to that of discontinuous quantity; it is also measured by the intensity of the act produced in the same object, and thus, it is more like continuous quantity. It is in this second way that charity increases, not the first.

Moreover, Richard of Middleton will affirm that the increase in charity results from the addition of a new charity to an existing charity[24]:

Venetiis mandato et expensis nobilis viri domini Octaviani scoti civis Modoetiensis. 1496. quarto kal. decemb. per Bonetum locatellum bergomensem.

[23] *Clarissimi Theologi Magistri* Ricardi de Mediavilla *super quatuor Libros Sententiarum Petri Lombardi, quæstiones subtiiissimæ*. Brixiæ, MDXCI.Lib. I, Dist. XVII, art. II, quæst. I: Utrum charitas possit augeri? Tom. I, p. 163.

[24] Ricardi de Mediavilla *Op. laud.*, Lib. I, Dist. XVII, quæst. II: Utrum charitas augeatur per additionem novæ charitatis? T. 1, pp. 162-164.

> The soul becomes more charitable because to charity which preexists in the soul, the divine power adds a new level of the essence which is charity; of this new degree of the preexisting degree of charity, an essence of more perfect charity is constituted; the first degree, in fact, was in potentiality to receive the higher grade, in the same way that an incomplete thing is in potentiality to the more complete degree.
>
> … If we contrast to this opinion the following objection: A single thing added to a simple thing gives nothing greater, I answer in these words: Though charity is simple in the sense that it does not have some quantity of mass, it however does have a quantity of force. Much more! It is, indeed, a certain quantity of force (*quantitas virtualis*). Similarly, a certain quantity of mass (*quantum mole*), added to a similar quantity, give something that is greater in mass; even a degree of a quantity of force added to a similar degree produces something that is greater in strength. We can equally say, according to the opinion that the Philosopher sets forth in book III of the *Metaphysics*: Although an indivisible added to an indivisible does not make something bigger, it does give it something more. Regarding charity, although what is added is simple and what is added to it the same as what is added, from this addition, however, results something that in essence is more, thus something which is better and consequently something which is larger; because, according to Augustine (VI *De Trinitate*, capp. VII and VIII): In the domain of things that are not large by mass, to be greater is to be better.

The English Franciscan William Vare or Varon certainly commentated on the *Sentences* toward the end of the 13th century; he was, indeed, the master of John of Duns Scotus. In his *Questions* on the writing of Peter Lombard[25], it is not necessary to search for the sharpness and vigor of thought that is marked in those of Richard of Middleton; prolix, confused, and little ordered, the debate of William Varon very often leads to tentative conclusions which are less a synthesis of the views expressed by various authors than a rough compromise between those views.

Does charity grow by the addition of some positive part? This is a question that William Varon discussed, as did his predecessors[26].

Favoring the affirmative response, some present this argument:

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[25] We read these *Questions* in manuscript n° 163 of the Municipal Library of Bordeaux. It is a beautiful manuscript of the 14th century written on parchment in two columns, decorated with capitals in reds and blues; the writing is very readable, despite many ligatures; unfortunately, the copyist, ignoring the Latin as well as the subject matter, sowed his work with a multitude of errors; a reader of the 14th century has corrected many of them by marginal annotations. The book bears no title; it starts (fol. I, col. *a*) in these terms:

> *Queritur utrum finis per se et proprias theologie ut est habitus scientificus perficiens viatorem sit cognitio veri vel dilectio boni. Quod cognitio boni videtur quia Johannis 3° dicitur.*

The last sentence of the work is:

> … *Quod non obstante quod sit cognocitivus qualitatum tangibilium, tamen patitur qualilatibus tangibilibus.*

It is followed by these words: *Explicit liber quartus Varonis*. A *Summa omnium questionum hujus libri* and a *Reduccio precedentium questionum per alfabetam* come next.

[26] Guillelmi Varonis *Quæstiones in libros Sententiarum*; quæst 67a: Queritur utrum charitas augetur per additionem alicujus partis positivæ? (Circa Lib. I, Dist. XII; ms. cit., fol. 54, col. *o*, to fol. 56, col. *a*.)

> the augmentation of qualities behaves with respect to the quality exactly as the augmentation of quantities behaves with respect to the quantity; the increase of qualities is done by addition.

The negative response is, however, common to both theories, which Varon described without naming the authors, but we recognize easily the doctrine of St. Thomas Aquinas and the doctrine of Giles of Rome.

According to this doctrine,

> when God created the first charity which he, first of all, infused in a man, he created in potentiality, in this charity, all the degrees that it is susceptible to take in actuality; when it pleases God to increase this charity, he draws into actuality one of these degrees of charity which were in potentiality and thus the total habit becomes more intense.

The supporters of the other countered this doctrine, saying that

> heat is not, by itself, in potentiality to a greater heat; this power for a greater heat is in the subject itself; if the subject did not have the power to change, it could not receive more heat; the greater heat is thus taken from the power of the subject, not from the power of heat.

Proponents of both doctrines refuse to see in the increase of charity or heat the addition of a new charity or heat to a preexisting charity or heat.

> Such an addition of a part to another part can only be done if charity becomes greater. Just as a warmth added to another warmth is not a more intense heat, similarly, a part of a warm charity added to another warm charity will not make it become greater.

To this argument, Varon replied as follows:

> What is said here of warmth added to warmth is worthless; here, indeed, is why a warmth added to another warmth is not a more intense heat: When one adds a warmth to another, one adds at the same time the subject of one of these warmths, water for example, to the subject of the other warmth; these subjects, added one to the other, prevent the heat from becoming more intense. If from a warm body we took what precisely is the heat, if we took the same heat which is in another warm body and place both heats in the same subject, I say it would make a greater heat.

This response is worth noting; soon we will hear John of Bassols resume it more precisely.

Among the various opinions that have been expressed regarding the addition of qualities, the reason of Varon remains singularly fleeting. He admits that the essence of a quality does not have *essential and formal parts*, but it admits of *material and accidental parts*; it is these parts which, added together, make quality more and more intense. On the other hand, he grants to Giles of Rome that the subject, more or less disposed to receive a determined quality, contributes to the greater or lesser intensity of this quality.

The *latitudo formæ*, according to Varon, is not in the form as this form is at its lowest degree or at its supreme degree; it is there because of the intermediate degrees between the first and the last; it is neither a potential latitude nor an actual latitude, but a *latitudo in consequenti*; by these words he means some attribute where the power and actuality simultaneously meet. When the form is at its highest degree, its latitude has nothing potential; it is the wholly reduced to actuality. These are thoughts that take us back to the Thomistic doctrine; it is thus, according to this doctrine, that one should conceive the latitude of the form.

Firmer and more coherent than his master William Varon, the opinion of John of Duns Scotus appears to have been inspired by the doctrine of Richard of Middleton, whose clearness, however, he does not equal.

John of Duns formally admits, firstly[27], "that this positive reality that existed in a lesser charity really remains the same in a greater charity." By that, the Subtle Doctor rejects the theory according to which the increase of a quality would be an uninterrupted series of destructions and generations, a quality being, at every moment, annihilated and replaced by a more intense quality.

Having thus rejected the system of Godfrey of Fontaines, Duns Scotus argued strongly against what Giles of Rome supported and concluded as follows:

> The positive reality that preexists in a lesser charity is not all the positive reality that exists in greater charity. Much more! I say that if this greater charity and that lesser charity were both separated from the subject where they are found, the greater would be, in it, the positive reality of the smaller and, in addition, another reality added to that one; and assuming this, *per impossible*, that any relationship with the subject was suppressed. Similarly, if we supposed that the quantity of mass (*quantitas molis*) was separate from its subject and, *per impossible*, that it had no inclination towards this subject, a large quantity would continue to be greater than another; the greater would contain all the positive reality of the smaller and, in addition, something that would be added to that reality.

Like Richard of Middleton, Duns Scotus admits that the qualitative form

> is endowed with the simplicity that is opposed to the quantity of mass; when adding such a form to a similar form, one obtains nothing greater in mass (*majus secundum motem*)… We thus grant to the form that simplicity which is opposed to the quantity of mass; there is nothing that contradicts the intensity because it is related to the quantity of perfection and of force (*quantitas perfectionis et virtutis*).

We see drawn in very firm contours the theory that Richard of Middleton and John Duns Scotus sketched, drawn by the favorite pupil of Duns Scotus, John of Bassols.

Straight off[28], the discussion of John of Bassols penetrates to the heart of the question; he defines the narrow sense of the term quantity in the Logic of Aristotle and the infinitely broader sense that Richard of Middleton and John of Duns attributed to it.

> I say, in the first place, that there are two kinds of quantities.
>
> There is, firstly, the *quantity of mass* (*quantitas molis*) which is a relation of extension[29], or the *discontinuous quantity* (*quantitas discretionis*); this quantity is a category; by the genus in which it ranks, it is a determination of being.

[27] *Primus liber* Joannis Duns Scoti Doctoris Subtilis *super Sententias*; Dist. XVII, quæst. III.

[28] *Opera* Joannis de Bassolis *Doctoris Subtilis Scoti (sua tempestate) fidelis Discipuli, Philosophi, ac Theologi profundissimi, In Quatuor Sententiurum Libros (credite) Aurea*… Venundantur a Francisco Regnault: Et Joanne Frellon. Parisiis. In fine: Anno JESU Aeterni Regis sesquimillesimo decimoseptimo Nono Idus Septembres. Lib. 1, Dist. XVII, quæst. II: Utrum charitas augeatur vel potest augeri? foll. CXIII-CXVII.

[29] Instead of: *extensionis* the very faulty text reads: *intensionis*.

There is, on the other hand, a transcendent quantity; it is *the quantity of perfection in the essence or the quantity of force in the action* (*quantitas perfectionis in essendo vel virtutis in agendo*); this quantity then is of no determined genus.

In support of this distinction, John of Bassols, as did Richard of Middleton, invokes the text of St. Augustine:

Dico quod in hiis quæ non sunt mole magna, illud est majus quod melius.

Then he goes on to say:

Just as there are two kinds of quantities, there are two sorts of movements of quantity.

One of these movements goes from an imperfect quantity of mass to a quantity of mass that is perfect or inversely; it is the movement called *augmentation* or *diminution*.

The other goes from an imperfect form that would attain a form in its essence or a form in its action to a perfect degree, or it goes in the opposite direction; it is properly named *intension* (*intensio*) or *remission* (*remissio*); but it also means the same name as the previous movement, namely *augmentation* or *diminution*.

After refuting the various views expressed, on the subject of the intension and remission of forms, by Giles of Rome, on one hand, and by Godfrey of Fontaine, on the other, our author formulates his own opinion:

Charity and likewise any form capable of intension or remission increases by the apposition of a new real degree, of the same kind as the preexisting degree; this new degree is added to the preexisting degree within the same subject; they form a single individual of the same form, but this individual is more perfect than what existed before.

Indeed,

in any specific form, in any natural quality susceptible to intension or remission, it is possible to mark multiple degrees which are the material parts, in the sense that Aristotle, in the seventh book of the *Metaphysics*, takes the word "material parts".

By degree of charity or of any form, I understand a certain individual of this form; this form is, in this individual, limited and quantitatively defined in the way that is proper to it, the way in which we can say that the form, in this individual, has this or that determined quantity. I thus give the same sense, in the proposition before me, to the words: *degree of form*, and to the words: *limited individual of that form*; it is the same to compare a subject that has a greater degree of that form to another subject that has a lesser degree of it, or to say that we are dealing with a more perfect individual of this form and with less perfect individual.

Immediately the following consequence results therefrom: Just as a single subject only has in itself a single individual of the form considered, so does it possess this form, at the same time, as under a single degree. When, therefore, in the increase we are talking about, a new individual of the same form is added to the degree of this form that preexisted in the subject, it is clear that a whole unique individual is constituted from the previous degree and the new degree, and it has the form in another degree.

An example will clarify for us the thought of John of Bassols.

Let us consider some heated bodies. In each of these subjects, the qualitative form that is the heat has a certain extension which depends on the size of the heated body, and a certain intensity, which says that such a body is hotter than another without accounting for their respective sizes. Each of these intensities is a specific individual of the same form that we call heat; it is also a *degree* of heat. These individual heats are, however, more or less strong, these degrees of heat are higher or lower, depending on the various subjects where we see them realized are more or less hot. But in the same subject, at the same time, there is one individual heat, one degree of heat.

If we take the individual heat or the degree of heat which was realized in a certain warm body; if we suppose them detached from the subject in which it is concretized for transporting it into another warm body, it will be joined to the individual heat in the degree of heat that preexisted in this latter subject, and of these two individual heats, a more perfect, unique, individual heat, more intense than each of the two individual components, will be formed; these two degrees of heat will constitute a unique degree higher than each of the two preexisting degrees; by adding warmth to warmth, one will produce heat.

Let one not make this objection to our author: Water added to warm water does not give hot water; William Varon has taught him not to fear this objection; he replies, quite rightly besides, that after this operation the two warms are not, any more than before, in the same subject:

> The two warm bodies here are something more than each of them; it is clear from the effect they produce, because, together, they generate in a third body a heat more intense than what each of them will generate in isolation; so if we add the heat of one to the heat of the other, we produce something of greater intensity, as the effect of these two heats is more intense than the effect of each in isolation. This can be seen clearly by taking the example of weight; two stones or two weights taken together weigh more than one of them, extensively; but if you added the weight or gravity of one of these bodies to the weight or gravity or of the other, so as to make a single weight or gravity by the union of the two weights or gravities, the result would be heavier in intensity than each of the two weights in isolation; and this is natural, although neither of these weights, considered separately, is more perfect than the other.

The choice of this last example seems especially suited to make the thought of John of Bassols accessible to our modern minds; under the influence of a text of St. Augustine, and in imitation of Richard of Middleton and Duns Scotus, Bassols distinguished two sorts of quantity, the quantity of mass and the quantity of force; now, here it is found that extension, which is a *quantitas molis*, corresponds precisely to what we call mass, and that the *quantitas virtutis* is what we call force.

The sharpness that we have come to admire in the doctrine of John of Bassols is not always found in the theories of his contemporaries and successors; moreover, among these, more than one, even among the Franciscans or the followers of Duns Scotus, tended to abandon the doctrine inaugurated by Richard of Middleton to return to the opinion nearer to that of St. Thomas.

Thus, Antonio of Andres, in his *Commentary on the Sentences*[30], admits that in a body that whitens, the existing degree of whiteness is not destroyed and that the increase in whiteness is due to the addition of a new reality, a new degree, which united to the previous one to compose a unique individual form; but his presenta-

[30] Ant. Andreæ *Conventualis Franciscani, ex Aragoniæ provincia ac Ioannis Scoti Doctoris Subtilis dbeipuli celeberrimi In quatuor Sentenliarum Libros opus longe absolutissimum*… Venetiis, Apud Damianum Zenarum. MDLXXVIII. In. I Lib. Distinct. XVII, quæst. III, foll. 36 v° et 37 r°.

tion is very concise, not very explicit, so we might as well apply it in the sense of Thomistic teaching than in the Scotist sense.

It is to the first of these teachings that Antonio of Andres seems to have leaned when he commentated on the *Book of the Six Principles* of Gilbert of Poitiers[31]. To this question:

> In the essence of an accidental form, are there intrinsic and essential degrees by which the increase or diminution of this form is produced?

he answers in these terms:

> The considered accidental form possesses such degrees. And I would add that the specific reason which allows the form to increase or diminish is the latitude of degrees (*latitudo graduum*) which is in it; this latitude is nothing other than an absence of limitation in the form that is susceptible to more or less.

This, it seems, is the Thomistic opinion which inspires the lines where the word *latitudo* seems employed in the same sense that Henry of Ghent gave it, which Durand of St. Pourçain retained.

The Franciscan Peter Auriol develops with clarity the opinion that Antonio of Andres briefly sketches in his second commentary on the first book of the *Sentences*, a commentary composed in 1318 or later in 1319[32].

Pierre Auriol admits, in the first place[33], with Duns Scotus, that any form whose intensity grows acquires some new reality; he admits, in the second place[34], of having encountered the opinion held by Godfrey of Fontaines, that the acquisition of a new reality does not involve the destruction of any reality contained in the existing form. But he does not admit in its fullness the doctrine that Richard of Middleton, John Duns Scotus, and John of Bassols supported.

> This reality,

he said[35],

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[31] *Quæstiones* Scoti *Super Universalia Porphy. necnon Aristotelis Predicamenta ac Periarmenias — Item super libros Elenchorum. — Et* Antonii Andree *super libro Sex principiorum — Item questiones* Joannis Angelici *super questiones universales eiusdem Scoti*. Colophon:

> Subtilissime questiones… féliciter expliciunt. Impresse Venetiib pet. Philippum pincium Mantuanum. Anno Domini 1512. die Decembris. — *Questiones clarissimi doctoris Antonii Andree super sex principiis Gilberti Porretani*. Quest. XVII: Utrum in essentia forme accidentalis sit dare gradus intrinsecos essentiales secundum quos possit suscipere magis et minus? fol. 61, coll. *c* et *d*.

[32] Noël Valois, *Pierre Auriol, frère mineur* (*Histoire littéraire de la France*, t. XXXIII, 1906 ; p. 485 et p. 50).

[33] *Commentariorum in primum librum Sententiarum. Pars prima. Auctore* Petro Aureolo Verberio *Ordinis Minorum Archiepiscopo Aquensi S. H. E. Cardinali*. Ad Clementem VIII. Pont. Opt. Max. Romæ. Ex Typograpliia Vaticana. MDXCVI. Lib. I, Dist. XVII, pars tertia, artic. secundus, p. 435.

[34] Petrus Aureoli, *loc. cit.*, p. 436.

[35] Petrus Aureoli, *loc. cit.*, p. 441.

> whereby a lesser charity becomes more perfect and intense is not a whole charity, which can be distinguished in a precise manner; it has not received in part the reality, the specific reason an individual charity possesses; it participates in the reality, the specific reason for the charity by reflection of a sort of reduction; it is, so to say, a *co-charity* (*concharitas*). It is a reality that is absolutely impossible, either in an effective way or by abstraction, to take separately. The Divine Power itself could not produce it in isolation; it can neither receive a distinct and determined existence nor be conceived by intuition; it is intelligible only insofar as it is conceived with another thing that terminates it. The intelligence itself of an angel could, intuitively, divide into two separate charities the charity which has undergone an increase. When charity increases, it behaves like a being to which one adds something that is not a charity, but is part of charity (*aliquid charitatis, non charitas*). The augmentation of whiteness, of heat, and of other forms must be understood in the same way.

The English Carmelite John Baconthorpe († 1346) uses the word *latitudo formæ*, defining it as Henry of Ghent and Antonio Andres did:

> The precise cause,

he said[36],

> for which a form is susceptible to more or less is the *latitude* that the form has, in its essence itself, to acquire or lose degrees. If you ask me why whiteness can be, in the same subject, sometimes intense and sometimes more weakened, I say that the precise cause is the following: Whiteness can sometimes affect its subject and sometimes leave it, so that there is a more or less intense existence.

Of the Thomistic theory,[37] the author seems to slide, in this passage, to the theory of Giles.

But when he comes to specify how this acquisition of new degrees occurs in a form that grows, Baconthorpe fully admits the theory of Peter Auriol, whose authority he invokes[38] and whose words he cites almost verbatim.

It is against this view of Peter Auriol, his confrere in the Franciscan Order, that William of Ockham argues with the clarity and harshness which is customary for

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[36] *En Lector Doctoris resoluti* Ioannis Bacconis Anglici *Carmelitæ radiantissimum opus super quatuor sententiarum libris* — Colophon of the first book:

> Theologi excellentissimi Joannis Bacconis Anglici Carmelitæ Questiones disputate in primum sententiarum. Explicite Mediolani. In officina libraria Leonardi Vegii anno MDX die XXIII Aprilis. Lib. I, Dist. XIV, quæst. I, art. V; fol. CVIII, col. *c*.

[37] Joannis Bacconis *Op. laud.*. Lib. I, Dist. XVI, quæst. I, art. III; fol. CXVII, col. *b*.

[38] Joannis Bacconis *Op. laud.*. Lib. I, Dist. XVI, quæst. I, art. III; fol. CXVII, col. *b*.

him[39]; and when he wants, before refuting it, to express this opinion, he reproduces the same terms of Auriol without changing anything.

> This reality that comes to the preexisting charity,

the Venerabilis Inceptor responds,

> is a real charity, just as a part of water is of true water, so a part of whiteness, irrespective of the place it occupies and the subject it informs, is a true whiteness.

When one adds the one to the other two realities which are in separate subjects, the sum has more *extension*, but not more *intensity* than the parts.

> But when two realities of the same species can exist in the same subject, the addition of one of these realities to another does not make one thing become greater in extension, but only in *intensity*; it is not said that this thing has become *bigger* (*majus late*), but that it has become *more in such a manner* (*magis tale*)…
>
> Between the augmentation of a quantity and the increase of quality, there is a similarity and difference. The difference consists is this: By increasing the quality, there is some absolutely and completely new reality which, with the previous reality, forms a single thing; it is not the same in the augmentation of a quantity…
>
> Against what we have said, a certain doctor argues thus: Like added to like does not increase. This is obvious, because if a warmth is added to another warmth, the heat is not increased. An increase thus cannot be the result of such an addition…
>
> To this argument, I answer thus: When you add a warmth to another warmth, the two attenuated heats remain in separate subjects, as before; also the heat in not increased; but it would be increased if the addition of the two warmths were made in the same subject.

Between the thought of John of Bassols and that of William of Ockham, the agreement is perfect.

Strong, at the same time, from the authority of Duns Scotus and that of William of Ockham, the theory which equates the growth of a quality to the increase of a quantity did not fail to impose itself on the most famous masters of the Paris School.

John the Canon teaches us[40] that in the opinion of certain doctors, any degree that is added to an existing form to strengthen the intensity of this form is more perfect, richer with actual existence than the previous degree. He disputed this opinion and, with William of Ockham, argues

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39 *Tabula ad diversas hujus operis Magistri* Guilhelmi de Ockam *super quatuor libros sententiarum annotationes et ad centilogii theologici ejusdem conclusiones facile reperiendas opprime conducibiles*. Colophon (at the end of the *Questiones super quatuor sententiarum libros*):

> Impressum est autem hoc opus Lugduni per M. Johannem Trechsel Alemannum: virum hujus artis solertissimum. Anno domini nostri MCCCCXCV. Die vero decima menais Novembris. Libri primi Dist. XVII; quæst.XVII: Item quæro utrum in augmentatione charitatis illud quod additur sit ejusdem speciei specialissime cum charitate præcedente separata ab ea?

40 Joannis Canonici *Quæstiones super VIII libros Physicorum Aristotelis perutiles*; In lib. V quæst. III; tertium dubium.

> that a form endowed with intensity comprises several degrees of the same species, such as the preceding and following degrees; that the following degree, taken in a precise way that distinguishes it from the preceding degree, is no more or less perfect than this one; if, on the contrary, we consider this degree to include in itself the lower degree, as taken in the same time as this lower degree, it is more perfect than this lower degree considered in isolation.

He admits that two tepidities make, when added together, a stronger heat, provided that the addition is done in the same subject.

Augustine Gregory of Rimini, in his famous commentary on the first two books of the *Sentences*, which he completed in 1344, also holds to the common doctrine of Duns Scotus and Ockham; he admits[41]

> that in any tension of a form, which occurs successively or takes place suddenly, the subject that becomes more so (*magis tale*) acquires a certain form it did not have before; similarly, in all remissions, the subject loses some of form that it previously contained.

Gregory uses all the resources of his subtle and powerful dialectic to refute the opinions contrary to this theory, particularly that of Giles of Rome and Walter Rurley. He finishes his presentation with these words, which are the direct contradiction of what St. Thomas said regarding the question at hand:

> If we say that a form is more imperfect the more composed it is, I deny this proposition; regarding the composition that I admit, I contend that a form is all the more perfect the more it is composed.

In the first half of 14th century, therefore, the most famous of Scotists and Nominalists conspired to completing the work that Richard of Middleton and John Duns Scotus inaugurated; abandoning the peripatetic doctrine, erasing the so entrenched distinction that it demarcated between the category of quantity and the category of quality, they established a close analogy between the increase of a quantity and the tension of a qualitative form; the increase of intensity, like the increase of a quantity, results from the addition of parts to other parts of the same species.

This theory leads at once to a corollary of extreme importance: The intensity of a quality is henceforth susceptible to measure, as is the magnitude of a quantity; just as they apply to such magnitudes, the reasonings and operations of Arithmetic can combine the various intensities of forms of the same species; we will be permitted to consider multiples and submultiples of *latitudes* of each other.

Without even bothering to formulate explicitly the principle that their doctrine justified, the Scholastics hastened to make constant use of it.

Already in 1344, Gregory of Rimini considered[42] *latitudes* which are duals of each other; already he talks about the speed with which the tension of a form is produced, distinguishing the case where this change is uniform (*uniformis*) and is done at a constant speed from the case where this speed changes with time; he used the same arithmetic language to treat the movement of alteration and local movement.

At the end of his *Tractatus Proportionum*, after treating local movement and the movement of dilation, Albert of Saxony treats the movement of alteration.

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41 Gregorius de Arimino *In primum Sententiarum*; Dist. XVII, quæst. IV.

42 Gregorii de Arimino *Op. laud.*, Lib. I, Dist. XVII, quæst. V.

> It must be known,

he said,

> that in alteration we can consider two types of successions, the succession in extension and the succession in intensity.

He admits, however, that,

> in the movement of alteration, the speed increases as the acquired quality over time… If, for example, unequal subjects gain in an hour equal qualities, they are altered with an equal speed; if the acquired qualities are unequal, these subjects are not altered with an equal speed.

The language that prevailed for treating local movement did not delay in spreading, so that it is possible to talk of qualitative forms. Walter Burley and Albert of Saxony have taught us that a movement should be called uniform (*uniformis*) when the speed is the same magnitude everywhere in the mobile; if it is not so, the movement is difform (*difformis*). We will soon see the qualifiers *uniformis* and *difformis* be used to designate a quality as it attains or does not attain the same intensity at all points of the subject it affects.

Arithmetic, however, does not lack at specifying the speed of certain difform qualities. Imagine the subject informed by a certain quality has the figure of a simple straight line; if the increase undergone by the intensity of the qualitative form, when going from one point to another of this line, is proportional to the increase in the distance between the affected point and the origin of the line, the quality is called *uniformly difform* (*uniformiter difformis*). Among uniformly difform *latitudes*, those that start at zero (*incipiens a non gradu*) are distinguished from those which start at such and such a degree.

This language will soon become common in the schools. The words "uniform heat", "uniformly difform heat" (*calor uniformis*, *calor uniformiter difformis*) are already encountered in one of the questions adjoined to the *Commentaries on the Sentences* composed by Robert Holkot[43]. However, the English Dominican Robert Holkot died in 1349, after teaching at Oxford and Paris. In truth, it is permissible to doubt the authenticity of the *Determinatæ quæstiones* attributed to him; in publishing them, Josse Bade prefaces them with the following warning:

> Many assume that these questions have been assembled by the disciples of Holkot or that, in the course of his education, he has professed them in a public gymnasium.

In any case, whether the question *On the Maximum and Minimum* is by Holkot or not, it only testified with these expressions: *qualitas uniformis*, *qualitas uniformiter*

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[43] Magistri Roberti Holkot *Super quatuor libros sententiarum questiones. Quedam conferentie. De imputabilitate peccati questio longa. Determinationes quarundam aliarum questionum. Tabule duplices omnium predictorum.* Colophon:

> Hujus operis diligenter impressi Lugduni a magistro Johanne Trechsel alemanno. anno salutis nostre. MCCCCXCVIJ. ad nonas Aprilis. Determinatio questionis I: De maximo et minimo.

difformis being commonly heard in the schools toward the middle of the 14th century; and these expressions assume the most evident way that the qualitative forms can, like quantities, be subjected to measurement and give rise to the operations of Arithmetic.

The reflections of modern physicists on the definition of certain properties, such as temperature, have taught us to follow the logical detour by which we can identify the intensity of such properties with the help of degrees and start to discuss it in mathematical language, without stripping them of their qualitative character, without making them out of quantities composed of parts and capable of addition and measurement. But this detour could not be offered, at first, to the minds of philosophers. It is natural that the faculty of subjecting *latitudes* of qualitative forms to arithmetic operations was the price of the hypothesis which assimilated the intensities of these forms to quantities. What Physics has gained all at once by using such a faculty we shall know by studying the work of Nicole Oresme.

Chapter 21
Nicole Oresme

From 1348, we see[1] Master Nicole Oresme, of the diocese of Bayeux, study theology in Paris. In 1356 he was Grandmaster of the College of Navarre. In 1362, already equipped with the rank of master in Theology, he was appointed canon of Rouen. On 18 March 1362, he is elevated to the rank of dean of the chapter. On 3 August 1377, he became Bishop of Lisieux. He died in Lisieux on 3 July 1382.

For Master Nicole Oresme, we have a very large number of works, some written in Latin and some composed in clear, concise, and savory French[2]. Of these works a good number were printed in the Renaissance. Others, no less important, remained unpublished; so it is, in particular, of the important writing on the latitudes of qualitative forms that will occupy us in the next two paragraphs.

But before analyzing this work, we should examine how far the thoughts of Oresme followed the trends that, of his time, appealed to the School of Paris. A little younger than Jean Buridan, a contemporary of Albert of Saxony, did Oresme share, regarding the various problems of Physics, the opinions of these two masters? We will be very accurately informed in this regard by reading two books that our author composed in French: The *Treatise of the Sphere* and the *Commentary on the books of the Heaven and the World of Aristotle*.

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[1] Denille et Châtelain, *Chartularium Universitatis Parisiensis*, tomus II, pars prior (1300-1350); pp. 638 and 641, in note.

[2] See, concerning the writings of Oresme: Francis Meunier, *Essai sur la vie et les ouvrages de Nicole Oresme*; thesis of Paris, 1857. — *Traictie de la première invention des monnoies de Nicole Oresme, textes français et latin d'après les manuscrits de la Bibliothèque impériale, et Traité de la monnoie de Copernic, texte latin et traduction française* published and annotated by M. L. Wolowski; Paris, Guillaumin, 1864. — Charles Jourdain. *Mémoire sur les commencements de l'Économie politique dans les Écoles du Moyjn-Age*. (*Mémoires de l'Académie des Inscriptions et Belles-Lettres*, t. XXVIII, 2e partie, 1874.) — Moritz Cantor, *Vorlesungen über die Geschichte der Mathematik*, 2ts Aufl, Leipzig, 1900. IIter Bd., pp. 128-137.

The *Treatise of the Heavens and the World*, of which the Bibliothèque Nationale possesses many contemporary manuscripts of Oresme[3], begins with these words[4]:

> In the name of God, I begin the book of Aristotle called on the Heavens and the World, which, from the commendation of the most sovereign and excellent prince Charles V, by the grace of God King of France, desiring and loving all noble sciences,
>
> I, Nicole Oresme, dean of the church of Rouen, propose to translate and present in French.

The end of the treatise is the following[5]:

> And so, for the praise of God, I completed the book on the Heavens and the World at the commendation of the very excellent prince Charles V of this name by the grace of God King of France, who, in so doing, made me bishop of Lisieux.
>
> And to enliven, excite, and move the hearts of the young men who have the subtility, noble skill, and desire for science, such that the student encounter and take it up for the love and affection of truth, I dare say, and you make me confident, that there is no mortal man who has seen a more beautiful or better book of natural philosophy than this one, neither in Hebrew, Greek, Arabic, Latin, or French.
>
> *Ecce librum celi Karolo pro rege peregi.*
> *Regi celesti gloria, laus et honor.*
> *Nam naturalis liber unquam philosophie*
> *Pulchrior aut potior nullus in orbe fuit.*

The end makes us know the date when the *Treatise of the Heavens and the World* was written; Oresme composed it when he was named Bishop of Lisieux in 1377; this was, without doubt, his last philosophical work; it has never been printed.[6]

The *Treatise of the Sphere* is older than the commentary on the books *On the Heavens and the World* of Aristotle; in this latter work, in fact, Oresme repeatedly[7] cited the first; thus, after having commentated on the second book of Aristotle, he wrote[8]:

> And so, for the honor of God and by his grace, I have finished the first and second books of De celo et mundo, and for better understanding them it is expedient to translate the Sphere, which I mentioned, into French. And it would good if they were put in one volume together with these II books, which seems to me will be a noble and very excellent book of natural philosophy.

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[3] One of these texts (French collection, n. 565), adorned with miniatures, bears the signature of the Duke of Berry, brother of Charles V, to whom it belonged; this is another text from the same era, and very correct (French collection, n. 1083), thanks to the kindness of Mr. Omont, Curator of Manuscripts at the Bibliothèque Nationale, we were able to study this work.

[4] Bibl. lat., fonds français, ms. n° 1083, fol. I, col. *a*.

[5] Ms. cit., fol. 122, coll. *a* et *b*.

[6] [It has now; cf. Oresme et al (1968).]

[7] Ms. cit., fol. 90, col. *c*.: « Et ce ai ge autrefois déclairé ou XXXIX chapitre du traictié en françois que je lis de l'espère. »

[8] Ms. cit., fol. 95, col. *d*.

This promise of Nicole Oresme is, moreover, granted in the manuscript where we studied the *Treatise of the Heavens and World*, because the copyist followed the book of the *Treatise of the Sphere*[9].

In this manuscript the *Treatise of the Sphere* is followed by a series of astrological treatises "translated from Latin into French", a series that begins with this preamble:

> *Here begins the book of the judgments of Astrology according to Aristole. The prologue of the last translator.*
>
> Aristotle made a book of the judgments of astrology which begins: Signorum alia sunt masculini generis alia femini etc.
>
> But in translating it from Latin into French for the very noble and mighty Prince Charles, firstborn son of the King of France, Duke of Normandy and Delphine of Vienna, we have ordered it differently.

Is this collection of astrological treatises, translated into French for the Delphine who was to be Charles VI[10], the work of Nicole Oresme? The style in which it is written, the place it occupies after the *Treatise of the Heavens and the World* and the *Treatise of the Sphere*, in a contemporary manuscript of Oresme, all seem to support this conclusion. If it were accurate, it would reveal to us a work of Oresme that the scholars have not yet attributed to him.

But let us return to the *Treatise of the Sphere*. More fortunate than the *Treatise of the Heavens and the World*, it was twice printed in Paris by Simon du Bois; the first edition is undated[11]; the second is dated 1508.

The intention Oresme proposed to follow in writing this treatise is defined in the preface:

> The figure and disposition of the world, the number and order of the elements, and the movements of celestial bodies appertains to making every man know what the condition and noble subtility of France is; and it is a beautiful and delectable thing, profitable and honest; and it is necessary for knowing philosophy and especially for astrology. But in order that human subtlety can more easily understand such a thing, the ancient sages composed among others an instrument that is called the material or artificial sphere, which you can watch all around, move and turn, and consider it part of the description and movement of the world and the heavens like an exemplar of which I want to say in French generally and plainly that which is suitable for any man to know, without the depth in the demonstrations and in the subtleties which appertain to the astrologers.

Oresme claimed that he united his *Treatise of the Sphere* to his *Treatise of the Heavens and the World*; "and it seems to me," he added, "that it will be a noble and very excellent book of natural philosophy." If we consider that the *Treatise of the Heavens and the World* supported the possibility of admitting the diurnal motion of

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[9] Ms. cit., fol. 126, col *a*, à fol. 145, col. *b*.

[10] The *Pronosticacions d'Aristote en françois* are found, in fact, in the Library of Charles VI (*Inventaire de la Bibliothèque du Roi Charles VI fait au Louvre en 1423 par ordre du régent*; Paris, 1807; n° 620, p. 161)

[11] *Le traicte de la sphère : translate de latin en françois par maistre* Nicole Oresme, *tres docte, et renomme philosophe*. In was sold in Paris on Judas street by Mr. Simon du Bois, printer: *In fine*: Imprime a Paris par maistre Simon du Bois. — This is the edition that we used.

the Earth[12], that he proved this possibility by arguments whose clarity and precision far surpass what Copernicus wrote on the same subject, we will think Oresme did not esteem too highly the value of his work.

[12] Pierre Duhem, *Un précurseur français de Copernic. Nicole Oresme (1377)* (*Revue générale des Sciences pures 15 appliquées*, nov. 1909).

Chapter 22
The Dynamics of Oresme and the Dynamics of Buridan

It is the French treatise of Natural Philosophy that we will read in order to find the traits of kinship that the doctrines of Oresme offered with those of Buridan and Albert of Saxony.

Moreover, we will not focus our attention on all the questions that were fashionable to dispute in the schools of Paris; we will only choose those whose importance was particularly pronounced in our previous studies; one concerns the explanation of projectile motion and accelerated falling bodies; the other treats the natural place of the earth.

We will know to what party Oresme aligned himself regarding the first question by reading the *Treatise on the Heavens and World*[1]; in this reading we will learn, at the same time, that Oresme held the same position in a commentary, now lost, which he composed on the *Physics* of Aristotle.

Oresme intends to commentate a text of Aristotle, a text that he translates as follows:

> If speed[2] were infinite, it would be fitting that gravity be infinite, and likewise for levity; because the longer a heavy thing descends, the greater is its speed, and so the greater the gravity the greater the speed. And, thus, if the addition of weight is infinite, the addition of the speed will be infinite.

Here is the "gloss" that the Dean of the Chapter of Rouen adds to this text[3]:

> From what he said about gravity being greater inasmuch as speed is greater, this is not to be understood of gravity taken for a natural quality that inclines it downwards.
>
> For if a stone descended from a book a league high and the movement was much faster at the end than at the beginning, nonetheless the stone would not have more natural gravity at one time than at another.
>
> But what one must understand by this gravity that grows in descending, an accidental quality, is that it is caused by the strengthening and increase of speed, as I have other times clarified in book VII of the Physics, and this quality can be called impetuosity.

[1] Nicole Oresme, *Traité du Ciel et du Monde*, livre I, ch. XVIII; ms. cit., fol. 16, col. *d*.

[2] *Isnelté* = *vitesse* [speed]; *isnel* = *rapide* [fast]; *isnelment* = *vivement* [lively or strongly].

[3] Nicole Oresme, *loc. cit.*, fol. 17, col. *a*.

> And it is not properly gravity; for if a hole were dug to the center of the earth and to the other side and a heavy thing descended through this hole, when it would come to the center it would pass beyond and rise by this acquired accidental quality and then descend and rise again and come back several times in the way we see a weight hanging from a long rope do, and thus it is not properly weight which makes it rise up.
>
> And this quality is in every natural and violent movement all the time that the speed increases, save the movement of the heavens.
>
> And such a quality is the cause of thrown things as they leave the hand or instrument, as as I have previously shown in book VI of the *Physics*.

We find in this passage all the principles of Dynamics that the writings of Buridan and Albert of Saxony profess and defend; we even find there considerations on the oscillations of a stone being dropped into a hole that pierces through the earth; these considerations, very similar to a remark made by Albert of Saxony[4], undoubtedly became classic at the University of Paris, where they quickly piqued the curiosity of the students; Desiderius Erasmus, who learned them at Montaigu, reproduced them in his *Colloquia*, and Maurolycus borrowed them from the *Colloquia* of Erasmus.

Master Nicole Oresme singularly liked them, moreover, because he developed them again in more detail.

> I propose,

he said[5],

> that the earth be pierced such that one can see, through a big hole, straight from one side to the other where the antipodes would be were the earth everywhere inhabited.
>
> I say first of all that if one drops a stone in the hole, it would descend beyond this center and immediately rise straight toward the other end and then immediately return past the center from below, and then it would descend back and pass the center less than before and go and come back several times, diminishing such oscillations, finally resting at the center.
>
> And the cause is the impetuosity and violent movement which it acquired by the increase of the speed of its movement according to what was said more plainly in chapter XIII.
>
> And this can be easily understood by something that we see notably; because if a heavy thing is hung on a long rope and is pushed forward, it swings and comes back, making several oscillations until it finally rests at the straightest and closest position to the center that it can.

We will not examine whether these considerations exerted some influence on Galileo himself and his contemporaries[6]; we have recognized, in any case, that Domingo Soto was not exempt from this influence[7].

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[4] *Léonard de Vinci et la pluralité des Mondes*, VIII: Commentaire aux réflexions sur la pluralité des Mondes données par Léonard de Vinci (*Étude sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu.* X; seconde série, p. 95).

[5] Nicole Oresme, *Op. laud.*, livre II, chap. XXXI; ms. cit., fol. 95, coll. *b* et *c*.

[6] To what extent the mechanical doctrines of Buridan and of the School of Paris were related to the theories in the School of Galileo, we see a particularly obvious when one reads the lesson of Torricelli *On the force of percussion* (*Lezioni Accademiche d'Evangelista Torricelli, Mattematico, e Filosofo del Sereniss. Ferdinando II. Gran Duca di Toscana, Lettore delle Mattematiche nello Studio di Firenze e Accademico della Crusca*. In Firenze MDCCXV, Nella Stamp. di S. A. R. Per Jacopo Guiducci, e Santi Franchi. — Della Forza della Percossa, Lezione terza, pp. 13-17 et pp. 19-21).

[7] See § VI: The Dynamics of Jean Buridan and the Dynamics of Soto [chapter 17].

Nicole Oresme does not seek from the Dynamics of Buridan any reflections on the oscillatory motion of a pendulum; he does borrow from him a profound thought on the movement of the celestial orbs.

Buridan dared advance that the movements of the celestial spheres required no driving intelligences to which Aristotle attributed to these circulations; God, creating the heavens, could have communicated to them an initial *impetus*, similar to what is put in a launched stone; and this *impetus* is indestructible because, by the nature of the heavens, he finds nothing that is contrary to it, leading each star on an undefined course[8]. We saw Albert of Saxony[9] receive this thought and transmit it through the teaching of Paris to Nicolas of Cusa[10] and to Kepler[11].

Nicole Oresme adopted this thought, but with a nuance.

The *impetus* impressed in a heavy projectile is violent because the natural gravity of the projectile opposes it. Albert of Saxony formally stated[12], and Marsilius of Inghen does not hesitate to declare[13], that in a heavy body *impetus* directed downward is natural. It is in virtue of this doctrine that Soto regards[14] gravity as a natural *impetus* communicated to the weight body by the cause which begot it.

In the nature of a celestial orb, nothing thwarts the *impetus* that God gave this orb when he created it; this *impetus* is therefore a natural motive force; this is the name that Nicole Oresme gave it, in fact, in the passage we will quote[15].

The canon of Rouen comes to examine some of the difficulties related to heavenly intelligences, the existence of which the peripatetic Physics admitted; he reasoned

> whether the heavens are moved by intelligences. Because,

Oresme continues,

> when by chance God created them, he put in them motive qualities and virtues such as he put gravity in terrestrial things, and he put in them resistances opposed to these motive virtues.
>
> And these virtues and resistances are of another nature and material than any sensitive thing or quality here below.
>
> And these virtues are against such moderate, ordered resistances, granted that the motions are done without violence.
>
> And except violence, it is in no way similar to when a man makes a clock and lets it go and be moved by itself; thus God left the heavens to be continually moved according to the

[8] *Jean I Buridan (of Béthune) and Leonardo da Vinci*, IV: The Dynamics of Jean Buridan [chapter 4]; p. 42, p. 52, and p. 53.

[9] *Nicolas de Cues et Léonard de Vinci*, IX: La Dynamique de Nicolas de Cues et les sources dont elle découle. (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*; seconde série, p. 199).

[10] *Ibid.*, p. 187.

[11] *Nicolas de Cues et Léonard de Vinci*, X: La Dynamique de Nicolas de Cues et la Dynamique de Kepler (*Op. laud.*, p. 208).

[12] *Nicolas de Cues et Léonard de Vinci*, IX: La Dynamique de Nicolas de Cues et les sources dont elle découle (*Op. laud.*, p. 194).

[13] *Ibid.*, p. 195.

[14] See § VI: The Dynamics of Jean Buridan and the Dynamics of Soto [chapter 17], p. 285.

[15] Nicole Oresme, *Traité du Ciel et du Monde*, livre II, chapitre II; ms. cit., fol. 40, col. *c*.

proportions that the motive virtues have to the resistances and according to the established order.

And thus when the Prophet said of God: *Laudate eum cæli cælorum*, he then said: *Statuit ea in æternam, et in sæculum sæculi præceptum posuit, et non præteribit.*

A simple Master of Arts, Jean Buridan humbly submitted his case to the judgment of the "Gentlemen Theologians." By the mouth of Nicole Oresme, the Theologians[16] declare this hypothesis admissible.

The hypothesis of *impetus* is put, in the School of Paris, at the foundation of the theory of the motion of projectiles. We know how this theory was developed by Jean Buridan[17], of which Albert of Saxony seems to have been, on this point, his most faithful disciple; Oresme deviates more, in some problems, from the tradition of the philosopher of Béthune; we first report the text[18], which makes his opinion known to us; we will then remark on it:

To properly understand this, one must know that there are four ways local movements have a beginning or end.

Some are purely natural, like a heavy thing descending downwards from a height.

The others are purely violent, like a heavy thing that rises upwards.

The others are not purely violent, like something that is thrown or carried across, like an arrow[19].

The others are by virtue of beast or man, like going, flying, swimming.

The first, or the first that is purely natural, will always strive to increase in speed, the other things being the same, as when a stone descends straight through the air.

The second, like an arrow that goes straight up, will initially strive and towards the end weakening and retarding.

And also the third, save that it goes longer in striving, has a virtue or force that is greater from the start than in that which is purely violent.

And the fourth is stronger toward the medium.

And to understand the causes of these things, I first say that any movement of something heavy or light of any kind starts in striving so that any degree of speed is in it, it should be that it had less speed, and less than any proportion, and it is what can only be called: To begin *a non gradu*,

And the cause is usually because the excess of the motive virtue over the resistance or application of it to the resistance cannot be suddenly done, but is is fitting that such things be made one part after another, and each part thus, and nothing can be suddenly made.

And if one objects that is a heavy millstone descended and found in its path a bean or small stone resting under it, this millstone would begin to move the stone by some great degree of speed, and not *a non gradu*:

I respond and say that by chance it would be slower than the millstone at the beginning, and it would begin *a non gradu* before the millstone touches it.

And given that it began at a certain degree, it would not be against what is said, because this small stone conjoined to the millstone has a mobile body with it, and a similar

16 At Paris, Oresme taught theology and commentated the *Sentences* of Peter Lombard. In fact, on the aforementioned chapter, he wrote: "Although I show piety on the *Sentences*…". (Nicole Oresme, *Traité du Ciel et du Monde*, livre Il, chapitre II; ms. cit., fol. 41, col. *d*.)

17 *Jean I Buridan (of Béthune) and Leonardo da Vinci*, IV: The Dynamics of Jean Buridan. [chapter 4]

18 Nicole Oresme, *Traité du Ciel et du Monde*, livre II, chapitre XIII; ms. cit., fol. 66, coll. *c* et *d*, fol. 67, coll. *a*, *b* et *c*.

19 *Saecte* = *flèche* [arrow] (*sagitta*).

movement is of the whole as its part, and this movement begins entirely *a non gradu* for the aforementioned causes.

Item, through the growth of this speed a new motive quality is acquired and caused in the moved thing, which we can call force or impetuosity; and this quality or impetuosity aids the natural movement, and it moves the violently moved thing as it is separated from the first mover or cause.

Item, the generation of this quality or impetuosity always grows and strives just as the increase of speed grows and strives; and as the increase of speed weakens, notwithstanding that no such increase so lasts, it lessens the increase of this quality, notwithstanding that it grew[20].

And thus violent movement has three states or three parts.

One is when the moved thing is conjoined with the instrument which does violence to it, and then the speed increases, and the generation or increase of speed[21] also increases, if there is no accidental hindrance; and by what is said, it follows that the increase of this quality or impetuosity will also increase.

Secondly, as the thing violently moved is separated from such an instrument or first mover, its speed will still increase; but the generation or forcing or increase of this speed comes to decrease and finally stops; and when the speed no longer grows, nor does the quality or impetuosity[22].

And the third state begins; and then, the natural quality of the moved thing, although it is gravity, decreases this quality or impetuosity that inclined against the natural movement of the thing, and the movement retarding and the violence decreasing, it finally stops.

And in this way, and not by any other, one can account for all the appearances and all the experiences that one sees in violent movements, either straight up or straight down or across or circular, as to their speed or tardiness, and going and returning, and as to all such things in which one cannot assign another sufficient cause, as I have at other times declared more plainly.

Item, it appears that the impact of a thing thrown or drawn is bigger not at the beginning nor at the end, and thus sometimes near the beginning, as if what is carried straight up, and sometimes more afar from the beginning and more toward the middle, as if what is carried across; because the impact is strongest where the speed is greater.

Item, and wherefore a thing which is compact and heavy, like a stone or iron or plum, gives a stronger kick and stronger throw than one less compact would, like cloth or wool, because the cause is that such a compact thing more receives the impression of this new quality that increases the speed, as said, than other things do.

Item, and wherefore the thing which can be thrown by a force better than any other thing is certainly peas, so that the virtue could so very well throw more weight, not less; and thus because a greater virtue requires a heavier thing for throwing better, and the less virtue, the less weight.

And the cause is: because if the thing is too small or too light, it can both receive the impression or new quality that I have before named impetuosity.

And if the thing thrown is too heavy, the virtue cannot make great violence on such a great weight, and thus to him who wants to throw a thing very well it is fitting that the virtue which throws and the thing are duly proportioned with each other.

Item, in natural movement, like when stone descends, this quality is always connected with the natural gravity, and this is the cause of why the generation of speed and of this quality is always growing, because the gravity and the new quality tend to an end.

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[20] This passage should be understood this way: Not only does the *impetus* grow along with the speed of the mobile, but the speed with which the *impetus* increases or decreases along with the acceleration of the mobile increases or decreases.

[21] Viz., the acceleration.

[22] Translated into modern language, this phrase becomes: “the speed and the *impetus* each attaining their maximum value when the acceleration is zero.”

> *Item*, and by this Aristotle said in chapter XVIII that if a heavy thing always descended without end, the speed would always grow without end, and so would its gravity. And by this gravity this new quality must be understood, because it is as accidental gravity, for which, in this case, it inclines to descend, as, in another case, it inclined upwards or across or in another way.
>
> Thus, we now have that no movement of a heavy or light thing can be entirely regular, because it is slower at the beginning than after—however much it is possible that, at least according to the imagination, the motive virtue and the resistance be so proportioned and moderated given that each part of such a movement would be regular, notwithstanding what the aforementioned quality dictates.

Compared to the doctrine of Buridan, the doctrine of Oresme, as this text presents it, offers numerous analogies with it, but it also offers a difference that attributes to Oresme the resumption of a serious mistake abandoned by his predecessors.

Aristotle believed that the speed of a projectile continues to grow for some time after the body has left the hand or instrument that launched it. Albert the Great and St. Thomas Aquinas did not hesitate to receive this erroneous opinion of the Stagirite[23].

Buridan and Albert of Saxony had been prudent to ignore this so-called initial acceleration of the movement of projectiles; one can think that they did not believe it.

Oresme believed it so well that he was not content to affirm the reality in the passage we have just reported; moreover, after quoting the text of Aristotle[24]:

> A heavy thing would not move quicker at the end of the movement than initially if it is moved by violence and propulsion, for all things moved by violence move slower when they are farther,

he adds this:

> That is to say, towards the end of the movement; since at the beginning, their speed goes on increasing, like a dart or vireton, as it is moved by violence, and is a certain distance where the speed is largest, and it most would be the greatest impact; and afterward the speed diminishes.

By granting this imaginary phenomenon his very authoritative trust, Oresme was, it seems, certified in the Parisian teaching; immediately after him, we see[25] Marsilius of Inghen try to explain how the *impetus*, by distributing itself in the best way within the mobile, begins by accelerating the progress of this body.

It must be admitted that Oresme thereby rendered a bad service to the progress of Dynamics. Convinced that the speed of a mobile continued to grow after the instant of projection, and, on the other hand, dissatisfied with the obviously insufficient theory of Marsilius of Inghen, the mechanists looked for any other explanation of

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[23] *Bernardino Baldi, Roberval et Descartes*, I: Une opinion de Bernardino Baldi touchant les mouvements accélérés (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, IV; première série, pp. 127-139).

[24] Nicole Oresme, *Traité du Ciel et du Monde*, livre I, chapitre XVIII; ms. cit., fol. 19, coll. *b* et *c*.

[25] *John Buridan I (of Béthune) and Leonardo da Vinci*, V: That the Dynamics Leonardo da Vinci proceeds, via Albert of Saxony, from that of Jean Buridan. — In what point it departs from it and why. — The various explanations of the accelerated fall of weights that have been proposed before Leonardo. [chapter 5]

this phenomenon, whose reality seemed to them beyond doubt; they were thus led to account for this alleged initial acceleration of the projectile by the disturbance of the air; then, naturally, they were tempted to attribute to the same cause the acceleration that is very really produced in the fall of a heavy body; they came in this way to disregard the fortunate and fruitful explanation of this acceleration that could be read in the writings of Jean Buridan, Albert of Saxony, and Nicole Oresme himself. We have seen how this unfortunate trend, to which the Dean of Rouen gave a boost, could influence Leonardo da Vinci[26], then Tartaglia, Cardan[27], and Domingo Soto[28].

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[26] *Ibid.*

[27] *The tradition of Buridan and the Italian Science in 16th century*, V: How, in the 16th century, the Dynamics of Jean Buridan spread in Italy. [chapter 10]

[28] See § VI: The Dynamics of Jean Buridan and the Dynamics of Soto [chapter 17].

Chapter 23
The center of gravity of the Earth and the center of the World

The earth is not everywhere of the same density, so that its center of gravity does not coincide with its center of magnitude.

The whole earth is at rest when its center of gravity coincides with the center of the world; hence, the surface which terminates it is not everywhere equidistant from the center of the World. As water is terminated by a spherical surface concentric to the World, one part of the land, whichever is less dense, can emerge, while the denser part is covered by water.

The displacements of weight that various causes—and, in particular, the erosion of rivers—produce on the surface of the earth determine a continual change of position of the center of gravity; the earth moves continually so that its center of gravity returns back to the center of the World.

By these incessant but very slow movements, continents and seas change places; the parts of the earth that are now submerged will eventually emerge and *vice versa*. In addition, the central parts of the Earth, at the end of many centuries, reach the surface.

These propositions that Albert of Saxony, if not imagined, at least formally taught, took on an extreme importance in the teaching of Parisian Scholasticism; they have strongly attracted the attention of those that this Scholasticism attracted and, especially, of Leonardo da Vinci, who deduced his entire Geology from it[1]; Soto did not ignore them[2].

However, we find these propositions in their entirety in the writings of Oresme; if they are not always affirmed categorically, if some of them are marked with an accent of doubt, that doubt is among those which also made Albert of Saxony hesitate; but often the hesitation will be more powerful in the spirit of the Norman doctor than in the spirit of the German master.

Here, first, in the *Treatise of the Sphere*[3], is a brief summary of the whole doctrine:

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[1] *Léonard de Vinci et les origines de la Géologie*, XI: Léonard de Vinci (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, XII: seconde série, pp. 332 seqq.).

[2] See § V: The equilibrium of the land and seas. [chapter 16]

[3] *Le Traicté de la Sphère, translaté de latin en françois par* Maistre Nicole Oresme. Chap. I: De la figure du monde et de ses parties principales.

> After the land is water or the sea, but it does not cover all the earth; because each part of the earth is not of so heavy a nature as the other. So we see that tin does not weigh as much as lead. And thus the less heavy part is higher and farther from the center; and not covered with water; in order that the beasts can live there; and it is also like the front and visage of the earth, completely uncovered; except among some small seas, sounds, and rivers; and all that remains is also covered, vested, and enveloped by the great sea.

In the *Treatise of the Heavens and the World*, this brief mention will be developed and completed, so that all parts of the theory of Albert of Saxony are successively presented to us.

Here, first, is the statement of the principle underlying this theory[4]:

> The center of the world is the place of the earth and of the whole mass of heavy things, because such mass is where it needs to be, and in its own natural place, because the center of its gravity is in the middle of the world, and such a center and center of the world are the same point, as this mass is or was surrounded and contained by water or air or both.

Is this the center of gravity of the earthen element only or the center of gravity of all the heavy mass that must be at the center of the World? Albert of Saxony hesitated between the two positions before choosing the second[5]. John of Jandun wrote a few lines that appear to relate to this debate[6], and Themo, the son of a Jew, clearly defined it[7] before taking the same position as Albert of Saxony.

It is towards the other position that Oresme seems to have a penchant in the passage just quoted, and more in this one, which is nearer to him[8]:

> And according to this, not solely the parts of elemental earth, but all heavy things tend to such a place so they are conjoined and united to the whole mass of the weight, of which the center of the world is the middle and center.

Oresme does not seem to have stopped there in a definitive manner; he seems to have abandoned it to explain, as did Albert of Saxony and Themo, the equilibrium of the earth and the seas; it is, in fact, this explanation which he indicates in the *Treatise of the Sphere*; he outlines it more fully in the following passage from the *Treatise on the Heavens and World*[9]:

> I say that, in this connection, three centers are to be considered, that is to say the center of the world, the center of the quantity of the earth, and the center of gravity; but if one part be of pure gold and the other a mixture of a lighter metal, the center and middle of its gravity will not be the center of its quantity; the center of its weight would be the center of world.

In a passage that the copyist has probably failed to reproduce, the Dean of the chapter of Rouen examined the hypothesis that the center of magnitude and the center of gravity of the earth coincide with each other and, hence, with the center of the world; he continued as follows:

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[4] Nicole Oresme, *Traité du Ciel et du Monde*, livre I, ch. XVII; ms. cit., fol. 15, col. *b*.

[5] Albert de Saxe et Léonard de Vinci, II: Quelques points de la Physique d'Albert de Saxe (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, 1; première série, pp. 14-15).

[6] P. Duhem, *Les origines de la Statique*, t. II, p. 15 [Duhem (1991, p. 269)].

[7] *Ibid.*, p. 51.

[8] Nicole Oresme, *loc. cit.*

[9] Nicole Oresme, *Traité du Ciel et du Monde*, livre II, chapitre XXXI; ms. cit., fol. 94, coll. *c* et *d*.

> And thus any part of its surface would not be lower than the other and, therefore, it would follow that it would all be covered with water, there not being, perchance, the tip of a high mountain.
>
> And because it is not so, it follows that the earth is dissimilar according to these parts, so that in the part that is not covered with water there is not so much gravity as in the other, for, perchance, it is not pure earth but has a mixture of other elements in it; and God and nature have ordained that it be uncovered so that men and beasts could live there; and thus this part is the most noble and also like the front and face or visage of the earth; and the rest or the other part is enveloped in water and vested and covered with sea like a hood or hat; and of this Scriptures said: *Abyssus sicut vestimentum amictus ejus*. And the center of the magnitude of the quantity of the earth [is A]; and the center of its gravity is lower, or center of the world, on line B, as one can imagine in the figure[10]; and the surface of the sea is concentric with the world, and the world and the sea have the same center.
>
> And because what was said is, if it follows that if God and nature made the earth, toward the habitable part, became and was made as heavy as it is toward the other part, or that the gravity of the other part diminished such that the earth was uniform and of similar gravity in all its parts, it would be fitting that the part that is habitable descended and that all the land was plunged in the sea and all covered with water, like how a man covers his face with his hat, and so it would be a deluge, and without rain.
>
> …I suppose that the elements can naturally, according to their parts, grow and decrease by generation or corruption… And so, given that by such generation it added a notable addition to each part of earth, as if, for example, in the part where we are, much under the meridian or line of noon, and that is part of land is signified by B; or that, by corruption, it made a diminution on the opposite part; I say that this fact, it appears to Aristotle, or the previous chapter, that the place where we are, called B, would descend towards the center of world, called A, as one can imagine in the figure.

Does the slightest weight added to one hemisphere suffice to determine a similar movement of the earth? To this question, here is the answer[11]:

> If the air which resists the movement of the earth did not exist, a bit of earth or another heavy thing cannot be added or engendered from one part of the earth more than the other, because it does not make any small movement as the center of gravity does in the center of the world.
>
> But because the air resists the movement of the earth, a small addition cannot make it move; but it could be so large that it would be greater than the resistance of the air that contains the earth; and then, certainly, the Earth would be moved all together until the middle of its gravity is at the center of the world.

Albert of Saxony, too, had worried[12] about the obstacle that the resistance of the atmosphere might bring to the small movements of the earth, caused by the displacement of weight on the surface; he expressed himself, in this regard, with the terms that Oresme used.

Aristotle held that in the sub-lunar world everything is subject to generation and destruction; he also held that an element cannot be corrupted if it is not in contact with another element endowed with a contrary quality. How can these two statements

[10] In the manuscript we consulted, the figures have not been drawn; the places reserved for them remained blank,

[11] Nicole Oresme, *Traité du Ciel et du Monde*, livre II, ch. XXX; ms. cit., fol. 93, coll. *c* and *d*.

[12] *Léonard de Vinci et les origines de la Géologie*, XII: Léonard de Vinci et la tradition parisienne en Italie (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, XII; seconde série, p. 345).

be reconciled? The central parts of the terrestrial element are shielded from contact with any other element; thus, it seems that they can never corrupt.

In his theory of the incessant movements of the earth, Albert of Saxony had found an answer to this perplexing question:

> The earth that is now in the center,

he said[13],

> will one day come to the surface and, therefore, to the place where it is corrupted; and, in fact, since some earthly particles are constantly driven by the rivers that flow to the sea, it follows that the earth becomes increasingly heavy in the hemisphere opposite to ours, while in this one it constantly lightens; and the center of gravity of the earth is continually changing place; what at one time was at the center of the earth is constantly pushed to the surface and will arise one day to the surface of the earth.

Nicole Oresme knows this solution proposed by Albert of Saxony, but he does not seem to be entirely convinced.

> And thus,

the Dean of the chapter of Rouen wrote[14],

> it can be that the earth, in each part of it, be corrupt and diminished and the other part grow, and so it will weigh more on one side than on the other. And as it will notably do so, it should be that all the mass of the earth moves so that the center of gravity of it, which would be outside of the center of the world for the aforementioned mutation, would come to the center of the world, and so the part of the earth that would be at the center would be carried toward the circumference, and by a similar transmutation in another time it will approach more from the circumference; and so by the process of time that part which was at the center will come to the circumference immediately to the place where alteration and corruption occurs, and it will corrupt, and thus other portions of earth by a long process for many thousands of years .

Having outlined in these terms the thesis of Albert of Saxony, Oresme made his doubts known to us[15]:

> I say that this is a beautiful imagining that I have at other times thought; but one can say that it proves the possibility and does not argue for the necessity of the corruption of the land that is towards the center; because, given that the part is now at the center goes out from the center according to that imagining, even if it could return in a similar way, it is not likely that such reduction of the mass of the earth is always from one part and from one side and the increase always from the other.
>
> And thus when the increase will be from the other side, the portion of land that issued and distanced from the center will turn to the center and will neither come immediately to the place of corruption nor near its contrary.
>
> And from the other part, if all the earth be each time thus moved as was said, it would seem that this would be contrary to what the Prophet said to God: *Qui fundasti terram super stabilitatem suam; non inclinabitur in sæculorum sæcula*.

[13] *Quæstiones subtilissimæ Magistri* Alberti de Saxonia *in libros de generatione et corruptione Aristotelis*. In lib. II quæst. VI.

[14] Nicole Oresme, *Traité du Ciel et du Monde*, livre I, chap. XXXVI; ms. cit., fol. 34, col. *d*, and fol. 35, col. *a*.

[15] *Ibid.*, fol. 35, coll. *c* and *d*.

In truth, this biblical text would have, for determining the conviction of Oresme, but little weight; to avoid the objection drawn from Scripture, "one would say that it conforms in this part to the manner of common human speaking," as our author puts it[16] regarding the diurnal movement of the earth.

[16] Nicole Oresme, *Le traité du Ciel et du Monde*, liv. II, ch. XXV; ms. cit., fol. 89, col. *a*. — Cf. *Un précurseur français de Copernic: Nicole Oresme (1377)* (*Revue générale des Sciences pures et appliquées*, 15 nov. 1909).

Chapter 24
The plurality of worlds and the natural place according to Nicole Oresme

We were able to recognize, by reading various texts just cited, firstly, that Nicole Oresme had a very accurate knowledge of the theory of the natural place of the earth, such as what Albert of Saxony taught; on the other hand, he did not accept this theory as free from all doubt. We will see that another doctrine sought, too, if it did not fully delight him, the consent of the Dean of the chapter of Rouen.

The doctrine of which we will speak is so anti-peripatetic that the theory of Albert of Saxony is in harmonious agreement with the Physics of Aristotle; it no longer makes the center of the World play any role in the explanation of gravity; he simply recognizes that heavy or light bodies tend to place themselves in a spherical mass, the heavier body occupying the center while the lighter residing at the surface; any movement that tends to disturb that order is violent movement, and all movement which tends to restore it is natural.

From this doctrine we perceived an indication, though still indecisive, in analyzing the Physics of Jean Buridan[1]; we will hear master Nicole Oresme present it with marked complacency.

It is the famous problem of the plurality of worlds which gives him the opportunity to do so.

The theory of natural place provided Aristotle with his strongest argument against the plurality of worlds. Each element has a unique proper place to which it naturally moves when perturbed by violence. If therefore elements similar to those in this world were found outside of it, they would naturally rush to their proper places as we know them, earth toward the center of our world, fire towards the concavity of the orb of our Moon.

William of Ockham rose vivaciously against this axiom: To an element of a given nature corresponds a numerically one place. He had tried to ruin it by an argument[2]

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[1] *Jean I Buridan (of Béthune) et Leonardo da Vinci*, III: That the theory of the center of gravity, taught by Albert of Saxony, is not borrowed from Jean Buridan [chapter 3], p. 31.

[2] Léonard de Vinci et la pluralité des mondes, IV: La pluralité des mondes et la toute-puissance de Dieu. Michel Scot; Saint Thomas d'Aquin; Étienne Tempier; Guillaume d'Ockam (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, X; seconde série, p. 77).

that Nicole Oresme takes up[3]. In imitation of this argument, the future bishop of Lisieux imagines this remark:

> And one could say similarly that if a portion of earth were at an equal distance between two worlds and if one could divide it, one part would go to the center of one world and the other to the center of the other world.
>
> And if it could not be divided, it would not move because of its indifference and would be like an iron between two equal and equally [strong] magnets.
>
> And if it would be closer to one world than the other, it would tend toward the center of the nearer.

Moreover, regarding equilibrium states that he considered: Our author recognized very clearly that the equilibrium of a sphere of fire whose center is at the center of the world and an equilibrium of one mass of earth equidistant from the centers of both worlds would be struck with instability:

> I imagine either true if the case is such as it was put before; but it could not by nature be such and last in such a condition, by variations or alterations or other common movements, like how a heavy spear cannot long stand on its tip.

Albert of Saxony also gave the remark of Oresme regarding the equilibrium of a stone equally removed from the centers of both worlds[4], but as an arbitrary result of a hypothesis he considered inadmissible; Leonardo da Vinci was to someday take it up[5].

Among the considerations that Oresme developed to enervate the arguments of Aristotle against the plurality of worlds, we find those that comprise a new theory of gravity and lightness; it is these that we now reproduce:

> It seems to me,

says our author[6],

> that these reasons do not obviously conclude; because the first and most principal is that if several such worlds existed, it would follow that the earth of the other world would be inclined to be moved to the center of this one and *e converso*…
>
> To show that this consequence is not necessary, firstly I say that how that both high and low are said in many ways, as will be said in the second book; however, as to the present purpose, they are said in one way or in regard to us, as if we say that a half or part of the sky above us is high and the other is low below us.
>
> But high and low are said otherwise with respect to heavy and light things, as when we say that heavy things tend downwards and light things upwards.
>
> I thus say that high and low, in this second way, are nothing outside of the natural ordinance of heavy and light things, which is such that all weights, according to what is possible for it, be in the midst of the light objects without determining them to another immobile place…

[3] Nicole Oresme, *Traité du Ciel et du Monde*; liv. I, ch. XVI; ms. cit., fol. 15, coll. *c* et *d*.

[4] *Léonard de Vinci et la pluralité des mondes*, V: La pluralité des mondes selon Albert de Saxe (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, X; seconde série, p. 82).

[5] *Loc. cit.*: I. Un texte de Léonard de Vinci (*Ibid.*, pp. 58-59).

[6] Nicole Oresme, *Traité du Ciel et du Monde*, livre I, ch. XIX; ms. cit., fol. 20, col. *d*, and fol. 21, coll. a, b, et c.

> I thus say that where a heavy thing would be and no light thing conjoined to it or to its totality, that heavy thing would not move, because in such a place, it would would not be high nor low for the reason that, being such a case, the ordinance "above" would not be dictated, neither consequently would there be low or high…
>
> And by this it clearly follows that if God by his power creates a portion of earth and puts it in the heavens where the stars are, or outside of the heavens, this earth would not have any inclination to be moved toward the center of this world. And thus it appears that the consequence of Aristotle, cited before, is not necessary.
>
> Then I say that if God created another world similar to this one, the earth and the elements of this other world would be in it as its own elements.
>
> But Aristotle confirms his consequence by another reason in chapter XVII, in this sentence: Because all parts of earth tend to a single location, which is numerically one, the land of another world would tend the center of this one.
>
> I respond that this reason has little appearance, considering what is now said and what was said in chapter XVII, because the truth is that, in this world, one part of earth does not tend to a center and the other to another center, denying all the heavy things of this is world tend to be conjoined in a mass such that the center [of this mass is united to the center] of this world, and all are one body in number, and for this they have a place according to number; and if a part of the earth of the other world were in this one, it would tend to be conjoined to the mass of this one and *e converso*.
>
> But, by this, it does not follow that the parts of the earth, or the heavy things of the other world, if there were, tend to the center of this one; because in their world they would make one mass that would be one body in number, and which would have a place according to number, and would be ordained according to up and down in the aforementioned manner.

Nicole Oresme formulates the principle of this new theory of gravity with perfect clarity:

> The natural ordinance of heavy and light things is such that all heavy things, as it is possible, be in the midst of light things *without determining them to any immobile place*.

Who does not see the consequences of such a principle? The gravity of the earth no longer requires, as in the Physics of Aristotle, that the earth remains motionless at the center of the world; surrounded by its elements of which the lighter envelop the heavier, it can move through space in the manner of a planet; and, on the other hand, nothing prevents that each planet be formed by a heavy earth that water, air, and fire similar to ours surround. The new doctrine compares the earth and planets, which the Peripatetic theory of gravity rigorously prohibited. Also, the opinion of Oresme will be adopted by all those who want to number the earth among the planets; it will be adopted by Nicolas of Cusa first, then by Leonardo da Vinci, then by Copernicus, and finally by Giordano Bruno, who will make it one of his favorite theories.

Moreover, the theory of gravity, so opposed to the Peripatetic theory, is not new in Physics; it is what Plato argued in *Timaeus*; and Plato drew, via natural movement, a different definition from what Aristotle would give; natural movement is not the movement that is directed toward the center of the World or the movement that is directed away from it, depending on whether the mobile is heavy or light; it is the movement by which a body tends to reach the ensemble of the element to which it belongs and from which it was violently detached to be placed in an element of another nature; and the air naturally descends when it is in the sphere of fire as it rises naturally when surrounded by water, because in both cases it seeks to get closer to the sphere of air; these two movements, contrary to each other, the inward movement and the centrifugal movement, are equally natural to the air or equally violent to it; to choose which of the two epithets should be attributed to one of them, we must know the medium in which the air is found.

This opinion, which is deduced in a forced way from the principles posed in *Timaeus*, is in formal contradiction with the Physics of Aristotle; because, according to this Physics, a simple body suits one natural motion, always circular, always centripetal or always centrifugal. Oresme fully admits this opinion; he carefully presents it and he likes to bring out the contrast it offers to the Peripatetic theory of natural movement.

The Dean of the chapter of Rouen expressed himself in these words[7]:

> Posed by the imagination that a pipe or canal of copper or another material is so long that, from the center of the earth, it reaches the end of the region of elements, that is, to the heavens:
>
> I say that if this pipe were filled with fire, save a little air above all at the top of the height, this air would descend straight to the center of the earth, because the less light always descends under the more light.
>
> And if this pipe were filled with water, considering that this tiny quantity of air be near the center, this air would rise toward the heavens, because air always naturally rises in water. And it appears that this can naturally descend and rise through the semi-diameter of the sphere of the elements. And these two movements are simple and contrary, and so a simple body is naturally movable by two simple and contrary movements.
>
> I respond that, perchance, one could say that the movement of this small quantity of air, for the aforementioned case, in descending, is immediately natural so long as this air is entitled to the natural place of air.
>
> And after this, this air descends downwards again by violence to the fire, which is lighter, compresses it, and puts it under itself, and so this descent is part natural and part violent.
>
> Similarly, the movement of this air in rising in the water is natural from when it rises from the center of the earth until the region of air, where its natural place is.
>
> And after that it rises by violence for what this is water rises this air and is launched under it by its gravity.
>
> And thus the entire descent and rising of this air, these two movements, being as they are contrary, one is natural and the other violent.

That a body cannot naturally take on two simple movements distinct from one another was, for Aristotle, one of the reasons that made the diurnal movement of the earth unacceptable. Oresme knows well that the ruin of the principle leads to the ruin of the consequence; and it is, no doubt, by shooting down the one that he undermined the other. Here, indeed, is how he responds[8] to the argument that Aristotle invoked in favor of the immobility of the earth:

> To the first argument where it is said that any simple body has one simple movement, I say that the earth, which is a simple body in its entirety, does not have any movement according to Aristotle…
>
> And who would say that such a body has one simple movement, not according to its entirety, but according to its parts, and only when they are out of their place, against this is the strong instance of the air which descends when it is in the region of fire, and rises when it is in the region of water, and these are two simple movements.

[7] Nicole Oresme, *Traité du Ciel et du Monde*, livre I, ch. IV; ms. cit., fol. 5, col. *d*.

[8] Nicole Oresme, *Traité du Ciel et du Monde*, livre II, ch, XXV; ms. cit., fol. 88, coll. *b* et *c*.

> And by this one can most reasonably say that every simple body or element of the world, except perchance the sovereign heaven[9], is moved in its heaven naturally with circular movement.
>
> And if any part of such a body is out of its place and its whole, it returns there straighter than it could, freed from obstacle.
>
> And it would also be a part of the heavens if it were outside of the heavens.
>
> And it is not a unfitting that a simple body according to its whole has a simple movement in its place, and another movement according to its parts, in returning to their place.

The same principles of mechanics allowed Nicole Oresme to maintain, against the opinion of Aristotle, that there might exist several worlds similar in form to our earth, surrounded by its elements, and that our earth could each day rotate on itself; these principles of Mechanics were those of the *Timaeus*, which a sort of vengeance exhumed from the long oblivion and where the triumph of peripatetic Physics had buried them; they are the ones that the precursors of Copernicus, Copernicus, and then the first supporters of the reformer of astronomy invoke in favor of their new system; but no one has given before or after Oresme as firm, as clear, and as complete an explanation than that of the passages that we just read. Oresme was not only the precursor of Copernicus in defending the diurnal motion of the earth[10] against the peripatetic arguments; he was also, and especially so, in formulating a theory of gravity that rendered the Copernican revolution possible. Boldly innovative, because he imposes identical axioms in celestial movements and the Mechanics of sublunary movements, this theory will be that of the astronomers of the new school, until the theory of universal gravitation, proposed for the first time by Kepler, will supplant it.

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[9] The immobile Empyrean.

[10] See the important fragment of the *Traité du Ciel et du Monde* we published in: *Un précurseur français de Copernic : Nicole Oresme (1877)* (*Revue générale des Sciences pures et appliquées*, 15 nov. 1909).

Chapter 25
Nicole Oresme, inventor of analytic Geometry

Nicole Oresme was not only the precursor of Copernicus; he was also the precursor of Descartes and the precursor of Galileo; he invented analytic Geometry; he established the law of the distances that a mobile travels in a varied movement.

These two great discoveries are recorded in a Latin writing that Oresme himself calls the *De difformitate qualitatum*. “As I once declared in a treatise named *De difformitate qualitatum*,” he wrote in his translation of the *Politics* of Aristotle[1]. This sentence tells us that the treatise in question was already old in the year 1371 when Oresme “translated from Latin into French and commented on” the *Politics*, at the request and expense of Charles V[2].

We have studied this treatise carefully in one of its handwritten texts[3] of the National Library.

In this text a more modern hand than that of the copyist gave this title: *De latitudinibus formarum ab Oresme*[4]; this title, which we will discuss again in paragraph XIX [section 26], is certainly not of the author.

The true title is: *Tractatus de figuratione potentiarum et mensurarum difformitatum*. It precedes a table of ninety-two chapters into which the book is divided.

The title itself is preceded by a short preamble that we transcribe[5]:

> *Assit ad inceptam Sancta Maria meum*
>
> *Cum ymaginationem veterum de difformitate et uniformitate intentionum ordinare cepissem, occurerunt mihi quedam alia que huic proposito sunt consona, ut iste tractatus non*

---

[1] Nicole Oresme, *Les Politiques d’Aristote*, livre VIII, ch. VIII et ch.XII. Cf. Francis Meunier, *La vie et les ouvrages de Nicole Oresme*, pp. 30-31.

[2] Francis Meunier, *Op. laud.*, p. 17 et p. 87.

[3] Bibl. Nat., fonds latin, ms. n° 7371 (autrefois, Golbertinus 4650). The National Library still has in its Latin collection two other texts of the same treatise. The one, entitled *De uniformitate et difformitate intentionum*, containing three principal parts, is found in manuscript n° 14579 (old Saint Victor collection, n° 111). The other, entitled: *De configuratione qualitatum*, is in manuscript n° 14580 (old Saint Victor collection, n° 100). We did not consult these two texts mentioned by F. Meunier, *Op. Laud.*, p. 30.

[4] Bibl. Nat., fonds latin, ms. n° 7871, fol. 214, r°.

[5] We had to correct or interpret certain words, some illegible, others meaningless.

solum excitatorie proceder et, sed etiam distinctive; in quo ea, que aliqui alii solent (?) circa hoc confuse sentire et obscure eloqui ac inconvenienter aptare, studui dearticulatim et clare tradere et quibusdam aliis materiis utiliter applicare.

At the end of chapter XIII of the third part[6], Oresme ends, in these words, his writing:

Multa quidem alia possunt ex predictis inferri. Sed hec, tanquam quedam elementa, sufficiunt, gracia exercii et exempli. Et hoc de uniformitate et difformitate dictum sit tantum. Et sic est finis hujus tractatus. Deo laus. Amen.

The copyist, no doubt, felt a great weariness of having transcribed this treatise because he expresses his satisfaction to have reached the end of his work:

Explicit tractatus magistri Nicholai Oresme de uniformitate et difformitate intensionum. Deo gratias. Amen. Amen. Qui plus scribere vult, scribat. Ego nolo plus.

The unfortunate scribe probably was not able to understand and admire the new and fruitful ideas which, in a perfect order and admirable clarity, presented themselves again and again throughout the pages that occupied him.

Oresme divided his book into three principal parts:

Prima pars: *De figurations et potentiarum uniformitate et difformitate*.

Segunda pars: *De figuratione potentiarum successivarurn*.

Tertia pars: *De acquisitione et mensura qualitatis et velocitatis*.

We will not analyze here the many chapters into which these three parts are divided; the most diverse problems are treated there; the author discusses the most varied issues; he lays the foundation of musical Aesthetics; he argues against the principles of Astrology and Magic. Leaving aside everything that does not contribute to our objective, we will focus only on what prepares the discovery that Soto will make.

The philosophers who, since Richard of Middleton, admitted that the increase of a quality is due to the addition of parts had, for the most part, absorbed the growth of a quality in the augmentation of a magnitude and, in particular, of length. This thought is one that will guide Oresme and serve as an introduction to his system.

With the exception of numbers,

he writes at the beginning of his treatise[7],

any measurable thing has to be imagined in the manner of a continuous quantity. To measure it, one must imagine points, surfaces, lines; in the opinion of Aristotle, in fact, these objects are where the measure or proportion meet immediately; in other objects, the measure or proportion is known only by analogy, insofar as the reason compares these objects to those…

So, any intensity that may be acquired in a successive manner should be imagined as a straight vertical line from each point of the space or subject that affects this intensity… Whatever the proportion between two intensities of the same species might be, a similar proportion must be found between the corresponding lines and *vice versa*. Just as a line is

[6] Ms. cit., fol. 266 r°.

[7] *Magistri* Nicholai Oresme *Tractatus de figuratione potentiarum*. Pars I, cap. I: De continuitate intensionis. Bibl. nat., fonds latin, ms. n° 7371, fol. 215 v°.

> commensurable with another line and incommensurable with a third line, so is it for intensities; there are some that are commensurable with each other and others which are incommensurable.

The various intensities of a quality of a given species can therefore be imagined as straight lengths;

> they can especially, and in the most appropriate way, be represented by straight lines attached to the subject and vertically raised from its various points. The consideration of these lines helps and naturally leads to the knowledge of each intensity. … Equal intensities are represented by equal lines, double intensities by lines where one is double the other, and so on, the intensities and lines always following the same ratio.
>
> And this representation extends, in a universal manner, to any imaginable intensity, whether the intensity of an active or non-active quality, whether the affected subject or object falls or does not fall under the senses…

"The intensity that the line in question designates should properly," according to the opinion of Oresme[8], "be named length or longitude (*longitudo*)." Our author supports this opinion with various reasons. He does not consider it appropriate to give this intensity the name of width or latitude (*latitudo*). "Many theologians," he remarks, "speak of the width (*latitudo*) of charity; in fact, by width they mean the intensity, so that one can have a width without length."

Therefore, it is not the intensity (*intensio*) of a quality that should be named width (*latitudo*), but the extension (*extensio*) of this same quality.

> It is fitting[9] to name the extension of this quality the width (*latitudo*) of an extensive quality; the said extension can be represented by a line drawn within the subject, a line at each point of which the line of intensity of the same quality rises perpendicularly. Thus, as any quality of this kind has intensity and extension, the measure of which must be taken into account, if one gives the intensity the name length (*longitudo*), one will give to the extension, which is the second dimension, the name of width (*latitudo*).

These are the names that Oresme would like to use; but he notes that

> according to the commonly used language, the extension is assigned to the first dimension, that is to say the length (*longitudo*), and the width (*latitudo*) to the intensity. But the imposition of different names or an impropriety of a phrase does nothing to the reality; one can, in both ways, express the same thing; I therefore want to follow the common fashion, lest a form of unusual language renders what I say less easy to understand.

Oresme will consider, first, a quality extended along a line, whether the subject affected by this quality is in reality linear or in a subject having two or three dimensions; he draws a line, and he proposes to study the intensity of the quality at the various points of this line. To such a quality, extended only along a line, he gives the name of *linear quality* (*qualitas linealis*)[10].

To represent it, he will draw, on a horizontal line, a length or *longitude* (*longitudo*) equal to the *extensio*; at each point of this line, he will raise a vertical whose height

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[8] Oresme, *Op. laud.*, Pars I, cap. II: De latitudine qualitatis. Ms. cit., fol. 216 r° et v°.

[9] Oresme, *Op. laud.*, Pars I, cap. III: De longitudine qualitatis. Ms. cit., fol. 216 v° et 217 r°.

[10] Oresme, *Op. laud.*, Pars I, cap. IV: De quantitate qualitatis. Ms. cit., fol. 217 r° et v°.

(*altitudo vel latitudo*) will be proportional to the intensity (*intensio*) of the quality at the corresponding point of the body. This will provide a geometric figure whose properties correspond exactly to the properties of the quality that he is studying. But, by this mode of representation, the study of this quality will be made remarkably easier; the properties

> will be examined more clearly and easily, inasmuch as something that is similar to them is drawn in a plane figure, and that this thing, made clear by a visible example, quickly and perfectly captures the imagination… For the imagination of figures helps greatly in the knowledge of the things themselves.

It is impossible to formulate more exactly than Oresme the principle of graphical representations based on the use of rectangular coordinates, or to better indicate the extreme convenience of such representations.

Any linear quality will thus be represented by a plane figure; conversely, any plane figure bounded from above by a line of which no point is projected beyond the base[11] can represent a linear quality. The geometric study of the arrangements which can affect a similar figure will allow for the classification of a variety of ways in which the intensity of a quality can consist.

Proceeding, in this study, from the simple to the commposite, Oresme first encounters[12] the case where the figure representing the quality is a right triangle and where the longitude is a leg of the right angle. The quality that such a triangle represents

> is commonly called a uniformly difform quality terminated at a null intensity. — *Qualitas uniformiter difformis terminata in intensione ad non gradum.*

Every other triangle[13] represents the set of two such qualities of the same species which succeed one another.

A rectangle[14] represents a quality whose intensity is the same at all points of the line which serves as its extension. “Such a quality is called uniform (*uniformis*) or of equal intensity in all its parts.”

If the representative figure is a trapezoid whose two bases are the both perpendiculars erected along the longitude at its two end points, the corresponding quality

> is said to be a uniformly difform quality terminated on both sides to a certain degree — *Qualitas uniformiter difformis utrinque terminata ad gradum.*
>
> Any other linear quality is called difformly difform (*difformiter difformis*)[15].

But in the multitude of these uniformly difform qualities, Oresme seeks to introduce a certain order. However, the choice of the principle that will be used to establish this classification presupposes a certain difficulty that we examined before. in this review, the logical sense of the author will appear to us singularly sure and refined.

[11] Oresme, *Op. laud.*, Pars I, cap. V: De figuratione qualitatis. Ms. cit., fol. 218 r°.

[12] Oresme, *Op. laud.*, Pars I, cap. VIII: De qualitate trianguli rectanguli. Ms. cit., fol. 219 r° et v°.

[13] Oresme, *Op. laud.*, Pars I, cap. IX: De qualitate aliter triangulari. Ms. cit., fol. 220 r°.

[14] Oresme, *Op. laud.*, Pars I, cap. X: De qualitate quadrangulari. Ms. cit., fol. 220 v°.

[15] Oresme, *Op. laud.*, Pars I, cap. XI: De qualitate uniformi et difformi. Ms. cit., fol. 220 v°.

> Any linear quality,

he says[16]

> can be represented by a figure raised perpendicularly to the line that serves as at extension of it, provided that the height of the figure is proportional to the intensity of the quality. A figure elevated on the line informed by the quality is said to be proportional in height to the intensity of the quality when all elevated lines, at a point of the base, perpendicularly to this base, and extended to the line which terminates the figure from above, has a height proportional to the intensity of the quality that affects the same point…
>
> But, on the same line AB, we can raise more plane figures that are, in height, proportional to each other, some of them being bigger and others smaller… The result is that the same quality of the line AB can be indifferently represented by any one of these figures.
>
> However, if this quality were represented using one of the figures in question, as long as we keep this representation, a quality whose intensity will be analogous to that of the first, but will be everywhere twice this first intensity, will be represented by a figure analogous to the preceding one, but twice as high; in whatever ratio the second quality is smaller or larger than the first, in this same ratio will the height of the second figure be to the height of the first.
>
> Nevertheless, at the start, the first quality could have been represented by a figure larger or smaller in such proportion as we would have liked to choose; these various figures could have been taken unequal in size and dissimilar in appearance; but they were, taken together, proportional in height.

In modern language, we translate this passage as saying that the length by which the unit intensity will be represented can be chosen arbitrarily; that, therefore, the same quality can be represented by an infinite number of different figures; that these figures can be deduced from one of them by an operation that leaves the abscissas invariable and multiplies all ordinates by the same arbitrary number.

So that a property of the figure which represents a quality can be regarded as a property of this quality, it is necessary that this property remains unchanged when the figure experiences the transformation we just defined.

This is what Master Nicole Oresme saw with perfect clarity; before concluding from a property of the representative figure to a property of the quality itself, he is always careful to ensure that the first property is invariant in the transformation by multiplication of the ordinates.

For example, he does not immediately declare that being represented by a right triangle, whose right angle has the longitude as its side, characterizes a certain manner of being of the quality, that which will be designated by the words: uniformly difform quality terminated at a zero intensity. He starts by establishing[17] that

> any quality representable by a right triangle whose right angle has the longitude as its side can be represented by any other triangle that would have a right angle similarly placed, and cannot be represented by any other figure.

He reasons similarly[18] before defining uniform quality.

[16] Oresme, *Op. laud.*, Pars I, cap. VII: De figurarum coaptatione. Ms. cit., fol. 218 v° et fol. 219 r°.

[17] Oresme, *Op. laud.*, Pars I, cap. VIII: De qualitate trianguli rectanguli. Ms. cit., fol. 219 r°.

[18] Oresme, *Op. laud.*, Pars I, cap. X: De qualitate quadrangulari. Ms. cit., fol. 220 v°.

The geometric properties do not remain invariable in the operation that increases or decreases all the ordinates by the same ratio; these properties can be a property of the represented quality.

Suppose, for example[19], that a quality were represented by a semicircle whose diameter is the line that this quality affects. One may equally represent this same quality with a higher figure than this semicircle, and higher in whatever proportion we like, or by a less high figure, and less in whatever proportion as we please.

These figures obtained by increasing or decreasing in some fixed relationship all the ordinates of a half-circumference are of semi-ellipses. Oresme was not enough of a geometer to discover this truth; he dared to assert and prove a less complete proposition:

> Is the figure, less high than the semi-circumference, by which this quality can be displayed, an arc? I leave this point up for discussion. But I say that it cannot be represented by any figure higher than the semicircle, and that it is a portion of a circle.

This proposition, however, suffices to justify the conclusion that our author formulates:

> The curve that terminates this higher figure is not circular and, however, it terminates a figure which is proportional in height to that which terminates a semi-circumference; thus, two figures, one of which has a circular curvature and the other a curvature of non-circular shape may be proportional to one another in height.

Being represented by a line that is a portion of a circle is not an intrinsic characteristic of the quality studied. Oresme will not appeal to them for classifying difformly difform qualities.

The *simple difform difformity* (*simplex difformis difformitas*) will be characterized[20] by the fact that the representative line is formed by a single curved line which, in its entire length, points its convexity in the same direction. Convex or concave, this line may be rational, that is to say circular, or irrational, that is to say non-circular; but the same quality can be represented equally well either by a rational line or by an irrational line.

If, leaving aside the intrinsic properties of quality, we consider only the geometric properties of the figurative representation, we have to distinguish four kinds of simple difform difformities:

1. The convex rational difformity,
2. The concave rational difformity,
3. The convex irrational difformity,
4. The concave irrational difformity.

If we add to them[21]:

[19] Oresme, *Op. laud.*, Pars I, cap. XIV: De simplici difformiter difformi. Ms. cit., fol. 222 v° et fol. 223 r°.

[20] Oresme, *Op. laud.*, Pars I, cap. XV: De quatuor generibus difformiter difformis. Ms. cit., fol. 223 r° et v°.

[21] Oresme, *Op. laud.*, Pars I, cap. XVI: De difformitate composita et qualitate hujusmodi secundum species. Ms. cit., fol. 223 v° et 224 r°.

5. Uniformity
6. Uniform difformity,

we see that the simple figurations are six in number.

But we can obtain composite representations, each of which follows two or more simple figurations.

Oresme classifies these composite representations into species as complex as it is necessary, to formulate them, borrowing simple representations from more numerous genera. Thus each of the less complex species will be formed by simple representations all borrowed from the same genus; to form a representation whose species belongs to the second degree of complexity, he will use simple representations of two different genera; and so on.

> Therefore, by the rules of arithmetic, the result is this: In every single genus in isolation, one can perform one and only one combination and composition, giving us 6 species of composed difform difformities. Using simple kinds taken in pairs, he forms combinations of composed species up to 15. From these genera taken three by three, 20 were born. From simple genera taken four by four, 15 were born. From these genera taken five by five, 6 result. Finally, of all these genres together, the result is a single one. So we have, in sum, 62 species of composed difform difformities.

We see that in the time of Oresme, the formula for the number of combinations was regarded as a common rule of Arithmetic[22]

So far we have seen Nicole Oresme study how we can represent graphically—using two rectangular coordinates, the longitude and latitude—the variations of a measurable property; but nothing, in what we have cited, suggests that he caught a glimpse of analytic Geometry, that he understood the equivalence that maps a certain graphical representation to a certain algebraic relationship among the simultaneously varying values of longitude and latitude. To reach the point where this insight can be grasped, further progress is needed.

That our author has at least taken the first steps in this direction is, we believe, hard to deny after reading the following lines[23] that come immediately after the geometric definitions of the terms “uniform” and “uniformly difform”:

> The said variations in intensity cannot be better, more clearly, or more easily explained and noted than by such images, relationships, and figures; we can give, however, other descriptions or notices which, incidentally, are also known by the figures that we so imagine. Thus,

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[22] Marsilius of Inghen was only a few years younger than Nicole Oresme. But, in his questions on the *De generatione*, Marsilius of Inghen gives the rule for the number of combinations of a number of terms two by two: *Tot sunt combinationes terminorum… quanta est medietas numeri qui surgit ex multitudine numeri terminorum in numerum immediate precedentem.* He demonstrates this rule exactly as we do today. (Egidius *cum* Marsilio *et* Alberto *de generatione*. Colophon:

> Impressum venetiis mandato et expensis Nobilis viri Luceantonii de giunta llorentini. Anno domini 1518 die 12 mensis Februarii. *Questiones clarissimi philosophi* Marsilii inguen *super libris de generatione et corruptione*.

Lib. II, quæst. XII, fol. 116, coll. *c* et *d*).

[23] Oresme, *Op. laud.*, Pars I, cap. XI: De qualitate uniformi et difformi. Ms. cit., fol. 220 v° et fol. 221 r°.

> we can say that the uniform quality is one that is equally strong in all parts of the subject; that the uniformly difform quality is such that, any three points [of the subject] being given, the ratio of the distance between the first and the second and the distance between the second and the third is as the ratio of the excess intensity of the first over the second to the excess of intensity of the second over the third[24].

And our author shows that the geometric representation of the uniformly difform intensity requires, in fact, that it be endowed with this property.

We translate into modern language the proposition that Oresme formulated and demonstrated; the translation can only be the following:

It amounts to saying: The intensity of a measure varies with the extension, so as to be represented by a straight line inclined on the axis of longitudes or abscissas. — Or to say: Given any three points M_1, M_2, M_3, whose x_1, x_2, x_3 are longitudes or abscissas, and y_1, y_2, y_3 the latitudes or ordinates, one constantly has the equality

$$\frac{x_1 - x_2}{x_2 - x_3} = \frac{y_1 - y_2}{y_2 - y_3}. \tag{25.1}$$

And what is it, if not the equation of a straight line, under one of the most used forms in modern analytic geometry? Therefore is it not fair to say that Oresme created analytic geometry in two dimensions?

He went further. He also conceived of the possibility of extending what he said of plane figures to figures drawn in space.

Instead of tracing only a line in the subject, one can trace a surface, for example a flat surface, and examine the quality that informs each of the points of this surface; and we will no longer deal with a linear quality but with a surface quality[25].

The quality of intensity will be represented by a line perpendicular to the informed surface[26]; to imagine how this intensity varies from one point to another of the surface in question, we will have to consider a geometric figure in three dimensions.

To the surface qualities thus represented, we can extend what has been said about linear qualities.

> Just as, among linear qualities, we encounter a uniform quality, a uniformly varying quality, a difformly difform quality, and this in many different ways, so it is, in a similar way, with surface qualities. Just as a uniform linear quality is represented by a rectangle, so will a uniform surface quality be represented by a body which has eight trirectangular trihedrals

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[24] Given the great interest that this passage seems to offer, we give here the Latin text, as it is in the manuscript:

> *Predicte differentie intentionum non melius nec clarius nec facilius declarari vel notari possunt quam per tales ymaginationes et relationes et figuras, quamvis quedam alie descriptiones seu notificationes dari possunt que etiam per hujusmodi figurarurn ymaginationes sunt note. Ut si diceretur: qualitas uniformis est que in omnibus partibus subjecti est equaliter intensa, qualitas vero uniformiter difformis est cujus omnium trium punctorum proportio distantie inter primum et secundum ad distantiam inter secundum et tertium est sicut proportio excessus primi super secundum ad excessum secundi super tertium in intentione.*

[25] Oresme, *Op. laud.*, Pars I, cap. IV: De quantitate qualitatis. Ms. cit., fol. 217 v°.

[26] Oresme, *Op. laud.*. Pars I, cap. XVII: De qualitate superficialis. Ms. cit., fol. 224 v° et 225 r°.

> [*trièdres trirectangles*] (*angulos rectos corporeos*); this quality, while remaining the same, can be represented by a body more or less high, according to what has been said of linear quality…
>
> What has been said about the uniform or difform linear quality can be repeated for the surface quality. Similarly, in fact, the summit of the figure which represents a uniform quality is a surface parallel to the base drawn in the subject, a base which was imagined planar. The summit of the figure, by which one can imagine a uniformly difform quality, is a flat surface not parallel to the base. The summit of the figure that represents a difformly difform quality is a curved surface, or is composed of surfaces which intersect at certain angles.

But surface quality does not exhaust our notion of quality. The subject informed by this quality is, in reality, neither a line nor a surface, but a body; we are still dealing, therefore, with a corporeal quality. Oresme, surely, would like[27] to imagine a fourth dimension of space, so that we could extend to tangible qualities the mode of representation that he used for linear and surface qualities:

> The surface quality is represented by a body, and there is no fourth dimension; we cannot even imagine one. Nevertheless, we must conceive the corporeal quality as having a twofold *corporeality*; it has a true extension, by the effect of the extension of the subject, which takes place according to all the dimensions; but it also has another, which is only imagined; it comes from the intensity of the quality, a quality that is repeated countless times by the multitude of surfaces that can be drawn within the subject.

One could without a doubt make the thought of Oresme much more precise than he had been able to do, but it seems it would not be distorted by expressing it thus: The subject itself, and each of the solids that are obtained in representing the surface quality of one of the surfaces, infinite in number, that can be drawn within the subject, are as many three-dimensional figures drawn in the same space, purely ideal, in four dimensions.

[27] Oresme, *Op. laud.*, Pars l, cap. IV: De quantitate qualitatum. Ms. cit., fol. 217 v° et fol. 218 °r.

Chapter 26
How Nicole Oresme established the law of uniformly varying motion

Not only did Nicole Oresme anticipate Copernicus arguing against the peripatetic Physics with the possibility of diurnal motion of the Earth; not only did he precede Descartes in making use of geometric representations using rectangular coordinates in two or three dimensions and in establishing the equation of the straight line; but he made another discovery that is commonly attributed to Galileo: He acknowledged the law according to which the distance traveled by a mobile of uniformly varying motion grows over time; it is this last part of his work that will now capture our attention.

The second part of the *Tractatus de difformitate qualitatum* was entitled: *De figuratione et potentiarum successivarum uniformitate et difformitate*. This part of the treatise is especially dedicated to the study of velocities.

The principles of kinematics which Oresme claims are no different from those that Albert of Saxony posed in his *Tractatus proportionum* and *Quæstiones in libros de Cælo et Mundo*, two works which, surely, were roughly contemporaneous with the *Tractatus de difformitate qualitatum*, either preceding or following it.

After Walter Burley, and almost exactly in the terms that Albert of Saxony employed, Oresme teaches us[1] that movement has two kinds of extensions, one which depends on the distribution of the velocity at the various points of the subject, i.e, on the mobile, and the other depending on the change of speed over time. Like Albert of Saxony, he would like the epithets “uniform” and “difform” to serve exclusively for characterizing the distribution that affects the speed within the subject, while the qualifiers “regular” and “irregular” to indicate how the values of speed succeed each other in time. But he observes that it is customary to use the words *uniform* and *difform* even to refer to the regularity and the irregularity in the time, and he declares that he will conform to this usage.

Our author then asks[2] how one should, in each type of movement, define the magnitude of the velocity; the speed of the local movement, the angular speed of rotation,

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[1] Oresme. *Op. laud.*, Pars II, cap. I: De difformitate motus. Ms. cit., fol. 336 r°. a.

[2] Oresme, *Op. laud.*, Pars II, cap. III: De quantitate velocitatis; cap. IV: De diversis modis velocitatis. Ms. cit., fol. 237 r° et fol. 238 r°.

the rate of descent, the speed of expansion or contraction, and the rate of alteration are successively considered and determined exactly as they are in the *Tractatus proportionum* of Albert of Saxony; here and there the same thoughts are proposed and clarified by means of the same examples.

Without dwelling on reproducing the considerations which are already known to us, we only indicate a precision that Oresme introduced into the definition of the speed of movement.

He says first[3], like Albert of Saxony:

> In local movement a degree of movement (*motus*) or speed (*velocitas*) is all the greater or more intense when a mobile traverses a greater space or a greater distance in equal time.

But this definition is insufficient to determine what we should call speed *at every moment*, in a movement whose speed changes from one moment to another; it should then be supplemented by adding the phrase: In assuming that, during all this time, the mobile continues to move with the speed it had at that instant. Our author does not formulate this addition in general; but it is in his thinking, and he happens to explain it:

> The degree of the speed of descent,

he said[4],

> is as much greater as in an equal time the moving subject descends more or would descend more *if the movement simply continued* (*magis descendit vel descenderet si continuaretur simpliciter*).

What Oresme added to the Kinematics of Albert of Saxony is the use of coordinates. He said, with his usual clarity, how rectangular coordinates will be used in such a study, at the beginning of the second part of his treatise[5]:

> One can imagine the two extensions in the manner of two lines which would intersect at right angles, so that the extension relative to the subject will be called latitude; the intensity of movement could then be called altitude in a point (*altitudo localis*) of the movement (*motus*) or *speed* (*velocitas*).
>
> But according to what has been said in the third chapter of the first part, the speed seen over time is commonly called latitude; then each of the two extensions, when one compares it with the intensity, may be named longitude; thus, the speed will have twice the longitude as it has double the extension.
>
> In each of these extensions, the intensity of the speed may vary according to multiple modes; as the difformity born of that intensity can be distributed in various ways according to the extension, it follows that the movement or speed could have two sorts of difformities and also two sorts of uniformities.

It is clear, therefore, that to each of the two kinds of difformities to which speed is susceptible, we can apply[6] all the denominations, all the methods of classification

[3] Oresme, *Op. laud.*, Pars II, cap. III. Ms. cit., fol. 237 r°.

[4] Oresme, *Op. laud.*, Pars II, cap. IV. Ms. cit., fol. 237 v°.

[5] Oresme, *Op. laud.*, Pars II, cap. I: De difformitate motus. Ms. cit., fol. 236 r°.

[6] Oresme, *Op. laud.*, Pars II, cap. VI: De difformitate velocitatis per partes quantitativas. Ms. cit., fol. 238 v°.

which we have used, in general, for intensities of any kind; both in relation to the duration and in relation to the extension, the speed will be uniformly difform or difformly difform; it might or might not start at the zeroth degree.

In any capacity, as well as in a motion, Oresme is not limited to considering the extension, represented by longitude, and the intensity, depicted by the latitude; he studied, moreover, what he calls the total amount (*quantitas totalis*)[7] or the measure (*mensura*). This is one of the main topics of the third part of the treatise, whose title is: *De acquisitione et mensura qualitatis et velocitatis*.

> In a universal way,

Oresme said[8],

> the measure or ratio of two qualities, or of two speeds, is equal to the ratio of the two figures, comparable with each other (*ad invicem comparatæ*), by which they are represented. I say: comparable between them, because of a note that has been made in the seventh chapter of the first part.

This remark, which we analyzed at the appropriate time, shows us what Oresme means by comparable figures; these are figures where equal intensities of a quality of the same species are represented by the same length.

The context is also responsible for teaching us what is meant by the ratio of two figures; it is the ratio of the areas of these two figures if they are planar, of their volumes if they are solid.

From the definition just given, the following corollary is immediately drawn: The ratio of the measures of two uniform qualities is the product of the ratio of the extensions with the ratio of the intensities.

> In the measurement mentioned above[9], it is always necessary to take the total extension of the quality, whether this quality is linear, a surface, or even corporeal. The same must be said of the measure of speed, except that, by extension, we must then understand the time during which this speed lasts, and by the intensity, the degree of speed… For example, a uniform speed that lasts for three days is equal to a speed three times as intense that lasts only for one day.

In this case the speed is uniform, the measurement amount or speed, such as Oresme just defined it, is evidently identical with the length as the mobile point has traveled during the time that here replaces the extension. The truth of the same proposition is no less clearly evident in our author in other cases where the movement, without being uniform, is a succession of uniform motions. This is what occurs in a problem that he resolves by a very elegant geometric demonstration[10].

Take the longitude of a figure which represents a linear quality and, according to the usual language in the Middle Ages, divide it into *proportional parts*. For that

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[7] Oresme, *Op. laud.*, Pars II, cap. III: De quantitate velocitatis. Ms. cit., fol. 237 r°.

[8] Oresme, *Op. laud.*, Pars III, cap. V: De mensura qualitatum uniformarum et velocitatum. Ms. cit., fol. 261 r°.

[9] Oresme, *Op. laud.*, Pars III, cap. VI: Adhuc de eodem. Ms. cit., fol. 261 v°.

[10] Oresme, *Op. laud.*, Pars III, cap. VIII: De mensura et extensione in infinitum quarundam qualitatum. Ms. cit., fol. 262 v° et fol. 263 r°.

purpose we split it first into two halves, the second half is then divided into two quarters, the last quarter into two eighths and so on. The longitude is formed from a sequence of segments placed end to end, and the lengths of these segments form a geometric progression of with ratio $\frac{1}{2}$. Those are the *proportional parts* of the longitude.

It is assumed that the first proportional part is affected by a uniform quality of a certain intensity; that said second proportional part is assigned a uniform quality of the same species and of double intensity; the third is assigned a uniform quality three times more intense than the first, etc. The intensities of the uniform qualities that affect the successive proportional parts are among themselves as the various integers are.

The representative figure is formed by a series of rectangles increasingly narrow and increasingly high. Although the heights of these rectangles grow beyond any limit, the sum of their areas is limited; it is four times the area of the first of these rectangles.

Oresme soon applied this theorem to the case where the quality is replaced by speed:

> If a certain time had been well divided into proportional parts; that in the first part of this time, a certain mobile is moved with a certain speed; in the second, it is moved two times faster, in the third three times as fast, and so forth, the speed increasing constantly, this speed would be exactly quadruple in height than the first part; so that in the entire hour, this mobile would traverse a path exactly quadruple that of the one that traversed in the first proportional part, that is to say, in the first half-hour; if, for example, in this first proportional part it has moved a length of a foot, while the rest of the time it will travel three feet, during the whole duration it will travel four feet.

In this case, the definition that Oresme provided of the intensity of the speed sufficed to prove to him that the area of the representative figure was the length described by the moving point. Did he know that it is the same in general? So that he could demonstrate it, he would have had to possess a precise definition of the instantaneous velocity, and he would have acquired the notions of derivative and integral. Surely, such a demonstration would by far surpass the means that his rudimentary knowledge of Mathematics provided him. But unable to demonstrate such a proposition, had he intuitively recognized the truth? We find, in his treatise, no sentence that stated it explicitly. It seems, however, that this silence is not the result of a doubt where the author would have remained, but rather of a perfect assurance in the accuracy of the proposition that it implies. He does not say that the area of the representative figure measures, in all circumstances, the path traveled by the mobile because he thinks that *this is self-evident*. We will find, moreover, in a moment, a passage which clearly implies this interpretation. We will see, too, that many of the disciples of Oresme and of his commentators have interpreted the thought of the master in this way, and without even thinking that one could interpret it otherwise.

It was important that this interpretation was reported, because it gives all its value to the passage that we will now translate[11]:

[11] Oresme, *Op. laud.*, Pars III, cap. VII: De mensura qualitatum et velocitatum difformarum. Ms. cit., fol. 262 r° et v°.

Any uniformly difform quality has the same quantity as if it uniformly informed the same subject according to the degree of the midpoint (*Omnis qualitas, si fœrit uniformiter difformis, secundum gradum puncti medii ipsa est tanta quanta qualitas ejusdem subjecti*). In saying: according to the degree of the midpoint, I imply: if the quality is linear; if it is a surface quality, it will be necessary to say: according to the degree of the average line…

We will demonstrate this proposition for a linear quality.

Let there be a quality that can be represented by a triangle ABC; (Figure 26.1) it is a

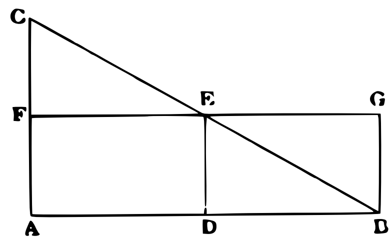

Figure 26.1 [Oresme's famous mean speed theorem diagram]

uniformly difform quality which, at point B, terminates at the zeroth degree; let D be the midpoint of the line that represents the subject (*subjectiva linea*); the degree or intensity that affects this point is illustrated by the line DE. The quality which would have the degree thus designated can be represented by the quadrilateral AFGB, as is apparent from chapter X of the first part. But by proposition XXVI the first book of Euclid, the two triangles EFC and EGB are equal. The triangle that represents the uniformly difform quality and the quadrilateral AFGB, which represents the uniform quality according to the degree of average point, are thus equal to each other; the two qualities that are conceivable, the one by the triangle and the other in the quadrilateral, are also equal to each other; and this is what we proposed to demonstrate.

We proceed in the same way regarding a uniformly varying quality which, on both sides, ends at a certain degree…

Regarding speed, we can say the exact same thing as of a linear quality, only instead of saying: midpoint, one must say: average time during which this speed lasts.

It is therefore evident that a quality or any uniformly difform speed is found equal to a quality or a uniform speed.

If, as we believe, the quantity or measure of a speed is identified in the mind of Oresme with the linear space that the moving point traverses, the result which our author has just attained is particularly deep; it may, indeed, be formulated as follows: When a mobile moves, for a time, with a uniformly varied motion, the path it travels is equal to the one it would travel in a uniform motion, of the same duration, whose speed would be equal to that which is taken in the average time of the first movement.

We will have the assurance that this is indeed the proposition that Oresme meant by reading one of the problems that our author treats.

As he did in a previous problem, Oresme takes[12] a certain longitude which he divides into proportional parts of ratio $\frac{1}{2}$; but, in each of the proportional parts, he no longer assumes that the longitude is uniform; he supposes it is only uniform in the parts of odd rank and uniformly difform in the parts of even rank. He therefore

[12] Oresme, *Op. laud.*, Pars III, cap. X: Quoddam aliud exemplum. Ms. cit., fol. 264 r° et v°.

admits that in the first part, the longitude uniformly maintains a certain degree; that in the second it grows uniformly from this degree to a double degree; that in the third it uniformly maintains this double degree; that in the fourth it grows uniformly from this double degree to a quadruple degree, and so on. He then states this theorem: The total measure of the quality is in the ratio $\frac{7}{2}$ to the measure of the quality that affects the first part. To demonstrate this theorem, he uses, of course, the rule that he posed regarding the measurement of a uniformly difform quality.

Once this theorem is demonstrated, Oresme adds:

> We can prove a similar proposition regarding the speed and apply it to the speed as was done in the previous chapter.

Now, in the ninth chapter, Oresme applied it to the speed the theorem that he demonstrated, and this application essentially assumed that the *measure* of the speed during a given time was the space the mobile actually traversed during this time. It is therefore clear that he admits the same supposition in his second chapter that he also admits in the rule on which the solution that this chapter discusses depends. He understands that the space traversed by a uniformly varied motion is equal to that which would be traversed by a uniform movement of the same duration, having for speed what the first reaches at its average time.

Now, it is customary to make this law one of the claims to fame of Galileo.

How was Oresme brought to conceive this fruitful thinking? We can, I believe, guess.

He sometimes insisted on the idea that speed has two kinds of extension, the extension according to the subject and the extension according to the duration; that each of these two extensions can be treated in the same way as the other; and that there are, for example, uniform, uniformly difform speeds according to the subject, as there are uniform, uniformly difform speeds in time.

However, wanting to give an example of a uniformly difform speed relative to the subject, and starting at the zeroth degree, he cites[13] the speed of a radius that turns around the center of the circle.

It is this speed that he treated in the little writing *De proportione motuum et magnitudinum*, the text of which was already known in the 13th century. The anonymous author of this treatise showed that a radius or a portion of a radius that rotates around the center of the circle sweeps out a space equal to what the same line would sweep out in a translation that would have the speed of its midpoint; the demonstration he gave, very similar to the one we found Oresme wrote, led him to look at the speed of the radius, varying from one point to the other, as equivalent to the speed of the average point; in short, he formulated, for the uniformly difform speed relative to the subject, the rule that Oresme was to formulate for the uniformly difform speed with respect to time.

Very certainly known to Bradwardine and very probably known to Albert of Saxony, Oresme without a doubt ignored the treatise *De proportione motuum et magnitudinum*; although this book had not come into his hands, the ideas it contained,

[13] Oresme, *Op. laud.*, Pars II, cap. VII: De quadam differentia inter motum localem et alterationem. Ms. cit., fol. 239 r°.

summarized in *Tractatus proportionum* of Bradwardine and Albert of Saxony, were certainly common in Paris when the Treatise *De difformitate qualitatum* was written. Directly or indirectly, thus, the small writing *De proportione motuum et magnitudinum* was able to inspire in the grand master of the College of Navarre the rule that we heard him formulate and that, from now on, we will name the *Rule of Oresme*. By that name, however, we do not mean to affirm that Oresme was the first to know of this rule; what we will say in paragraph XXIII [chapter 28] will show that this assertion is by no means assured.

In 1368, Albert of Saxony wrote his *Quæstiones in libros de Cælo et Mundo*; in 1371, Nicole Oresme already considered his treatise *De difformitate qualitatum* as old. Before the year 1370, therefore, two great truths had, the one, been seen, the other, discovered; it was speculated that falling bodies have a uniformly accelerated motion; the law that, in such a movement, binds the space traversed to the time taken to traverse it had been formulated. It sufficed to give the first proposition as assured and compare it to the second so that the two essential laws of falling bodies would be formulated. The fruit, it seems, was ripe; the slightest touch would undo it.

However, despite this prediction, more than a century and a half will elapse before this fruit will be picked; it is only in the writings of Domingo Soto that the assumption of Albert of Saxony, on the one hand, and the discovery of Oresme, on the other, will join and complement each other; until the day when they will reunite through the Dominican savant, these ideas will be transmitted from age to age and from school to school, but will remain separated from each other. These are the vicissitudes through which this long tradition is maintained that we must now retrace.

The influence of Nicole Oresme at the University of Paris. — The treatise *De latitudinibus formarum*. Albert of Saxony. Marsilius of Inghen.

The handwritten text we studied in the two preceding paragraphs is entitled: *Tractatus de figuratione potentiarum et mensurarum difformitatum*. But a hand, less ancient than that of the copyist, attributed to it this other title: *De latitudinibus formarum ab Oresme*.

This latter title is of another work, a text of which Maximilian Gurtze found, probably dating from the late 14thcentury, in a manuscript of the library of the Royal Gymnasium of Thorn[14].

[14] Maximilian Gurtze, *Ueber die Handschrift R. 4°. 2, Problematum Euclidis explicatio der Königl. Gymnasialbibliothek zu Thorn* (*Zeitschrift fur Mathemalik und Physik*, KIII° Jahrgang, 1868. Supplement, pp. 92-97).

This writing was printed several times, at the end of the 15th century and at the beginning of the 16th century[15].

The 1505 edition seems to attribute this treatise to Oresme himself; but the 1486 edition is limited to saying that it is composed *secundum Nicholaum Horen*, and the 1515 edition indicates, more explicitly, that it was written *secundum doctrinam Magistri Nicolai Horem*. It is certain, indeed, that we do not find an original work of the great master of the College of Navarre, but a summary, written by a disciple, of the treatise *De difformitate qualitatum*.

Reduced almost exclusively to definitions and statements of propositions which no reasoning accompanies, this dry compendium gives only a very poor idea of the work that inspired it; however, such is the power of this work that we can still guess something in the mediocre imitation of it given in the treatise *De latitudinibus formarum*; Maximilian Gurtze and Maurice Cantor[16] who have known the thought of

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1. *lncipit perutilis tractatus de latitudinibus formarum secundum Reverendum doctorem magistrum Nicholæum Horen*. Die decima Januarij — (in fol. n r°) *Tractatus de latitudinibns formarum a venerabili doctore magistro Nicolao horen editus fuit fœliciter. Impressus ac diligenti cura emendatus padue per magistrum Matheum cerdonis de vuindisgrech. Anno domini 1486. Die vero 18 mensis Februarij.* — (in fol. 12 r°) *Incipiunt questiones super tractatu de latitudinibus formarum determinate per venerandum doctorem magistrum* blasium de parma de pelicanis. — (fol. 19, r°) Expliciunt questiones super tractatum de latitudinibus formarum magistri Johannis (*sic*) Horen determinate per venerandum doctorem artium: magistrum Blasium de parma de pelicanis. Impressum Padue Die: mense et anno supradictis. In laude dei summi.
2. *Questio de modalibus* Bassani Politi. — *Tractatus proportionum introductorius ad calculationes Suiset.* — *Tractatus proportionum* Thome Braduardini.— *Tractatus proportionum* Nicholai Horen. — *Tractatus de latitudinibus formarum ejusdem* Nicholai. — *Tractatus de latitudinibus formarum Blasii de Parma.* — *Auctor sex inconvenientibus.* — *Questio subtilis doctoris* Johannis de Casali *de velocitate motus alterationis.* — *Questio* Blasii de Parma *de tactu corporum durorum.* Colophon:

   > Venetiis mandato et sumptibus heredum quondam nobilis Viri D. Octaviani scoti Civis Modoetiensis per Bonetum locatellum bergomensem presbyterum Kal. Septembris 1505.

3. *Contenta in hoc libello. Arithmetica communis.* — *Proportiones breves.* — *De latitudinibus formarum.* — *Algorithmus* M. Georgii Peurbachii *in integris.* — *Algorithmus* Magistri Joanis de Gmunden *de minuciis phisicis.* Colophon:

   > Impressum Viennæ per Joannem Singrenium Expensis vero Leonardi et Luceæ Alantse fratrum Anno domini MCCCCCXV. Decimonono die Maii.

   In the body of the volume, the first three treatises are entitled:

   > *Incipit Arithmetica communis ex* divi Severini Boetii *Arithmetica per* M. Joannem de muris *compendiose excerpta.*
   >
   > *Tractatus brevis proportionum: abbraviatus ex libro de Proportionibus* D. Thome Braguardini Anglici.
   >
   > *Tractatus de latitudinibus formarum secundum doctrinam* magistri Nicolai Horem.

16 Moritz Cantor, *Vorlesungen über die Geschichte der Mathematik*. Bd. II, von 1200-1668, 2le Aufl., Leipzig, 1900; pp. 129-131.

Oresme by the small writing his disciple, did not hesitate, however, to look at the future Bishop of Lisieux as the precursor of Descartes.

They had not, in any case, greeted him with the title of precursor of Galileo; the proposition that we called the rule of Oresme is passed over in silence in the treatise *De latitudinibus formarum*; we only find a quick indication of the proportionality between the *quantities* of two qualities of the same species and the areas of the figures that represent these qualities:

> *Eadem est proportio formæ ad formam quæ est figuras ad figuram.*

That a similar manual has been written, and apparently before the 14th century, is clear evidence for us that the methods of Oresme—the use of the latitude and longitude, that is to say, of rectangular coordinates, to figure the variations of the diverse measurable properties—were quickly propagated in the schools, at least in Paris.

Of this rapid dissemination of doctrines offered by the great master of the College of Navarre, we will find two contemporary witnesses: Albert of Saxony and Marsilius of Inghen.

Albert of Saxony wrote in one of his questions on Physics[17]:

> Let there be a line on which a semicircle is described. Suppose that each marked point on this line is white, and that the whitenesses of any two of these points be as the lines drawn from these points to the circumference; the difformity of this whiteness will be similar to the semicircle; this semicircle, described on the line [that affects this whiteness], defines (*causat*) the radius that can represent the intensity of the whiteness at the midpoint of this line.

It is clear that Albert of Saxony employs here the rectangular coordinates according to the principles that Oresme established; the last sentence is obviously inspired by the thought on which the great master of the College of Navarre insisted: A quality, represented by a semicircle when one chooses in a certain way the length which must represent the unit of intensity of the quality, ceases to be represented in this way if we change this length.

The printed work where[18] the writings of Giles of Rome, Albert of Saxony, and Marsilius of Inghen on *De generatione et corruptione* reunited ends with a table

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[17] Egidius *cum* marsilio *et alberto de generatione. Commentaria fidelissimi expositoris* D. Egidii Romani *in libros de generatione et corruptione Aristotelis cum textu intercluso singulis locis. — Questiones item subtilissime eiusdem doctoris super primo libro de generatione: nunc quidam primum in publicum prodeuntes. — Questiones quoque clarissimi doctoris* Marsilii Inguem *in prefatos libros de generatione. — Item questiones subtilissime magistri* Alberti *de saxonia in eosdem libros de gene. nusquam alias impresse. — Omnia accuratissime revisa: atque castigata: ac quantum ars eniti potuit Fideliter impressa.* Colophon:

> Impressum venetiis mandato et expensis Nobilis viri Luceantonii de giunta florentini. Anno domini 1518. die 12 mensis Februarii.

[18] *Acutissime Questiones super libros de Physica auscultatione ab* Alberto de Saxonia *edite*… Venetiis sumptibus heredum q. D. Octaviani Scoti Modoetiensis: ac Sociorum. 21 Augusti 1516. Lib. VII, quæst. VI, fol. 74, col. *a*.

of questions that these various authors addressed; this table has the following date: *1385, die 13 Aprilis*; this date is obviously that of the manuscript that the printer has reproduced.

So, before the year 1382, when death overtook the bishop of Lisieux, or, at the latest, in the time immediately following the death, Marsilius of Inghen compiled his *Quæstiones in libros de generatione et corruptione*. However, in these *Questiones* he used longitude and latitude in a way that imitated Nicole Oresme.

We indicate in brief the theory regarding how this is done.

This theory, which is quite unique, had been imagined by Jean Buridan[19].

Let us conceive a certain subject unevenly hot in its various points. Buridan supposed that each point was both hot and cold, that the intensity of the cold at one point, added to the intensity of the heat at the same point, gave the same amount everywhere, which our author designated as being the *gradus summus caloris*.

He did not need to modify it much to transform it into this: The intensity of the cold is only the intensity of the heat *changed in sign*; this opinion, we say, strongly attracted the attention of scholastics of Paris.

Albert of Saxony presents[20] this opinion carefully and, soon after, the contrary view, that at various points of an unevenly hot subject, only unevenly intense heats exist, without any admixture of cold; then he adds, by way of conclusion:

> I think this second opinion is more accurate, but the former is more widespread.

Between these two opinions, Oresme will not discuss where the true doctrine is located[21]; he intends only to show how his method enables representing the theory of Buridan geometrically.

He supposes that the heated subject is reduced to a straight line. At each point of this line, he raises a latitude proportional to the intensity of heat in this point; he extends this line of a proportional length to the intensity of cold at the same point; the resulting total latitude has, at any point, the same length. A rectangular figure on the longitude which represents the extension is thus drawn. a line divides this rectangle into two parts which represent, respectively, the two contrary qualities associated with one another within the subject.

> This opinion,

Marsilius of Inghen said[22],

> seems likely to me; I do not know if this is because I have a passion for the opinion of my Master, Jean Buridan, who proposed it.

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[19] *Magistri Joannis Buridam Quæstiones super octo Physicorum libros*; lib. III, Quaest. III.

[20] Alberti de Saxonia *Quæstiones in libros Physicorum*; lib. V, quæst. IX; ed. cit., fol. 62, coll. *a* and *b*.

[21] Magistri Nicholai Oresme *Tractatus de difformitate qualitatum*; Pars I, cap. XIX: De figuratione contrariorum; ms. cit., fol. 225, v°, et fol. 226, r°.

[22] *Questiones clarissimi philosophi Marsilii inguen super libris de generatione et corruptione*. Lib. II, quæst. VI; ed. cit., fol. 106, coll. *c* et *d*, et fol. 107, col. *a*.

It is in the middle of the geometric representation devised by Oresme that Marsilius presents the theory that pleases him so much[23].

Marsilius of Inghen does not only make use of the rectangular coordinates of longitude and latitude; he also knows and uses the rule of Oresme; he cites it as an unquestioned truth, of common use, which is invoked as an argument for or against a proposition submitted to the discussion. It is thus that this rule is recalled[24] in an equation on the *De generatione et corruptione*; "If it were not so," we read in an argument, "a uniformly difform latitude would not correspond to its average degree."

The *Abridgment of the Book of the Physics* was certainly composed by Marsilius of Inghen in Paris, starting before the year 1386, when the author was rector of Heidelberg. Yet, we find several allusions to the rule of Nicole Oresme.

In this abstract, for example, we read, on the speeds of the various movements, of considerations that are mostly borrowed from the *Tractatus proportionum* of Albert of Saxony. However, they differ in one point; against Bradwardine and Albertutius, Marsilius resumed the opinion sustained in the treatise *De proportionalitate motuum et magnitudinum*; he admits that in a body with different parts that move unequally, the velocity must be measured by the length that a midpoint describes; now, in support of this view, the author invokes[25] the reason:

> The midpoint, not the most intense point, should refer to a difform latitude.

Moreover, Marsilius wonders how the proportionality, accepted by the peripatetic Dynamics, between the power that moves a body and the speed of this body, in the case where the power varies from one moment to the next, must be understood; he answers in these terms[26]:

> In this case, there is no uniform power[27] that always remains the same, but there is a difform power constantly the same, known by its average degree; similarly, there is not a speed which remains uniform, but a difform speed, known by its average degree, or by another degree if it is not uniformly difform.

In his *Questions on the Physics*, Marsilius of Inghen returns to the opinion of Bradwardine and Albert of Saxony; he wants the speed of a body to be the velocity of the point that moves the quickest. The rule of Oresme can no longer serve as an argument in favor of such an opinion; but, against this view, it becomes an objection that must be examined. Marsilius is careful to formulate[28] this objection:

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[23] Marsilius again makes use, in another part of the same treatise, of the representation by rectangular coordinates (Marsilii Inguen, *Op. laud.*, lib. I, quæst. XVIII; ed. cit., fol. 77, col. *c*).

[24] Marsilius of Inghen, *Op. laud.*, lib. I, quæst. XX; ed. cit., fol. 90, col. *c*.

[25] Marsilius of Inghen, *Op. laud.*, fol. marked i 3, col. 6.

[26] *Incipiunt subtiles doctrinaque plene abbreviation.es libri phisicorum edite a prestantissimo philosopho Marsilio inguen doctore parisiensi* (S. 1. n. d.) (Pavia, Antonius de Carcano, ca. 1490), 3rd fol. (unpaginated) after the fol. marked g 4, col. *d*.

[27] The text, instead of power (*potentiæ*), said proportion (*proportio*).

[28] *Questiones subtilissime* Johannis Marcilii Inguen; *super octo libros Physicorum secundum nominalium viam*. Lib. VI, quæst. V: Utrum velocitas motus sit attendenda penes spatium in tante tempore pertransitum.

The uniformly difform whiteness is not more intense than its average degree.

This objection summarily dismissed, the question our author addressed is very similar, in its substance and form, to the *Tractatus proportionum* of Albert of Saxony.

The various indications that we have collected show us that when Nicole Oresme, Bishop of Lisieux, lived his last days, the use of rectangular coordinates, which he had imagined and recommended, spread into the schools of Paris; in particular, the rule relating to uniformly difform latitudes, which justified the use of these coordinates, was commonly invoked in discussions of Physics.

We will see that, about the same time, this rule was not ignored at the University of Oxford; perhaps it even knew about it before Nicole Oresme presented it in Paris.

The Oxford School in the middle of the 14th century. — William Heytesbury. — John Dumbleton. — Swineshead. — The Calculator. — The treatise *De sex inconvenientibus*. — William of Colligham.

In the preamble of his treatise *De figuratione potentiarum et difformitate qualitatum*, Oresme does not attribute to himself the role of its inventor, but the more modest role of the one who brings, into a subject already treated, order and clarity; this order and clarity results from the use of geometric representations which it seems he was the first to have imagined to use in similar matters; but the considerations on the measure of intensities, their uniformity or difformity, were certainly familiar before him to those he names the *veteres*.

Where should we look for these *veteres*? We did not encounter them at the University of Paris among those, such as Jean Buridan, who immediately preceded Oresme; it seems that we should rather expect to find them at the University of Oxford.

At the University of Oxford, toward the mid-14th century, we see bunch of writings appear where the intensity of the forms, their longitude and latitude, and their uniformity and difformity are disputed. That some of these writings are earlier than the treatise of Oresme and that the grand master of the College of Navarre may have knowledge of them is extremely likely, although it is still very difficult to specify more exactly this too vague assertion. The treatise of Oresme is not dated and the writings, emanating from the School of Oxford, that we have to compare to it, are not extant. When these writings are not anonymous, which often happens, their authors are, mostly, men of whom we know little or nothing; it is difficult to decide whether these writings could have inspired the author of such another and, in particular, Nicole Oresme.

So after we have described the progress made by certain ideas in the School of Paris toward the mid-14th century, we will follow the course that these ideas have made about the same time in the School of Oxford, without it being possible for us to say what the mutual reactions of these two movements were.

The School of the logicians of Oxford, in the middle of 14th century, is dominated and personified by William Heytesbury; this dialectician seems to enjoy, with the fellows of Merton College, or of Queen's College, a prestige similar to that which surrounded, a half-century before him, the person of Thomas Bradwardine.

The fame of this personage passed, in the 15th century, from the University of Oxford to the Universities in the continent; his name became the most celebrated in the schools; but as it spread, it would be distorted more and more. English documents, contemporary to the life of our logician, call him [29] Hethelbury, Hegterbury, Hegtelbury; the Scholastics of the continent, Latinizing that name, made it Hentisberus and, frequently, Tisberus; it is in this form that the Averroists and the Italian Humanists took it most often in their diatribes against the Logic of Oxford.

The facts authentically known of the life of William Heytesbury are reduced to very little.

In 1330 he is mentioned as a fellow of Merton College; in 1338 he is its bursar[30]; in 1338 and 1339 we find his name in the lists of the examinations of this college[31].

In 1340, among the first fellows of Queen's College, there is a William Heightilbury[32] who is probably none other than Heytesbury.

From 1340 to 1371, documents no longer mention his name; but in 1371, we find[33] William Heighterbury or Hetisbury, doctor in Theology and chancellor of the University of Oxford.

We only have the Logic works of this chancellor of Oxford; these works are five in number:

1. The first, very short, carries the title: *De sensu composito et diviso*.
2. The second is entitled: *Regulæ solvendi sophismata*; very famous in the schools, it was simply designated by the name of *Regulæ*. It consists, in reality, of six small treatises, which are designated: *De insolubilibus*. *De scire et dubitare*. *De relativis*. *De incipit et desinit*. *De maximo et minimo*. *De tribus prædicamentis*. The last of these treatises is itself divided into three parts: *De motu locali*. *De motu augmentationis*. *De motu alterationis*.
3. In his *Regulæ*, Heytesbury advances a number of propositions which he does not demonstrate; as a result, he completed his first book with a second writing in which the proofs of the assertions made in the *Regulæ* are given; the second writing is entitled: *Probationes profundissimse conclusionum regulis positarum*.
4. A very concise opuscule treats *De veritate et falsitate propositionis*.
5. Finally, the largest work of the Chancellor of Oxford has for its objective the *Sophismata*. It is dedicated to the discussion of a series of twenty-two fallacies. The

[29] R. L. Poole, art.: *Heytesbury (William)* in *Dictionary of National Biography*, edited by Sidney Lee; vol. XXVI, pp. 337-328.

[30] G. C. Broderick, *Memorial of Merton College*, Oxford, 1885; p. 207. Cf. R. L. Poole, art. cit.

[31] J. E. Therold Rogers, *History of Agriculture and Prices*, vol. II, pp. 670-671; Oxford, 1866. Cf. R. L. Poole, art. cit.

[32] Wood, *History and Antiquities of Oxford: College and Halls*; ed. Gutch, p. 139. Cf. R. L. Poole, art. cit.

[33] Wood, *Fasti Oxonienses*, ed. Gutch, p. 28. Cf. R. L. Poole, art. cit,

study of a text manuscript preserved at the National Library[34] makes us believe that a first draft contained only thirty sophisms; the author had later added the last two: *Necesse est aliquid condensari si aliquid rarefiat. — Impossibile est aliquid calefieri nisi aliquid frigefiat.*

The printing press has reproduced, several times, various treatises of Hentisberus; but a single edition brings them all together; at the same time, it gives some important commentaries that they provoked, in the 15th century, in Italy; this edition, to which we have constantly refered, was printed in Venice in 1494[35].

At the beginning of the treatise *De insolubilibus*, which work opens the *Regulæ*, Heytesbury lists[36] three opinions relating to the nature of sophisms; he did not name the authors of these opinions, since no name is ever found in his writing; but Cajetan of Tiene, commenting on the *Regulæ* makes these names known to us[37]:

> The first of these positions, he says, is that of Suisset; the second is admitted by Dulmenton; the third is of Richard Clienton in his *Sophismata*.

Suisset, Dulmenton, and Richard Clienton were the names of three logicians who were, without a doubt, among the predecessors of Heytesbury. What do we know of these men, experts in subtle dialectics?

"This Clienton is totally unknown to us," wrote Prantl[38]. Prantl was misinformed; we have the manuscript text of the *Sophismata* to which Heytesbury and Cajetan of Tiene alluded; truly, the author's name was not Clymeton nor Clienton. The scribe—who, after copying the *Sophismata* of Albert of Saxony and before reproducing the last *Sophismata* of Heylesbury, has transcribed the *Sophismata* of Clymeton in a notebook now preserved at the National Library[39]—was called John; he took care to date his copy, not without ambiguity, though; he ended it, in fact, in these terms:

> *Et sic est finis horum sophismatum scriptorum per manum cujusdam Johannis C. Et fuerunt completa die lune post dominicam septuagesime anno domini* M° CCC° LXXXIXI° (sic).

[34] Bibl. Nat., fonds latin, ms. n° 16134; fol. 81, col. *a*, to fol. 146, col. *a*.

[35] *Tractatus* gulielmi Hentisberi *de sensu composito et diviso. — Regule eiusdem cum sophismatibus. — Declaratio* gaetani *supra easdem. — Expositio litteralis supra tractatum de tribus. — Questio* messini *de motu locali cum expletione* gaetani. — *Scriptum supra eodem* angeli de fosambruno. — Bernardi torni *annotata supra eodem*. — Simon de lendenaria *supra sex sophismata*. — Tractatus hentisberi *de veritate et falsitate propositions. — Conclusiones eiusdem.* — Colophon:

> Expliciunt probationes conclusionum acutissimi doctoris Gulielmi hentisberi una cum ceteris opusculis ut in prima facie huius voluminis habetur. Que quidem omnia emendata ac in unum redacta fuere per preclarum virum dominum Joannem Mariam Mapellum vincentinum philosophum egregium accuratissimumque medicum. Impressa venetiis per Bonetum locatellum bergomensem: sumptibus Nobilis viri Octaviani scoti Modoetiensis. Millesimo quadringentesimo nonagesimo quarto sexto Kalendas iunias.

[36] Hentisberi *De insolubilibus*; ed. cit., fol. 4, col. *c*.

[37] Gætani de Thienis Vicentini *In regulas Gulielmi Hesburi recollecte*; ed. cit., fol. 7, col. *c*.

[38] Cari Prantl, *Geschichte der Logik im Abendlande*, IVier Bd., p. 90.

[39] Bibl. Nat., fonds latin, ms. n° 16134; fol. 56, col. *b*, inc.: Ad utrumque dubitare potentes facile speculabuntur verum et falsum…; fol. 73, col. *a*, des.: Per hoc satis faciliter potest ad alia insolubilia, in quocunque fuerint genere, respondere.

> *Explicit hoc totum; pro pena da mihi potum.*
> *Expliciunt sophismata Clymelonis, Deo gratias, per manum cujusdam Johannis.*

This Clymeton Langley (which was, it seems, his real name) was famous in the Scholasticism of the 15th century and the beginning of the 16th century; the Scotsman John Majoris, regent of the College of Montaigu in the early 16th century, places him[40] among the monuments of the University of Oxford.

> This University,

he said,

> once gave very famous philosophers and theologians, such as Alexander of Hales, Middilton[41]; John Duns, the Subtle Doctor; Ockham, Adam Hibernicus, Ro. Holkot, Bokinkam, Eliphat, Climiton Langley, Jean Roditon, the English monk; Suisset, the very penetrating calculator; Hentisber, the well-practiced dialectician; Strodus, Bravardin, and a host of others.

Of Climiton of Langley—as they call him, following John Majoris—Conrad Gessner[42] and Pitse[43] make a brief mention. They lived about 1350 and attributed to him, besides his *Sophismata*, the *Replicationes scholasticæ* and a treatise *De orbibus astrologicis*.

The author who in France and in Italy was called Dulmenton was was in fact named John Dumbleton.

At Merton College[44], at Oxford, since 1324, Thomas Dumbleton is found; but the name of John Dumbleton did not appear until 1331 in the records of the College. On 27 September 1332, John Dumbleton is put in charge of Rotherfield Peppard, near Henley, in the archdeaconry of Oxford; in 1334 he resigned that office. In 1338 and 1339, we see him take part in the meetings of Merton College. In February 1340 (1341 in the current style), he was appointed among the first fellows of Queen's College, in the original statutes of the college. We find him again, in 1344 and 1349, at the Merton College.

We cite two treatises by John Dumbleton that we possess, which have not been printed.

One of these treatises, entitled *De logica intellectuali*, is preserved in manuscript form at Merton College, Oxford.

The other, which was the more famous, is entitled *Summa logicæ et naturalis philosophiæ* or *Summa de logicis et naturalibus*; sometimes divided into nine books,

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40 *Historia maioris britannise, tam Angliæ quam Scotiæ, per* Ioannem Maiorem, *nomine quidem Scotum, professione autem Theologum, e veterum monumentis concinnata*. Vænundatur Iodoco Badio Ascensio. In fine: Ex officina Ascensiana ad Idus Aprilis MDXXI. Lib. I, cap. V, fol. VIII, recto.

41 That is, Richard of Middleton.

42 *Bibliotheca universalis,…* authore Conrado Gesnero Tigurino doctore medico. Tiguri, apud Christophorum Froschoverum, Mense Septembri, anno MDXLV.

43 Thorold Rogers. *History of Agriculture and Prices*, vol. II, pp. 670-674; Oxford, 1866. — Cf. R. L. Poole, *art. cit.*

44 R. L. Poole, art. *Dumbleton (John of) in Dictionary of National Biography*, edited by Sidney Lee; vol. XVI, p. 146.

sometimes into eight books, it is preserved in manuscript form in various libraries of Oxford, notably at Merton College and Magdalen College; a manuscript of Magdalen College gives it the title, in little conformity with its content, of *Summa de theologia major*.

The number of manuscripts of the *Summa* of Dumbleton found in English libraries reflects the popularity this work enjoyed in the 14th century.

This popularity spread to the summary of the *Summa* which was later composed by John Chilmark.

John Chilmark[45] was a member of the Merton College and master of arts; an account, kept in the archives of the Exeter College, Oxford, tells us[46] that in 1386 he was paid ten shillings "*in parte solutionis scolarum bassarum iuxta scholas ubi Scammum situatur in medio*". Between Merton College and Exeter College, there was a continual exchange of teachers; in 1386, John Chilmark, member of Merton, had given lessons in schools that depended on Exeter.

The various libraries of Oxford have the handwritten texts of various works of John Chilmark; one of them is entitled: *Compendium de actione elementorum*; others treat the *De motu*, *De augmentatione*, and *De alteratione*. However, the first of these writings is a summary of a portion of the *Summa* of Dumbleton; in a manuscript in the Bodleian Library (cod. Digby 77), indeed, it bears the title: *Compendium de actione elementorum abslractum de quarta parte J. Dumbletoni*. It would be interesting to see if the treatises *De motu*, *De augmentatione*, *De alteratione* are not themselves, too, extracts from the *Summa* of Dumbleton, because this *Summa* contained chapters likewise titled.

The manuscript n° 16621 of the Latin collection of the National Library is a collection of books where, towards the end of the 14th century, a student at the University of Paris has recorded many notes; the disorder of these notes is immense and the writing is executed with little care; they provide, however, valuable information to those who take the patience to decipher them; whoever wrote them, indeed, has united there all the information he could gather on the doctrines in vogue at the School of Oxford. Among these pieces of information are, in particular, very extensive excerpts from the *Summa* of Dulmenton; it is to these extracts that we owe, first of all, the knowledge of certain theories developed in this *Summa*.

We were able to complete this knowledge by reading the text itself of the *Summa*.

This very extensive text fills one hundred and forty pages of a large format, two column manuscript[47] written on parchment, a writing whose form indicates the English nationality of the copyist.

At the beginning of the prologue, the author presents his book to readers in a few sentences, where he finds the occasion to mention the name of Oxford; here is the beginning[48]:

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45 R. L. Poole, art. *Chilmark or Chylmark (John)* in *Dictionary of National Biography* edited by Sidney Lee; vol. X, p. 257.

46 Wood, *History and Antiquities of the University of Oxford* , ed. Gutch, vol. II, pt, II, p. 742. — Cf. R. L. Poole. *art. cit.*

47 Bibl. Nat., fonds latin, ms. n° 16146.

48 Ms. cit., fol. 2, col. *a*.

> *Plurimorum scribentium grati laboris dignique memoria particeps, ad mensuram mee facultatis doni, ex logicali materia communi et philosophica quandam summam, veluti spicarum dispersarum manipulum quoquomodo materiatum et incompositum recolatum, recolegi, nequaquam, tanto beneficio libato, ut remuneratione eadem munificum me arbitratus, verum moderatam discretionem non alta tenentibus et lectione potius privata contentis ut degestam utilemque sensui offeram*[49]*. Itineranti via recta Oxoniam tendens a pluribus edocetur, precisus pedum spacii numerus nequaquam ostenditur.*

In the preamble John Dumbleton tells us that his *Summa* is divided into ten parts[50]: "*Hujus summule divisio decimembris*." But the manuscript that we consulted contains only nine, because it is incomplete or because the author did not finish his work. At the end of the ninth part and before the table of chapters, we read[51]: *Explicit nona pars Magistri Johannis Dombilton*.

In listing the logicians of the School of Oxford whose opinions William Heytesbury discussed, before appointing Dulmenton and Richard Glienton, Cajetan Tiene quoted Suisset. This name was, from the time of Cajetan and, especially, in the 15th century and in the 16th century, one of the most well-known in France and Italy; even more than that of Hentisberus, he evoked the thought of the subtle dialectics of Oxford, so strongly admired by some, so bitterly denigrated by others. However, we will see how difficult it is to know anything specific about the personage who held that name.

The name (or nickname) that should be attributed to him is not Suisset, but Swineshead. This name, which the English manuscripts often spell Swynshed, became, on the Continent, first Suincet, then Suicet, Suisset, Suiseth, etc.

The first authentic information we find regarding the personage holding this name is the following[52]: In 1345, a Swineshead, a member of Merton College, is one of the leaders of a riot caused by the election of the chancellor.

A second piece of information is provided by the manuscripts of works composed by Swineshead[53]. He cites the *Quaestiones super Sententias* stored at Oriel College; a treatise, entitled *Descriptiones motuum* or *De molu cœli et similibus*, of which Caius College keeps an exemplar; and a book *De insolubilibus* the one to which Cajetan of Tiene alluded.

The book *De insolubilibus* is not, without doubt, the only writing of Logic that the author has composed. In a manuscript[54] whose last page is dated 1 March 1878, the National Library has—in addition to the logic of Albert of Saxony, the *De sensu composito et diviso* of Richard of Belingham, and the *De prædestinatione* of William

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49 Wood, *History and Antiquities of Oxford*, I, p. 448. — Cf. G. L. Kingsford, art. *Swineshead (Richard) in Dictionary of National Biography*, edited by Sidney Lee, vol. LV, p. 231.

50 Ms. cit., fol. 2, col. *a*.

51 Ms. cit., fol. 141, col. *a*.

52 C. L. Kingsford, *art. cit.*

53 Bibl. Nat., fonds latin, ms. n° 14715 (ancien S. Victor 717).

54 Fol. 86, col. *c*, inc.: Cum in singulis secundum materiam subjectam sit certitudo querenda, primo Ethycorum… Fol. 90, col. *d*, expl.: Igitur male respondet, igitur non est *a*.

of Ockham—a treatise *De obligationibus*[55] at the end of which we read[56]: *Et in hoc terminantur obligationes Reverendi Magistri Jo. Swiinsed de Anglia doctoris in sacra theologia.*

If we believe this colophon, Master Swineshead, to whom we owe various treatises of logic, would have received the name of John.

The notebooks of Philosophy where a Parisian student, toward the late 14th century, copied fragments of the *Summa* of Dulmenton also contain numerous and extended extracts of a book that our student attributes to Suincet; for this book, he constantly gives[57] this title: *De primo motore*. It seems likely that this work does not differ from what the Oxford manuscripts title *Descriptiones motuum* or *De motu cœli et similibus*. This treatise of Swineshead, which consists of eight *differentiæ*, treats, as our student remarked[58], many subjects that the Summa of Dumbleton also treats.

However, the final extract of the *Opus primo motore* is followed by this statement[59]: *Explicit tractatus M. Rogero Suincet datas eximio.*

The first name of Swineshead would thus not be John, but Roger.

The simplest solution to this contradiction would be, it seems, to admit that there have been two Swinesheads, a John Swineshead who was the author of Logic treatises *De insolubilibus* and *De obligationibus* and a Roger Swineshead who composed the *De primo motore*. One can also assume that these various works are by the same author and to leave it to the copyists for these changes in first names.

Moreover, we have not yet noted these variations.

At the beginning of his *Tractatus de reactione*[60] Cajetan of Tiene said: "*Nuper tractatas quidam in eadem materia recenter compilatus ad manus meas pervenit.*" This recently compiled treatise does not name its author.

In his commentaries on the *Physics* of Aristotle, Cajetan discusses an opinion which is emitted in the same work he calls[61] the author *Calculator*, the Calculator, without mentioning whose nickname it is.

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[55] Ms. cit., fol. 90, col. *d*.

[56] Bibl. Nat., fonds latin, ms. n° 16621.

[57] Ms. cit., fol. 13, V; fol. 35, v°; fol. 64, v°.

[58] Ms. cit., fol. 195, r°.

[59] Ms. cit., fol. 84, v°.

[60] *Habes solertissime lector in hoc codice libros Metheororum* Aristotelis Stagirite *peripatheticorum principis cum commentariis fidelissimi expositoris* Gaietani de Thienis *noviter impressos: ac mendis erroribusque purgatos. Tractatum de reactione. Et tractatum de intensione et remissione eiusdem* Gaietani. *Questiones perspicacissimi philosophi Thimonis saper quattuor libros metheororum* (s. 1. n d. — ca. 1505). — A second edition, issued under the same title, has the following colophon: *Opuscula impressa fuerunt Venetiis nutu ac impendio heredum quondam nobilis viri domini Octaviani Scoti civis Modoetiensis: ac sociorum. Anno salutis 1522. Die 20 Novembris.*

[61] *Recollecte Gaietani super octo libros physicorum cum annotationibus textuum.* Colophon:

> Impressum est hoc opus Venetiis per Bonetum Locatellum iussu et expensis nobilis viri domini Octaviani Scoti civis Modoetiensis. Anno salutis 1496. Nonis sextilibus. Augustino Barbadico Serenissimo Venetiarum Duce.

Fol. 41, col. *d*.

In his commentaries on the *Regulæ* of William Heytesbury, Cajetan of Tiene, who cited Suisset without giving him the nickname of Calculator, quotes the Calculator in another place[62], without giving him any other name.

The name constantly published in the 15th and 16th centuries for the epithet Calculator, to designate the author that Cajetan was the first to discuss, is the name of Suisset. Thus, in his opuscule *De distributionibus ac de proportione motuum*, which was printed for the first time in 1494, Alexander Achillini cites[63]: "Thomas Braduardin and, after him, Suiset the Calculator and Nicole Orem."

In fact, about 1480[64], appeared a book devoid of any title, but which bore this colophon:

> *Subtilissimi Doctoris Anglici Sidset Calculationum liber. Per Egregium Artium et Medicine Doctorem Magistrum lohanem de Cipro diligentissime emendatus. fœliciter Explicit. DEO GRATIAS. PADUE.*

An arsenal of subtleties, which then delighted the dialectic of the Schools, the *Calculationum liber* spread everywhere the renown of Suisset the Calculator. It was reprinted in 1488[65], in 1498[66], and in 1527[67].

Now, the titles of the 1488 and 1520 editions give Suisset the Calculator the first name of Richard; the colophon of the edition of 1520 transforms this name into

[62] *Tractatus* gulielmi Hentisberi *de sensu cornposito et diviso*… Venetiis, 1494, fol. 39, col. *b*.

[63] Alexandri Achillini Bononiensis *Opera omnia*. Venetiis, apud Hieronymum Scotum, MDXLV, fol. 185, col. *c*.

[64] The copy that I own carries, in the margins of one of its pages, some annotations and drawings from a student who found the analysis of local movement boring. Among these annotations, one reads the following: *Anno domini MCCCCLXXXI die XVI° Decembris*; this is the date when they were drawn.

[65] Subtilissimi Anglici Doctoris Ricardi Suiseth. Opus aureum calculationum. Papie, 1488. In his *Repertorium bibliographicum* (vol. II, pars II, p. 368, col. *a*, n° 16137), Hain cites this incunabulum without having seen it. In the *Guide du Libraire et de l'Amateur de livres* (5e édition, t. V, 1864; col. 587), Brunet cites the edition of 1498 as the first dated edition; thus, he regards that of 1488 as not longer extant.

[66] *Calculationes* Suiseth Anglici. Colophon:

> Subtilissimi doctoris anglici Suiseth Calculationum liber. Per egregium artium et medicine doctorem magistrum Ioannem tollentinum veronensem diligentissime emendatum fœliciter explicit. Papie per Franciscum gyrardengum. MCCCCLXXXXVIII. die IIII. Ianuarii.

[67] *Calculator*. Subtilissimi Ricardi Suiseth Anglici *calculationes noviter emendate atque revise. Questio insuper de reactione juxta Aristotelis sententiam et commentarios.* Colophon:

> … Magistri Raymundi Suiseth noviter impressus. Venetiis aere ac sollerti cura hæredum Octaviani Scoti et sociorum 1520. (According to Brücker in: Jacohi Bruckcri *Historia critica Philosophiæ*, tomus III, Lipsiæ, MDCCXLIII, p. 852).

Brunet (*loc. cit.*) cites from the colophon of this edition:

> Explicit questio de reactione edita ab … domino Victore Trincavello … noviter impresse Venetiis ere ac sollerti cura heredum Octaviani Scoti … ac sociorum anno … millesimo quingentesimo vigesimo decimo Kal. Aprilis.

that of Raymond. John, Roger, Richard, Raymond—among these four names, the biographers of Swineshead will have many choices, but the embarrassment will be great.

It is the work of Raymond Suiseth that the Dominican Isidoro Isolani quotes at the end of the *Tractatus proportionum* of Albert of Saxony, giving it a new wording[68]. Luis Vives accuses[69] the Englishman Roger Suicet of having greatly developed the calculations which he abhors. In book XVI of the *De Sublilitate*, Cardan classes the geniuses that humanity honored; the third place is occupied by Euclid, Duns Scotus, and the Scottish "Jean Suisset, whom the vernacular calls the Calculator."

Conrad Gesner[70] and John Leland[71], who have not, regarding our author, any other documents than the diatribes of Luis Vives, named him Roger Suicet; Leland speaks of Swineshead[72], a member of Merton College and commentator of Peter Lombard; but he does not identify this *Suineshevedus* with *Rogerus Suicetus*; only the editor who drew up the table of his work indicated[73] this likening as probable.

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[68] *De velocitate motuum. Preclara dogmata de omnium motuum velocitate; ingenuo Epitomate digesta a fratre* Isidoro de Isolanis Mediolanense: *ordinis predicatorum.* Colophon:

> Expliciunt proportiones fratris Alberti de Saxonia ordinis predicatorum breviate. Qui a Thoma berduardi excipiens a nobis est breviatus: nihil minus: sed aliquid amplius dicentes. Scito quod hunc Thomam vocat Raymundus Suiseth calculator in tractatu primo de intensione et remissione: Venerabilem magistrum Thomam de Berduerdino: cujus dicta veneratur et recipit.

This work, along with various other opuscules of Isidoro Isolani, is adjoined to the work entitled:

> *Clarissimi sacre Theologie doctoris Fratris* Pauli Soncinatis *vite regularie ordinis predicatorum: Divinum Epitoma Questionum in quatuor libros Sententiarum a principe Thomistarum Joanne Capreolo Tholosano disputatarum. His additis: que idem morte preventus perficere nequivit; per fratrem* Isidorum de Isolanis Mediolanensem *ejusdem predicatorie professionis.*

Colophon:

> … Lugdunique exactissima cura impressum persolertem virum Joannem Crespinum Annodomini Mcccccxxviij.

[69] Joannis Ludovici Vivis *De causis corruptarum artium liber V: De philosophiæ naturæ et medicinæ et artium corruptione*; Brugis, MDXXXI (Jo. Ludovici Vivis *Opera*, Basilæ, MDLV; tomus I, pp. 412-413).

[70] *Bibliotheca universalis*… authore Conrado Gesnero; Tiguri, MDXLV; p. 588, recto.

[71] *Commentarii de Scriptoribus Britannicis*, auctore Joanne Lelando Londinate. Tomus secundus, Oxonii, MDCCIX; p. 38a, cap. CDXXXI. De Rogero Suiceto.

[72] Leland, *Op. laud.*, tom. II, p. 373, cap. CDXVI. De Suineshevedo.

[73] Leland, *Op. laud.*, index, art. Rogerus Suicetus.

The identity of Roger Suiset, Suicet, or Suinset with Swinsete or Suinshed is admitted by Gabriel Naudé[74], Visch[75], Pitse[76], Bay[77], and Fabricius[78]. They make Roger Swineshead into a Cistercian monk, but it is not very clear what information leads them to do so.

The first name of John, which Cardan gave to the Calculator, finds some other supporters[79]; but it is from the "very subtle Englishman Richard Suisset" that Casaubon commends himself[80] with having been able to read, at Oxford, the *Calculationes*; Brücker, who has dedicated to the Calculator an extremely documented article[81], boasts of having established that the first name of the author was indeed Richard; the authors of the *Dictionary of National Biography* adopted this opinion[82].

John, Roger, Raymond, or Richard Swineshead was, thanks to the book entitled *Calculationes*, one of the most famous, most admired, and most maligned men in the 15th and 16th centuries; his subtlety was extolled by the adepts of the Dialectic of Oxford and Paris; his meticulous arguments, the *quisquiliæ Suiceticæ*, excited to fury the aversion that the Humanists professed for the sterile quarrels of the Schools. And the popularity of the *Calculationes* perdured, since Leibniz even gave them the honor of writing to Wallis[83] and to wish to have them reprinted[84].

Now this *Calculationum liber*, this *Opus aureum calculationum*, these *Calculationes* which earned a great reputation for Swineshead dubbed the Calculator, do not bear the title of *Calculationes* and did not have Swineshead for their author.

None of the books that we have read report the existence of the manuscript text which was printed under this title; of this text, however, there is a copy, to our knowledge; this copy is kept, under number 6558, in the Latin collection of the National Library; written at the end of the 14th or the beginning of 15th century, this text differs only in insignificant variations from what was printed around 1480.

But at the end of this treatise[85], the scribe who copied it wrote:

Explicit tractatus datas a Magistro Riccardo de Ghlymi Eshedi.

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[74] Naudæus, Additiones ad *Historiam Ludovici XI*, p. 214.

[75] Car. de Visch., *De Scriptoribus Ordinis Cisterciensis*, p. 292.

[76] Ioannis Pitsei Angli *Relationum Historicarum de Rebus Anglicis Tomus primus*, Parisiis, MDCXIX; n° 575, p. 677.

[77] *Scriptorum illustrium Maioris Brytaniæ (sic), quam nunc Angliam et Scotiam vocant: Catalogus*… Authore Ioanne Baleo. Basileæ, MDLIX. Pars I, Centuria sexta, cap. II: Rogerus Swinsete, p. 456.

[78] Jo. Alberti Fabricii Lipsiensis *Bibliotheca latina medise et infirme ætatis*. Tomus V; Florentin, MDCCCLVIII; p. 418: Rogerius Suiset.

[79] Vossius, *De Scientiis mathematicis*, cap. XVIII, p. 78.
Gaddius, *De Scriptoribus non-ecclesiasticis*, t. II, p. 326.

[80] Wolfius, *Casauboniana*, p. 24.

[81] Jacobi Bruckeri *Historia critica Philosophiæ*, Tomus III, Lipsiæ, MDCCXLIII; p. 849.

[82] G. L. Kingsford, art. *Swineshead (Richard) in Dictionary of National Biography* edited by by Sidney Lee; t. LV, p. 231.

[83] Letter from Leibniz to Wallis (Jo. Wallisii *Opera*, t, III, p. 673).

[84] *Leibniziana*, p. 42. — Cf. Brücker, *Op. laud.*, *loc. cit.*

[85] Bibl. Nat., fonds latin, ms. n° 6558, fol. 70, col. *c*.

Later, another hand added:

> *De Intensione et remissione formarum, de actione et reactione, et de velocitate et tarditate motus.*

The letters *hly* in the word *Ghlymi* have a horizontal line drawn above, assuring it is an abbreviation [$\overline{hly}$*mi*]. What is the full name that should replace the abbreviated word Ghlymi? We have guessed, and many others before us have not been more fortunate. On the verso of the first folio (unnumbered), three readers have, successively, reproduced the title of the treatise that would follow. The first simply wrote:

Tractatus de intensione et remissione per Riccardum.

The second has put:

De intensione et remissione etc. Riccardi de Ghlymi Eshedi.

The third, more verbose, composed this title:

Tractatus de intensione et remissione formarum, de actione et reactione, de velocitate et tarditate motus per Magistrum Ghlymum Eshedum editus.

The last two have, moreover, reproduced the horizontal line drawn above the letters *hly*.

The authors of the Catalog of Latin manuscripts from the Royal Library have failed to explain the abbreviation that this line indicates, for the manuscript of which we speak is described by them in these terms:

> *Codex membranaceus, quo continetur Richardi de Ghlymi Eshedi tractatus de intensione et remissione formarum, de actione et reactione, de velocitate et tarditate motus. Is codex decimo quarto sæculo videtur exaratus.*

Thus, it seems that the author has given no title to this treatise, and that the first readers did not consider *Calculationes*; additionally, if the first name of the author were Richard, as admitted by some printers, his last name was not Swineshead.

In fact, the comparison of this work with the treatise *De primo motore*, which itself is undoubtedly by Swineshead, shows, at first glance, that these two works cannot be from the same author. The treatise of Riccardus of Ghlymi Eshedi covers a number of questions, all of which are also considered in the treatise of Swineshead; the same author would not write two books that so obviously treat the same subject-matter but differ so completely in every detail of writing. The work composed by Riccardus of Ghlymi Eshedi belongs to the family whose types are the *De primo motore* of Swineshead and the *Summa* of Dumbleton; but it seems to have been written after the works of Swineshead and Dumbleton; one can, in particular, note the obvious borrowings from the treatise *De difformitate qualitatum* of Nicole Oresme; reading the *De primo motore* and the *Summa* reveals no borrowings of this kind.

Moreover, a particularly competent judge in the matter, Peter Pomponazzi—who at the beginning of 16th century wrote, as we shall see, several treatises on the doctrines of William Heytesbury and the Calculator—has well discerned that one had to come after the other:

> The second reason, and the most powerful of all,

he says somewhere[86],

> was the one made by the Calculator, although before him (as I believe) Hentisberus gave the same reason; he [the Calculator] seemed to follow, indeed, a position that had already been held, while being driven simultaneously by contrary motives, as one could quite clearly deduce.

We tried to find some information about this Riccardus of Ghlymi Eshedi, whose work, under the false name of Suiseth the Calculator, was destined for such great fame; all our efforts have been in vain. We hardly dare report a parallel that seems doubtful; the library of Charles VI contained a treatise of Astrology[87] entitled: *Summa Eskilde Anglici de judiciis*; does it identify *Eshilde* and *Eshedi*?

Clymeton, Dumbleton, and Swineshead represent for us the opinion of the School of Oxford just before the time when William Heytesbury developed there the subtle agility of his Dialectic; an anonymous writing will acquaint us with the thought of a disciple of this logician.

Under the title of *Tractatus de sex inconvenientibus*, whose adaptation to the subject of the book escapes us, this anonymous writing was printed; it was in Venice, in 1505, in a collection where the *Tractatus de latitudinibus formarum* inspired by Nicole Oresme is found; in § XIX [section 26] we have given the description of this edition.

It is not this edition, but two manuscript texts, which we consulted.

Of these two manuscripts, one tells us more completely than the other about the book it reproduces.

The first text is part of a collection of pieces[88] that were all composed by masters of Oxford University; presumably, if we judge by the spelling of names, the copyist or copyists were English.

In this collection the treatise with which we are concerned has no title; it starts immediately[89] with this question: *Utrum in generatione formarum sit certa ponenda velocitas*. In its current state, however, it is incomplete; it stops abruptly in the middle

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[86] Petri Pomponatii Mantuani *Tractatus de reaction*e, sect. I, cap. XIV (Petri Pomponatii Mantuani. *Tractatus acutissimi, utilissimi, et mere peripatetici. De intensione et remissione formarum ac de parvitate et magnitudine. De reactione. De modo agendi primarum qualitatum. De immortalitate anime. Apologie libri tres. Contradictoris tractatus doctissimus. Defensorium autoris. Approbationes rationum defensorii, per* Fratrem Chrysostomum *Theologum ordinis predicatorii divinum. De nutritione et augmentatione.* Colophon:

> Venetiis impressum arte et sumptibus tueredum quondam domini Octaviani Scoti, civis ac patritii Modeotiensis: et sociorum. Anno ab incarnatione dominica MDXXV calendis Martii.

Fol. 26, col. *d*.).

[87] *Inventaire de la bibliothèque du Roi Charles VI fait au Louvre en 1523 par ordre du Régent, Duc de Bedford.* Paris, 1867; p. 187, n° 721.

[88] Bibliothèque Nationale, fonds latin, ms. n° 6559 (*olim* Colbert. 2094, Regius 38113).

[89] Ms. cit., fol. 1, col. *a*.

of a question[90] and the appeal that follows the last words[91] reveals the absence of the book that was to follow. But when the collection was formed, the treatise was complete, and the copyist had composed a table of contents[92] that makes its content known to us. The entire work consisted of eleven questions; in each of the first four is included, in addition, under the title of articles, some subsidiary questions which are formed as parentheses. What we possess today contains the first four questions and part of the fifth; this is little more than half of the book, since this fragment ends with the fol. 48, and the last question, the table tells us, began on folio 82.

The other manuscript copy owned by the National Library[93] is far from filling this large lacuna; it was copied on a text where it already existed; the copyist, eager to reproduce only comprehensive questions, abolished the beginning of the fifth question and kept only the first four. He placed his titles so that the items subordinate to the questions appear to have the same importance as the questions themselves. Also, under the title: *Incipit tabula questionum 6 inconvenientium*, a copyist, giving the same priority to articles and questions, listed sixteen questions grouped in fours under these headings: *De generatione*. *De alteratione*. *De quantitate*. *De motu locali*. Pushing the error further, the catalog of Latin manuscripts from the Royal Library has named the book in question: *Tractatus de sexdecim inconvenientibus*. More precisely, the scribe who copied it had given the actual title in this strange *explicit*:

> *Explicit tractatus de sex inconvenientibus.*
> *Finito libro sit laus et gloria Cristo.*
> *Dabitur pro pena scriptori pulchra puella.*

The copyist was not English like the one of the first text; he crippled several proper English names that he encountered; sometimes he deleted them.

Is the printed text of the *Tractatus de sex inconvenientibus* more comprehensive than the manuscripts that we read? This is what we have been able to ensure.

That the treatise *De sex inconvenientibus* emanates from the School of Oxford is clearly evident by the fact that only this School and the teachers who were in honor there are cited by the author.

> If it is necessary, in the movement of alteration, to define a certain speed,

he said[94],

> this speed should be taken because of the latitudes of intensities, as the School of Oxford and Aristotle in book VII of the *Physics* admit, comm. 41. It is this supposition… that must be, I think, regarded as preferable to others, and truth even prefers it.

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90 Ms. cit., fol. 48, col. *d*.

91 This appeal is: *in movendo orbes*; the fol. 49, which in the full collection was page 109, begins with these words: *et per consequens*.

92 Ms. cit., fol. 196, verso.

93 Bibl. Nat., fonds latin, ms. n° 6527.

94 *Tractatus de sex inconvenientibus*. Quæst. II: Utrum in motu alterationis velocitas sit signanda vel tarditas. Bibl. Nat., fonds latin, ms. n° 6559, fol. 16, col. *b*.

The authority of the School of Oxford is here treated on the same footing as that of the Philosopher.

Several times the views espoused by Master Thomas Bradwardine in his *Treatise of Proportions* are invoked[95]. We learn, moreover, that theories of Mechanics outlined in the treatise were developed by other masters of arts, including a certain master Adam Pipewell or Pippewell[96].

Not only has the author of the treatise *De sex inconvenientibus* written at the School of Oxford, but he wrote there after Magister Willelmus Hethysbyry whose treatise *De motu*[97] he cites; one can suppose he was a disciple of this subtle logician when reading the admiring epithets he bestows[98] on the name of this Master:

> Unus solemnis Magister, potissimus et famosus Hethysbyry.

One of the manuscripts in the National Library, where the *Tractatus de sex inconvenientibus* is found, contains, additionally, the *Tractatus de proportionibus* of Thomas Bradwardine, then a series[99], although incomplete[100], of eleven questions concerning the *De generatione et corruptione*; the first ten questions bear no name of the author, but the eleventh ends with this colophon[101]:

> *Et sic finitur questio prima Magistri Willelmi de Colymgam Oxoniensis.*

Following this we read a explanation of text of Aristotle that opens the first book of the *Physics*, to which Averroes devoted his first commentary on this work; this new fragment bears, in turn, the following colophon[102]:

> *Et sic finis est questionum Colligham cum explanatione commentarii primi primi Phisicorum.*

The drafting of the final colophon, no less than the reading of eleven questions related to the *De generatione et corruptione* of Aristotle, convinced us that they were all of the same author, of this William Colligham or Colymgam, Master of Arts of the University of Oxford; only the disorder of the copyists ended up putting the first in last place. These questions are not unlike various parts of the *De primo motore* or *Summa* of Dumbleton; they could be contemporaries of these two works; in their content we have found nothing that could provide us any information in this regard except for the names of Aristotle and Averroes; the only proper name that these fragments have presented to us is that of *Lynconiensis*, i.e., Robert Grosseteste, Bishop of

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95 Ms. cit., fol. 28, col. *c*, et fol. 34, col. *b*.

96 Ms. cit., fol. 28, col. *c*, et fol. 33, col. *b*. — The ms.ms. n° 6527 of the Latin collection of the Biblio. Nat. wrote, first (fol. 158, col. *c*): *Magister Adam Palpavie*, and then (fol. 161, col. *c*): *Magister Adam*.

97 Bibl. Nat., fonds latin, ms. n° 6559, fol. 36, col. *a*.

98 Ms. cit., fol. 22, col. *c*.

99 Bibl. Nat., fonds latin, ms. n° 6559, fol. 61. col. *a* to fol. 153, col. *b*.

100 The register that is at the bottom of fol. 132 (verso) does not correspond to the words that begin fol. 133; it lacks one or more notebooks.

101 Ms. cit., fol. 153, col. *b*.

102 Ms. cit., fol. 190, col. *c*.

Lincoln; the writing of this author on the *Posterior Analytics* is mentioned twice[103] in the commentary at the beginning of the *Physics* of Aristotle.

[103] Ms. cit., fol. 162, col. *c*, et fol. 183, col. *b*.

Chapter 27
The spirit of the Oxford School in the middle of the 14th century

Physics

Before searching, in the various treatises of which we have been speaking, what they teach concerning the questions discussed in this study, it will not be useless to seek in them some information of a more general nature; through this information we will try to unravel the trends that most strongly appealed, toward the mid-14th century, to the logicians of the School of Oxford; we also try to see how the doctrines that prevailed in this University resembled or differed from those which, around the same time, were fashionable in Paris.

Among the peculiarities that distinguish the emulated teachings of the two schools, we can indicate, first, the usage, much more common in Oxford than in Paris, of the various treatises of mechanics composed by Jordanus of Nemore nd his disciples.

No doubt, in the 14th century, Parisian masters such as Albert of Saxony do not ignore the work of the *Auctores de ponderibus*, and they sometimes refer to it in their own writings; but they invoke it only in rare circumstances, while some Oxford teachers seem to have made continual use of it.

This vogue was very old in England; how else can we explain the fact that Roger Bacon already knew and readily cited many of the *De ponderibus* treatises which his contemporaries on the continent seemed to ignore? Because Roger Bacon, in the *Opus majus*, cites[1] Jordanus and his Commentator; in the *Communia naturalium* he mentions[2] the treatise *De ponderibus* attributed to Euclid and the treatise Thabit ibn Qurra wrote.

Already Bradwardine cites[3] the first conclusion of the treatise *De ponderibus*, attributed to Jordanus of Nemore without mentioning, however, the name of the

[1] *Fr. Rogeri Bacon, Opus majus, Pars IV, Dist. IV, cap. XV. De motu libræ* (Ed. Jebb, pp. 105-108; ed. Bridges, vol. I, pp. 169-174).

[2] *Liber primus communium naturalium* Fratris Rogeri Bacon; Prima pars principalis; Prima distinctio; cap. II (Bibliothèque Mazarine, ms. n° 3576, fol. 2, col. *b*. — *Liber primus communium naturalium* Fratris Rogeri. Partes prima et secunda. Edidit Robert Steele, p. 6).

[3] *Tractatus de proportionibus a* Magistro Thoma de Bradwardin *editus*; capituli II[i] pars IV[a].

author. He also cites[4], but without naming the author anymore, the treatise *De proportionalitate motuum et magnitudinum* which is sometimes found associated with the writings of the School of Jordanus, which we treated in § VIII [section 19].

The *Tractatus de sex inconvenientibus* repeatedly cites[5] the treatise *De ponderibus* or *De pensis ponderibus*; he spells the same of the author of this treatise "Jordanis." As we have seen in § VIII [section 19], he also attributes to a certain Ricardus of Versellis or Usellis a writing which was identical to the *De proportionalitate motuum et magnitudinum*, or at least supported the same conclusions as this latter writing.

But if there is, at the University of Oxford, a master who seems to have attentively read most pamphlets attributed to the *Auctores de ponderibus*, it is certainly John Dumbleton.

In his *Summa* he devotes a chapter[6] to discussing this question: "Since the proportion of movement is accomplished according to the proportion of the greatest inequality, one wonders (*dubitatur*) if the finite can act on the infinite." The considerations, although very confused, to which Dumbleton is engaged concern especially the theory of the lever, where one sees a very light weight lifting a very heavy weight. On this occasion, the author cites[7] the *Auctores de ponderibus*.

But it is not in his *Summa* that Dumbleton best shows us his knowledge of the writings produced by the mechanists of the School of Jordanus. This knowledge is especially stated in another work which was not reported by the biographers of the author of the *Summa*.

In this notebook of Philosophy[8], where a Parisian student has collected a wealth of documents relating to the doctrines of Oxford, the extracts of the *Summa* of Dumbleton are accompanied with a fragment that is not from this *Summa*, but which the copyist also gives[9] as a work of John Dumbleton. This fragment consists of three parts. The first part[10], which precedes the title: *De motu locali demonstrata per Dul-*

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[4] Bradwardine, *Op. laud.*, capituli II[i] pars IV[a].

[5] *Tractatus de sex inconvenientibus*, Quæst. I, Quæst. IV, Art. I quæstionis IV.

[6] Johannis de Dumbleton *Summa*, Pars tertia, Cap. XII°. Bibl. Nat., fonds latin, ms. n° 16146, fol. 30, col. *b*. — ms. n° 16621, fol. 120, v°.

[7] John Dumbleton, *loc. cit.*; ms. n° 16146, fol. 30, col. *c*; ras. n° 16621, fol. 121, r°. The author of the extracts that contained in this collection put this footnote on the bottom of the page: *Et vocatur gravius secundum situm.*

[8] Bibl. Nat., fonds latin, ms. n° 16621.

[9] In the first table of contents found in fol. 13 v°, the copyist describes this fragment:

> *Item de Dulmenton de uniformiter difformi varia cum quodam sophismate forti de uniformiter difformi in sequenti cisterno* [for *sexterno*]. *Item de maximo spacio lineari pertransito questio, una cum articulis notabilibus. Hec in duobus cisternis.*

In another table of contents in fol. 64 v°, the same fragment is defined thus:

> *Dulmenton de proportionibus motuum, gradu medio et similibus; unum sophisma de alteratione uniformiter difformi; questio una de maximo spacio lineari cum quibusdam similis materie.*

[10] Ms. cit., fol. 11 4, v°, to fol. 116, v°.

menton, discusses the rule that the speed of a mobile depends on the magnitude of the power and the magnitude of the resistance. The second part[11], which ends with this colophon: *Explicit sophisma. Deo Gratias*, examines this "fallacy": *Uniformiter continue variabitur alteratio uniformis.* The third part[12], announced by these words: *Incipit alla questio*, deals with this problem:

> Must the speed of any local movement be evaluated by the maximum linear space that a point of the mobile describes in its movement?

The problem thus formulated is none other, it is clear, than the very subject of the treatise *De proportionalitate motuum et magnitudinum*. The discussion of Dumbleton is long and confusing, which would have stopped us if it did not present an interesting peculiarity. In favor of the opinions he wants to support, for meeting those he wants to combat, the author invokes a host of arguments he draws from the laws of Statics, from the method of virtual displacements, and from the notion of *gravitas secundum situm*. He is careful to indicate the books from which he borrows these arguments. In these books, the most frequently cited is the treatise of Jordanus of Nemore to which Dumbleton sometimes[13] gives the full title as found in ancient manuscripts: *Elementa Jordanis super demonstrationem ponderis*; sometimes[14], also, he briefly names: *Jordanis de ponderibus*, *Jordanis super demonstrationem ponderis*, *Elementa Jordanis*, or even *Elementa Euclidis et Jordanis*; many axioms and propositions of this treatise are thus explicitly stated. But Dumbleton cites not only the authority of Jordanus of Nemore he borrows[15] two theorems from the *Auctor* or from the *Liber de canonio*[16]. Finally, he invokes[17] the authority of a certain *Magister de ponderibus* who demonstrates, at the beginning of his treatise, this proposition: A larger portion of a larger circle is less curved; we recognize immediately the author that we named[18] the peripatetic Commentator of Jordanus.

Dumbleton, we see, knew the *Liber de canonio*, which probably occasioned Jordanus to write his treatise; he knew this treatise as well as the *Commentary*, subsequently composed, to which Bacon also alluded; of the various works that illustrate, in Statics, the activity of the School of Jordanus, he mentioned not one; it is truly the most beautiful, whose author, not yet known, we called the Precursor of Leonardo da Vinci[19].

Statics was not the only part of the mechanics which occupied the minds of the Oxford masters; willingly, they also disputed on Dynamics and sought the relationship that united power, resistance, and speed of a mobile. In this regard the peripatetic

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[11] Ms. cit., fol. 124, r°, to fol. 130, r°.

[12] Ms. cit., fol. 130, v°, to fol. 139, r°; at the bottom of this last leaf, we read: *Explicit questio*.

[13] Ms. cit., fol. 133, v°, and fol. 134, r°.

[14] Ms. cit., fol. 131, v°; fol. 132, r°; fol. 132, v°; fol. 133, r°.

[15] Ms. cit., fol. 134, v°.

[16] On this work, see Duhem *Les Origines de la Statique*, ch. V, § 3; tome I. pp. 93-97 [Duhem (1991, 71-74)].

[17] Ms. cit., fol. 131, v°.

[18] *Les Origines de la Statique*, ch, VII, § 2; tome I, pp. 128-134 [Duhem (1991, 95-98)].

[19] *Les Origines de la Statique*, ch. VII, § 3; tome I, pp. 134-147 [Duhem (1991, 98-107)].

doctrine that Bradwardine endorsed in his *Tractatus de proportionibus* was generally accepted; he was attached only to perfecting its explanation and deducing various corollaries.

If we believe the author of the treatise *De sex inconvenientibus*[20], Master Adam Pippewell supported the theory of Thomas Bradwardine with subtle demonstrations.

In the *De primo motore*, Swineshead[21] presents, without adding anything essential, this same theory.

In his *Summa*, John Dumbleton, too, examines[22] what the various opinions are that have been expressed regarding the law that links the speed of a mobile to the magnitude of the power and the magnitude of the resistance. "We will treat," he said, "some opinions so that the knowledge of the false ones leads us, as by a dilemma (*per viam divisionis*), to the truth." And the truth is this:

> The third opinion is that of Aristotle and the Commentator; this is the one necessary to hold; it is the following: The movement intensifies or weakens according to a geometric proportion…

This view is well supported by Bradwardine.

To facilitate the understanding of this law, John Dumbleton devotes a chapter of his *Summa*[23] to show the rules of relationships and proportions "to those who are not proficient in Geometry, such that, through coarse and sensible methods, they would penetrate the truth and see the cause."

Our author uses the dynamical law formulated by Aristotle, Averroes, and Bradwardine to solve some problems, such as this one[24]:

> A mobile moves in a uniform and invariable medium under the action of a power that increases with uniform velocity; what is the law according to which the speed of the mobile varies?

The masters of the School of Oxford will passionately apply themselves to problems of this sort.

The mysterious Calculator, Riccardus of Ghlymi Eshedi, takes for granted the principle established by "the Venerable Master Thomas Bradwardine"[25]. Besides Aristotle and Averroes, he is the only author whose name he pronounces. From this principle, a long series of rules is deduced[26] on the change in the speed of a mobile

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[20] *Tractatus de sex inconvenientibus*, quæst. IV: Utrum in motu locali sit certa formanda velocitas. Bibl. Nat., fonds latin, ms. n° 6559, fol. 28, col. *c*; ms. n° 6527, fol. 158, col. *c*.

[21] Suincet *De primo motore*, Differentia VII^a, cap. I. Bibl. Nat., fonds latin, ms. n° 16621, fol. 76, r°.

[22] Joannis de Dumbleton *Summa*, Pars tertia, capp. IV^m et V^m. Biblio. Nat., fonds latin, ms. n° 16146, fol. 27, col. *a*, to fol. 28, col. *at*.

[23] Joannis de Dumbleton *Summa*, Pars tertia, Cap. VI^m; ms. n° 161 46, fol. 38, col. *a*; ms. n° 16621, fol. 14, v°.

[24] Johannis de Dumbleton *Summa*, Pars tertia, Cap. XI^m; ms. n° 16146, fol. 30, col. *b*; ms. n° 16621, fol. 119 v°.

[25] Subtilissimi Doctoris Anglici Suiset *Calculationum Liber*; ed. Paduæ, ca. 1480; col. *c* of the third folio (the folios have neither pagination nor signature).

[26] Suiset *Op. laud.*, Cap. XIV: De motu locali; ed. cit., fol. 53, v° seqq.

when we increase or decrease the power without changing the resistance, or when we vary the resistance without changing the power; the influence exerted on the magnitude of this variation by the initial values of power and resistance is, on its part, the subject of special attention.

We find these rules formulated by the Calculator almost literally reproduced and followed by curious applications to theological problems, a fragment[27] that inflated the Philosophy notebook of our Parisian student.

This fragment bears no name of its author; but maybe can we guess how he who preserved it for us had the original.

Our student, in fact, transcribed[28] "some information necessary to understand the English words", and he tells us[29] that "this information to understand what the English say about the increase of powers in relation to the resistances were given by Master Clay, *Magister Claius*."

This Master Clay, who, no doubt, taught in Paris towards the end of the 14th century after studying at Oxford, taught the Parisians the doctrines favored at the great English university. After having spoken to our student on questions of Dynamics, he discoursed to him on the movement of magnets[30]. However, what Master Clay teaches concerning the increase of the power or the resistance are some of the rules that you can read in the *Liber calculationum* or in the fragment copied by our student; and he remarks: "These two rules are stated differently above", he wrote[31] in the margin where he summarized the conversation of Master Clay; perhaps this Master Clay held the original pages where they are stated.

In any case, our student, in one of the tables of contents sprinkled throughout his notebook[32], describes this fragment:

> *Aliqua dubia theologica per extraneum audita et cogitata ab aliis.*

Thus we know that a stranger gave it to him.

The information Master Clay provided taught us that toward the end of 14th century, the University of Oxford had generally acquired peripatetic Dynamics such as the *Tractatus proportionibus* of Thomas Bradwardine taught it and such as the rules formulated in the book of the Calculator developed it. Clay, however, admitted, at least as a working hypothesis, a different doctrine; the notes of our student recount[33] the presentation of this doctrine, the doubts that made the English teacher hesitate, and the reasons for or against the theory that his listeners presented to him; they give us a summary account of this controversy, something like the minutes of a meeting that the Society of Physics held in the late 14th century.

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[27] Bibl. Nat., fonds latin, ms. n° 16621, fol. 52, r° et v°, et fol. 65, r° et v°.

[28] Ms. cit., fol. 212, v°.

[29] See the table of the notebook, in ms. cit., fol. 195, r°.

[30] Ms. cit., fol. 213, v°.

[31] Ms. cit., fol. 212, v°.

[32] Ms. cit., fol. 64, v°.

[33] Ms. cit., fol. 213, r°.

The opinion of Master Clay is the following: Applied to a given mobile, a given power would communicate to it, in the absence of any resistant medium, a determinate speed. In a resisting medium, the speed of the mobile would be less than that speed; it would be less by an amount proportional to the resistance of the medium. If one increasingly rarefied the medium, the power remaining constant, the speed of the mobile would not grow beyond all limits as Aristotle claimed; it would tend toward the determinate value which was first discussed. According to this hypothesis, therefore, a mobile would move successively in a vacuum, and one of the listeners of Master Clay objected to him that there is contradiction between this corollary and Aristotelian physics.

The English master is concerned with another problem, and this concern brought great honor to his perspicacity. In the absence of any resistance, the power would instantly give the mobile the determined speed about which we have spoken:

> the mobile would pass infinitely fast from the zeroth degree of movement to total movement;

Master Clay considered the proposition difficult to accept.

The opinion of Master Clay undoubtedly had to find favor in Paris. Indeed, we see that it was received in the 16^{th} century by Dominic Soto, whose Physics the Parisian teaching has so greatly influenced.

Soto admits[34] that in a vacuum a mobile does not instantly move; then he encounters this objection formulated by Gregory of Rimini and by other authors: Removing the medium, suppressing what more or less retards the movement of the various bodies, all the weights would fall, in the void, with the same speed; “a morsel of very heavy iron would descend in exactly the same time as a very light sponge.”

This proposition is, for us, the statement of a fundamental law of falling bodies. For Soto and many of his contemporaries, it appears as an inadmissible statement, capable of ruining any theory from which it would necessarily follow. To show that it does not necessarily follow from his own theory, Soto invokes the principle of Dynamics that seduced Master Clay:

> It is necessary,

he said,

> to admit the rule that a certain quickness or slowness corresponds to each natural motive power; this slowness can grow as a result of the resistance of the medium; this resistance suppressed, the mobile will move in the void with the same speed that corresponds to the power. This is why a heavier body will descend faster than a lighter body.

Master Clay, however, was not, towards the end of 14^{th} century, the only Englishman who recognized the inadequacy of the Dynamics of Aristotle; the author of the *Treatise on the Six Inconveniences* addresses some critiques[35] of this Dynam-

[34] Reverendi Patris Domini Soto Segobiensis… *Super octo libros Physicorum Aristotelis Quæstiones*. Salmanticæ. In ædibus Dominici a Portonariis. MDLXXII. Lib. IV, quæst. III, fol. 67, coll. *b* et *c*.

[35] *Tractatus de sex inconvenientibus*, Quæst. IV: Utrtim in motu locali sit certa assignanda velocitas; Bibl. Nat., fonds latin, ms. n° 6559, fol. 28, coll. *c* seqq.; ms. n° 6527, fol. 158, coll. *c* seqq.

ics analogous to those that Jean Buridan addressed; it seems, however, that for the principles of this Dynamics, Oxford is trusted longer and more firmly than Paris.

There is one question in which Oxford appears to have remained far behind Paris; we wish to speak of the acceleration of falling bodies. The explanation of this acceleration aided by a gradually increasing *impetus* seems to have found little favor in the English University, if we judge, at least, by the words of the *Treatise on the Six Inconveniences*.

An important article[36] of this treatise is devoted to the consideration of this issue: Does the acceleration of the motion of a weight come from a certain cause?

The author lists the various causes that can be and actually have been invoked to explain this acceleration: The decrease in the resistance of the medium, the continuation of the movement, the growing proximity of the mobile to its natural place, the impulsion of the shaken medium, the accidental gravity that the descending weight gains, and finally the appetite by which it desires its place.

Contrary to each of these hypotheses stand objections that the *Treatise on the Six Inconveniences* examines and discusses.

This discussion is not free of paralogisms; in particular, the principles of Statics formulated by Jordanus of Nemore lay a role that only verbal confusion permits. Thus, to demonstrate that the gravity of a weight cannot grow when this weight, descending, nears its natural place, our author Jordanus borrows this proposition: The *gravitas secundum situm* of a weight hanging on the end of a balance beam decreases when the beam is raised. Elsewhere he formally identifies the *gravitas secundum situm* of Jordanus with accidental gravity, which Parisians also called *impetus*; the same proposition is then used to prove that accidental gravity can only grow while the weight descends, as claimed by those who cite this increase in accidental gravity in order to explain the acceleration.

This discussion, confused and illogical, leads to the following conclusion:

> As a conclusion to this article, here is what I answer to this question: Does the acceleration of the motion of a weight depend on a certain cause? If this term *some* is understood with such a precision that it signifies: there is one sole cause of the acceleration of the weight, then to the posed question I answer: no. Indeed, the acceleration that a weight experiences in its descent depends on several causes. But it is one cause that prevails over the others; so I say, with Master Adam Pippewell, that the principal cause is the decrease in resistance; the continuation of the movement, surrounding medium, accidental gravity, and the natural inclination that is the appetite are partial causes; each is a partial and auxiliary cause; but none of them is a cause necessarily required for the acceleration of the movement of a weight.

The author of the *Liber sex inconvenientium* is prevented from giving a definite conclusion; it is conceivable that he is too much on his guard and that he would have acted more wisely by concluding clearly in the sense that Buridan and Albert of Saxony prescribed for him.

However, there is a point where he would have been well-advised to imitate the wise reserve of these two authors; he does not hesitate to believe, indeed, that an

[36] *Op. cit.*, quæst. cit., Articulus: Utrum velocitatio motus gravis sit ab aliqua causa certa. ms. n° 6559, fol. 31, col. *d*, in fol. 33, col. *d*.

arrow accelerates its movement after it has left the bow; here is how he ends the article we are analyzing:

> I grant an arrow hits an object harder at a greater distance than an object placed at a lesser distance; in this case the continuation of the movement would contribute much to this effect; the power that moves the arrow would be larger at a greater distance and would increase by the continuation of the movement.

Adam Pippewell and the *Treatise on the Six Inconveniences* do not make the gradually acquired *impetus* the essential cause of the acceleration of a falling weight; they ignore the ideas by which Buridan, Albert of Saxony, and Nicole Oresme prepared modern Dynamics; these ideas appear to have been completely ignored or unrecognized at the time, perhaps contemporary to Adam Pippewell but certainly prior to the drafting of the *Treatise on the Six Inconveniences*, when John Dumbleton taught at Oxford.

In his *Summa* Dumbleton devotes a long chapter to the explanation of projectile motion[37]. He addresses this explanation by this question, which seems strange:

> One wonders if the water or ambient air moves naturally in the projection of stones and other projectiles.

It begins, however, with considerations similar to those to which Nicolas of Cusa and Leonardo da Vinci will later devote much attention[38].

> In this regard,

he said,

> be aware, first of all, that any violent movement is reduced to natural movement. This is also seen: That A is moved with violent movement; as in every movement the mover accompanies the mobile, A has a certain mover; call this mover B. This mover is moved with natural or unnatural movement; if not, A has a certain mover; let C be this mover. As a series of distinct movements cannot proceed to infinity, it is clear that there exists a [naturally moved] mover by which all intermediate movers are moved violently. We see that in any violent movement, we must come ultimately to a natural mover; and not only to a natural mover as the form of an element [heaviness or lightness] would be, but to an mover that is natural and voluntary.

Does the air or water surrounding the projectile thus move naturally? Dumbleton has no difficulty in proving that it is not so. Do they move the projectile violently? We must admit that a certain form was induced into the fluid by what first launched the stone. It seems this is, Dumbleton notes quite rightly, the opinion of the Commentator. He also incidentally shows that such a form could be induced into a projectile, not to the medium; but he pays no attention to this theory that the School of Paris will develop and upon which it will lay the foundations of modern Dynamics. He is content to refute the opinion of Averroes and show that the mover of a projectile is

[37] Johannis de Dumbleton *Summa*, Pars sexta, cap. IV^m^. Biblio. Nat., fonds latin, ms. n° 16146, fol. 61, col. *b*, to fol. 62, col. *a*. — No extract of this chapter is found in manuscript n° 16621.

[38] P. Duhem, *Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, XI: *Nicolas de Cues et Léonard de Vinci*, XII; seconde série, pp. 222 seqq.

not an infused form at the beginning of its movement in the medium. Where is he going to discover the cause that keeps the projectile moving after it has left the hand or ballistic machine? In the same case that prevents the production of a vacuum in nature:

> A natural body,

he said in treating the void[39],

> can have two kinds of movement.
>
> One such movement happens because it is of such a species; thus fire, as long as it is fire, happens to be moved by its form toward the concavity of the lunar orb.
>
> The second movement belongs to it insofar as it is a natural body; and, in this respect, all bodies behave the same…
>
> To understand the second proposition, it is necessary to presume that principle from experience. Every body, even when it would be in its natural place, wishes to be conjoined to another body. And this is proved as follows: It is repugnant that a vacuum exist, while it is not repugnant that a body is outside its proper place; in fact, it often happens that a natural body is found outside of its proper place. It is therefore natural that a body moves to remain in close contact with another body rather than gaining its own place; the nature of a body is to be conjoined to another body before being in its proper place. This movement, by which a body remains in close contact with another body, does not happen to an element inasmuch as it is an element, but inasmuch as it is simply a natural body. In this way any natural body is mobile toward any place, whether this place be up or down; every element is indifferently mobile toward any place so that it stays conjoined to a natural body. As a magnet induces in the iron a form whereby the iron follows the movement of the magnet and stops when the magnet stops, so the body which follows another by this movement stops when that other body remains at rest.

Was Dumbleton the author of this doctrine? We cannot tell. But it had, after this master, a very large and lasting fortune. We find a witness to it, among many others, in Domingo Soto, who would tell us[40] "of the universal appetite to fill the void, lest the harmony of the Universe be dissolved." We find the same doctrine fully developed, later, by Julius Caesar Scaliger[41]. It was the hypothesis of the *abhorrence of a vacuum* that, alone, the memorable experiments of Torricelli and Pascal were able to ruin, and which was put, after it was abandoned, into a ridiculous form that was not his.

Yet it is this abhorrence of the void that John Dumbleton will invoke for explaining the motion of projectiles.

> Projectiles follow the air[42], thanks to the shape given to them, so that the vacuum does not produce such a movement; indeed, following what has been demonstrated, any body is naturally mobile so that it remains in contact with another natural body… As water follows water, smoke, which is an igneous body, follows smoke, and the flame follows the flame,

[39] Johannis de Dumbleton, *Summa* Pars sexta, cap. III^{m}. Ms. cit., fol. 60, col. *c*.

[40] Dominici Soto Segobiensis *Super octo libros Physicorum Aristotelis quæstiones*. Lib. IV, quæst. 3^{a}: Utrum si quid moveretur per vacuum moveretur in instanti. Ed. cit., fol. 65, col. *d*.

[41] Julii Cæsaris Scaligeri *Exotericarum exercitationum liber XV. De Subtilitate ad Hieronymum Cardanum*. Lutetiar, apud Vascosanum, 1557. Exercitatio V: De materia. De vacuo.

[42] Johannis de Dumbleton *Summa*, Pars sexta, Cap. IV^{m}; ms. cit., fol. 61, coll. *c* et *d*.

> so do projectiles follow the air or any other body which is moved in front of them, like how iron follows a magnet…
>
> All natural bodies have a double movement: A first movement that belongs to the body as it is in such a species, and a second movement by which this body follows another body. It is through this second movement that projectiles are moved by following the water or air launched before them; then, water or air follows the projectile from behind and thus helps to push it. This stone has a surface which is immediately contiguous to the air; when the air which is located in front of the stone was shaken by hand and the hand is removed, the air continues to move; if the stone remained motionless, the air could not, in an instant, rush to its full extent to the anterior face of the stone; therefore, so that the stone does not cease to be immediately adjacent to another body, it must move.

At the end of his presentation, John Dumbleton lists some observations, highly questionable though, which seem to claim, regarding the motion of projectiles, a different explanation from the one he gave. "But," he adds[43]"to explain how the medium moves when the impulsion has stopped, we must give another answer, namely the last one, which is the most common." Thus it was common, in the School of Oxford, to assign projectile motion to the abhorrence of the void.

These thoughts of a contemporary of Jean Buridan help us measure the intellectual height of the Parisian master.

The doctrines of the Parisian Dynamics, unknown from Dumbleton, seem to have been ignored by the Calculator.

Jean Buridan, Albert of Saxony, and Nicole Oresme have managed very cleverly to use the term *impetus* to explain how a weight oscillates on either side of its equilibrium position when it is pushed aside once; Soto will not fail to gather what they taught. Albert of Saxony and Oresme described in detail how a weight pushed away from the center of the world would execute oscillations on either side of the center.

The Earth would be immobile if its center coincided with the center of the World; deviated from this position, it would move so that the center of gravity would come back to the center of the World; do these two points ever coincide? This is the question that the Calculator formulates in these terms[44]: *Utrum omni elemento locus naturalis aliquis conveniat, omnibusque elementis ejusdem speciei*. He arrives at this conclusion that the center of the terrestrial mass would be indefinitely close to the center of the World without ever reaching it; instead of being periodic, as admitted by Albert of Saxony and Nicole Oresme, the movement of the Earth would be aperiodic. If Magister Riccardus of Ghlymi Eshedi obtained a result that contradicted the teaching of the Parisians, it is because he takes no account of the *impetus*.

In the part of the *Tractatus de sex inconvenientibus* that our manuscripts do not contain, a question, the seventh, was titled[45]: *Utrum omne corpus naturale habeat locum naturalem*. To this question did the author himself reply like the Calculator answers the question that he formulates almost in the same terms? One is permitted to believe so.

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[43] Jean de Dumbleton, *loc. cit.*; ms. cit., fol. 62, col. *a*.

[44] *Subtilissimi Doctoris Anglici Suiset Calculationum Liber*. Ed. Paduæ, ca. 1480; 43° fol.

[45] See the table in fol. 194, v°, du ms. n° 6559 of the Latin collection of the *Bibliothèque nationale*.

These various pieces of information and all their corroborations permit, we think, one to make this statement: The Dynamics that was taught at Oxford in the second half of 14th century differed greatly from what is professed in Paris around the same time; the notion of *impetus*, which dominated in the one, played almost no role in the other.

Other theories of Physics, on the contrary, found an equally favorable reception with the two universities. Thus it seems that the doctors of Oxford commonly admitted the movements of the Earth, very slow but constant, to which Albert of Saxony attributed such great importance.

Guillaume Heytesbury considers[46] the following supposition:

> Any part of an element such as earth or fire may be corrupted, because something cannot be brought into contact with a contrary element, and it probably will one day be brought into contact. Suppose, indeed, as is quite likely, that the earth is in continual motion, or, at least, that it moves frequently, so that this portion of earth that is now near the center might, in the course of eternal time, be a large number of miles distant from it; then, in fact, a body that is contrary to it can approach it close enough to be able to corrupt it.

When it wants to prove that the continuation of the movement is not enough to accelerate this movement, the *Tractatus de sex inconvenientibus* expresses itself thus[47]:

> If the continuation of the movement were the cause that accelerates the fall a weight like the Earth—since it began to exist and the Sun, too, began to exist—is in continual motion because of the heat from the Sun, it would have, from the beginning, accelerated its movement; now, it would move so quickly, and its movement would be sensible; the Earth would therefore have a continual and significant movement that would overthrow the great monuments, houses, and castles.

Among the information that Master Clay gave to the students in Paris on the doctrines of the School of Oxford, there are various considerations related to the actions of magnets[48]. These considerations begin with a phrase that is worth noting.

> If the center of the world were a point, as some think, and it was moving, it is certain that any weight, however great, would follow this point with a speed equal to that of its movement, as this point is the universal place of weights.

The same place that this reasoning occupies shows that the proponents of this opinion equated this movement of the weight toward the center with the movement of iron that a magnet displaces.

Certainly well known at the School of Oxford, this view was not universally accepted. John Dumbleton is careful to reject it[49]. He makes a profound distinction between the movement of weights to the center of the world and the movement of iron toward a magnet.

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[46] *Sophismata* Hentisberi; Sophismatum sextum. Ed. Venetiis, 1494, fol. 89, col. *b*.

[47] *Tractatus de sex inconvenientibus*; Quæst. IV: Utrum in motu locali sit certa assignanda velocitas; art. I; Utrum velocitatio motus gravis sit ab aliqua causa certa.

[48] Bibl. Nat., fonds latin, ms. n° 16621, fol. 213, v°.

[49] Johannis de Dumbleton *Summa*, Pars VI, cap. X. Bibl. Nat., ms. n° 16146. fol. 65, col. *c*.

> Those bodies,

he said, speaking of weights,

> do not follow the direction in which they are moved like how iron follows the magnet when the latter moves. Even when this point which is the center of the world would move, the earth would not follow it.

When he stated or reported this opinion, Master Clay could undoubtedly glimpse the fortune to which it was called. Forced to abandon the Aristotelian theory of gravity, Copernicus would one day conceive, in each star, a point which moves with this star; he had to admit that all parts of this star constantly tended to this point; later, when this view of Copernicus was adopted by a large number of physicists and William Gilbert had to assimilate this tendency that brings the parts of a star toward a point of this star with the tendency that brings iron toward a magnet, he thus had to construct his magnetic Philosophy, destined to win the support of Francis Bacon and Otto von Guericke; but all this magnetic Philosophy was inchoate in the thinking of Professor Clay.

Logic

It is easy, at least where the documents do not fail us, to say what physical theories were simultaneously admitted at Oxford and Paris and which doctrines were received in one of the Universities and rejected in the other. It is more difficult to describe the nuances by which the two universities were distinguished from each other when discoursing on logic; these nuances, however, seem to have contrasted them very vividly.

The essential character of Logic at Oxford seems to be marked in these terms: It gave an almost exclusive place and, consequently, an exaggerated importance to the solution of sophisms.

In the study of any science, teaching the general principles would be, in itself, insufficient; it is necessary that some skillfully chosen exercises accustom the student in the use of these principles, accustoming him to invoking the rule that is necessary in the right place. For practice, therefore, the moralist discusses matters of conscience, the lawyer pleads cases, and the mathematician solves problems. And it does not matter if the exercises are purely artificial, that the questions demanding a response were never presented and must never be presented; if they have increased the reliability of the knowledge the mind knows regarding the principle to be employed, they have reached their goal; they are similar to a gymnastics which forces the body to make unusual movements, but designed to give the members more strength and flexibility.

What gymnastics is to the body, discussing cases of conscience is to the moralist, and solving problems is to the mathematician, the solution of sophisms is to the logician; in the presence of a false proposition that seems to justify a captious reasoning, it accustoms them to discern the rule that this reasoning violates, the employment of which will destroy the false paralogism.

The solution of sophisms presents itself as a legitimate exercise of logic, as long as it remains an exercise. But gymnastics which no longer merely strengthens and relaxes the body, a gymnastics that ceases to be a means and is taken for an end, becomes acrobatics; similarly, in all studies the artificial exercise that loses sight of the real purpose for which it was combined becomes an acrobatics; as moral or legal casuistry can degenerate into acrobatics, so the solution of the problems can lead to mathematical acrobatics and the solution of sophisms to logical acrobatics.

At the time of William Heytesbury, this logical acrobatics was the sport in vogue at the School of Oxford.

The idea of collecting *sophismata* and *insolubilia* for exercising young logicians, like collecting problems to exercise young geometers, is too natural not to be very old. By the second half of the 13th century, collections of this kind were made. In effect, this is the *Impossibilia* of Siger of Brabant that Fr. Mandonnet has published[50], which Mr. Clemens Bauemrer has also published[51], but he misunderstood it in a strange a manner, following Bartholomew Haureau, regarding their true nature[52]. It is equally a *Sophisma* as this question of Siger of Brabant[53]: *Utrum hæc sit vera: Homo est animal, nullo homine existente*.

At the time of Siger of Brabant, moreover, in the University of Paris, the prevailing fashion was the discussion of paralogical statements[54]; various manuscripts retain a collection of sophisms analyzed by Pierre d'Auvergne and some spare sophistical questions due to Pierre de Saint-Amour, Boethius of Dacia, Bonus Dacus, and Nicholas of Normandy[55]. In 1270 Albert the Great complained[56] that "many Parisians left Philosophy to indulge in sophistry."

Having become, by 1252 in the English Nation of the University of Paris, one of the obligatory scholarly exercises[57], the discussion of sophisms greatly sought, in the 14th century, the activity of Parisian masters. In the first half of this century, a master who, after teaching at Oxford, taught at Paris, Walter Burley, gathered a large collection of *Sophismata insolubilia*[58]. He was probably not the only one, at that

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[50] Pierre Mandonnet, O.P., *Siger de Brabant*, IIe Partie (Textes inédits); pp. 71-94. (*Les Philosophes Belges. Textes et études*, t. VII. Louvain, 1908).

[51] Glemens Bauemker, *Die Impossibilia der Siger von Brabant, eine philosophische Streitschrift aus dem XIII Jahrhundert*. Münster, 1898.

[52] Pierre Mandonnet, O.P., *Siger de Brabant*, Ire Partie (*Étude critique*); pp. 127-128, en note (*Les Philosophes Belges*, t. VI. Louvain, 1911).

[53] Pierre Mandonnet, O.P., *Siger de Brabant*, IIe Partie (*Textes inédits*); pp. 63-70.

[54] Pierre Mandonnet, O.P., *Siger de Brabant*, Ire Partie (*Étude critique*); p. 123.

[55] Pierre Mandonnet, O.P., *loc. cit.*, pp. 123-124 en note.

[56] Pierre Mandonnet, O.P., *Op. laud.*, IIe partie, p. 35.

[57] H. Denifle et E. Châtelain, *Chartularium Universitatis Parisiensis*, t. I, p. 228.

[58] Bibl. Nat., fonds latin, ms. 16621; fol. 243, r°: Circa insolubilia queritur primo circa insolubile… fol. 247, v°: Explicit (*sic*) sophismata insolubilia magistri Gualterii de burlay anglici magistri theologie. Prantl (*Geschichte der Logik in Abendlande*, IIIter Band, pp. 297 seqq.) does not know about this writing of Burley.

time, who maintained, at the University of Paris, the fashion of collecting sophisms; we can, in any case, ensure that it thereafter developed greatly; we have as a witness the work that Albert of Saxony titled *Sophismata*. In the manuscript copy that we saw, the book ends with this sentence[59], which seems to be by the same author:

> *Et sic est finis hujus tractatus in quo continentur 259[a] sophismata principalia preter minus principalia que interposita sunt, quorum numerum nescio invenire.*

This prodigious collection of sophisms is however, according to Albert of Saxony, only a basic work; the trained dialectician, eager to solve the most specious sophisms, must look to the treatises of the *Insolubilia* or *Obligationes* contained in the Logic of Albertutius, because it goes on to say:

> *Si aillent aliquis voluerit videre sophismata alterius materie, perlegat tractatus de insolubilibas et de obligationibus quos alias scripsi, et in eis inveniet sophismata difficiliora et subtiliora sophismatibus predictis. Et hic finis. Deo gratias.*

The treatises of Albert of Saxony mark in what honor the exercises of Logic were held at the University of Paris towards the middle of the 14th century; it does not seem, however, that these exercises overtook all other studies. A logician such as Albertutius does not dedicate himself exclusively to Dialectical skills; his *Questions* on the *Physics*, the *De Cælo*, and the *De generatione et corruptione* show him as a man greatly concerned about the problems of Physics; he does not bring into the consideration of these problems the spirit of subtle chicanery that the continual analysis of sophisms easily develops. Beside him, Nicole Oresme devotes the power of his genius to Theology, Morals, economic Science, Physics, and Mathematics; it does not appear that he composed any treatise on pure Logic.

At Oxford, on the contrary, we willingly believe that no master of some renown has failed to write on the *Sophismata*, the *Insolubilia*, the *Consequentiæ*, and the *Obligationes*. Before William Heytesbury, we encountered Swineshead, Dumbleton, and Glymeton Langley; almost immediately after Heytesbury, we found Rudolph Strodus and Richard Ferabrich. Not only do all students devote much of their activity to the most subtle exercises of Logic, but the most prominent personage of the University, the one it chose for chancellor, the one we call the "*Solemnis Magister, potissimus et famosissimus Hethysbery*", wrote nothing that is not dedicated to the solution of sophisms; his *Regulæ* themselves, indeed, under titles that seem to be of Physics, are only rules specifically for untying the sophisms that one can weave regarding certain questions of Physics.

And, indeed, the desire to discover everywhere some opportunities for displaying himself as a skillful dialectician of untying complicated sophisms will soon invade all studies. The Scholastic method was only too favorable to that provision of mind. Born of the *Sic et non* of Abelard, it only speaks about the demonstration of a proposition that has carefully laid out all the opinions that go against it as well as all the opinions that lean toward it; then one must refute one by one all the objections of opponents and in turn raise objections against each of the opinions that must be rejected; the direct demonstration of a truth is thus framed as a host of small, accessory

[59] Bibl. Nat., fonds latin, ms. n° 16134 (*olim* fonds Sorbonne, n° 848); fol. 56, col. *b*.

quarrels. Surely, such a method, when properly practiced, bears the stamp of a clear loyalty; it leaves nothing that can be opposed to the position that one takes; it is not permitted to take it up until one has cleared it of all charges. But this method presents dangers; in this multitude of singular combats that includes all demonstration, the champion of the truth is greatly tempted to prove that he is a skilled dueler; when opponents are lacking, he will create one for the pleasure of beating him; against the opinion which he holds, he will invent from scratch some sophistical objections to show that he can solve them.

The greatest scholastics have not avoided this failing. It is easy to guess to what excesses this intellectual vice must have been carried in a School whose dialectical dexterity seems to have been the only concern. All problems of Theology, Morality, and Physics become an excuse to imagine captious difficulties and to overcome them by subtle tricks. Soon, the direct demonstration, intended to give the truth of immediate and face-to-face apperception, has completely disappeared; one imagined he established an opinion when he refuted—by forcing the opinions, real or fictitious, which were enumerated against it—into some *inconvenientia*; one only used this sort of *reductio ad absurdum*, by no means convincing, because, of course, the enumeration of possible opinions was never complete; all reasoning was nothing more than chicanery.

The very fruitful idea that the intensities of the various forms and qualities can be measured, or at least represented by numbers, further increased the thorny subtleties of scholastic Dialectics; by introducing the *gradus*, *formæ uniformes*, and *formæ uniformiter difformes*, it has given to this Dialectics a kind of mathematical accoutrement and has provided new processes for forging sophisms as well as for breaking them; these quibbles coated with an arithmetic adornment were given the name of *calculationes*. The *calculationes* are already numerous in the *Questions* of William of Colligham, the *De primo motore* of Swineshead, and in the *Summa* of Dumbleton; they invade everything, carrying their false precision and apparent rigor everywhere, in the *Liber sex inconvenientium* and the treatise of Riccardus of Ghlymi Eshedi, the Calculator *par excellence*.

The *calculationes* penetrate everywhere, we say; they penetrate even and especially in areas that seem, inherently, to escape the grips of calculation, such as Theology. Besides, is not it in discussing the increase of grace in the soul of a Christian that the commentators of Peter Lombard conceived the thought of representing by numbers the various degrees of intensity of a form or quality? Naturally, therefore, the Oxford masters, faithful to the tradition of Richard of Middleton, took to building a mathematical Morality and Theology where the fervor of grace and the gravity of sin is measured in numbers like how we evaluate the degree of the temperature or the weight of a body.

Take, for example, some questions on the *Books of the Sentences*[60] which ends with the following formula:

> *Expliciunt questiones magistri Richardi Kyluxuton super librum sententiarum.*
> *Vinum scriptori debetur de meliori.*

[60] Bibliothèque nationale, fonds latin, ms. n° 14576, fol. 117, col. *a*, to fol. 199, col. *d*.

The author, whom the copyist called Richardua Kyluxuton, is called Rioardus Cliqueton by another scribe who compiled a table of contents[61] of the manuscript collection; he can be none other than Richard Langley Clienton or Clymeton whome we met among the Logicians.

Let us open this work at random. There we have[62] that "merit is measured by the latitude that grace has gained, not only by the more or lesser degree of grace." We see there[63] a love of God and a love of neighbor, both decreasing in geometric progression with ratio $1/2$.

He goes about proving that in a certain case, does Plato not sin more seriously than *Sortes*? Here is how the argument[64] begins:

> Suppose that Plato, in the given case, sins more gravely than *Sortes*; suppose that *Sortes* sins to the degree A and Plato to the degree B, more grave than the degree A. The excess of B over A is either divisible or indivisible. But it is not indivisible, because some excess, in the matter of mortal sin, would be indivisible, and we will prove later that this cannot be. The excess of B over A is thus divisible. I then take a degree of sin which is the average degree between A and B; let C be this average degree. Someone could, therefore, sin precisely in the degree C…

Constantly, there are comparisons between the degree of merit or demerit of an act and the speed of a local movement[65]; also we frequently encounter phrases such as these:[66]

> If two vicious acts are continued uniformly for the duration of a natural day, they will grow equally during this day…

We do not believe that Master Kyluxuton was, in Oxford, the only theologian who gave himself up to this mathematical casuistry; others came after him, who only made it more learned and more complicated.

Let us again examine these disorderly notebooks where a Parisian student has preserved for us so much precious information on the Oxford School. We find a short fragment[67] whose origin is not specified. This fragment first presents a series of rules, taken from the peripatetic Dynamics, regarding the relationship between power, resistance, and speed of a mobile; these rules are formulated in almost identical terms to those in the treatise of the Calculator; immediately afterwards, the uniformly difform latitude is defined; we recall that as regards the distance traveled, uniformly difform motion corresponds to its average degree; he adds that "these statements are general because they can be applied in a more general manner to

[61] Ms. cit., verso of the flyleaf, unnumbered.

[62] Magistri Richardi Kyluxuton *Quæstiones*; quæst. I, 3° ad principale; ms. cit., fol. 123, col. *d*.

[63] Magistri Richardi Kyluxuton *Quæstiones*; quæst. I, 5° ad principale; ms. cit., fol. 126, col. *d*.

[64] Magistri Richardi Kyluxuton *Quæstiones*; quæst. II; ms. cit., fol. 140, col. *b*.

[65] Magistri Richardi Kyluxuton *Quæstiones*; quæst. V; ms. cit., fol. 169, col. *d*.

[66] Magistri Richardi Kyluxuton *Quæstiones*, quæst. V; ms. cit., fol. 188, col. *d*.

[67] Bibl. Nat., fonds latin, ms. n° 16621; fol. 5a, r° and v° and fol. 65, r° and v°. We said, in the previous § [section 27], that this fragment had probably been brought from Oxford to Paris by Master Clay or by some other Englishman.

increases and decreases that occur in any movement." Now these preambles of Mechanics are intended to discuss this conclusion: Every sin is voluntary; so the more it is voluntary, the more it is sin. During this discussion we hear questions such as this: Can the intensity of sin be acquired in a uniformly difform manner? We have before us a remarkable example of the what applied *calculatio* gave to casuistry.

An artifice might have made these *calculationes* less messy, less difficult to follow; it consisted in using the geometrical representation by coordinates of which Nicole Oresme so happily marked the advantages. We do not see that one ever used this representation at the School of Oxford; the *calculationes* have always kept a purely arithmetic form; in any case, they were replaced by geometric constructions.

Not only do we not find any allusion to the representation by coordinates in the writings of those who were able to be the elders of Nicole Oresme or his contemporaries, such as Swineshead, Dumbleton, or Heytesbury, but we do not find a trace of the representation in the *Tractatus de sex inconvenientibus*, whose author, coming after Heytesbury, is certainly posterior to Oresme; nay, we encounter it neither in the treatise of Riccardus of Ghlymi Eshedi nor in an anonymous pamphlet titled *A est unum calidum*, which we will discuss later on; now, we will acquire the certitude that the authors of these latter two writings read the *De difformitate qualitatum* of Oresme.

The use of these geometric representations would, however, have greatly helped in following the *calculationes* of the English masters; also, the French copyists frequently drew in the margins of the manuscripts their own figures to illuminate the text; so it is for the manuscript, preserved in the National Library, of the treatise of Riccardus of Ghlymi Eshedi; but it is enough to read the text carefully to recognize that these figures were neither appropriate nor intended by the author and that he only appeals to the procedures of Arithmetic.

The Scholasticism of Oxford—which found in all subjects the opportunity to invent strange sophisms for the fun of solving them and of developing many unnecessary *calculationes*—greatly offended, first of all, the Parisian masters; they did not find there discussions conducted, for the truth, following the method of *sic et non*, but discussions that were sober, clear, ordered, free of unnecessary baffles and subtle tricks, with which Jean Buridan, Nicole Oresme, Albert of Saxony, and Marsilius of Inghen were accustomed; between the Scholasticism of Paris and the Scholasticism of Oxford, it was difficult for them not to give preference to the former.

We have encountered the testimony of this sentiment. The Parisian student whose notebooks have so often served us in this study on Oxford Scholasticism copies[68] what the *Summa* of Dumbleton said on this question: Can and should we compare, from the point of view of perfection, something of one species with something of another species? On the bottom of the page, he wrote:

> You who possess what Master Nicole Oresme said, compare: *Vos habentes dicta M. N. Orem, comparate.*

After having surprised and, perhaps, scandalized the Parisians, the Scholasticism of Oxford ends up being in great vogue at the Sorbonne and the Rue du Fourre. What

[68] Bibl. Nat., fonds latin, ms. n° 16621, fol. 181, r°.

was the cause of this triumph? Who ever will make sense of the whims of fashion? It is permissible, in any case, to note that the quodlibetal discussions, the essential tests of many an examination, singularly promote this last invasion of English Dialectics; he who was competent to bind and unbind the sophistical arguments played well in these sophistical tournaments; also, we learn from many a testimony that the chicaneries and *calculationes* in the style of Suiseth were in continual use in these logical games.

It also happened that the method of Oxford was characteristic of the School of Paris in the 15th century. When Averroists or humanists in the Renaissance were attacking the Parisian Scholasticism, they ridiculed the patterns borrowed from the School of Oxford; Giovanni Pico della Mirandola abhors the *quisquiliæ Suiceticæ*; in coining a nickname that ridicules the Parisians, Nifo transforms the title *calculatores* into the epithet *captiunculatores*; it is Suiseth who takes most readily the sarcastic verve of Luis Vives. What is criticized most strongly in the Parisians is that they have set themselves in the fashion of Oxford; their old doctors, who comport themselves in the French manner, almost always escape derision.

Opponents of Parisian Scholasticism, moreover, were not all wrong; many did not hesitate to point the finger at the true inventors of the new form that Logic took. Listen to[69] Leonardo Bruni d'Arezzo († 1444):

> What shall we say of Dialectics, the art so necessary for discussion? Is its reign flourishing? Does it entirely escape the calamity of war that ignorance conducts? Not at all, because the barbarian who lives across the Ocean has pounced on it. But what people, great God! Their very names fill me with horror: Ferabrich, Tysber[70], Ockham, Suisset, and others of the same kind; they all seem to have borrowed their nicknames from the troop of Radamanthe… What is it, I say, in the Dialectics which has not been thoroughly scrambled from top to bottom by English sophisms?

Pomponazzi, who calls William Heytesbury "the greatest of the sophists", who incessantly fights the opinions of the Calculator, also knows to which country he should direct his attacks:

> In the proposition in question,

he wrote[71] in 1515, in the preamble of his treatise *De reactione*,

> none of the Greeks expressed doubt, nor were any of the ancients among our countrymen. But those who came later, especially the British, have formulated subtle doubts; in meeting the commonly accepted proposition, they imagined such difficult arguments which a host of famous men have struggled to solve; and however, in my view, they did not perfectly satisfy this task.

[69] Leonardi Arretini *De disputationum usa*, Nürnberg, Keuerlin, 1734, p. 26; quoted by Prantl, *Geschichle der Logik im Abendlande*, IVter Bd, Leipzig, 1870; note 39, p. 160.

[70] The text says: *Busser*; we corrected it according to the information of Prantl. It is unlikely that Leonardo d'Arezzo intends to speak about William Bucer, who was in Paris at the time of Albert of Saxony.

[71] Petri Pomponatii Mantuani *Tractatus acutissimi, utilissimi, et mere peripatetici*… Venetiis, MDXXV; fol. 21, col. *a*.

Since the Renaissance, therefore, discerning minds would have subscribed to this judgment: The decadence of Parisian Scholasticism began the day it forgot its own traditions and adopted the Dialectic of the University of Oxford.

Chapter 28
The law of uniformly varied movement at the School of Oxford

The *De primo motore* of Swineshead and the *Dubia parisiensia*

After attempting to trace, in a rapid sketch, the physiognomy of the School of Oxford in the mid-14th century, we try to summarize what the school taught about the latitude of forms and, particularly, of the uniformly difform latitude. For this purpose, let us successively review the various writings whose existence we have reported.

We start with the *De primo motore* of Swineshead; it will present, in a way, the type of the family of treatises that we will read.

Our Parisian student will still, in his precious drafts, teach us to obtain at Oxford the information we need.

This student had the very fortunate idea to give us[1] a detailed table of contents of the treatise of Swineshead.

The *De primo motore* comprises eight parts or "differences".

The *first difference* is formed by the preamble.

The *second difference* "presents some hardly widespread truths, but no news yet on generation." Neither one nor the other of these first two differences has subdivisions.

The *third difference* is divided into three chapters. Chapter I deals with the generation of simple elements, Chapter II of mixed generation; Chapter III sets out how generation occurs for simple substances.

The *fourth difference* is devoted to the solution of objections. Among the issues discussed there, these are the two main ones:

1. Are primary qualities effects produced by the ethereal Heaven?
2. Are the four elements corruptible bodies?

The *fifth difference* is composed of three parts.

> The first part outlines the misconceptions regarding the intensity and remission of form. The second part shows what the truth about this subject is. The third part shows how the velocity is measured in a movement of alteration.

[1] Bibliothèque Nationale, fonds latin, ms. n° 16621, fol. 35, v°.

Incidentally, in this difference, it is proven that motion is a cause of heat, which leads to speaking of light, and it treats of the movement of augmentation.

Movement of augmentation and of diminution is the proper object of the *sixth difference*, which is divided into two parts.

The first part examines in detail in what manner augmentation and dimunation is performed. The main issue addressed therein is this: In an object that grows, does each part grow? In this connection, the movement of food towards each member of the body is examined.

Two chapters follow in the *seventh difference*.

The first chapter deals with powers that produce local movement and their relations with the bodies that move; a first part examines the power that creates a natural movement, and a second part examines the power that generates a violent movement.

The second chapter deals with the speed and slowness of local motion.

There are also two chapters in the *eighth difference*. The first chapter outlines the various kinds of maxima and minima that should be considered in the study of active and passive powers. The second chapter examines how and to what extent these distinctions may extend to other cases.

Our Parisian reproduces nothing from the *Proœmium* of Swineshead, but he copied[2] the invocation by which this author ended his book:

> *Sola enim potentia potentiarum, accidentia non quoquomodo passiva, infinita, totarumque potentiarum principium est et finis; solum igitur ejus Principium optimum et unum impassibile consistit, cui per infinita sæcula sæculorum sit honor et gloria. Amen.*

He has, moreover, made insignificant extracts of the first three differences[3]; his interesting extracts only begin[4] in the fourth.

The fifth, sixth, and seventh difference, entirely or almost entirely copied by our student of Paris, are those which should above all capture our attention. There, three kinds of motion that the peripatetic Physics recognized are studied: movement of alteration, movement of augmentation, and locomotion. The examination of these *three predicaments* in which movement is possible was already the main purpose of the *Tractatus proportionum* of Albert of Saxony, whose three differences of which we have already spoken sometimes offer an analogy. The eighth and final difference also deals with a question that has greatly occupied Albert, that of the maxima and minima *in quod sic* and *in quod non*[5]; but in this matter, he does not bring the extreme rigor and extreme precision with which the Parisian Master prided himself.

It is the *fifth difference*, dedicated to the intensity of the form and movement of alteration, that Swineshead examines the properties of the uniformly difform latitude[6]. Should such a latitude be evaluated using its average degree or its extreme degree? There can be no hesitation, it seems to him, between these two assumptions:

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[2] Ms. cit., fol. 84, v°.

[3] Ms. cit., fol. 3g, r° et v° fol. 40, r°.

[4] Ms. cit., fol. 40, v°.

[5] *Léonard de Vinci et les deux infinis*, II : L'infiniment petit dans la Scolastique (*Études sur Léonard de Vinci*, IX ; seconde série, pp. 26 seqq.).

[6] Ms. cit., fol. 62, r°.

> *Igitur conclusio sequitur: Ista intensio vel remissio latitudinis penes gradum medium vel extremum intensionis opportet altendi.*

But, he continues, it cannot be measured by its average degree because then all uniformly difform latitudes that have the same average degree are equal. It is by its extreme degree that it will be measured.

This solution is evidently permitted, in the spirit of Swineshead, of the opinion expressed by Bradwardine and adopted by Albert of Saxony, that the speed of a body animated by a movement of rotation is the speed of the point which moves the fastest. Swineshead endorsed this opinion[7]; he said that the truth appears sufficiently to whomever reads a certain chapter entitled *De proportionibus*.

In his discussion of the maximum and minimum, he considers[8] a uniformly difform motion relative to the subject, and he asserts that "this movement has the same speed as the degree that terminates it". To justify this assertion, he takes an example from a line that rotates around one of its points; according to the previous proposition, the speed of this line is the speed of the fastest moving end.

How far we are from the considerations we admired in the treatise of Nicole Oresme!

The passages we have analyzed do not seem to express what the final thought of Swineshead was.

The student or the Parisian master who tells us about the work of this author has scribbled on a page of his notebook[9] a list of the writings that are reproduced or summarized there. In this list, immediately before mentioning the *In primo motore*, he mentions a "quatern"[10] dedicated to Suincet, *unus quaternus de Suincet*, where we find "a question about the average degree and two determinations on the maximum and minimum."

The three questions thus stated are, in fact, copied successively in the manuscript we are examining.

Of these questions the second is formulated in these terms[11]:

> *Utrum sit dare maximum pondus quod Sortes potest portare.*

It was one of the problems that all Parisian Scholastics treated; it was, in fact, the concept of limit that they deepened under this form; we elsewhere[12] marked the importance of their considerations on this issue; we also saw that they had attracted the attention of Leonardo da Vinci.

This issue is closely linked to the concepts of *maximum in quod non* and of *minimum in quod sic*, of which Swineshead has already spoken in the last "difference"

[7] Ms. cit., fol. 78, v°: Penes quid attendatur velocitas in motu locali.

[8] Ms. cit., fol. 81 v°.

[9] Ms. cit., fol. 64, v°.

[10] Group of four sheets.

[11] Ms. cit., fol. 87, r°.

[12] *Léonard de Vinci et les deux infinis*, II: L'infiniment petit dans la Scolastique (*Études sur Léonard de Vinci*, IX; seconde série, pp. 28-29 et pp. 52-53).

of the treatise *De primo motore*; he returns to the last of the three "doubts"[13] which preoccupy us.

The first of these doubts is formulated in these terms[14]:

> *Utrum omnis motus uniformiter difformis correspondeat suo gradui medio.*

All at once the author presents one reason in favor of the affirmative and another reason in favor of the negative.

That it is necessary to answer yes results from this proposition: A uniform movement, corresponding to the average degree, acquired as much space as the movement considered.

That it is necessary to answer no is suggested by this remark: The movement of the radius of the circle is a uniformly difform motion through the various points of that radius; however, it does not correspond to its average degree. Bradwardine, and Albert of Saxony after him, wanted to take for the speed of this rotational movement the speed of the most quickly moving point; our author cites neither Bradwardine nor, of course, Albert of Saxony, but he takes their opinions as assured.

After a fairly lengthy discussion, the author concludes in the affirmative[15]. His entire demonstration ultimately rests on the first of the reasons he invoked, which he regards as an established truth; he takes it up and ranks it sixth[16] among the suppositions that he admits for constructing his deduction.

In the *De primo motore*, Swineshead formally rejected this proposition: A uniformly difform latitude is measured by its average degree. It seems that, later, writing the three questions we just discussed, he changed his opinion; and this change of opinion was dictated by this proposition, which he considered certain: Two movements of the same duration, the one uniformly difform and the other uniform, the degree of the second being the average of the degree of the first, make the mobiles that they displace travel in equal spaces.

These three questions that our student seems, we have seen, to attribute to Swineshead, he calls[17] the three *Doubts of Paris*; he tells us, indeed, that one will find in his notebook:

> The *In primo motore* of Suincet in four quaterns, with three doubts of Paris (*cum tribus dubiis parisiensibus*), one on the uniformly difform and two on the maximum and minimum.

We must therefore assume that Swineshead, or perhaps one of his disciples after him, followed the *In primo motore* of the three questions we have just analyzed, but that he held them as problems imported from Paris to Oxford. By this, we are, it seems, allowed to think that the University of Paris taught the law of the space covered by a uniformly varied movement to the University of Oxford. The name of the *Rule Nicole Oresme*, which we have previously given it, would be far from

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[13] Ms. cit., fol. 88, v°, to fol. 92, v°.

[14] Ms. cit., fol. 85, r°.

[15] Ms. cit., fol 86, v°.

[16] Ms. cit., fol. 85, r°.

[17] Ms. cit., fol. 13, v°.

being condemned by a similar conclusion. However, it seems difficult to place, in time, Swineshead after Oresme; we must admit, without a doubt, that before the time when the latter composed the *De difformitate qualitatum*, the reduction to uniformity of the uniformly difform latitudes was already discussed in Paris.

However, reading the *Questions on the Physics* composed by master Jean Buridan confirmed this supposition. Here, indeed, is the remarkable passage we met in these *Questions*[18]:

> I suppose that a column is as long on one side as on the other, so that it may be, on both sides, ten feet long; I suppose that another column be of difform length, that is to say it has ten feet on one side and nine feet on the other; the first column will be a half foot longer than the other, because the length of the body resides not only in its right side or its left side or in the middle, but it resides, at the same time, in its right, middle, and left sides; therefore, we should not say that such a body is long or has such a length in considering purely and simply its right side or its left side, but by considering jointly its right side, left side, and midpoint; and if there is no uniformity of length, it is necessary to compare the longer side to the shorter side, taking something from the longer side and adding it to the shorter side, to find the average (*et si non sit uniformitas longitudinis, oportet inferre longius ad minas longum, auferendo de longiori latere et apponendo minus longo, ut inveniatur medium*).

Buridan then cited other examples that provide it with luminous intensity and color, and then he goes on to say:

> So, just to name [a difform magnitude,] one must compensate between the parts so that the simple denomination results from the average; so it is clear that they who make measurements to determine the size of a surface or a body, reduce difformities to uniformity. (*Ergo ad simpliciter denominandum oportet recompensare inter partes ut a medio fiat simpliciter denominatio, et ideo manifestum est quod mensurantes superficiem quanta sit, vel corpus quantum sit, reducunt difformitates ad uniformitatem.*)
>
> This is why it seems good to conclude this, as a corollary: It is not the speed of the point on the circumference moving the quickest that must simply be referred to as the total speed of a sphere [moving rotationally]; many people, however, commonly express themselves thus, leaving aside, in this denomination, all the rest of the sphere, whereas what remains surpasses infinitely in magnitude [what they do take into account].

We have here, there is no doubt, the first sketch of the considerations that Nicole Oresme was, later, developing with so much artifice. We also have the evidence that before Nicole Oresme, the reduction of difform magnitudes to uniformity was discussed. But there is more. Immediately after the above-quoted passage, *in the same question*, Buridan considers how one should define the upper limit of the effects

[18] *Acutissimi philosophi reverendi Magistri* Johannis Buridani *subtilissime questiones super octo phisicorum libros Aristotelis diligenter recognite et revise a magistro Johanne dullaert de gandavo antea nusquam impresse. Venum exponuntur in edibus dionisii roce parisius in vico divi Jacobi sub divi martini intersignio.* — Colophon:

> Hic finem accipiunt questiones reverendi magistri Johannis buridani super octo phisicorum libros impresse parhisiis opera ac industria Magistri Petri ledru Impensis vero honesti bibliopole Dionisii roce sub divo martino in via ad divum Jacobum Anno millesimo quingentesimo nono octavo calendas novembres.

Lib. I, quæst. XII: Utrum omnia entia naturalia sint determinata ad maximum, fol XV, col. *c*.

of which an active power is capable. This review leads him to resolve this question: Can we assign a maximum weight among those that a man is able to bear? Thus we find, following one another, in the same question of the *Physics* of Buridan, the subjects of the three *Doubts of Paris*, and, for both, these subjects are classified in the same order. If we observe that the subject of the first of the *Dubia parisiensia* has, by itself, no relation to the subjects of the last two *Dubia*, we cannot fail to be struck by such a coincidence; uneasily one will defend the formulation of the following conclusion: The three *Doubts of Paris* that Swineshead took the trouble to discuss in Oxford issued from the teaching of Jean Buridan.

Let us leave aside the three *Doubts of Paris* and return to the *In primo motore*.

In the beginning of the seventh difference, which is devoted to the study of local movement, Swineshead wrote the following[19]:

> To study speed and slowness in local movements, I will introduce five latitudes that reason alone distinguishes:
>
> The first is the latitude of the local movement; the second is the speed of the first latitude; the third is the slowness of that first latitude; the fourth is the latitude of the acquisition of latitude of local motion (*latitudo acquisitionis latitudinis motus localis*); the fifth is the latitude of decrease in the same latitude (*latitudo deperditions ejusdem latitudinis*).

What are these two new latitudes that Swineshead adjoins to the speed and slowness of local motion? The names themselves which serve to designate them make us guess that they are what we call positive acceleration and negative acceleration. So from the time when the *De primo motore* was composed, the importance of the concept of acceleration was manifested to the logicians of Oxford. The writings of William Heytesbury will affirm the importance of this even better.

The *Summa* of John Dumbleton

The notebooks of Philosophy from where the preceding information was extracted gave us the table of contents of the *De primo motore*; they do not reproduce the table of the *Summa* of Dumbleton; to piece it together according to the excerpts contained in these notebooks would be a difficult task; fortunately, we were able, in addition to these extracts, to consult the book itself.

To present an overview of subjects covered by the review, we cannot do better, we believe, than to reproduce the analysis given by the author in the preamble of his *Summa*.

This *Summa*, he tells us[20], is divided into ten parts.

[19] Ms. cit., fol. 74 v°.

[20] Johannis de Dumbleton *Summa*, Prœemium. Bibliothèque Nationale, fonds latin, ms. n° 16146, fol. 2, coll. *a* and *b*.

The *First part*[21] treats four articles.

The first article shows there is some natural cause of the signification of the term and its imposition on the subject; it deals with various incidental questions.

In the second section, it examines what it is for a truth to precede another, being more easily knowable by nature or by us; how one can know in a more confused or more distinct manner; how universal truths are better known than particular truths; it compares the knowledge of the definition to that of the defined and its parts.

The third article states some conclusions concerning the principles of our science and the intensity of knowledge and belief.

The *second part*[22] quickly demonstrates some propositions regarding first principles, what matter and form are; regarding the many opinions that have been expressed concerning substantial forms and the intensities of primary and secondary qualities; regarding the intensity or remission of a quality that is said to be uniform in reality or in name only; about finally regarding the description of the mixed intensity.

The *Third Part*[23] puts forth some conclusions concerning movement relative to three predicaments; it shows how much movement results from the configuration and distance; it decides on what way the speed of the local movement, movement of alteration, movement of augmentation, and movement relative to the latitude of density or rarity should be evaluated.

Finally, it researches through a variety of reasons what movement, time, and their properties are; it demonstrates, in the same part, that uniformly acquired movement equals its average degree, plus a few other conclusions.

The *Fourth part*[24], examining in a first article the nature of the elements, seeks to show if the extreme elements possess the highest degree each of the qualities and how the primary qualities act.

A second article deals with the reaction among these same qualities; it defines how the primary qualities naturally result from primary forms, the extremely intense or extremely weak density or rarity of bodies; finally, it examines whether these primary qualities are really distinct from the other qualities.

In a third article, the fourth part shows how the powers of bodies depend on their size; it examines whether mixtures will affect them and if they are heavier than the pure elements.

The *Fifth Part*[25] has spiritual action for its object; it sets forth whether light is peculiar to an element, if it is a single quality or a resultant quality.

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[21] The first part has thirty-nine chapters. The first chapter begins on fol. 2, col. *b*, in the cited ms., with the words: “Incipiendum est a primis. Minimus error in principio, in fine est maxima et maxime causa.” The last chapter ends at the bottom of col. *b* of fol. 14.

[22] The second part of the *Summa* contains forty-one chapters. The first chapter begins on col. c of fol. 14 with these words: “Post logicalia, naturalia aggredientes dubia…” The last chapter ends on col. *b* of the fol. 26.

[23] This third section is divided into thirty-eight chapters; in fol. 26, col. *b*, of the ms. cit., the first chapter begins in these terms: “Quia singulorum noticia motu, tanquam signo naturali, nobis primum inesse [constat], superest aliquid de eodem dicere et de ejusdem principiis pertractare.” This part concludes on col. *d* of fol. 39.

[24] The fourth part of the *Summa* of Dumbleton comprises seventeen chapters. In fol. 39, col. *d*, the first chapter begins: “Peracta determinacione materie communis, ad particularia descendamus, et de primis corporibus, scilicet elementis, pertractemus.” This part ends at the top of col. *b* of fol. 51.

[25] The fifth part has, in the ms. cited, six numbered chapters, which may need to join, as of an unnumbered chapter, the development that begins at fol. 50, col. *a*, by: “Quedam conclusiones in diversis materiis, admisso contrario principio, restant probande.” The first chapter begins at fol. 51, col. *b*, in the following way: “Completa determinacione de actione reali inter formas et qualitates sensibiles communiter, de actione spirituali inquiramus duobus requisitis.” This part ends at the top of col. *a* of fol. 67.

In addition, this part examines the doubts that one can conceive related to the difference between the higher forms and the lower forms capable of producing light, and touching their uniform or difform action, either in respect of the agent or in respect of the patient.

The *Sixth Part*[26], which deals with the terms assigned to powers, firstly teaches how to determine an active power in a defined way.

Secondly, among the other parts, the sixth particularly expresses, regarding the action and term taken in a universal manner, what the forms of rest and motion are; it deduced if such form is properly mobile, and if its shape and location are attributed equally to the generated body.

Then the same part of asks questions relating to how the Philosopher proceeds in the study on the movements and movers of the heavens; it determines how natural bodies are limited in volume and whether they must be subtracted from the primary movement; it adds which are those that move themselves and which are unable to do so.

The *Seventh part*[27] indicates what the cause is that assigns a minimum to the individuals and species subject to generation and corruption, which determines the order of the powers of matter and agents; it also sees if it can be proved by philosophical reasons that there exists a primary Mover of infinite force, and that the world began.

The *Eighth part*[28] treats, firstly, of the generation of a substance from a similar substance; it also deals with the generation of perfect animals and those arising from putrefaction.

This part achieves its job in establishing the numerical unity of the soul in an animate being with both sense and intellect, and it examines the operations of the intellective faculty.

The *Ninth part*[29] continues the order in which the work proceeds, resolving doubts concerning the soul and the five senses; it also examines a number of questions related to the same subject matter.

The *Tenth and final part*[30] addresses universals that are called ideas in Plato; it studies the simple and complex passivity of human intelligence, touching the extension that its own operation can receive; in concluding with a sort of summary of these subjects, this *Summa* itself ends.

This summary that Dulmenton gives us of his *Summa* suffices to leave us a glimpse of a host of diverse topics that this works studies; it also makes us foresee that the order in which they succeed will, in many cases, be neither rational nor very rigorous; unfortunately, the reading of the treatise itself does not deny this last hunch.

This lack of order is especially marked in what the Oxford logician teaches on uniformly difform latitude and its equivalence to the average agree; we must look in two different locations of the *Summa* for the presentation of his thought; still, reading

[26] Fourteen chapters form this sixth part. The first chapter begins, in fol. 57, col. *a*, with this sentence: "Cum omnia finem appetunt, ideo de finibus potentiarum activarum et passivarum est equaliter determinandum ut, cum natura scire desideramus, in istis potentiis activis et passivis, veritatem, que finis est, attingamus." The last chapter, which is not numbered, ends at fol. 70, col. *b*.

[27] The seventh part has eighteen chapters, of which only three—Chapters I, XV, and XVI—are numbered. The first chapter begins on fol. 70, col. *b*, with these words: "De primo principio et nobilissimo motore…" The last chapter ends at the bottom of col. *c* of fol. 85.

[28] The eighth part, which begins with the col. *d* of fol. 85, consists of eighteen unnumbered chapters. The beginning of the first chapter is: "De actione et de motu naturali corporum taliter exposito…" The end of the last chapter is at fol. 112, col. *a*

[29] The ninth part comprises forty unnumbered chapters. It begins with these words: "De virtute animali cognitiva que post vegetativam ponitur…" The last chapter ends at the bottom of col. *a* of fol. 141. It is followed by the table that occupies the three other columns of fol. 141.

[30] This tenth part is missing in the manuscript that we consulted.

this double presentation does not prevent all incertitude regarding the sense of the author.

The first of the two discussions we have just mentioned is in the second part of the *Summa*; it is preceded by a general study on the intensity of qualities.

> We need to examine,

the author says[31],

> how primary qualities can intensify or relax; concerning this matter, there are many opinions.

He devoted, in fact, five chapters[32] to presenting three opinions that he will reject. Then he goes on to say:[33]

> The fourth opinion, which is necessary to hold, is this: No quality becomes more intense or less intense; it is the subject where this quality resides that becomes more intense or less intense by an acquisition or loss of quality, just as quantity increases or decreases by the affixing or cutting off of parts.

Neither Richard of Middlelon nor William of Ockham had formally articulated this doctrine that John Dumbleton develops in five chapters[34].

Following this development he addresses the problem that interests us particularly:

> These principles put forth, we have to consider,

he said[35],

> how difform qualities are intense or attenuated; to see how the latitude of these qualities, in its nature, properly and by itself, is less intense; to research whether it corresponds to some degree that is intrinsic to it.
>
> There are, on this subject, three opinions.
>
> The first says that the intensity of a latitude or difform quality depends on how it is distributed in its subject; as a result of this extension, it can be equaled in intensity to each of the degrees that are in it.
>
> The second claims that, properly and of itself, it corresponds to the average degree, viz., to its half.
>
> The third says: All qualities of the same species, either uniform or difform, are latitudes, viz., qualitative distances, and are by their nature of the same intensity.

According to the scholastic custom, the views that are listed first are those that the author proposes to reject.

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31 Johannis de Dumbleton *Summa*, Pars II, cap. XXI^{m}; ms. cit., fol. 21, col. *c*.

32 Johannis de Dumbleton *Summa*, Pars II, capp. XXI^{m}, XXIl^{m}, XXlI^{m}, XXlV^{m} et XXV^{m}; ms. cit, fol. 20, col. *c*, in fol. 21, col. *c*.

33 Johannis de Dumbleton *Summa*, Pars II, cap. XXVI^{m}; ms. cit., fol. 21, col. *c*.

34 Johannis de Dumbleton *Summa*, Pars II, capp. XXVI^{m}, XXVII^{m}, XXVIII^{m}, XXIX^{m} et XXX^{m}; ms. cit., fol. 21, col. *c*, to fol. 22, col. *d*.

35 Johannis de Dumbleton *Summa*, Pars II, cap. XXXI^{m}; ms. n° 16146, fol. 22, col. *d*. — Cf. ms. n° 16621, fol. 174, r° (Entitled: De correspondentia difformis cum uniformi).

Nothing equals the weakness of the argument[36] by which John Dumbleton claims to refute the second opinion; to give an idea of it, we cite one of the arguments that seems convincing to him[37].

> No movement of difform quality can provide the acquisition of a sum equal to that which would be gained with the help of uniform motion to which this difform motion results in its most intense extremity, supposing that for the movement considered, a uniform part terminates the difform part. Such movements are not and cannot be equivalent in quality, if the quality is necessarily weakened by the quantity or extension; the first of the two movements is necessarily weaker than the second, because the speed of a movement is measured by the acquired space.

The impatient reader cannot hold back from exclaiming: But what does that prove? The Parisian master to whom we owe some excerpts of the *Summa* obviously felt this impatience. After having reproduced what we just quoted, he hastily wrote[38]:

> We prove, however, that a uniformly difform motion suffices for covering as much space as the uniform movement defined by its average degree.

His very confused demonstration ends with these words:

> This movement is equivalent to its average degree because [when replaced by uniform motion] it is as much augmented toward its weakest extremity as it is diminished toward its strongest extremity.

This sentence is a brief but clear allusion to the demonstration of Nicole Oresme, which annotator knew, as discussed earlier.

John Dumbleton now comes to demonstrating the opinion he holds true and that, in his enumeration, was third[39]. In this regard he poses some clarifications that, pushed further, dispelled many misunderstandings and brought the thought of the master of Oxford in accordance with that of Nicole Oresme.

> We now explain,

he said,

> the third opinion, which is the truth. Regarding this view we must show that, in accordance with custom, we understand in two different ways this proposition: There is a latitude in a difform quality. One of these senses is the proper sense, and the other the improper sense.
>
> We speak in the proper sense when we mean to say that it contains both, intensively, without relating to any extension or some magnitude taken in the matter; when we simply want to say that there is such a qualitative distance between the degrees by which the movement of alteration is evaluated, such that a line of two feet is a line whose ends are spaced two feet; in this sense the latitude considered, taken in its entirety, is the highest degree of its kind.

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36 Johannis de Dumbleton *Summa*, Pars II, cap. XXXII$^{\text{m}}$; ms. n° 16146, fol. 23, col. *a*.

37 John Dumbleton, *loc. cit.*, ms. cit., fol. 23, col. *b*. — Cf. ms. n° 16621, fol. 175, r°.

38 ms. n° 16621, fol. 175, v°.

39 Johannis de Dumbleton *Summa*, Pars II, cap. XXXIII$^{\text{m}}$. ms. n° 16146, fol. 23, col. *b*; ms. 16621, fol. 176, r°.

It is, on the contrary, improper to speak of the latitude of a quality whose parts are unequally intense within the subject; and it is only in this way that those who speak of it, considering a difform quality, say it has a certain intensity, that it acquires a special intensity according to the variable manner in which it is coextensive with the subject, or that it is equivalent to some degree intrinsically proper to it.

What John Dumbleton called here *latitude properly so-called* of a quality is what Nicole Oresme also called *latitude*; what the master of Oxford calls *latitude improperly so-called* is what the master of Paris called the *measure* of the quality. If the one had posed these distinctions with the same sharpness as the other, his views would have become much clearer and more easily acceptable.

One would have admitted then, as perfectly clear, what he says regarding the latitude properly so-called[40]:

As a line of two feet, however curved it is, provided it experiences neither rarefaction nor condensation, remains in itself equally long, because it always contains two feet put end to end; so does a difform heat, in any manner that it is extended within the subject, if it keeps an equal latitude, neither become more nor less intense. As all lines that contain an equal distance between their ends are equal in length to the first of these, so all the qualities of the same species which contain, in themselves, the same qualitative distance are equally intense and exist under the same degree—because this degree is nothing else than this qualitative distance, and the length of a line is the distance between the ends of the line.

Latitude thus understood, it is no longer surprising to hear John Dumbleton declare[41] "that a uniformly difform quality is not equal to its average degree."

After the explanations we have collected in the *Summa*, we will not accuse the author of contradicting himself, he who stated the proposition we have just mentioned, where we will see him, in the part of his work where he discusses local movement, devote two chapters to demonstrating that "the latitude of a uniformly difform motion corresponds to its average degree"[42]. The author takes the word latitude here in the sense that he himself declared unfit; he identifies it with the space that the mobile travels during the movement.

He develops at length[43] a first demonstration where he ridicules the inevitable *Sortes*; he is not satisfied, because he gives a second[44]; but the second demonstration assumes that in the first half of the duration, *Sortes*, by its uniformly difform motion, traveled the quarter of the way he travels in the whole duration; it is right to suppose what is in question, as Dumbleton notes[45]. "*Vos habentes dicta Magistri Nicolai Orem, comparate*," our copyist said; he cannot help to make this comparison for his own account; in the margin of the *calculationes* of Dumbleton, he sometimes

[40] ms. n° 16146, fol. 23, col. *c*; ms. n° 16621, fol. 176, r°.

[41] John Dumbleton, *ibid.*

[42] Johannis de Dumbleton *Summa*, Pars III, cap. IX^{m}; ms. n° 16146, fol. 29, col. *c*; ms. n° 16621, fol. 117, v°.

[43] Johannis de Dumbleton *Summa*, Pars III, cap. X^{m}; ms. n° 16146, fol. 29, col. *c*; ms. n° 16621, fol. 118, r° et v°.

[44] Johannis de Dumbleton *Summa*, Pars III. cap. X^{m}; ms. n° 16146, fol. 29, col. *d*; ms. n° 16621, fol. 119, r°.

[45] Ms. n° 16146, fol. 30, col. *a*; ms. n° 16146, fol. 119, v°.

draws his own figure to clarify them; much more, in some lines which accompany a geometric sketch[46], he summarizes the demonstration given by Oresme of this proposition, which appears to be a stumbling-block for the whole Logic of Oxford.

The *Regulæ solvendi sophismata* and the *Probationes* of William Heytesbury

We said, in Article XXI [section 27], which chapters formed the *Regulæ solvendi sophismata* of William Heytesbury. The chapter devoted to local movement is the one that should detain us here.

With Thomas Bradwardine, Hentisberus holds as certain[47] that the speed of a body animated by a rotational movement is nothing other than the speed of the most quickly moving point; his authority contributed greatly to spreading and strengthening this view.

This opinion, however, does not prevent him from admitting the following proposition: When in a movement the speed increases with time in such a way that it is uniformly difform, the mobile moved with this movement travels, in a given time, the same path as if it had moved uniformly with the speed it gained during half this time.

He repeats this proposition twice[48]; he uses it as an indisputable truth; but he gives it, in his *Regulæ*, no demonstration.

The most important among the propositions that William Heytesbury has invoked in his *Regulæ*, are demonstrated, as we have said, in a pamphlet entitled *Probationes conclusionum in regulis positarum*; so it is, in particular, with the proposition we are treating. The demonstration that Heytesbury presents on this occasion[49] is, roughly, the first that Dumbleton has given, which he mixed with considerations on the intensity of forms; it is also accompanied by lemmas and corollaries, many of which are almost identical to those in the first *Doubt of Paris*; so it would seem that Heytesbury, to construct his deduction, has combined information borrowed from the *Summa* of John Dumbleton with other information taken from these *Dubia parisiensia* that Swineshead might have added to the treatise *In primo motore*. So are we more and more strongly tempted to see, in this evaluation of the path traveled by a mobile moved by a uniformly difform motion, a loan that the University of Oxford had contracted with the University of Paris.

The writings of William Heytesbury are worthy of remark in that, alongside the notion of the speed of a varied movement, we see in them, though still confused, the notion of acceleration of such a movement.

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[46] Ms. n° 16621, fol. 118, v°.

[47] *Tractatus* Gulielmi Hentisberi *de sensu composito et diviso…* Venetiis, 1494; fol. 38, col. *d*.

[48] Hentisberi *Op. laud.*, ed. cit., fol. 40, col. a et col. *d*.

[49] Gulielmi Hentisberi *Probationes conclusionum in regulis positarum*. Conclusiones declarative de motu locali. cap. 1, art. 9 (*Tractatus* Gulielmi Hentisberi *de sensu composito et diviso…* Venetiis, 1494; fol. 198, col. *d*, and fol. 199, col. *a*)

In his treatise *De tribus prædicamentis*, William constructed various fallacies related to the acceleration (*intensio*) of movement; to resolve them, he distinguishes[50] between the *latitudo motus*, which is velocity, and the *velocitas intensionis vel remissionis motus*; the one is evaluated by the acquisition or loss of the other; this *velocitas intensionis vel remissionis motus* is none other than positive or negative acceleration.

On this subject, he wrote the following remarkable passage[51]:

> One body can move more quickly and another more slowly; one body can accelerate (*intendere*) its movement and another slow it down; so it happens that one mobile accelerates its movement faster (*intendit velocius*) and another more slowly; the same can happen to bodies that slow down their movement. Just as, therefore, in a mobile which starts from rest, one can imagine a latitude of speed (*latitudo velocitatis*) rising indefinitely, so one can imagine there is a latitude of acceleration or deceleration (*latitudo intensionis et remissionis*) according to which a mover can accelerate or slow down its movement with an infinitely variable speed or slowness. This latitude then behaves with regard to the latitude of movement as the movement behaves with regard to the magnitude or quantity that may be traversed successively in a truly continuous manner (*Et illa latitudo consimiliter se habet respectu latitudinis motus sicut se habet motus respectu magnitudinis et quantitatis continuæ vere pertransibilis successive*).

Often one defines acceleration as the speed of the speed; thus, one merely repeats the idea that we have heard William Heytesbury express.

Moreover, in his *Probationes conclusionum*, he never speaks of a uniformly difform motion, but of a movement whose intensity increases evenly (*uniformiter intenditur*); nor does he speak of a uniformly difform latitude, but of a *latitudo uniformiter acquisita vel deperdita*; the idea of uniform acceleration seems to precede in his mind that of uniformly variable motion.

But this difference in language that can be noted here between the *Regulæ solvendi sophismata* and the *Probationes conclusionum* might suggest a doubt: Are both of these writings by William Heytesbury?

The *Probationes* constitute a commentary followed by the *Regulæ*. That the Chancellor of Oxford is commentating on himself is already a just matter of astonishment. It is another, and much more powerful, to see a vast difference between the ways of reasoning and writing which the same author used in composing the *Regulæ* or the *Probationes*. The *Regulæ* are a type of that messy, tangled, sophistical argumentation that was fashionable at Oxford, from which Heytesbury has not departed in his other writings; by its order, clarity, sobriety, and rigor, the *Probationes* are reminiscent of the writings of Buridan and Albert of Saxony; most of the time, they borrow their reasoning and style from these masters. It seems very difficult for us not to regard the *Probationes conclusionum* as a commentary composed by a Parisian master, by any disciple of Albert of Saxony, on the *Regulæ solvendi sophismata* due to William Heytesbury.

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50 *Tractatus* Gulielmi Hentisberi *de sensu composito et diviso…* Venetiis, 1494; fol. 42, col. *d*.

51 William Heytesbury, *loc. cit.*, ed. cit., fol, 44, col. *b*.

Regardless of the assumption that we have to make, the Italian commentators take care to make the information relating to the idea of acceleration, which the Chancellor of Oxford gave, more precise.

The *Tractatus de sex inconvenientibus*

Never, at the University of Oxford, has the evaluation of the path traveled in a uniformly varied movement been in such a clear and precise a form as what Nicole Oresme gave it by using coordinates.

Take, for instance, the *Tractatus de sex inconvenientibus*, whose author wrote after Heytesbury and therefore certainly after Oresme.

This treatise belongs to the same family as the *De primo motore* of Swineshead and the *Summa* of Dumbleton; to convince ourselves, we need only browse the contents of the entire book, which one of the manuscripts from the National Library conserves[52].

Here is the table where several principal questions are accompanied by articles, devoted to related topics, which are inserted there:

Prima quæstio: Utrum in generatione formæ sit certa ponenda velocitas.
Articulus I: Utrum generans tantum loci contribuat quantum formæ.
Art. II: Utrum ex coloribus extremis intermedii generentur colores.
Art. III: Utrum cælestia corpora generent qualitates primarias, lumine mediantæ.
Secunda quæstio: Utrum in motu alterationis velocitas sit signanda vel tarditas.
Art. I: Utrum magnes suppositum sibi ferum sufficiat attrahere.
Art. II: Utrum alteratio medii luminosi sit subita in distanti.
Art. III: Utrum quodlibet alterans in agendo repatiatur.
Tertia quaestio: Utrum augmentatum continuum in augendo velocitet motum suum.
Art. I: Utrum rarefactio sit possibilis.
Art. II: Utrum rarefactio sit motus ad aliquam quantitatem.
Art. III: Utrum rarefactio sit per rarum et densum.
Quarta quæstio: Utrum in motu locali sit certa servanda velocitas.
Art. I: Utrum velocitatio motus gravis sit ab aliqua causa certa.
Art. II: Utrum velocitas motus sphæræ cujuslibet penes punctum vel spatium aliquod attendatur.
Art. III: Utrum velocitas omnis motus uniformiter difformis incipiens *a non gradu* sit æqualis suo medio gradui.
Quinta qœestio: Utrum cælum possit suo motu et lumine inferiora corpora transmutare.
Quæstio sexta: Utrum corpora gravia et levia in suis motibus requirant medium.
Quæstio septima: Utrum omne corpus naturale habeat locum naturalem.
Quæstio octava: Utrum tempus sit consequens motum.
Quæstio nona: Utrum tempus sit numerus motus secundum prius et posterius.
Quæstio decima: Utrum motus reperiatur in tribus generibus tantum.
Quæstio undecima: Utrum omnis motus sit de contrario in contrarium.

[52] Bibl. Nat., fonds latin, ms. n° 6559, fol. 194, v°.

As we said in article XXI [section 27], the two manuscripts that we had on hand are incomplete; one[53] contains the first four questions; the other[54] presents, in addition, the beginning of the fifth question.

It is the fourth question that will, for a moment, hold our attention.

The second section is devoted to the consideration of the problem that concerned almost all Scholastics of Oxford: What is meant by the speed of a body moving with a rotational movement? The author of the *Treatise of the Six Inconveniences* lists the various opinions that came before him. He cited in particular the opinion of Magister Ricardus of Versellis or Uselis: The speed of the radius of a circle or a portion of this radius, in rotation around the center, is the speed of the midpoint of the segment which rotates. But he does not consider this opinion as demonstrated by the master who proposes it; he prefers the position taken by Master Thomas Bradwardine in his *Tractatus proportionibus*: The speed of the body moving with a rotational movement is the speed of the point of the body that is farthest from the axis.

The solution that the author of the *Treatise of the Six Inconveniences* gave to this first problem contrasts with what he will give in his third article to this other problem:

> Is the speed of all uniformly difform local movement equivalent to its average degree?

Those who want to understand the extreme difference which distinguishes, at that time, the Logic of Oxford from the Logic of Paris could not find anything more proper than the comparison between what the *Tractatus de sex inconvenientibus* and the *Tractatus de difformitate qualitatum* said about this problem. The argument of the first of these treatises is but a pitiful heap of *sophismata*. It takes as its starting point this supposed dilemma[55]:

> If the speed of any local movement is not equivalent to its average degree, it is equivalent to its most intense degree.

By an accumulation of *inconvenientia*, it makes the second position untenable and concludes that the former is correct.

So this author, coming after William Heytesbury, has made no progress in the demonstration of this proposition[56]:

> In any uniformly difform motion that starts at the zeroth degree and grows incessantly, the space traveled during a certain time is equal to that which it would travel at its average degree of speed during the same time or during an equal time.

Quite the contrary! The semblances of demonstration of the *Dubia parisiensia* or of John Dumbleton, as inadequate as they were, offered, however, a reflection of truth; it is vain to seek this reflection in the obscure dialectic of the *Tractatus de sex inconvenientibus*.

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[53] Bibl. Nat., fonds latin, ms. n° 6527.

[54] Bibl. Nat., fonds latin, ms. n° 6559.

[55] Bibl. Nat., fonds latin, ms. n° 6559, fol. 38, col. *c*.

[56] Ms. cit., fol. 39, coll. *a* et *b*.

The opuscule entitled: *A est unum calidum*.

The author of the *Treatise of the Six Inconveniences* read the *Tractatus de figuratione intensionum* of master Nicole Oresme, but had he read it indeed? If so, he took so little fruit from this reading that nothing in his writings recalls it. But the School of Oxford will introduce other works in which the influence of Nicole Oresme left a recognizable mark.

In a manuscript preserved in the National Library[57], a certain John assembled some of the most celebrated treatises on the *Sophismata*; the *Sophismata* of Albert of Saxony occupy the beginning of the collection; then come the *Sophismata* of Clymelon; its copy was completed the Monday of Septuagesima in the year MCCCLXXXIXI (*sic*). To these copies, probably made in Paris, John joined a notebook, probably from Oxford and wrote, like the table he put at the end of his work[58], *in littera anglicana veteri*; this book contains the first thirty sophisms of Heytesbury; the last two have been transcribed by John.

However, immediately after the *Sophismata* of Clymeton and before the *Sophismata* of Heytesbury, this collection presents[59], written in the hand of John, a series of twenty-two sophisms. No authorial name is attached to this treatise, which does not have a title; it starts immediately with this statement of the first sophism:

> *A est unum calidum per totum quod per horam alterabitur e gradu uniformi, et tamen per illam [horam] nec alterabitur uniformiter quoad tempus nec quoad partes subjecti.*

The first words of this first sophism served as the title of the entire collection, as evidenced by the point[60] by which John ends his transcript:

> *Explicit iste liber qui intitulatur A est unum calidum. Deo gratias.*

This collection of sophisms is a perfect model of the kind of logic that was in vogue at the School of Oxford; the most litigious *calculationes* were all too common.

The twenty-second and final sophism is formulated thus[61]:

> *In aliquo instanti, extremo remissiori [subjecti] correspondent gradus summus caliditatis; et, immediate ante illud instans, terminabitur latitudo caliditatis ad non gradum.*

It is in discussing this sophism that the author is required to make the following proposition[62] whose demonstration ends his treatise:

> A mobile moves for one hour which was divided into proportional parts, and its movement is of this sort: Throughout the first proportional part, it moves with a certain speed; during the second proportional part, it continually accelerates its movement, to a double degree, so that at the end of the second proportional part, it reaches a speed double that of the first

[57] Bibliothèque Nationale, fonds latin, ms. n° 16134 (ancien fonds Sorbonne, ms. n° 848).

[58] Ms. cit., fol. 146, col. *a*.

[59] Ms. cit., fol. 73, col. *b*, to fol. 80, col. *d*.

[60] Ms. cit., fol. 80, col. *d*.

[61] Ms. cit., fol. 79, col. *d*.

[62] Ms. cit., fol. 80, col. *b*.

> part; during the third proportional part, it moves continuously in a uniform manner with this double degree of speed; at the beginning of the fourth part, it begins to accelerate its movement, and, during this fourth part, it continually increases its speed in a uniformly difform way so that it has at the end a speed double of what it was in the third part and four times of that which corresponded to the first part; during the fifth proportional part, it moves with a uniform speed; during the sixth it uniformly accelerates its movement, as before, up to a double speed; during the seventh, it moves uniformly; and so alternately without end. I say that in the whole time, the mobile will travel a path that is three and two-thirds times the path traveled in the first proportional part.

We recognize one of the problems solved by Nicole Oresme in his *Tractatus de figuratione intensionum*. The solution given by the master of Oxford is equivalent, needless to say, to that given by the Parisian master; we could say more exactly that it is, fundamentally, identical; but Oresme made, in presenting it, a very happy use of the representation by coordinates; the English logician does not use this geometric representation; he wants his deduction to retain a purely arithmetic aspect; so he translated into arithmetic language the geometrical reasoning that Oresme gave.

The development of this argument requires, of course, the evaluation of the space that a mobile travels during a certain time when it moves with a uniformly varied motion; all what we have said shows that this assessment was very familiar to the logicians of Oxford; hence our author confines himself to recalling it as as banal truth:

> *Ipsa est uniformiter difformis; ergo est æqualis suo gradui medio.*

The *Liber calculationum* of Riccardus of Ghlymi Eshedi

We finally come to the writings engendered by the logic of Oxford which had, perhaps, the strongest and most widespread popularity, to the book whose author, regarded as the Calculator *par excellence*, lost his real name of Riccardus of Ghlymi Eshedi to take, we know not how, that of Swineshead or Suiseth.

The treatise that will occupy us is divided into chapters; in the draft manuscript we had on hand and in the oldest printed editions, these chapters are untitled; the particular edition published in Pavia in 1498 by Franciscus Gyrardengus attributed them to him; here is the complete list of these chapters:

I. *De intensione et remissione.* — II. *De difformibus.* — III. *De intensione elementi.* — IV. *De intensione mixtorum.* — V. *De augmentatione.* — VI. *De reactione.* — VII. *De potentia rei.* — VIII. *De difficultate actionis.* — IX. *De maximo et minimo.* — X. *De loco elementi.* — XI. *De luminosis.* — XII. *De actione luminosi.* — XIII. *De motu locali.* — XIV. *De medio non resistente.* — XV. *De medio uniformiter difformi.* — XVI. *De inductione gradus summi.* — XVII. *De acquisitione alterationis.*

Reading of this table alone shows the analogy between the plan of the treatise of the Calculator and those of the three books described above: the *Tractatus de primo motore* of Swineshead, the *Summa* of John Dumbleton, and finally the *Tractatus* *de sex inconvenientibus*; we are faced with four treatises of the same family. The comparison between the table of contents of the *Liber calculationum* and that of the *Tractatus de primo motore* also suffices to demonstrate, in the absence of direct evidence, that these two works cannot be of the same Swineshead; a single author does not write two books so similar in purpose and so different in their composition.

The *Liber calculationum* presents us, having reached their full development, all the faults of the School of Oxford; the sophistical discussions form the constant background; they impressed the hairsplitters, for whom Philosophy had no other object than to furnish matter for dispute; in this book they found an arsenal of tricks and chicaneries; a mediocre and unoriginal book, moreover, where one cannot find a thought that has not been repeatedly discussed, turned over, and examined from all angles by the doctors of Paris or Oxford, the *Liber calculationum* is the work of a senile Science that starts to ramble; the prodigious success that this work will meet in Paris and the great vogue which it will enjoy from all Italian masters really signaled the decrepitude of Scholasticism; the Humanists will not be deceived, and when they wish to riddle the universities and teach what they teach, they will know where to aim; the *calculationes* of Suiseth will be the vulnerable point to which they will preferentially direct their fire.

However, the annoying remarks that an old man rehashes may be good to hear and precious to remember; they transmit to us the knowledge acquired when the old man was young; they are the tradition without which no progress would be possible; even in this *Liber calculationum*, whose complicated quibbles they rejected, the students of the Renaissance would have found precious truths, the heritage of the Nominalist masters of the 14th century; they had recognized in it, especially, the legacy of Nicole Oresme.

Indeed, like the collection of sophisms entitled *A est unum calidum*, the treatise of Riccardus of Ghlymi Eshedi bears the recognizable trace of the influence of the *Tractatus de figuratione intensionum*.

In the chapter *De difformibus*, which is the second of the whole work, the author is led to formulate[63] the following proposition:

> If we assume that the first proportional part of a certain quality had a determined intensity, the second proportional part had a double intensity, the third had a triple intensity, and so on to infinity, the whole would have an [average] intensity precisely equal to that of the second proportional part; which, first of all, does not seem true, because this quality seems infinite.

This proposition is one of those that Oresme established in the treatise *De difformitate qualitatum*[64]. The demonstration given by Riccardus of Ghlymi Eshedi is the translation into arithmetical language of the geometrical demonstration of Oresme; the Master of Oxford, indeed, like all his countrymen, refused to use the coordinate representation; but the translation is literal, at the point that the reader is inclined to draw the figure illuminating the deduction; and this is what a reader of the manuscript

[63] Bibl. Nat., fonds latin, ms. n° 6558, fol. 6, col. *b*. — Subtilissimi Doctoris Anglici Suiset *Calculationum Liber*, Paduæ (ca. 1480), 5th printed fol., col. *d*.

[64] See § XVIII [chapter 26].

preserved in the National Library did; but reading the text easily shows that the drawing of this figure was not at what the author intended.

The chapter *De difformibus*, where the problem we just discussed is treated, begins with the examination of this question: Does a uniformly difform latitude corresponds to its average degree? The author reproduces in these terms[65] the argument which concludes in the affirmative:

> When one takes such a latitude or such a heat, when one attenuates one half to the average degree, and when, equivalently, the intensity increases in the other half to the average degree; the whole thing becomes neither more nor less intense, because it acquires on one side just as great a latitude as it loses on the other side; and now it is uniformly intense in a degree equal to the average degree; it thus corresponds to this average degree.

We will not dwell on the endless discussion, the entangled sophistries, whereby the Calculator challenges the general value of this proposition; it suffices to remark: He did not call this proposition into question when the latitude considered is the speed of a local movement; he invoked it then as a commonly accepted truth.

For example, treating in the 15th chapter the movement of a mobile in a resisting medium, the Calculator expresses himself thus[66]:

> If the mobile uniformly accelerated its movement, as it had commenced to accelerate from the zeroth degree, it would travel in the second half of time three times farther than in the first.

This phrase supposes that we know the law that links, in a uniformly varying motion, the path traversed to the time taken to travel it.

No one was ignorant of this law at the School of Oxford when Swineshead, John Dumbleton, and William Heytesbury taught there; among the disciples of these masters, no one was ignorant of it. Was it discovered at Oxford or, rather, did it come from Paris, like the “doubts” by which the *Treatise on the First Mover* of Swineshead seems to have been completed? This is a question to which any peremptory answer would certainly be strongly ill-founded. In any case, ignorant or contemptuous of the representation by coordinates, the masters of Oxford were not able to give to their arguments in favor of this proposition the sharpness of the deductions of Oresme. Not that these deductions are, here, truly demonstrative; they assume, in fact, this serious postulate: When in a rectangular coordinate system the times are taken for abscissas and the speeds for ordinates, the area of the figure represents the path traversed by the mobile. But to justify this assumption, it will be necessary to resort to infinitesimal calculus; until the invention of this calculus, Physics will have, of the law of uniformly varied motion, no better demonstration than that of Oresme.

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[65] Ms., cit., fol. 5, col. *a*; ed. Paduæ, ca. 1480, fol. sign. a 5, col. *d*.

[66] Ms. cit., fol. 58, col. *a*; ed. Paduæ, ca. 1480, fol. sign. k 2, col. *d*.

Chapter 29
How the doctrines of Nicole Oresme spread in Italy

We saw what Nicole Oresme taught in Paris regarding the latitude of forms; we saw how Albert of Saxony and especially Marsilius of Inghen had used this teaching; then we tried to trace the importance that the doctrine of the latitude of forms took at Oxford and the particular form which had coated the excessive habit of discussing sophisms; we will try now to assess how it spread in the teaching of Italian Universities.

The mathematical theories developed regarding the intensity of forms did not all at once invade the Universities of Padua and Bologna; similar to a tide, they advanced through a succession of waves; a first wave brought the Parisian ideas of Nicole Oresme; a second wave pushed the sophisticated dialectic of William Heytesbury; a third wave brought, into all the schools, the complicated arguments of the Calculator.

The main initiator of the Italian Universities in the Logic of Paris seems to have been Paul Nicoletti of Venice, who died in Padua on 15 June 1429. Also, we find in his writings indubitable marks of Oresme and his disciples.

In his commentary on the *De generatione et corruptione*, Paul of Venice cites[1] very frequently the names of Jean Buridan and Marsilius of Inghen. In particular, he knows and discusses[2] the opinion of these masters, according to which, in an unevenly heated body, the latitude of heat and the latitude of cold have a sum whose value is the same in all parts of the body; but to present this theory, which he rejects, he does not use the geometric figuration that Oresme had imagined and that Marsilius adopted.

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[1] *Expositio Magistri* Pauli Veneti *super libros de generatione et corruptione Aristotelis. Eiusdem de compositione mundi cum figuris.* Colophon:

> Impressus Venetiis mandato et expensis nobilis Viri Dornini Octaviani Scoti Civis Modoetiensis duodecimo kalendas Junias 1498. Per Bonetum Locatellum Bergomensem.

Fol. 33, col. *a*; fol. 34, col. *a*; fol. 35, col. *a*; fol. 43, col. *b*; fol. 45, col. *b*; fol. 49, col. *d*; fol. 50, col. *a*; fol. 54, col. *a*.

[2] Pauli Veneti *Op. laud.*, fol. 72, col. *c*; fol. 84, col. *c*; fol. 87, col. *b*.

The voluminous *Expositio super octo libros Physicorum* published by Paul of Venice[3], is dated; it was completed on 30 June 1409. Almost always a loyal supporter of Averroist Physics, the author of this book shows, however, that he also knows Parisian Physics. Thus only twice[4] we will hear him invoke this rule: A uniformly difform latitude corresponds to its average degree.

The *Summa totius Physicæ* of Paul of Venice is undoubtedly posterior to the *Expositio super octo libros Physicorum*; in a large number of questions, the author is now seen converted to the doctrines of Paris; we will not be surprised to learn that invokes[5], as an undisputed truth, this rule:

> Omnis latitudo uniformiter difformis correspondet suo gradui medio.

Reading the *Summa*, like that of the *Expositio*, we learn that the knowledge of the rule of Nicole Oresme was common among the auditors of Paul of Venice, around the year 1420. A manuscript, indeed, copied in 1421 in Rimini by J. of Beylario, already contains the *Summa naturalium*, *De generatione et corruptione*, *Logica*, and *De Cælo et Mundo* of Paul of Venice[6].

Biagio Pelacani, Blaise of Parma said, was roughly contemporary with Paul of Venice; doctor of the University of Pavia in 1374, he taught astronomy at Bologna from 1378 to 1384; he then professed in Padua until 1388 and again in Bologna; in 1404, 1406, and 1407, we find him at Pavia; in 1407 he taught at Padua[7], but he leaves his chair that same year; he is thought to have gone to Paris about this time; from 1408 to 1411, he resumed his chair at Padua; on 15 May 1409, he was among the judges who conferred on Prosdocimo de' Beldomandi the title of master of arts[8]; he died in Parma, his hometown, on 23 April 1416.

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[3] *Expositio* Pauli Veneti *super octo libros phisicorum Aristotelis necnon super comento Averois cum dubiis eiusdem* Colophon:

> Explicit liber Phisicorum aristotelis: expositus per me fratrem Paulum de Venetiis: artium liberalium et sacre theologie doctorem: ordinis fratrum heremitarum beatissimi Augustini. Anno domini. Mccccix. die ultima mensis Junii: qua festum celebratur commemorationis doctoris gentium et christianorum apostoli Pauli. Impressum Venetijs per providum virum dominum Gregorium de Gregoriis. Anno nativitatis domini. Mccccxcix. die xxiij mensis Aprilis.

[4] Pauli Veneti *Op. laud.*, col. d fol. which immediately follows the fol. sign. Oiiij; col. *d* of the fol. sign. Pij.

[5] Pauli Veneti *Summa* totius Physicæ, Pars I, cap. XXXVIII.

[6] *Catalogue de Manuscrits, autographes, incunables et livres rares* of the library T. de Marinis and G., Florence, 1911, p. 23, no. 71. — On the verso of fol. 174 of the ms., we read:

> Scriptum Arimini per me fratrem Johannem de beylario colonie provincie in studio Ariminj sub anno domini M°cccc°xxj°. ultima die decembr. completum. Finito libro sit laus et gloria christo.

[7] Antonio Favaro, *Intorno alla vita ed alle opere di Prosdocimo de' Beldomandi* (*Bulletino di Bibliografia e di Storia delle Scienze matematiche e fisiche* pubblicato da B. Boncompagni, t. XII, 1879, pp. 24-25).

[8] Antonio Favaro, *Op. laud.*, p. 22.

We owe to Blaise of Parma the *Quæstiones super tractatu de latitudinibus formarum*. Twice, in 1486 and in 1505, these *Questiones* were printed[9] following the *Tractatus de latitudinibus formarum* falsely attributed to Nicole Oresme. Recently, they have been studied by F. Amodeo[10].

There are three of the questions:

1. Is the latitude of any form necessarily uniform or difform?
2. Is there a uniformly difform form that starts *a non gradu*?
3. Does all uniformly difform latitude correspond to its average degree?

No, Blaise of Parma responds to the first question; all forms are not necessarily uniform or difform. Taking, in effect, the scholastic notion of form in all its generality, he distinguishes forms into essential and accidental; depending on whether it is susceptible to attaining various degrees or not, an accidental form is, in turn, *gradual* or *not gradual*; it can be *divisible* or *indivisible*; only accidental, gradual, and divisible forms are susceptible to being uniform or difform.

The treatise *De latitudinibus formarum* composed *ad mentem Oresme* considers only forms endowed with longitude and latitude, susceptible, consequently, to being represented by a plane figure; Blaise of Parma rises to a greater generality; he also considers shapes that have length, width, and depth, forms that are represented using three-dimensional figures; Nicole Oresme, we have seen, had long considered such forms; Pelacani appears to us here in the guise of a man who has read the *Tractatus de difformitate qualitatum* and who uses it to complete the *Tractatus de latitudinibus formarum*.

A similar impression emerges from reading the second question.

The *Tractatus de latitudinibus formarum* gave the following definition of uniformly difform latitude:

> *Latitudo uniformiter difformis est illa cujus est æqualis excessus graduum inter se æqualiter distantium.*

Blaise of Parma criticizes this definition, as well as two other definitions whose authors he does not name, and concludes by proposing the following:

> *Latitudo uniformiter difformis est latitudo difformis cujus quarumlibet trium partium extensive æqualium ab invicem æque distantium situantur ut primæ ad secundam sicut secundæ ad tertiam æquales intensive sunt excessus; talis est primæ ad secundam sicut secundæ ad tertiam, loquendo de partibus totalibus quantitatis intensive.*

This definition clearly resembles what Oresme gave in the *Tractatus de difformitatibus qualitatum*; but it attained neither clarity nor generality.

The third question that Blaise of Parma addressed is the one that interests us most; according to the analysis Amodeo gives[11], the thought of Pelacani is very confusing:

[9] These two editions have been described above, in § XIX [section 26].

[10] F. Amodeo. *Appunti su Biagio Pelacani da Parma* [*Atti del IV Congresso internazionale dei Matematici (Roma 6-11 Aprile 1608)*, vol. III, pp. 549-553.] — It is according to this work that we speak of the *Questions* of Blaise of Parma; we could not consult them directly.

[11] F. Amodeo, *loc. cit.*, p. 553.

> He poses, first, some premises that relate to the various classes of latitudes he characterized in the beginning; we do not believe it is necessary to follow them. Then he endeavors to develop very simple geometrical considerations and to demonstrate that the line joining the two sides of a triangle is half of the third side; that the parallelogram that has this line for sides and the third side of the triangle is equivalent to the triangle; that the triangle detached from the total triangle by this line is a quarter of the total triangle.
>
> He then formulates eight conclusions, the third of which we mention: In any uniformly difform latitude that begins *a non gradu* or terminates *ad non gradum*, the average degree is half of the most intense degree. We also mention the fifth conclusion: In all uniformly varying latitudes, there are an infinite number of parts that have the same average degree. These findings essentially seek to show that the average degree does not always exist in the form.

Of the rule: The uniformly difform latitude corresponds to its average degree, there is no question in the *Tractatus de latitudinibus formarum*. There is no doubt, in reading the *Tractatus de difformitate qualitatum*, that Blaise of Parma was aware of it; from this reading, however, one must, it seems, recognize its trace in the geometrical demonstration which he excessively diluted.

We learn, in any case, both by the teaching of Biago Pelacani as by the teaching of Paolo Nicoletti, that the Italian Universities, around the year 1420, were aware of the doctrines of Nicole Oresme; in particular, one knew the law that links, in a uniformly variable motion, the path traversed to the time taken to travel it.

The hesitations in the discussion of Blaise of Parma already seem to indicate the influence of the Logic at Oxford; this same influence has undoubtedly exercised some influence on a writer who was a contemporary of Pelacani, Jacopo da Forlì.

Giacomo della Torre, born in Forlì, and named, in the Latin writings of the 15th century, Jacopo da Forlì[12], is a doctor in Padua in 1402; he left this town for some time, returning in 1407[13]; in 1409 and 1411, he teaches medicine at the University; on 15 May 1409, he is, with Blaise of Parma, numbered among the examiners who tested Prosdocimo de' Beldomandi for Master of Arts[14]; on 15 April 1411, he is one of the judges who conferred the doctorate in medicine on the same Prosdocimo[15]; he died in Padua on 12 February of a year which, beginning at Easter, bore the date of 1413 and which should, today, be designated as the year 1414.

Jacopo da Forlì composed a treatise entitled *De intensione et remissione formarum*; the object of this treatise was to discuss and combat the doctrines that Walter Burley supported in a writing of the same title; also, the book of Walter Burley and of Jacopo da Forlì were printed together in Venice in 1496[16].

[12] We must not confuse the author of which we speak with Jacopo da Forlì who taught philosophy in Bologna in 1347.

[13] Antonio Favaro, *Intorno alla vita ed alle opere di Prosdocimo de' Beldomandi* (*Bulletino di Bibliografia e di Sloria delle Scienze matematiche e fisiche*, t. XII, 1879, pp. 27-28).

[14] Antonio Favaro, *Op. laud.*, p. 22.

[15] Antonio Favaro, *Op. laud.*, p. 23.

[16] This edition has been described in § XII [section 20].

To refute the opinions of Burley, Jacopo da Forlì uses[17] all that had been said, in the second half of the 14th century, on the latitude of forms, the degrees of this latitude, and the uniformity and difformity of qualities; many theories, dear to the physicists at Paris, are invoked by him; thus, regarding the coexistence of hot and cold in each point of an unevenly heated subject, he admits, which Paul of Venice did not, the opinion of John Buridan, which had so strongly seduced Marsilius of Inghen.

Jacopo da Forlì gives the following definition of uniformly difform quality:

> *Qualitas uniformiter difformis est illa cujus, quibuscunque partibus duobus datis sequalibus, per tantam distantiam excedit extremum intensius in una extremum remissius ejusdem, per quantam in alia extremum intensius excedit extremum remissius ipsius.*

Clearer than the definition proposed by Blaise of Parma, it is, fundamentally, identical to it.

Jacopo da Forlì wants this uniformly difform latitude to be as intense as the most intense degree that it contains or which serves as its term; "exactly," Luis Coronel remarks[18], "as Hentisber holds in his treatise on local movement, as a mobile moves with the same quickness as its most rapidly moved point". The position that Jacopo da Forlì holds is, as we saw in the previous article [chapter 28], the one that Swineshead held in his *De primo motore*. According to the fair observation of Luis Coronel, this position draws its main strength from this proposition: The velocity of a body moving rotationally is the speed of the point of this body that moves the quickest. We saw that this proposition, formulated by Bradwardine, won the support not only of the whole School of Oxford, but also of Albert of Saxony.

The influence of Oxford does not seem to have exerted only on Jacopo da Forlì, in pressing him to adhere to any particular opinion; it seems to have inspired in him, by a more general action, an immoderate taste for *calculationes*.

Jacopo da Forlì was a physician, and he wrote extensively on medicine. We have from him a commentary[19] of passages where the *Canons* of Avicenna treat embryology. But three books have especially made the name of Giacomo della Torre famous among physicians in the mid-16th century. These three works are: a commentary followed by questions on the *Aphorisms* of Hippocrates[20]; a commentary followed by

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[17] We could not consult the work of Jacopo da Forlì; what we say of it is extracted from the *Perscrutationes physicæ* of Luis Coronel; we had many chances to check the perfect accuracy of the information on this author.

[18] *Physicæ perscrutationes* magistri Ludovici Coronel Hispani Segoviensis; lib. III, cap.: *De difformibus*. Ed. Parrhisiis, 151, fol. LXVI, col. *a*.

[19] Jacobi de Forlivio *Expositio in Avicennse capitulum de generatione embrii ac de extensione graduum formationis fœtus in utero.* Hain, in his Repertorium bibliographicum, cites from this book two incunabula, one published at Pavia in 1479, the other in Bologna in 1485.

[20] Jacobi de Forlivio *Expositio in aphorismos Hippocratis*. The *Repertorium bibliographicum* of Hain cites, as prior to 1500, an edition without any typographical indication; two editions, with no indication of place or printer, one dated 1473 and the other from 1477; then the editions published in Pavia in 1485 and in Venice in 1490 and 1495. The one we consulted is entitled:

> *Super aphorismos*, Iacobi Foroliviensis *In Hippocratis aphorismos, et Galeni super eisdem commentarios expositio et quæstiones quamendatissimæ. Additis* Marsilii Sancta Sophia *in-*

questions on the treatise of Galen entitled Μιχροτέκνη[21], and finally a commentary and some questions on the first book of the Canon of Avicenna[22].

It is hard not to judge these medical treatises how Luis Vives did, and the judgment he gave is quite harsh:

> It is necessary to see,

he wrote about the decline of Medicine[23],

> the chicaneries and complications introduced by Jacopo da Forlì; they are no less thorny nor less useless than the discussions of Suicet; they yield to it neither in prolixity nor in ennui.

The *cavillationes*, the *tricæ* of which Luis Vives complains, are still, in the *Questions on the Aphorism of Hippocrates*, contained within certain limits; they spill over into the writings of Jacopo da Forlì devoted to Galen; there, the *calculationes* which had so strangely invaded and corrupted the Logic, Physics, and Theology of the School of Oxford began to take hold of Italian Medicine. We simply need to open the *Explanation of the Μιχροτέκνη of Galen* to read arguments like this:[24]

> Suppose that *Sortes* passes from A, which is the extreme degree of his health, to C, which is the extreme degree of the closest disease, of a fever, for example; let B be the degree equidistant from the two extremes A and C. It is obvious that before reaching B, *Sortes* will reach the average arrangement between A and B; it is also clear that once the degree B is acquired, it will acquire, before reaching C, the average arrangement between B and C…

This is the device of false rigor, the language futilely made up in mathematical style which renders his reading of Swineshead, Dumbleton, or the Calculator insupportable.

The *calculationes* could break into the field of Medicine if the concepts specific to this science were supposed measurable, if one pretended to express them in numbers, and if one attributed to health and disease latitudes divisible into degrees; Jacopo da Forlì attributes it to them thus:

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terpretationibus in Aphorismos eos, qui a Iacobo expositi no fuerant. Venetiis apud Iuntas MDXLVII.

[21] Jacobi de Forlivio *Super I, II et III tegni Galeni*. Also, an edition which has no typographical indication and was probably published at Padua or Venice, the *Repertorium bibliographicum*, mentions three other incunabula: Venetiis, 1470; Paduæ, 1475; Papiæ, 1487. The edition that we read is the following: Iacobi Foroliviensis Medici *Singularis expositio et quæstiones in artem medicinalem Galeni quæ vulgo techni appellatur quamemendatissime* (sic). Venetiis apud Iuntas MDXLVII.

[22] Jacobi de Forlivio *Expositio in primum librum Canonis Avicennæ*. Hain lists the following incunabula: edition published in Milan lacking a typographical indication; undated edition published at Pavia; Venice, 1479; Pavia, 1488; without an indication of place, 1495; Venice, 1495. This is the title of the one we consulted: Iacobi Foroliviensis Medici *Singularis expositio et quæstiones in primum canonem Avicennæ adjecta* Iacobi de partibus *in VII et VIII cap. Doct. ij. Fen. iij. explanatione, ac* Ugonis *quæstione, de malitia complexionis diversæ*. Venetiis apud Iuntas MDXLVII.

[23] Joannis Ludovici Vivis *De causis corruptarum artium liber V*[us]. De philosophia naturæ, medicina et artibus corruptis. De medicina (Io. Ludovici Vivis *Opera*, Basileæ, MDLV, p. 415).

[24] Iacobi Foroliviensis *Expositio super libros techni Galeni*, lib. I, text. 6; ed. cit., fol. 6, col. *d*.

> Here is obviously[25] how the order according to which bodies should be placed in the latitude of health proceeds; in the first order is placed the still healthy body; in the second order, the body healthy most of the time; in the third, the body is, mostly, in the neutral state; in the fourth, the body is always in the neutral state; in the fifth, the one who is sick most of the time; in the sixth, the always sick body.

Health and disease are therefore endowed with a latitude that can attain varying degrees, as are the other qualities, hot and cold, dry and wet; arithmetic reasoning took on those as it has taken on these; also we now see him introduce many questions composed on the Μιχροτέκνη of Galen, on the *Canon* of Avicenna.

What, by the use of latitudes, physicists from Paris or Oxford said of qualities may also extend to health and disease; this is what leads Jacopo da Forlì, in one of his *Questions on the Canon of Avicenna*, to recall[26] a famous theory of Buridan: In an unevenly heated body, the most intense degree of heat coexists with the lowest degree of cold, the average degree of heat with the average degree of cold.

This theory, applied to physiology in one of his *Questions on Galen*, leads him to quote his own treatise *In intensione formarum*:

> The members which are immediately contiguous,

he wrote[27],

> can therefore react on each other in a positive manner following the contrary qualities; to receive, on this subject, a more complete teaching, see my *Treatise on the Intensity of Forms*, where I have touched on the probable way of saving the reaction using qualities endowed with intensity.

Luis Vives accuses Jacopo da Forlì of having been the first to introduce into medicine these thorny problems analogous to the *calculationes* of Oxford[28]. It seems

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[25] Iacobi Foroliviensis *Quæstiones super libros techni Galeni*; liber I, quæstio XI; ed. cit., fol. 91, col. *a*. — Cf. quæst. XII; ed. cit., fol. 92, col. *a*.

[26] Iacobi Foroliviensis *Quæstiones saper duas primas fen primi canonis Abi halyabin sceni*, quæst. VI; ed. cit., fol. 190, col. *d*.

[27] Iacobi Foroliviensis *Quæstiones in librum techni Galeni*; lib. II, quæst. XXXIII; ed. cit., fol. 142, col. *c*.

[28] Sometimes, the opinions of Jacopo da Forlì lend themselves to certain reconciliations with the doctrines prevailing at Oxford; so are the opinions that he professes regarding the abhorrence of the vacuum:

> The vacuum produces no attraction, if not in the sense… that a certain attraction occurs to prevent a vacuum. One could argue in a contrary sense and say that this attraction, whose effect is positive, must be some positive quality; and as it is not a manifest elemental quality, it should be an occult principle or occult property which must be named form or specific virtue. To this argument we will respond that any principle or any occult property must not be named form or specific virtue because the specific form, as commonly understood, concerns a determinate agent and a determinate patient; but it is not so for the attraction that occurs to prevent a vacuum; indeed, it is proper to any body; although to this attraction an occult principle contributed which a heavenly virtue has impressed on all being, the principle by which the nature of this being is taken to save the continuity of the parts of the Universe—for by this continuity the universal order of the bodies that make up the universe is saved—that principle, however, does not properly merit the name of specific form.

that this criticism is not entirely fair. Before Giacomo della Torre, Italian doctors were accustomed to reason about the latitude of health and disease; the physician of Forlì had undoubtedly exaggerated the false rigor of his predecessors and more completely mimicked the form of mathematical reasoning. The testimony of Jacopo da Forlì himself can inform us in this regard. Here[29], he tells us that the "old Bolognese" distinguished, by their natural dispositions, a distance of latitude and a distance of nature; "by the first they understood the distance assigned to degrees of which we spoke above, and by the second, the distance in perfection." There[30] we see similar considerations attributed "to Gentilis and to the Paduans".

Jacopo da Forlì frequently cites the School of Padua and, in an incessant manner, the views of Gentilis.

A certain Gentile of Foligno was the physician of John XXII; another Gentile of Foligno, who may be son of the former, and who practiced medicine in Padua, died at Perugia on 12 June 1348; it is the latter one whose name arises so often in the writings of Jacopo da Forlì.

This Gentile of Foligno wrote extensively on matters of medicine, and his writings remained celebrated for a long time[31]. One has from him an *Explanation on the second book of the canon of Avicenna*, an *Explanation* composed in 1346 on the first *fen* of the fourth book of the *Canon* of Avicenna, a writing *On the fifth book of this Canon*, a treatise *De majoritate morbi* which is dated 1344, a *Treatise on the proportions in which it is necessary to mix medicines*, a *Treatise on baths*, and a book *On the uses of the bath water of Porretta*. It seems that this prolific writer has been, at least in part, the introducer, into the study of medicine, of the subtle discussions that pleased Jacopo da Forlì. However, the quibbles of Gentilis are far less complicated than those of Giacomo della Torre, and, above all, they do not seem to be in mathematical form; the taste for *calculationes* had not yet moved from Oxford to Italy.

If the *cavillationes* and the *tricæ* which delighted Giacomo della Torre often seem to us to merit the sarcasms with which the humanists condemned them and with which Luis Vives armed himself in their regard, they were far from having appeared useless and tedious to the Italian doctors of the 15th century; a good number of them, on the contrary, were singuarly pleasing to them; the views which this author had given his support were often later embraced by the crowd of doctors, "tota medicorum caterva," in the words of Luis Coronel[32].

It is, without a doubt, among these physicians, admirers of Jacopo da Forlì, that we must place John of Casal (*Johannes de Casali*), of whom we know nothing, except a

(Jacobi Foroliviensis *Expositio super duas primas fen primi canonis Avicennæ*; Can. 1, fen. I, doct. VI; ed. cit., fol. G3, col. *a*). — This is exactly the doctrine that Dumbleton presents in his *Summa*.

[29] Iacobi Foroliviensis *Quæstiones in librum techni Galeni*; lib. I, quæst. XII; ed. cit., fol. 92, col. *a*.

[30] Iacobi Foroliviensis *Op. laud.*, lib. I, quæst. XVI; ed. cit., fol. 96, col. *a*.

[31] The *Repertorium bibliographicum* of Hain lists multiple incunabula editions of these writings.

[32] Ludovici Coronel *Op. laud.*, lib. III, cap.: De compossibilitate qualitatum; ed. cit., fol. LX, col. *c*.

Quæstio subtilis de velocitate motu alterationis which was printed in 1505[33] with the treatise *De latitudinibus formarum* attributed to Oresme, the *Quæstiones* composed on the same subject by Blaise of Parma, and the *Tractatus de sex inconvenientibus*.

Despite the disparaging reviews of Vives, the favor with which the Italian doctors welcomed the *calculationes* of Jacopo da Forlì stemmed from a legitimate desire; these doctors were eager to place into their discussions the accuracy and rigor of mathematical reasoning; their attempt at trying to achieve this was certainly premature; it would still be so for most of the subjects they were discussing; at least they can be grateful for having clearly perceived the truth that any part of the Science of Nature makes considerable progress when it becomes capable of taking on a mathematical form; their only crime was having believed all to easily in the realization of an ideal that even today seems immensely distant.

[33] This edition was described in § XIX [section 26].

Chapter 30

How the doctrines of the Oxford school spread into Italy

If Oxford trends have, perhaps, already appealed to Jacopo da Forlì, the doctrines of the great English University seem to have waited a little longer before entering on the same level in Italian Science; their triumph was soon marked by the extraordinary popularity of the various treatises due to William Heytesbury.

Towards the middle of the 15th century and in the years that fill the second half of this century, a large number of philosophers and physicians sought to commentate on the various works of the chancellor of Oxford; unfortunately, the lives of most of these commentators is almost or entirely unknown to us.

Thus we know nothing of a Messino who undertook reviewing the treatise *De tribus prædicamentis* inserted by Heytesbury into his *Regulæ solvendi sophismata*. Messino died without completing his commentary; he left it interrupted in the middle of the chapter devoted to the movement of alteration; Cajetan of Tiene finished it; the treatise of Messino, thus completed, was printed in 1494[1] in the collection of the works of Hentisberus.

Cajetan of Tiene, who finished the treatises that Messino had been unable to complete, was, of the Italian Universities toward the mid-15th century, one of the most famous masters. Born in Vicenza from an illustrious family, Cajetan was, in Padua, the pupil of Paul of Venice; he taught brilliantly for a long time in the same city of Padua, where he died in 1465. Proud of the luster he had thrown on it, the family of Tiene often subsequently gave the name of Cajetan to those born to them; so another Cajetan of Tiene was born in 1480; after founding the order of Theatines, he died in 1547; he was honored with canonization.

The philosopher Cajetan of Tiene had spent much of his tireless activity on commentating on the various treatises of William Heytesbury.

Not content with finishing the booklet *De tribus prædicamentis* that Messino had written, Cajetan composed, under the name of the *Recollectæ*, an extensive work in which he comments very closely, and often sentence by sentence, on the *Regulæ*

[1] Tractatus Gulielmi Hentisberi *de sensu composito et diviso…*, Venetiis, 1494; fol. 62, col. *c*, to fol. 62, col. *d*. — This edition has been described in paragraph XX.

solvendi sophismata of the English Dialectician; the commentary was printed with the *Regulæ* in 1494, in the collection of the works of William Heytesbury[2].

Cajetan of Tiene also commentated, sophism by sophism, on the *Sophismata* of Hentisberus. Printed for the first time in Venice in 1483, this commentary, attached to the work he proposed to clarify, was joined, in 1494, to the edition of treatises of Heytesbury[3].

This edition makes known, moreover, a number of other commentaries that the writings of the Logician of Oxford spawned in 15th century Italy.

We see[4], for example, that a certain Simon of Lendinara (*de Lendenaria*) has, like Cajetan of Tiene, commentated, article by article, on the thirty-two *Sophismata* of the Master.

We also read[5] a treatise *On local movement*, composed by a man named Angel of Fossombrone (*Angelus Forsempronìensis*) regarding what this Hentisberus wrote on the same subject.

This treatise of Angel of Fossombrone had already been printed[6]; but as we learn from the second edition[7], this first edition added, to the treatise of local motion, a second treatise on the movement of augmentation that was purely and simply borrowed from the work of Messino.

A physician from Florence who died in 1500, Bernard of Torni or Tornio, having read this treatise by Angel of Fossombrone, discovered there some assertions that seemed wrong to him; to correct these defects, he composed, in turn, the *Annotata* on the treatise *De motu locali* of Heytesbury; in these *Annotata* he was not only content to discuss the words of Angel of Fossombrone, but also those of Jacopo da Forlì; although already old, the assertions of the latter were already controversial, because Bernard Torni talks about the discussions he had regarding them with Jean-Pierre Apollinaire Arculis[8] and the famous John Marliano, whom we will meet again in a moment.

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[2] Tractatus Gulielmi Hentisberi *de sensu composito et diviso*…, ed. cit., fol. 7, col. *b*, to fol. 52, col. *b*

[3] Tractatus Gulielmi Hentisberi *de sensu composito et diviso*…, ed. cit., fol. 81, col. *b*, to fol. 170, col. *d*.

[4] Tractatus Gulielmi Hentisberi *de sensu composito et diviso*…, ed. cit., fol. 171, col. *a*, to fol. 183, col. *c*.

[5] Tractatus Gulielmi Hentisberi *de sensu composito et diviso*…, ed. cit., fol. 64, col. *a*, to fol. 73, col. *a*.

[6] Angeli de Fossambruno *Tractatus de velocitate motus*. Colophon: Finis secundi tractatus de vellocitate motus augmentationis secundum angelum de fosambruno… s. 1. a et typ. nom. (Pavia, Hieronymus de Durantibus, circa 1485) (Hain, *Repertorium bibliographicum*, n° 7309).

[7] *Tractatus* Gulielmi Hentisberi *de sensu composito et diviso*…, fol. 73, col. *b*.

[8] This is without a doubt the same Apolinaris who had, with Peter of Mantua, a controversy over the initial instant and final state, and composed, on this, a writing dated 2 December 1450 [*Illustris philosophi et medici Apolinaris Offredi Cremonensis de primo et ultimo instanti in defensionem communis opinionis adversus Petrum Mantuanum.* Printed in Colle in 1478 by Master Bonus Gallus, and perhaps in Pavia in 1482 by an unknown typographer (Hain, Repertorium bibliographicum, n° 12005, and T. de Marinis, catalogue de *Manuscrits, autographes, incunables et livres rares*, Florence, 1911, n°° 295 and 296.)]

The *Annotata* of Bernard Torni were first printed in Pisa[9] in 1484 together with a writing by another Florentine, Francis Raphael, entitled *Verificatio universalis in regulas Aristotelis de motu*; the treatise of Francis Raphael was a discussion of the Dynamics that Aristotle presents in book VII of the *Physics*.

The *Annotata* of Bernard Torni were printed again in 1494 in the collection of works of Hentisberus[10].

Although specifically dedicated to the commentary on the writings of Heytesbury, the various treatises we have just cited have, for the most part, proved not only the influence of the Chancellor of Oxford, but also that of the Calculator; the popularity of the one, in fact, closely followed that of the other; Cajetan of Tiene, who has so greatly contributed to spreading in the Italian universities the study of Hentisberus, seems to have introduced into these universities the treatise of the Calculator, who would be confused with Swineshead.

"*Penes quid habeant intensio et remissio qualitatis attendi?* On what basis is it necessary to determine the intensity or the remission of a quality?", Riccardus of Ghlymi Eshedi opened his treatise with this question. One of the chapters of this treatise had for its object the study of the reaction of qualities that are contrary to each other, of the hot on cold, dry on wet. The opinions accepted by the Calculator—concerning the intensity and remission, on the one hand, and the reaction, on the other hand—had a gift for attracting, with singular force, the attention of Italian philosophers.

Cajetan of Tiene had written a treatise *De intensione et remissione formarum*[11], at the end of which he also addressed the problem of the reaction between contrary qualities; it does not appear that when he drafted this treatise he had knowledge of the work of the Calculator, as there is no allusion to it; his whole argument is directed at the treatise of the same title written by Jacopo da Forlì.

Cajetan remarks, during this argument, that Giacomo della Torre suffered the influence of the School of Oxford; the physician of Forlì maintained, regarding on the warming of bodies, a complicated opinion, "is," said Cajetan[12], "an English objection, *sed hæc oppositio est britannica*." Peter Pomponazzi, moreover, later dis-

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[9] *Verificatio universalis in Regulas Aristotelis de motu non recedens a communi mathematicorum doctrina;* præced.: *Auctoris* Raphælis Francisci Florentini *ad Casparem Elephantucium Patricium Rononiensem scripta epistola* — Bernardi Tornij Florentini *Medici ac Philosophi in Capitulum de Motu Locali Hentisberi quedam annotata incipiunt.* — Colophon:

> Finis quorundam dictorum supra capitulo de motu locali Hentisberi cum quibusdam conclusionibus per Bernardum Tornium Florentinum pisis impressa anno domini Mccccl xxxiiij.

[10] *Tractatus* Gulielmi Hentisberi *de sensu composito et diviso*…, Ed. cit., fol. 73, col. *c*, to fol. 77, col. *c*.

[11] We have described, in § XX [section 26], the two editions of which we are aware of this treatise and of the treatise *De reactione*.

[12] Gaietani de Thienis *Tractatus de intensione et remissione formarum*; cap. III; ed. 1522, fol. 86, coll. *c* and *d*.

cussing some of the opinions of Jacopo da Forlì, also remarks[13] that they are identified with those that the Calculator maintained on the same subject.

Shortly after having published his *Tractatus de intensione et remissione formarum*, Cajetan of Tiene composed a *Tractatus de reactione*; this time the Vicentine philosopher knew the writing of Riccardus of Ghlymi Eshedi:

> In the question of the reaction,

he wrote at the beginning of his pamphlet,

> the ancients as well as the moderns conceived various theses. A certain treatise recently composed on this matter came into my possession; after I had finished reading it, it prompted me to write something regarding what I think of reaction. In this booklet I do not intend to treat fully the views of all philosophers, criticizing each of the assertions they have made, as many are trying to do. I only want to discuss two opinions: the first is the view that has emerged in the aforesaid treatise; the second is what I have followed in the commentaries that I have given on the third book of *Physics*.

In his *Tractatus de reactione*, Cajetan of Tiene gives no name to the author of the treatise he discusses; but in the works he composed afterwards, he always refers to the nickname of Calculator.

The discussion conducted against the Calculator, in his *Tractatus de reactione*, by Cajetan of Tiene had confronted one of the most famous physicians of that time; we wish to speak of John Marliano, who was the physician of John Galeasz Sforza, who died in Milan, his hometown, in 1483.

In the *Tractatus de reactione* of Cajetan, Marliano opposed—and this was the first writing of the young physician—a treatise of the same title; he held some of the arguments that the Calculator proposed and fought the doctrine of Cajetan of Tiene. It seems that this writing was the first where the mysterious Ricardus of Ghlymi Eshedi had received the nickname of Calculator. "This man," Peter Pomponazzi says[14] speaking of Marliano, "with his Calculator" (because this is also what he continually calls him) "holds the following view:..." Cajetan replied with an opuscule as he tried to defend his theory against the attacks of Marliano. The latter, in turn, replied[15].

This controversy between two of the most famous philosophers of Italy very strongly attracted the attention of all those concerned with scholastic problems; it

[13] *Petri Pomponatii Mantuani Tractatus de reactione*; sectio I, cap. II; fol. 21, col. *c* of the 1525 edition which will be described later.

[14] Petri Pomponatii Mantuani *Tractatus de reactione*, sectio I, cap. I; fol. 24, col. *b*. of the 1525 edition, described a little further.

[15] Both writings of Marliano, with the response of Cajetan of Tiene to the first of these writings, are printed in the following collection: *Clarissimi philosophi et medici* Iohannis marliani mediolanensis *disputatio cum Magistro Ioanne de Arculis in diversis materiis ad philosophiam et utramque partem medicinæ pertinentibus — Clarissimi philosophi ac medici* Iohannis Marliani *de reactione subtilissimus tractatus et iuventutis sue opus primum — Clarissimi philosophi* Gaietani de tienis *tractatus subtilissimus quo conatur improbatam suam in materia de reactione opinionem deffendere — Clarissimi philosophi et medici* Iohannis Marliani *secundus tractatus in materia de reactione ab eodem editus in Prestantissimi philosophi Gaietani de tienis opinionem in eadem materia maie in precedenti eiusdem tractatu corroboratam esse ostenderet suamque opinionem defensaret. — Difjîcultates quedam misse per subtilissimi* (sic) *doctorem ac philosophorum monarcham d. M. Io. de Marliano de philippo adjute veneto potentem* (sic) *ab eo dari responsiones.* Colophon:

contributed greatly to spreading among them the renown of the book written by the Calculator. Moreover, the debate on the various theories of reaction lasted well after the death of Cajetan of Tiene and John Marliano; it was still ardent in the 16th century. In 1515 Peter Pomponazzi gave[16] a treatise *De reactione* whose principal object was to discuss some doctrines of the Calculator and of John Marliano. It is also against "a philosopher, English in origin, named Suiset and nicknamed the Calculator" that the same Peter Pomponazzi had, in 1514, composed a *Tractatus de intensione et remissione formarum*. Neither Marliano nor Cajetan confused the Calculator with Swineshead. But, since 1480, the printing press had popularized this confusion.

If the chapters that the Calculator devoted to the intensity and remission of forms and to the reaction of contrary qualities have particularly attracted the attention of the Italian masters, it should not be thought that they ignored the other chapters written by the same author and, especially, the one that treats local movement.

In this regard, it is true, not only in the rest of the book composed by Riccardus of Ghlymi Eshedi, that there is no allusion in the treatise *De tribus prædicamentis* that Messino wrote; it is permissible to think that he is not aware of the Calculator.

Cajetan of Tiene already read this author when he commentated on the *Regulæ* of Heytesbury; indeed, in presenting the treatise entitled *De incipit et desinit*, he invokes[17] an opinion of the Calculator regarding the intensity of forms; when he deals with the movement of augmentation and diminution, he made known[18] a certain opinion of the Chancellor of Oxford and adds:

> It should be noted that the Calculator is of a contrary opinion… He argues in a number of ways against the opinion of Tisberus.

However, with what Cajetan said on local movement, we do not recognize anything that is borrowed from Riccardus of Ghlymi Eshedi.

John Marliano was greatly interested in the chapter the Calculator devoted to the study of local movement. He took advantage of this in the opuscule where he treated

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> Expliciunt opera subtilissima Clarissimi artium ac medicine doctoris Johannis Marliani ducalis phisici primi sue etatis omnium philosophorum principis. Scilicet Questio de proportionibus. De reductione aque calide. Probatio cujusdam consequentie calculatoris in de motu locali. Uterque tractatus de reactione cum tractatu Gaietani. Conclusiones quedam cum responsionibus ac replicationibus domini Philippi adiute. Laus deo. S. 1. a. et typ. n. (Papiæ, Damianus Confalonierus).

[16] Petri Pomponatii Mantuani *Tractatus acutissimi, utilissimi, et mere peripatetici. De intensione et remissione formarum ac de parvitate et magnitudine. De reactione. De modo agendi primarum qualitatum. De immortalitate anime. Apologie libri tres. Contradictoris tractatus doctissimus. Defensorium autoris. Approbationes rationum defensorii, per* Fratrem Chrysostomum *Theologum ordinis predicatorii divinum. De nutritione et augmentatione.* Colophon:

> Venetiis impressum arte et sumptibus heredum quondam domini Octaviani Scoti, civis ac patritii Modoetiensis: et sociorum. Anno ab incarnatione dominica. MDXXV. calendis Martij.

[17] Tractatus Gulielmi Hentisberi *de sensu composilo et diviso*…, ed. Venetiis, 1494, fol. 29, col. *b*.

[18] Tractatus Gulielmi Hentisberi *de sensu composilo et diviso*…, ed. cit., fol. 52, col. *b*.

the relationship, a constant object of research for the mechanists of this time, between the power that moves a mobile, the resistance that retains it, and the speed of the movement of the mobile; printed in Pavia in 1482[19], this opuscule was subsequently reproduced in the collection of the writings of Marliano. This collection contains, besides, another piece where the Milanese physician attempts to prove a proposition that the Calculator had advanced in his chapter *De motu locali*.

The name of the Calculator arises, no more than any other name, in the treatise *De motu locali* composed by Angel of Fossombrone; but this author formulates[20] whole series of rules that the speed of a mobile experiences when varying either the power or strength; these rules are precisely those to which Riccardus of Ghlymi Eshedi devoted, in his work, the chapter on local movement.

Bernard Torni, in his opuscule *De motu locali*, repeatedly cites[21] the Calculator; moreover, no more than Cajetan of Tiene nor John Marliano does he associate the name Suiseth with this nickname.

We find, instead, that name and nickname united together in a writing by a famous Averroist, the illustrious professor University of Padua, Alessandro Achillini of Bologna (1463-1512). This writing, entitled *De distributionibus ac de proportione motuum*, was printed at Bologna by Benedictus Hectoris in 1494; under the title *De proportionibus motuum*, it was included in the editions of *Alexandri Achillini Opera* which Hieronymus Scotus published at Venice in 1545, 1551, and 1568[22]. In this study on the relationship between the speed of a mobile and the magnitudes of the power and resistance, Achillini repeatedly cites[23] the Calculator; but in one circumstance[24], he names Suiset the Calculator; in this instance he associates him with Nicole Oresme and makes both teachers be under the influence of Thomas Bradwardine. Very learned, Achillini still attached to these names those of Tisberus[25] (Heytesbury) and Marliano[26].

Achillini, we have said, gave the name of Nicole Oresme; but he has only referred to the *Treatise of Proportions* by this author. Bernard Torni himself knew of the

[19] Johannis Marliani *sua etate philosophorum et medicorum principis et ducalis physici primi de Proportione motuum in velocitate questio subtilisima incipit*… Colophon:

> Impressum Papiæ per Damianum de comphalonerii de binascho. 16 die Decembris anni M. 482. Amen.

This piece is entitled: *Questio de proportionibus* in the collection of works by John Marliano.

[20] Tractatus Gulielmi Hentisberi *de sensu composito et diviso*…, ed. Venetiis, 1494, fol. 69, col. *c*, to fol. 70, col. *d*.-

[21] Tractatus Gulielmi Hentisberi *de sensu composito et diviso*…, ed. cit., fol. 73, col. *d*, and fol. 76, col. *a*.

[22] The edition of these same *Opera* in Venice, lacking the name of the publisher, in 1508, does not contain the opuscule *De proportionibus motuum*.

[23] Alexandri Achillini Bononiensis *philosophi celeberrimi Opera omnia in unum collecta*… Venetijs apud Hieronymum Scotum MDXLV; fol. 190, col. *c*; fol. 191, col. *a*; fol. 193, col. *b*; fol. 195, col. *b*.

[24] Alessandro Achillini, *ibid.*, fol. 185, col. *c*.

[25] Alessandro Achillini, *ibid.*, fol. 192, col. *d*.

[26] Alessandro Achillini, *ibid.*, fol. 192, col. *c*.

treatise *De difformitate qualitatum*, yet he refers to it under the inaccurate title of *Sophismata*. At the end of his treatise *De motu locali*, he writes[27]:

> These days, as I was on vacation, I was reminded of a certain conclusion that Nicole Oresme demonstrated in his *Sophismata* and that he said is surprising. The conclusion is beautiful, I would say, but the demonstration is extremely beautiful.

The conclusion, or rather the two findings of Nicole Oresme that aroused so much admiration for Bernard Torni, are the ones we have summarized in article XVIII [chapter 26]; an hour was divided into *proportional parts* of ratio $\frac{1}{2}$; for each of these parts, a mobile moves with uniform motion or, alternatively, with a uniform movement and with a uniformly accelerated movement; from one part to the next, the speed of this movement grows according to a certain law; Oresme assesses the path that the mobile describes in the entire time.

Bernard Torni takes up the demonstrations of these conclusions and modifies them in order to give them a purely mathematical form, without any use of coordinates; he solves, in addition, by a similar method, two similar problems: one where the time is divided into *proportional parts* of ratio $\frac{1}{3}$, and the other where it is split into proportional parts of ratio $\frac{2}{3}$.

> On the foundation that Oresme established,

Bernard Torni said,

> I will base a few new conclusions, and I will demonstrate them by other means; but I think the principle alone is more than half of the work; also, rather than thinking that everything came from me, I would rather you believe that everything came from him.

This modesty befitted Bernard Torni even better as he was not the first to put in purely arithmetical form the demonstrations of Nicole Oresme; the Calculator had accomplished this task for the first problem, and the second we saw carried out in the opuscule entitled *A est unum calidum*.

Now, Bernard Torni, who like all his contemporaries had studied the first book, had also read the second; in his treatise *De motu locali*, he cited[28]: "*Illud sophisma: A est unum calidum.*"

Through the example of Bernard Torni, we see how the Italians during the second half of the *Quattrocento* were interested in all the Parisian and English writings that dealt with the latitude of forms; we will now look at what they gathered from the fertile ideas in these writings.

While some of them, like Bernard Torni, knew of the treatise *De difformitate qualitatum* by Nicole Oresme, we do not see any of them, in their reasonings, following the geometric method inaugurated by this treatise. Like the Oxford masters, the Italians still conducted their arguments in a purely arithmetical way that does not require the use of any figures.

Sometimes, however, the authors of the treatises—or, at least, the copyists or the printers who, in the 15$^{\text{th}}$ century, reproduced these treatises—trace, next to the

[27] Tractatus Gulielmi Hentisberi *de sensu composito et diviso*…, ed. Venetiis, 1494; fol. 76, col. *d*.

[28] Tractatus Gulielmi Hentisberi *de sensu composito et diviso*…, ed. cit., fol. 76, col. *a*.

arithmetical deduction, a figure that would allow representing it according to the method of Oresme; this figure becomes a true illustration that, while not essential to the understanding of the text, makes the imagination work toward this understanding.

The illustrations of this kind abound in the edition that was published at Venice in 1494, of the commentary composed by Cajetan of Tiene on the *Regulæ* of Heytesbury; they are attached not only to the clarifications written by Cajetan, but also to the text of Heytesbury itself, whose original manuscripts certainly contained no figures.

An example will show us what kind of relationship was established between the argument and the illustration.

In his *Rules*, on the treatise *De tribus prædicamentis*, Heytesbury spoke in these terms[29]:

> As for the space that a mobile should traverse which uniformly acquires a latitude of movement beginning at zero and ending at a certain final degree, we said earlier that all this movement and all this acquisition corresponds to its average degree.

Cajetan of Tiene adds:

> The Master said here that one can, with the help of what preceded, prove and make evident the following rule: Let there be a mobile that moves with an increasingly intense and uniformly difform movement, from the zeroth degree up to a certain degree; it travels the same path as if, during the same time, it was moved uniformly, with a movement equal to the average degree of this uniformly difform latitude that starts at zero and ends at the degree that must terminate it. The Master does not prove this rule, but he said it can be proven, and this is true; I demonstrate it thus: The average degree between 0 and 4 is equal to 2, as has been shown above; now add all the degrees that exceed 2 to the other parts that do not attain 2 and you will have 2.

This reasoning or, rather, this semblance of reasoning does not appeal to any figure; the printer, however, immediately places below the following drawing (Figure 30.1).

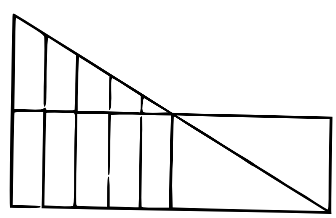

Figure 30.1 [Figure not drawn in the commentary of Cajetan of Tiene on the *Regulæ* of Heytesbury]

We recognize in this sketch what should be drawn when deducing the reasoning of Oresme; and, in fact, what Cajetan said is a kind of short, rough sketch of the argument by Nicole Oresme.

[29] Tractatus Gulielmi Hentiberi *de sensu composito et diviso*…, ed. cit., fol. 40, col. *d*.

Without being instruments of reasoning, such figures speak to the eyes and force them to support the work of the intellect. Its use became common in Italy; thus they abound in the treatise *De motu locali* by Angel of Fossombrone; and we see Achillini using some in the fourth *Quodlibet*[30] of his treatise *De intelligentiis* and in the third book[31] of his treatise *De elementis*.

What is meant by speed, at any moment, in a non-uniform movement? Clarifying some vague information from Heytesbury[32], Messino tries[33] to answer this question; Angel of Fossombrone takes up[34], in a more explicit and clear way, what Messino said. Therefore we reproduce here the essentials of the remarks of Angel of Fossombrone:

> In a movement that is constantly difform, the speed should not be evaluated by the space that the mobile traverses the entire time that this movement lasts; but at each of the moments of time which this movement measures, the mobile moves with such and such a speed. The speed of such a mobile [at one instant] must be assessed by means of the space it would travel in as much time as if, during that time, it moved uniformly with the same degree as in this moment.

There is no surprise, moreover, that our logicians did not glimpse the idea of defining the instantaneous velocity as the derivative of the traversed path with respect to time taken to traverse it; such thinking was still very far from their reasoning.

In studying the speed of local movement, the *In primo motore* by Swineshead introduced[35] five distinct latitudes he referred to as:

1. *Latitudo motus localti;*
2. *Latitudo velocitatis latitudinis primæ;*
3. *Latitudo tarditatis ejusdem;*
4. *Latitudo acquisitionis latitudinis motus localis;*
5. *Latitudo deperditionis ejusdem latitudinis.*

We have said[36] how these last two latitudes would seem to have to correspond to the positive and negative acceleration, and we understood more clearly how William Heytesbury defined these accelerations.

In his commentary on the treatise *De tribus prædicamentis* of William Heytesbury, Cajetan of Tiene distinguished[37], as the Chancellor of Oxford, two latitudes which he called *latitudo motus* and *latitudo intensionis motus*; in what he says of the first, we easily recognize instantaneous speed; of the second, it is less easy to give a precise definition; but we do not doubt much that the notion of acceleration is the

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30 Alexandri Achillini Opera, Venetiis, 1545; fol. 21, col. *a*.

31 *Ibid.*, fol. 132, col. *b*.

32 Tractatus Gulielmi Hentisberi *de sensu composito et diviso*…, Venetiis, 1494; fol. 38, col. *d*.

33 *Ibid.*, fol. 54, col. *a*.

34 *Ibid.*, fol. 66, col. *c*, to fol. 67, col. *a*.

35 Bibliothèque Nationale, fonds latin, manuscriptms. n° 16621, fol. 74, v°.

36 See § XXIII [chapter 28].

37 Tractatus Gulielmi Hentisberi *de Sensu composito et diviso*…, ed. cit., fol. 43, coll. *a* and *b*.

one he has in mind when we hear him declare that in a uniformly difform motion, the *intensio motus* is uniform, or even when we hear him say:

> *Latitudo motus attenditur penes spatium tanquam penes effectum; latitudo intensionis motus attenditur penes latitudinem motus partibiliter acquisitam.*

It follows, in effect, from this last formula that the *latitudo intensionis motus* is to the *latitudo motus* as this one is to the distance traversed; in other words, that the *latitudo intensionis motus* is the speed of the speed.

Cajetan of Tiene takes up, however, a little further[38], these considerations on the *latitudo motus* and *latitudo intensionis motus*; he seeks to demonstrate these two conclusions:

1. In a movement where the *latitudo intensionis motus* is uniform, the *latitudo motus* and hence the movement itself are uniformly difform.
2. In a movement where the *latitudo intensionis motus* is uniformly difform, the *latitudo motus* and the movement are difformly difform.

More clearly than Cajetan of Tiene does Messino clarify[39] the distinction that must be drawn between *latitudo motus* and *latitudo intensionis motus*; in addition, he gives the first as a synonym for speed (*velocitas motus*) and the second as a synonym for the acceleration (*velocitatio motus*); let us listen to him:

> Like everything that moves, it moves in a uniform or difform manner, and so all mobiles that accelerate (*intendit*) their movement accelerate it in a uniform or difform manner. He [Heytesbury] thus defines[40] what uniformly accelerated motion is; he says that a mobile uniformly accelerates a motion when, in any equal portion of time, it acquires an equal latitude of movement or speed, and, as was said above, that a mobile moves uniformly if it traverses an equal space in each equal part of time. In the case at hand, we deal with the *intensio motus* such that the *intensio* behaves with respect to the movement or the latitude of movement exactly as the movement or the latitude of movement behaves with respect to the actual space.
>
> Also it should be noted that the *intensio motus* is not named speed of movement (*velocitas motus*) but acceleration or acquisition of motion (*velocitatio*[41] *vel acquisitio motus*)… When such an acquisition exists, we say that the movement is growing in intensity, because it is then faster and faster (*velocior et velocior*), so that it is accelerated (*velocitatur*). That is why we distinguish between the speed of a movement (*velocitas motus*) and the acceleration (*velocitatio*) of this same movement. As I have shown elsewhere, the speed of a movement can be constantly greater and greater while the acceleration becomes smaller.

Angel of Fosombrone clearly distinguishes[42] between the *latitudo motus* and *latitudo intensionis motus* in his treatise *De motu locali*; we translate some passages of this treatise:

[38] *Ibid.*, fol. 44, coll. *c* and *d*.

[39] *Ibid.*, fol. 54, coll. *a* and *b*.

[40] In reality, we do not find in the treatise of Heytesbury any of the details that Messino so happily lends to him.

[41] In this place, the printer, by an obvious error, put *velocitas* for *velocitatio*; the word *velocitatio* is used correctly a little further down.

[42] *Ibid.*, fol. 67, coll. *c* and *d*.

> To understand what follows, it must be known that the movement (*motus*) differs from *intensio motus*,… and that the speed of movement (*velocitas motus*) also differs from the *velocitas intensionis motus*. The movement and the *intensio motus* differ because, sometimes, there is movement without *intensio motus*; this is what occurs in uniform motion, where the movement does not become more intense. Similarly, the speed of movement and the *velocitas intensionis motus* are different; it is seen, indeed, that where there is speed of movement, there cannot be *velocitas intensionis motus*; so is it in uniform motion, where the movement does not grow in intensity.
>
> They are still different for another reason: The effect of the speed of movement is the space that has been traversed; but the effect of the *velocitas intensionis motus* is the *latitudo motus* which was acquired…
>
> We note in this regard that a mobile is said to move with uniform local movement when, all things being equal, in equal parts of time, it travels equal distances; similarly, one says that it moves with a uniform *motus intensionis* or that it accelerates (*intenditur*) uniformly when in equal parts and any time during which the movement lasts, it acquires equal latitudes of movement…
>
> Conversely, it is said that the *intensio motus* is difform or that the movement accelerates (*intenditur*) in a difform way if it acquires, in equal times, unequal latitudes of movement…
>
> Therefore, we must imagine that the uniformly difform *latitudo motus* corresponds to the uniform *latitudo intensionis motus* and conversely; there is, in fact, a uniform latitude of *intensio* and a uniformly difform latitude of movement.

The Italian masters themselves allow us to substitute the words "uniformly accelerated motion" for the words "uniformly difform motion".

Did these teachers know of the law which, in a uniformly accelerated motion, links the path traveled by the mobile to the time employed in traversing it? This law, as we have seen, was regarded as a truth that Paul Nicoletti of Venice acquired; we will not be surprised to see that his successors knew of it and admitted its accuracy.

As student of Paul of Venice, Cajetan of Tiene was informed of this rule early on; we have seen how, in the commentary of the *Regulæ* of Heytesbury, he outlined a demonstration that seemed inspired by Nicole Oresme; but he already invoked it in a writing that seems to be among his first, in his *Commentary on the Physics of Aristotle*[43]; he rejected a mode of definition proposed for a quality, "because the uniformly difform latitude would not correspond to its average degree."

Messino also admits[44] the accuracy of this rule.

> The only reason

he said[45],

> for asserting that a uniformly difform latitude corresponds to its average degree is this: Its average degree is equivalent in regards to the path traversed… It is not necessary to give

[43] *Recollecte* Gaietani *super octo libros physicorum cum annolationibus textuum*. Colophon:

> Impressum est hoc opus Venetiis per Bonetum Locatellum iussu et expensis nobilis viri domini Octaviani Scoti civis Modoetiensis. Anno salutis 1496. Nonis sextilibus. Augustino Barbadico Serenissimo Venetiarum Duce. Lib. Vil, text. commenti 32, fol. 43, col. *d*.

[44] Tractatus Gulielmi Hentisberi *de sensu composito et diviso*…, Venetiis, 1494; fol. 54, col. *a*; fol. 55, col. *c*.

[45] *Ibid.*, fol. 54, col. *c*.

> here the proof of this principle because I have sufficiently proved it in the second principal doubt of the first conclusion.

The demonstration which Messino recalls is little more than an obscure paraphrase[46] of the reasoning of William Heytesbury.

Angel of Fossombrone wrote[47]:

> It is a commonly received principle in this matter that any latitude of uniformly difform movement—whether it starts at zero and ends at a certain degree, whether it is acquired or lost uniformly—corresponds to its average degree…
>
> By that, here is what actually needs to be understood: The mobile thus moved travels as far as the same or another mobile if it would move, during the same time, with a uniform motion whose degree is the average of the first.

Angel of Fossombrone does not attempt any demonstration of this "*commune principium in illa materia*".

> From the foregoing,

Bernard Torni wrote[48] to Mariano Romano, to whom his treatise is dedicated,

> you easily deduce that any latitude of uniformly difform motion effectively corresponds to its average degree; still, in fact, the mobile that moves under a similar latitude will move, in the second half-hour, with a movement that surpasses the average degree; it will move with a uniformly difform motion whose average degree will be said to be its zeroth degree; it will thus move up to a degree that exceeds the average degree in as much as it surpasses the initial degree of movement that was accomplished in the first half-hour. But all these things are commonly accepted and very well-known to you.

Clearly, Bernard Torni wants summarize here in plain language the demonstration of Nicole Oresme, which he read.

Thanks to Nicole Oresme, William Heytesbury, and the Calculator, the Italian masters all know, in the middle of the *Quattrocento*, the laws of uniformly accelerated or uniformly retarded motion; but it does not appear that any of them had the idea of admitting that the fall of bodies was uniformly accelerated nor, therefore, the thought of applying these laws to it.

Leonardo, on the contrary, knew and said that the fall of bodies was a uniformly accelerated motion; but he did not think to look in this motion for the properties, well-known when he lived, of the uniformly difform latitude.

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[46] *Ibid.*, fol. 53, coll. *b* and *c*.

[47] *Ibid.*, fol. 68, col. *a*.

[48] *Ibid.*, fol. 75, col. *d*.

Chapter 31
Leonardo da Vinci and the laws of falling bodies

Leonardo lived at a time when the study of the local movement was, in the schools and among the learned, a classic topic of discussion; passionate for Mechanics, he could take the greatest interest in this discussion; and he took it, indeed, because we see that he has read almost all treatises where the laws of various movements were sought, almost all the books of which we have spoken in this writing.

Let us review his notes and discover the names of the authors he has consulted or whose works he seeks to obtain.

First,[1] a list of "books of Venice"; there we read:

> Albertuccio et Marliano, *De calculatione*.
> Albert, *De Cælo et Mundo*.

We have ample evidence proving[2] this latter book, one of those which most often inspired Leonardo, is the *Quæstiones subtilissimæ in libros de Cælo et Mundo* composed by Albert of Saxony.

As for the two *De calculatione* treatises, the mention of which precedes that of the *De Cælo et Mundo*, there are the *Tractatus proportionum* by Albert of Saxony, nicknamed Albertutius, and presumably the *Qœestio subtilissima de proportione motuum in velocitate* by John Marliano.

The passage we have just reported is not the only one that alludes to these two books.

There is one[3] with these words:

> *El chaluo de li Alberti* — The calculation of Albert.

Elsewhere[4], the same writing is designated more explicitly:

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[1] *The manuscripts of Leonardo da Vinci*, published by Ch. Ravaisson Mollien; ms. F of the Bibliothèque de l'Institut, verso of the cover.

[2] *Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, l: Albert de Saxe et Léonard de Vinci.

[3] *Codice Atlantico*, 11 b, 37 b. — Cf. J. P. Richter, *The Literary Works of Leonardo da Vinci*, vol. II, § 1439.

[4] *The manuscripts of* Leonardo da Vinci; ms. I of the Bibliothèque de l'Institut, fol. 120, recto.

> On movement. Albert of Saxony, in his *On Proportions*, said…

A sheet of the *Codice Atlantico* bares[5]:

> Show yourself the *De proportione* by Messer Fatio… The *Proportions* of Alchino with the considerations of Marliano, of Messer Fatio.

This Messer Fatio is none other than Gardano Fazio, the father of the illustrious Jerome Cardan[6]. Leonardo wants to borrow the *Proportions* of Alchino, that is to say the *De proportione motuum in velocitate* by Achillini; he also wants to see, by the same person, the considerations of Marliano on this subject, that is to say, without doubt, the *Probatio cujusdam consequentiæ Calculatoris in de motu locali*.

This latter writing taught Leonardo the name, so often repeated around him in the schools, of Calculator. Surely he consulted the treatise of this author and also those that William Heytesbury and Angel of Fossombrone composed on local movement; here, in fact, is a list[7] where the names we just mentioned are found reconciled to that of Albert of Saxony:

> Du mouvement local.
> Suisset, i.e., the Calculator.
> Tisber. Angel of Fossombrone.
> Albert.

Finally, Leonardo, who has repeatedly cited the *De ponderibus* of Blaise of Parma, was able to read the *Questions on the Latitude of Forms* by the same author, because he knew where to find the works of Biagio Pelacani[8]: "The heirs of Master Giovanni Ghiringallo have the works of Pelacano," he wrote in his notes.

What use did Da Vinci make of this extensive documentation? One can, we believe, characterize it as follows:

Leonardo specified in the most fortunate way the information that he had found in the *Quæstiones in libros de Cælo et Mundo* composed by Albert of Saxony. These two laws of falling bodies were presented as equally likely:

1. The speed increases proportionally to the time elapsed since the beginning of the fall.
2. The speed increases proportionally to the distance traveled from the origin of the fall.

He even insisted more strongly on the second law than on the first.

Leonardo was able to see, after some hesitation, that the first law was the exact law of falling bodies; he formulated it with precision and insistence.

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[5] *Codice Atlantico*, 222 a. 664 a — J. P. Richter, *Op. laud.*, t. II, § 1448.

[6] *Léonard de Vinci, Cardan et Bernard Palissy*, I (*Études sur Léonard de Vinci, ceux qu'il a lus et ceux qui l'ont lu*, VI ; première série, pp. 227-228).

[7] *The manuscripts of Leonardo da Vinci*; ms. M of the Bibliothèque de l'Institut, fol. 8, recto.

[8] Leonardo da Vinci, *Manuscript* III *of the Forster Library, South Kensington Museum in London*, 3 *b*. — J. P. Richter, *Op. laud.*, t. II, § 1496.

However, Da Vinci did not grasp the scope of the considerations on the latitude of forms. The proposition which we have named the *Rule of Oresme*, which proposition Angel of Fossombrone called the "*commune principium in illa materia*", which Bernard Torni qualifies with "*communis et notissima*", would have made known to him how the path traveled by a falling weight increases with the time of fall. Leonardo da Vinci did not have the idea to appeal to this rule, so common among the learned of his time. He preferred to divide the time of fall into a number of equal parts and, during each of these parts, to treat the movement as a uniform movement accomplished with a speed equal to that of the varying movement should take at the end of this part. So that a similar method could lead to an exact result, the number of divisions performed in the period of fall would have had to grow indefinitely, at the same time that each of them was indefinitely shortened, and perform a limit. This infinitesimal reasoning seems not to have presented itself to the mind of Da Vinci. So he constantly taught that in equal parts of equal times which follow from the beginning of the fall, a weight traverses paths that grow as the integers 1, 2, 3, 4. He could read, however, in the *Treatise on Local Motion* by William Heytesbury, the following proposition[9]:

> When the acceleration (*intensio*) of a movement is uniform and this movement leaves at the zeroth degree to end up at certain degree, the path traveled during the first half of the time is precisely one-third of what is traversed during the second half.

Cajetan of Tiene developed[10] the calculation that justifies this proposition. Messino[11], Angel of Fossombrone[12], and Bernard Torni[13] had, at will, reproduced and commented on the theorem of Heytesbury. It was enough to repeat indefinitely the reasoning they had used to prove that the paths traveled by a weight, in successive and equal times, are to each other as the odd numbers 1, 3, 5, 7… These truths and the books that Leonardo read shouted, as it were, in his ears. He did not hear them.

Like all the authors whose writings we have read in this study, Leonardo always speaks, as two separate magnitudes, of movement, which the Scholastics named *motus* and he called *moto*, and speed, which the Latin of the Scholastics was first to name *velocitas* and the Italian of Da Vinci then called *velocità*; still, like the Scholastics, he implicitly admits that, for a given mobile, these two quantities are proportional to each other, such that the same laws govern the one as the other; one must think that the movement is the product of speed by the quantity of matter of the mobile; this is the relationship that Buridan already seemed to admit[14] between *impetus* and *velocitas*; it is that which, later, Galileo will keep between the *impeto* or *moto* and the *velocità*, which Descartes will maintain between the *quantité de mouvement* and *vitesse*. This remark will illuminate the texts of Vinci that we will report; it will allow

[9] Tractatus Gulielmi Hentisberi *de sensu composito et diviso*…, Venetiis, 1494, fol. 40, col. *d*.

[10] *Ibid.*, fol. 41, col. *a*.

[11] *Ibid.*, fol. 55, coll. *a* and *b*.

[12] *Ibid.*, fol. 68, col. *d*.

[13] *Ibid.*, fol. 75, col. *d*.

[14] *Jean I Buridan (of Béthune) and Leonardo da Vinci*, IV: The Dynamics of Jean Buridan [chapter 4].

the reader to recognize in these texts, without any difficulty, the opinions which we have attributed to their author.

The first of the texts that we will quote[15] is preceded by the words: "In a place in the air of uniform thickness," i.e., of uniform density; Leonardo therefore did not imagine that anyone, it seems, had conceived before Descartes, Beckman, and Galileo, that only in the vacuum falling bodies would be uniformly accelerated.

Here are, gathered together, the various passages in which Leonardo formulated the laws of falling bodies:

> In a place in the air of uniform thickness.
>
> The weight that descends acquires at each degree of time a degree of movement more than the degree of time passed, and similarly a degree of speed higher than the previous degree of movement. Thus, in each doubled amount of time, the length of the descent is doubled, and also the speed of movement.
>
> Here it is shown (Figure 31.1) how such a proportion that has a quantity of time with

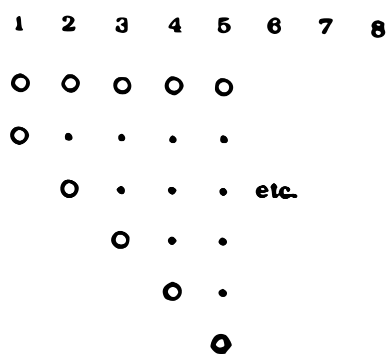

Figure 31.1 [Leonardo da Vinci's figure explaining the relation between time, quantity of movement, and quantity of time]

> another, such will have a quantity of movement with the other, and a quantity of speed with the other.

> The proof[16] that the proportion of the time and of movement in the same time is as of the speed that is in the descent of heavy bodies is found in the pyramidal figure (Figure 31.2), because the aforesaid powers are all pyramidal, since they begin at nothing and grow by degrees in arithmetical proportion.

The figure drawn by Leonardo reminds us that the treatises of the time did not lack drawing whenever there is a question of a uniformly difform latitude.

> *On movement*[17]. The weight descending freely acquires at each moment a degree of movement and, at each degree of motion, it acquires a degree of speed.

---

[15] *The manuscripts of* Leonardo da Vinci, published by Ch. Ravaisson Mollien; ms. M. of the Bibliothèque de l'Institut, fol. 44, verso.

[16] *Ibid.*, fol. 44, recto.

[17] *Ibid.*, fol. 45, recto.

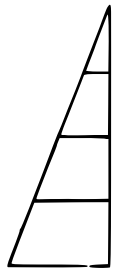

Figure 31.2 [Leonardo da Vinci's pyramidal figure illustrating an arithmetical proportion]

Let us say that for the first degree of the time, it acquires a degree of movement and a degree of speed; for the second degree of time, it will acquire two degrees of movement and two degrees of speed, and so on, as described above.

If two bodies equal in weight and shape[18] fall one after another from a height, for each degree of time, one will be a degree farther from the other.

See (Figure 31.3) that when Q has the movement PQ, T had not yet budged from its

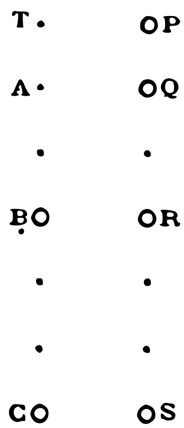

Figure 31.3 [Leonardo da Vinci's comparison of two descents]

place; and when the weight T acquired the space up to A, i.e., a degree of movement, Q acquired two up to R; and when A, at the same time, descended into B and acquired its two degrees of movement, Q already descended into S and had, in such a time, acquired three degrees.

The weight that freely descends[19] acquires a degree of speed for each degree of motion.

And the part of movement done at each degree of time is always longer, successively, the than its immediate antecedent.

If many bodies equal in weight and shape[20] are dropped one after another in equal times, the excesses of their intervals will be equal between them. — *Demonstration*: By the fifth of

[18] *Ibid.*, fol. 48, recto.
[19] *Ibid.*, fol. 49, recto.
[20] *Ibid.*, fol. 57, verso.

> the first which says how the thing descends, for each degree of movement it acquires equal degrees of speed.
>
> Thus, for that purpose, the movement of the latter becomes more rapid at the bottom than the former at the top.
>
> And by the eighth of the first that says: The upper pair will have in its interval such proportion with the interval of the lower pair that is the speed of the lower pair with the upper; and conversely, the speed with the distances as the distances with the speed.
>
> The experiment[21] of the aforementioned conclusion of movement should be done in this way, i.e.: Take two balls of equal weight and shape, and let them fall from a great height, so that at the beginning of their movement, they touch each other, and the experimenter be on the earth to see if their fall has still kept them in contact or not. And when this experiment is done repeatedly, so that some accident does not adversely impede or disrupt such an event, the experiment can be false and mislead or not mislead its observer.

Leonardo applies[22] the rules thus formulated affecting the spaces traversed by falling bodies to a stream of water which tapers in its fall and whose successive drops eventually sever to become more and more distant.

The passage that contains this application starts on the verso of a sheet and continues on the recto of the same page; Leonardo, in fact, was not content to write from right to left; often, when he recorded his notes into a notebook, he turned the pages in the opposite direction to what we follow, so that the notebook began for him where it ends for us. To find, in a similar notebook, the order of the thoughts of the great painter, one must read backwards. Now, if we thus read the various fragments that we have just mentioned, we will be struck by the fact that the statement of the law of falling bodies reveals itself there more and more clearly, as if Leonardo had caught a glimpse first and then recognized more clearly that the speed grows in proportion to the duration of the fall.

There is more; in thus following the manuscript M of the Bibliothèque de l'Institut, we will find a fragment[23] which we are led to put before those we have cited; however, in this fragment Leonardo seems to admit that the speed of fall of a weight is proportional not to the time elapsed since the beginning of the fall, but to the path traveled during this time. Here is this fragment:

> Why the natural movement of heavy things acquires at each degree of descent a degree of speed.
>
> And for that reason such a movement is represented, in that which it acquires power, by a pyramidal figure, because the pyramid similarly acquires at each degree of its length a degree of width. Thus, such a proportion of gain is found to be in arithmetical proportion, whereas the surpluses are always equal.

So before we recognize the true law of falling bodies, Da Vinci would have, first, admitted the inexact law of the proportionality between the speed and the distance traveled; a very natural mistake, if we think that Albert of Saxony, without giving formal preference to the false law, put it in sharper focus than the true law.

Galileo, too, subscribed to this false law, until he proved the absurdity of it and firmly attached himself to the exact law. Leonardo had surely adopted it before the

[21] *Ibid.*, fol. 57, recto.
[22] *Ibid.*, fol. 47, verso and recto.
[23] *Ibid.*, fol. 59, verso.

time when the aforementioned notes were written; they are, for us, like witnesses of his conversion.

Here, on the contrary, is a passage[24] which clearly testifies of the first belief that the great artist professed:

> To define the descent or the inequality of the intervals of balls. I say in the first place, by the ninth of this book, that, *the descent of each ball having been divided into degrees equal to each other by the height, at each degree of this movement, this ball acquires a degree of speed in arithmetical proportion*, because the excesses or differences of speeds are all proportionate to each other; from which I conclude that such spaces are always equal, because they always exceed or surpass each other by equal increments.

If the end of this passage is an obscure confusion, the lines that have been put by us in italics are as definite as may be desired.

Therefore, regarding the law of the speeds of falling bodies, the mind of Leonardo followed an approach similar to what the mind of Galileo had to follow; he came to the knowledge of the truth only by passing through error.

Unfortunately, if he was able, by a path analogous to that which Galileo will follow, to discover that the fall of bodies was a uniformly accelerated motion, he did not have the happy inspiration that Galileo will have; he did not apply to this movement the rule that the calculators formulated for all uniformly difform latitudes, in reproducing the demonstration that Oresme gave of this rule.

It is, however, a point where Leonardo is still ahead of Galileo; he knew the relationship between the speed of movement of a weight that slides on an inclined plane and the speed that this same weight falling in free fall would have; he clearly expressed that the fall of a weight down an inclined plane was a uniformly accelerated motion; to that statement he joined a drawing where he clearly indicates that the speed has the same latitude when the weight, starting from the same point, reaches the same level, either by a vertical drop or an oblique fall. Here is the text[25] and drawing (Figure 31.4):

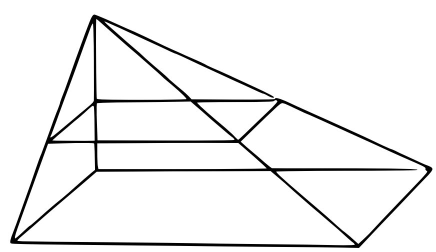

Figure 31.4 [Leonardo da Vinci’s three-dimensional pyramid]

[24] *Codice Atlantico*, fol. 145. cited par Libri, *Histoire des Sciences mathématiques en Italie*, t. III, note V, p. 212.

[25] *The Manuscripts* of Leonardo da Vinci, published by Ch. Ravaisson Mollien; ms. M of the Bibliothèque de l’Institut, fol. 42, verso.

> Although the movement is oblique, it observes in each of its degrees an increase of movement and speed in arithmetical proportion.

Of this law, it is true, Leonardo could not have reached the position that Galileo would; he was not able to use it to verify that the fall of weights is uniformly accelerated, since he made use of an erroneous rule for determining the path traversed in such a fall.

Chapter 32
The study of latitude forms at the University of Paris at the beginning of the 16th century

John Majoris, John Dullaert of Ghent

We left off the University of Paris when Marsilius of Inghen left it; this is the time when disputes concerning the Great Schism will substitute for peaceful discussions of Logic and Physics, and lessen the prestige hitherto unchallenged, of the *Alma Mater*; this is also the time when the Hundred Years War, the rivalry between the Armagnacs and Burgundians, and deadly epidemics will pitilessly grieve Paris as is in all the kingdom of France. We crossed the sea for introducing ourselves to the doctrines that Oxford University professed in the 14th century; then we came to follow, in Italy, the fortune that the teachings of France and England met during the *Quattrocento*. It is time to return to Paris and inquire into the fate of the truths discovered in the 14th century.

Since the start of the Great Schism in the beginning of 15th century, the intellectual life of the University Paris is very poorly known to us for more than a century; the few documents we have consulted provided us some rare and insufficient information.

The middle of the 14th century undoubtedly already passed when Master John Hennon, bachelor of Theology, wrote a treatise of Philosophy[1] where he successively presented the issues addressed in the following works of Aristotle: the *Physics*, *De Cælo et Mundo*, *De generatione et corruptione*, *Meteorology*, *De anima*, *De sensu et sensato*, *De memoria et reminiscentia*, *De somno et vigilia*, *De causis longitudinis et brevitatis vitæ*, and, finally, the first six books of the *Metaphysics*.

Francis Fine, a student of the College of Navarre and of the Faculty of Arts, who copied this writing and ingeniously illuminated the titles of the various parts which compose it, dated his work twice.

At the end of his presentation of the *De anima*[2], he wrote:

> *Explicit liber 3us de anima per me franciscum fine die prima octobris anno domini 1463.*

[1] Bibl. Nat., fonds latin, ms. n° 6539.
[2] Ms. cit., fol. 281, v°.

On the last sheet of the manuscript[3], we read:

> *Completus est presens liber philosophie Aristotelis in alma Parisius universitate conditus ab eximio viro doctissimo magistro Johanne hennon In sacra pagina pro tunc baccalaureo formato. Scriptus per me franciscum fine in preclara arcium facultate eo tunc studentem in collegio provincie navarre in monte Sancte genovese virginis. anno domini nostri Jhesu christi millesimo CCCC° LXIII°. Die vero prima octobris. In fine cujus laudes extolle terno et uni viventi in secula seculorum amen.*

In any debate related to Metaphysics, John Hennon is significantly Scotist; almost always does he assent to the opinion of *Doctor Subtilis*.

In all that concerns Physics and Mechanics, on the contrary, he follows, preferentially, the opinion of the Parisian Nominalists of the 14th century; he seems, above all, to make great use of the treatises of Albert of Saxony, some of whose questions he reproduces almost verbatim.

In particular, master John Hennon fully admits the Dynamics that Jean Buridan and Albert of Saxony professed.

At the end of the *Physics*, for example, he examines[4] this difficulty: What moves projectiles? Having outlined and discussed the peripatetic opinion which attributes the continuation of the movement of the body to the air that shook it, he goes on to say:

> A second opinion said that this first explanation is false. This second opinion is this: He who launches a projectile imprints on it an *impetus* or an impulsive virtue situated in the projectile; the gravity and the resistance of the medium oppose this *impetus*; the projectile therefore moves continuously until this *impetus* is corrupted.
>
> And indeed, as this opinion says, it seems impossible that a sabot, blacksmith wheel, or other mobile animated in place with rotational movement be moved by the air surrounding it; it seems impossible that the arrow or heavy stone that a war machine launches can be moved by the air as strongly as it is moved, nor can it it be supported in the air so long, if not by a such an *impetus*.

John Hennon is aware, however, that in taking this view, he goes directly counter to the doctrine of Aristotle.

> Although this opinion is probable,

he said,

> it is simply and clearly contrary to the Philosopher and false according to him.

He did not refute the objections that the Peripatetics were wont to raise against the theory of the *impetus*.

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[3] Ms. cit., fol. 327, r°.

[4] Magistri Johannis Hennon *Op. laud.*; Physicorum lib. VIII, quæst. III: Quæritur utrum primus motor qui simpliciter est immobilis et nullam habet magnitudinem, sit infinitæ virtutis. Difficultas secunda: A quo moventur projecta post recessum a primo motore projiciente? Ms. cit., Fol. 146, coll. *b* and *c*.

The presentation of the *De Cælo* leads our author to research[5] why natural motion is faster at the end than at the beginning. Having made and rejected all other explanations for the acceleration of falling bodies, he goes on to say:

> Thus they say that what causes the greatest speed taken towards the end by the natural movement is the *impetus* that is acquired within the mobile; so that, by its movement, the weight gains some accidental gravity that helps the essential and natural gravity to move this weight faster; it is similar for levity. Indeed, by the very fact that the body moves longer, it acquires a larger *impetus* and, consequently, it continually moves faster, unless it is hindered by a resistance that grows stronger than the *impetus* acquired by the mobile. Such an *impetus* is a quality of impetus of the second species; the substantial form of the mobile, through the intermediary of movement, generates this quality; this quality is corrupted by the absence of what it has produced, i.e., of the movement.

These two quotations show us that in the 15th century, the Scotist John Hennon keeps the essential principles of Dynamics formulated in the 16th century by the Parisian Nominalist School. But we find no trace in the treatise of Philosophy that we analyze of what this School and Nicole Oresme in particular taught concerning the latitude of forms; perhaps the problems on the uniformly difform were regarded as too complicated for what should be mentioned in an elementary work.

The *Commentarii in libros Philosophiæ naturalis et Metaphysicæ Aristotelis*, published by Pierre Tataret, whose first edition appeared in 1494[6], proceed in exactly the same spirit as the treatise of John Hennon. Under the influence of Duns Scotus in all issues we would call today metaphysical, the author follows the views of the Nominalists whenever he debates a problem we would attribute to Physics. Like John Hennon, Pierre Tataret gladly draws from Albert of Saxony; he even goes so far as to borrow whole pages verbatim; it is by a borrowing of this kind that the considerations of Albertutius on the law of accelerated fall of weights have, we stated in Article XI [chapter 20], entered into the treatise of our Scotist and benefited from the extreme popularity of this treatise.

But in the *Commentaries* of Pierre Tataret, or in the *Commentaries* of John Hennon, we find nothing that reminds us of the teachings of Nicole Oresme on the difformity of qualities.

Besides this School—Scotist in Metaphysics, but largely welcoming of the nominalist Physics, of which Hennon and Tataret are the representatives—the University

[5] Johannis Hennon *Op. laud.*, De Cælo et Mundo lib. II, dubium III: Utrum omnis motus naturalis sit velocior in fine quam in principio. Ms. cit., fol. 164, coll. *a*, *b*, and *c*.

[6] *Clarissima singularisque totius philosophie necnon metaphisice Aristotelis: magistri Petri tatareti expositio.* Colophon:

> Fructuosum facileque opus introductorium in logicam philosophiam necnon metaphisicam aristotelis doctissimi viri magistri petri tataret diligentissime castigatum impensis prudentis viri Iacobi bezanceau mercatoris pictavensis consummatum parisii cura pervigili magistri andree bocard. Anno domini millesimo CCCC nonagesimo quarto, decima die februarij.

of Paris has, in the 15th century, a Thomist School whose most prolific writer seems to have been Johannes Versoris[7], who died in 1480.

Like Hennon and Tataret, Versoris commentated on the *Physics* of Aristotle, *De Cœlo et Mundo*, *De generatione et corruptione*, *Meteorology*, *De anima*, *Parva naturalia*, and *Metaphysics*; like Tataret, he outlined the *Summulæ* of Petrus Hispanus; but the spirit that guides him is very different from that which animates his Scotist followers. We cannot surpass his Thomism in intolerant narrow-mindedness. He does not care about the progress accomplished, in several chapters of Physics, since the time of the Angel of the School; doctrines like those of *impetus* do not even merit the honor of a mention. Blinded by his prejudice, Versoris believes without a doubt that he resurrected St. Thomas Aquinas; and, indeed, he does make him come out of his grave, but he does not restore his soul; he shows us but the withered mummy of this genius who had such an intense and beautiful life.

Surely it is not in the *Commentaries* of Versoris, well worthy of disputing at the expense of routine with the treatises of Italian Averroists, that we will be able to meet any trace of the teachings of Albert of Saxony on the law of falling bodies, or of Nicole Oresme on the difformity of qualities.

Thus, during the 15th century, we have received no thought, issued or reproduced at the University of Paris, concerning uniformly difform latitudes. It is only at the beginning of 16th century that books were written which we will read, and where we will hear from the Parisian masters treating, in great detail, latitudes and related problems. In these treatises, the names of those who taught in Paris in the 14th century will be cited often; Hentisberus and the Calculator will also be cited; finally, the authors will have many occasions to name Paul of Venice, Cajetan of Tiene, Jacopo da Forlì, Angel of Fossombrone, or Bernard Torni; but not once, in their writings, will we find any mention of a young Parisian master younger than Marsilius of Inghen. Thus, while the School of Oxford, firstly, then the Italian Schools were impassioned with the newly discovered methods that allowed one to submit to calculation the latitudes of forms, it seems that the University of Paris, forgetting the tradition of Albert of Saxony and Nicole Oresme, neglected these issues since the beginning of the Great Schism until the 15th century.

In the early 16th century, however, the diatribes of Erasmus and Vives sufficed to be necessary for learning it, the Faculties and Colleges of Paris becoming more like academies of dialectical fencing where the *Calculationes*, copied from Heytesbury, Suiseth, and Jacopo da Forlì, were in continual use for attacking and responding;

[7] And not Johannes Versor, as he is commonly called. An edition of: Johannis Versoris *Quæstiones super Metaphysicam Arestotelis*, published in Lyon, circa 1490, by an unknown typographer, bears, on the first page, an epitaph of the author; in this epitaph we read:

Parisee jacet hic urbis studiique Johannes
Versoris decus eximium doctissimus omnium.

This *epitaphium* is preceded by an *exortatio* which reads: "…*a divo precepiore nostro Johanne Versoris.*"

This edition of the *Metaphysics* of Johannes Versoris is described by the learned bookseller Mr. Joseph Baer of Frankfurt am Main, under no. 673 in his *Lagercatalog 585* (*Incunabilia xylographica et typographica*, 1455-1500).

Spanish masters showed themselves, in these duels, particularly fierce and skillful. Many documents will confirm the accuracy of the words of Desiderius Erasmus and Luis Vives.

Let us first go to the College of Montaigu, of which Erasmus was resident and where Vives will be student, and which will remain an object of horror for these two humanists. At Montaigu in the early 15th century, the most honored regent is the Scottish theologian John Majoris.

Until the Theology of Majoris, we find considerations on the latitude of forms, uniformly difform forms, and their reduction to uniformity.

In his commentary on the first book of the *Sentences* of Peter Lombard[8], the Scottish Regent is led to define uniformly difform latitude[9]. He then poses, regarding this latitude, various conclusions of which this is the second:

> The intensity of a uniformly varying quality is measured by the average degree of this intensity. For example: Consider a uniformly difform quality—heat, if you like—which is spread, from degree 0 to degree 8, in a subject A two feet long. I say that A has a heat equal to 4. I prove it. Suppose the heat whose intensity is between 0 and 4 increases in intensity until being uniformly equal to 4; at the end of this operation, half of the body where the heat is found will be uniformly heated to degree 4. Suppose that, during this time, the heat of the second half attenuates until it is uniform and equal to 4. In the end, the whole body is warm to the degree 4; now, however much heat is gained in one half is lost in the other; the heat of such a body is therefore equivalent to 4…
>
> Similarly, when our masters place into the hands of the chancellor, regarding licentiate candidates, notes that are not uniform, it is necessary to reduce them to uniformity; half of the notes would assign to Sortes the first rank; the other half would give him third place; then there is much reason for his occupying the first rank as the third rank; he is reduced to the second rank.

John Majoris would be a skillful popularizer; for some Theology students, probably with little concern for Geometry, he knows how to introduce in a concrete form the substance of the reasoning of Nicole Oresme.

Among the objections drawn up against the rule he formulated, John Majoris encounters this one: The speed of a wheel is the velocity of the point which moves the fastest. This was, as we know, the teaching of Bradwardine, Albert of Saxony, and Heytesbury. Our theologian rejects this teaching in order to stick to the old opinion of the *Liber de proportionalitate motuum et magnitudinum*:

> The wheel of a blacksmith,

he said,

> moves with the same speed as the point which lies in the middle of the length of the radius from the circumference; and it is the same for all bodies between the various parts of which the movement is distributed in a uniformly difform manner.

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8 Joannes Major *In primum sententiarum ex recognitione Jo. Badii*. Venundantur apud eundem Badium. On the back of the title page, Epistola: Joannes Major Georgio Hepburnensi. This letter is dated from Montaigu, day 7 of the kalends of June 1509. It is followed by these words: Impressit autem jam Badius anno MDXIX. This edition of 1519 seems to reproduce a previous edition of 1509, which we were unable to consult.

9 Joannis Majoris *Op. laud.*, ed. cit., lib. I, dist. XVII, quæst. XVIII, fol. LXXX, coll. *b*, *c* and *d*.

The theological problems did not lend themselves to debating at length the properties of uniform and difform latitudes; Master John Majoris discussed it more fully when dealing with Physics; we will undoubtedly know what he said by reading the writings of his disciples.

One of his most significant students appears to have been John Dullaert of Ghent, as his teacher and at the same time as his master, regent at Montaigu. There, John Dullaert liked to develop the *calculationes* of Suiseth, to the great annoyance of the student Luis Vives.

That the argumentation of John Dullaert is often tedious, we willingly grant to Vives when reading the *Questions on the Physics of Aristotle* that the Ghent master published in 1506[10]. These questions, however, will bring us valuable information about who gave lessons in Montaigu on the latitudes of forms.

To comment on what Aristotle, in the third book of the *Physics*, said regarding movement, Dullaert says[11]

> one must examine various questions. We must consider, first, whether movement is a successive entity really distinct from any permanent thing; it must be sought, secondly, in relation to what the speed of local movement should be evaluated to be; thirdly, in relation to what the speed of movement of augmentation should be evaluated to be; fourthly, in relation to what the speed of movement of alteration should be evaluated to be.

Let us leave aside the first question, which does not relate to our subject. The last three will constitute a *Tractatus de tribus prædicamentis*, a treatise on the speed in the three kinds of movement that the peripatetic Physics recognizes. If we add that this treatise is preceded[12] by a mathematical introduction on ratios and proportions, we will have enough stated that it will be built according to the same plan as the *Tractatus proportionum* by Albert of Saxony.

Of the various chapters that comprise the small treatise on Mechanics written by Albertutius, only one does not have its analogue here; this is the first one which studies the relationship of movement with the causes that produce it; Dullaert reserves the examination of this question for the commentary on book VII of the *Physics*.

If the influence of the *Tractatus proportionum* of Albert of Saxony is recognizable in the composition of our Ghent Philosopher, another influence has, deeper still, impressed its mark; it is that of the *Tractatus de tribus prædicamentis* of William Heytesbury; the name of Hentisberus, incidentally, appears often in the discussions conducted by John Dullaert[13] and, sometimes, it appears near that of Albert of Sax-

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[10] Johannis Dullaert *questiones in libros phisicorum Aristotelis*. Colophon:

> Hic finera accipiunt questiones phisicales Magistri iohannis dullaert de gandavo quas edidit in cursu artium regentando parisius in collegio montisacuti impensis honesti viri Oliverii senant solertia vero ac caracteribus Nicolai depratis viri hujus artis impressorie solertissimi prout caracteres indicant anno domini millesimequingentesimo sexto vigesima tertia martii.

[11] Johannis Dullaert *Op. laud.*, lib. III, Quaest. I, fol. sign. fj, col. *c*.

[12] Johannis Dullaert *Op. laud.*, *loc. cit.*, fol. sign. gj, col. *c*.

[13] Johannis Dullaert *Op. laud.*, *loc. cit.*, fol. sign. giij, col. *b* and *c*; fol. sign. iiij, col. *d*; following fol., col. *a*.

ony[14]. It is the influence of Heytesbury, that of the Calculator, whose name is also pronounced[15], which introduced, into the argumentation of the Regent of Montaigu, some incessant *sophismata*; drawn up as objections against each of the opinions among which it is necessary to choose, these sophisms and their solutions gave, in the consideration of any question, hopeless confusion; these are bundles of thorns which impede the mind willing to run to encounter the truth.

Dullaert first examines the problems related to the distribution of movement in the subject. For him, as for Albert of Saxony, this examination is reduced to the study of the movement of translation and to the study of rotational movement.

To define the speed of rotational movement, he refused to adopt the stance that John Majoris took; returning to the opinion of Thomas Bradwardine and Albert of Saxony, he wants this speed to be that of the fasting moving point among those belonging to the mobile. "It is," he said[16], "the opinion of Hentisber, and almost all the calculators consider subtle." It especially gave Heytesbury the opportunity to invent and solve puerile *sophismata* that our Ghent delights to reproduce. He is most happily inspired when he borrows[17] from Albert of Saxony the distinction between the speed of the parts of a mobile in rotational movement and the angular speed of rotation.

What deserves the best of our attention in the *Tractatus de tribus prædicamentis*, of which Dullaert explains the successive articles, is the chapter devoted[18] to difform rectilinear or circular movement relative to time.

To represent the various kinds of difformities that the movement can have, the Regent of Montaigu readily uses geometric figures that he constructed using the longitude and latitude as coordinates; but he never takes advantage of this representation as Oresme advised to do; he never uses them to replace geometric reasoning with arithmetical reasoning on the degrees of intensity of qualities; in his book, as in many texts, handwritten or printed, published previously, coordinates are used to construct graphical representations; they are not used to establish an equivalence between algebra and geometric constructions, an equivalence that is the essence of analytic Geometry.

Dullaert, therefore, does not do analytic Geometry.

This is clear when he proposes[19] to establish "some rules that are common to all calculators."

The first rule is formulated as:

> Any uniformly difform latitude, whether it starts at a certain degree or whether it starts at zero and ends at a certain degree, corresponds to its average degree.

Here is the demonstration:

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[14] Johannis Dullaert *Op. laud.*, *loc. cit.*, fol. sign. giij, col. *a*.

[15] Johannis Dullaert *Op. laud.*, *loc. cit.*, fol. sign. iiij, col. *d*.

[16] Johannis Dullaert *Op. laud.*, *loc. cit.*, fol. sign. giij, col. *c*.

[17] Johannis Dullaert *Op. laud.*, *loc. cit.*, preceding fol. the fol. sign. hj, col. *b*.

[18] Johannis Dullaert *Op. laud.*, *loc. cit.*, fol. sign. hij, col. *a* in fol. sign. iiij, col. *c*.

[19] Johannis Dullaert *Op. laud.*, *loc. cit.*, fol. sign. hij, col. *d*.

> I mean this: Let there be two mobiles A and B; for one hour, A moves uniformly to a movement 4, while B moves with a uniformly difform motion that increases from 0 to 8. I say that these two mobiles will travel equal spaces, although, throughout the second half-hour, B moves faster than A; and the reason is the following: As much as B moves faster than A in the second half-hour, A moved faster than B in the first half-hour.

No doubt, the demonstration of Oresme was not, in fact, more convincing than this; but how much clearer was it, and how much, above all, better oriented to ideas that would one day illuminate all of Kinematics!

Following what has been reported, Dullaert demonstrates at length various rules with a childish ease; these are all almost verbatim borrowings from the *Tractatus de tribus prædicamentis* and the *Probationes conclusionum* of William Heytesbury.

Many sophisticated discussions also find place in the end of the considerations of Dullaert on local movement; in these discussions, copied from the Chancellor of Oxford, uniformly difform motion is always designated as the movement "*qui uniformiter intenditur vel uniformiter remittitur*"; implicitly, therefore, it is recognized that this movement is identical to uniformly accelerated or uniformly retarded motion; but from the complicated argument of our Ghent, we do not see the concept of acceleration emerge as it emerged from the *Regulæ* of Heytesbury, as it is specified by the Italian commentaries; the Italian masters introduced order and clarity into the English work that they analyzed; Dullaert has rather increased the obscurity and confusion.

Yet Dullaert read these Italian commentaries or, at least, the most recent of them, that of Bernard Torni; we will have proof of it.

"We will," our author says[20] "insert here and some conclusions and, firstly, four conclusions of Nicole Oresme (*Orem*), whose demonstrations are very beautiful and very ingenious."

It is these problems where, for times which follow one another in decreasing geometric progression, the mobile moves with speeds which increase according to certain laws.

Of the four conclusions that Dullaert attributed to Nicole Oresme, only the first two are of this master; the other two are the ones that Bernard Torni imagined. Even for those of Oresme, the demonstrations presented by the Ghent have the arithmetic form with which the Italian coated them, not the geometric form proposed by the inventor. We can therefore ensure that Dullaert read the *Tractatus de motu locali* of Bernard Torni; but we, moreover, assert that he had not read the *De difformitate qualitatum* of Oresme; this is a remark that we merely indicate here, only to return to it in its time.

After he solved the four problems borrowed from Bernard Torni, "here are," Dullaert writes[21], "these four conclusions of Nicole Oresme, to which I will add a few more."

Oresme considered the "proportional parts" whose durations formed a geometric progression with ratio $1/2$; Bernard Torni had taken the ratio to be either $1/3$

[20] Johannis Dullaert *Op. laud., loc. cit.*, fol. following the fol. sign. hiij, col. *d*.

[21] Johannis Dullaert *Op. laud., loc. cit.*, second fol. after the fol. sign. hiij, col. *d*.

or $2/3$; the Regent of Montaigu formulates it, in turn, following geometric progressions which have ratio $1/4$, $1/5$, $1/6$; these are not generalizations, but new particular cases, though similar to those the inventor treated; the satisfaction that Dullaert seems to have experienced in solving these problems does not giving us a very high opinion of his mathematical genius.

We will find in a Portuguese teacher, Alvaro Thomas, who taught in Paris at the same time as Dullaert, a more penetrating understanding of the science of numbers.

Alvaro Thomas of Lisbon

If we believe Luis Vives, the subtlest, most abstruse disputants of the University of Paris at the beginning of 16th century were the masters from Spain; in them, the Dialectics combined with Oxford found its strongest champions.

To the meticulous chicaneries of the Calculator, the Portuguese Scholastics were not less attractive than the Spanish Scholastics, if we judge by Master Alvarez Thomé or Alvaro Thomas of Lisbon.

This master was, at the beginning of the 16th century, regent at the little-known College of Coqueret in Paris[22]. He composed a treatise on the three movements: local movement, movement of augmentation, and movement of alteration. In the thought of the author, the main purpose of the *Book on Triple Movement*[23] was to elucidate the *Calculationes* of the one whom common error called Suiseth; and, indeed, it was a real commentary on the *Opus aureum calculationum*. Completed by its author on 11 February 1509, the *Book on Triple Movement* was, without a doubt, printed in Paris immediately afterwards[24]. One hundred sixty-two sheets covered, in two columns, with a very fine Gothic text are devoted to these *calculationes* which had the knack of driving the humanists into a fury.

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[22] The impasse *Coqueret* or *Coquerie* opened at the corner of the Rue des Juifs and of the Rue des Rosiers (Le Roux de Lincy et Tisserand, *Paris et ses historiens aux XIVe et XVe siècles*, Paris, 1867, p. 217, en note).

[23] [Alvarus Thomas and Trzeciok (2016a) and Alvarus Thomas and Trzeciok (2016b)]

[24] *Liber de triplici motu proportionibus annexis magistri* Aluari Thome. Ulixbonensis *philosophicas Suiseth calculationes ex parte declarans*. Venundantur parrhisius et a ponceto le preux eiusdem civitatis bibliopola ad signum potti stannei in vico sancti iacobi prope divi yvonis edem commorante. — First colophon at the end of the text of the author:

> Explicit liber de triplici motu compositus per Magistrum Aluarum Thomam ulixbonensem Regentem Parrhisius in Collegio Coquereti. Anno domini 1509. Die Februarii 11.

Second colophon, on the verso of the final leaf:

> Impressum parrisius per Guillermum Anabat commorantem apud parvum pontem ante hospitium dei prope intersignium Imperatoris expensis ponseti le preux eiusdem civitatis bibliopole. Omnia pro meliori.

The *Tractatus de proportionibus* of Thomas Bradwardine was in reality a treatise of local movement; the *Tractatus proportionum* by Albert of Saxony was a treatise of the three movements, the first of which we have met. Each of these two treatises of Mechanics was preceded by an introduction, purely mathematical, where the reader found the concepts of Arithmetic useful for reading the rest of the book. Such an introduction was missing in the book of the Calculator; Riccardus of Ghlymi Eshedi assumed that his disciple had learned the theory of proportions elsewhere, such as in the opuscule of Bradwardine, to which he explicitly referred.

Some teachers judged the *Opus calculationum* would be more perfect if it was preceded by an arithmetical introduction where the rules of ratios and proportions would be established, and they began to compose such an introduction. Among these was a Bassanus Politius; his *Tractatus proportionum introductorius ad calculationes Suisset* was printed in Venice in 1505 in a collection[25] which also contained the *Tractatus proportionem* of Thomas Bradwardine and Nicole Oresme, the *Tractatus latitudinibus formarum* falsely attributed to Oresme, and the writing on the same subject that Blaise of Parma composed.

Master Alvaro Thomas finds that Bassanus Politius has not at all succeeded in his endeavor to write an introduction to *Calculationes* of Suiseth; he directs some sharp critiques toward this introduction[26].

> In his introduction,

he said,

> the author professes that his treatise on proportions is an introduction to Suisethian calculations; but regarding the proportionality of ratios, the Suiseth Calculator thinks completely differently than he and deviates extremely from him… He therefore did not understand the intent of the Calculator; his treatise, far from introducing us to the intelligence of this author, rather leads us away from it.

Alvaro Thomas tries, in turn, to write this arithmetical introduction that he blames Bassanus Politius of having poorly made, and he devotes the first two parts of his book to it. He is very aware of the various treatises, both ancient and modern, on proportions; he cites those of Thomas Bradwardine[27] and of Nicole Oresme, whom he calls Horen[28]; he uses the *Elementa Jordanis*[29], viz., the Arithmetic of Jordanus Nemorarius, so much in vogue, which Lefèvre d'Étaples had printed in Paris in 1496. Even when he has recourse to the authors of antiquity, he intends to address the good editions. "Mind you," he said[30], "that every time I invoke Euclid, I use the new translation of Bartolomeo Zamberti."

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[25] We have described this collection in § XIX [section 26].

[26] Alvari Thomæ *Op. laud.*, pars I, capitulum quintum in quo recitatur paucis et impugnatur opinio Basani Politi de proportione sive commensurabilitate proportionum; fol. sign. diii, col. *d*; fol. sign. diii, recto and verso; next fol., col. *a*.

[27] Alvari Thomæ *Op. laud.*, fol. sign. eii, col. *a*.

[28] Alvari Thomæ *Op. laud.*, fol. following the fol. sign. diii, col. *d*.

[29] Alvari Thomæ *Op. laud.*, fol. sign. diii, col. *c*.

[30] Alvari Thomæ *Op. laud.*, fol. following the fol. sign. diii, col. *b*.

The study of triple movement is the subject only of the third part of the book; this part is, it is true, by far the most extensive. Intended primarily to review the work of the Calculator, this study is not, however, built on the plan of the treatise by Riccardus of Ghlymi Eshedi; it is the *Tractatus proportionum* by Albert of Saxony that continues to show Master Thomas Alvarus the order that he will follow, as it marked the order followed by William Heytesbury in the *Tractatus de tribus prædicamentis*, and, more recently, the order adopted by John Dullaert in his study of movement. The second part of the *Liber de triplici motu* is divided into four treatises that the following titles characterize:

Tractatus I^us: *De motu locali quoad causam.*
Tractatus II^us: *De motu locali quoad effectum.*
Tractatus III^us: *De motu augmentationis.*
Tractatus IV^us: *De motu alterationis.*

Not only did the Portuguese Master substitute the plan adopted by the Calculator for a more logically designed plan, but he had put, into his discussions, much more clarity than the logician of Oxford had introduced; no doubt, we would gladly reproach many of these discussions for being too complacent and too complicated; often, however, one can follow them without experiencing this feeling of deadly ennui that reading the *Opus aureum calculationum* causes.

The most logical order adopted by Alvaro Thomas allows him to be more complete than the Calculator has been; thus in his fourth treatise, he examines the problem of the intension and remission of forms in a wholly different way than Riccardus of Ghlymi Eshedi had done. He distinguishes[31] three theories: that of St. Thomas Aquinas, that of Burley, and finally that which Duns Scotus and the Nominalists have developed, that the intensity of a form grows by adding new degrees to degrees of the same species.

When he proposes to present the Thomist theory, he invokes not only the authority of the Angel of the School, but also that of his commentator Du Chevreul (*Capreolus*)[32]. His erudition, moreover, is very extensive; the various discussions related to movement of alteration give him occasion to cite not only St. Thomas Aquinas, Duns Scotus, Gregory of Rimini, Walter Burley, and Robert Holkot[33], not only the *Tractatus proportionum* of Albert of Saxony[34], the *Sophismata* of Heytesbury[35] and the *Calculationes* of the so-called Suiseth, but also the *De generatione et corruptione* of Marsilius of Inghen[36] and the *Summa philosophiæ* of Paul of Venice[37], the treatise that Jacopo da Forlì entitled *De intensione et remissione formarum*[38] and

[31] Alvari Thomæ *Op. laud.*, pars III, tract. IV, capitulum secundum in quo agitur de intensione et remissione formarum.

[32] Alvari Thomæ *Op. laud.*, *loc. cit..*, fol. sign. A. i, coll. *a* and *b*.

[33] Alvari Thomæ *Op. laud.*, fol. sign. A i., col. a; fol. sign. B 2, col. a.

[34] Alvari Thomæ *Op. laud.*, first fol. after the fol. sign. yii, col. *b*.

[35] Alvari Thomæ *Op. laud.*, fol. sign. B 1, col. *a*.

[36] Alvari Thomæ *Op. laud.*, fol. sign. C 1, col. *b*.

[37] Alvari Thomæ *Op. laud.*, first fol. after the fol. sign. yii, coll. *a* and *b*.

[38] Alvari Thomæ *Op. laud.*, first fol. after the fol. sign. B. 3, col. *d*; third fol. after B. 3, col. *a*.

his commentaries he composed on the *Canons* of Avicenna[39], the opuscule *De motu alterationis* written by John of Casal[40], and the book *De primo et ultimo instanti* of Peter of Mantua[41].

When he quotes either the *De motu locali*[42] or the *Sophismata*[43] of William Heytesbury, Alvaro Thomas sometimes says: "*Hentisberus cum suo commentatore*". The commentator to whom he refers he also happens to refer to by name, rather strangely distorted[44]; it is Cajetan of Tiene that he calls *Gaythanus de Thebis*.

As for Nicole Oresme, we have seen that our author knows and cites him; later he himself will tell us what he owes him.

This list of authors cited, which would be easy to lengthen, tells us enough of what the erudition of Master Alvaro Thomas was; his eclecticism is no less. If he commentates on the Calculator, it is not for blindly following all his opinions; quite to the contrary; of these opinions there is much that he condemns, and severely. If he has studied Heytesbury closely, it is not, far from it, for adopting the opinion of the Oxford logician. Finally, despite his admiration for Nicole Oresme, when he meets in reading this author a demonstration which seems insufficient to him, he points out this defect and corrects it[45].

Movement is capable of two kinds of uniformity or difformity; the one with respect to the subject and the other with respect to time. This classic distinction traces for our Portuguese master the plan of his study of local movement considered as an effect; it is the difformity with respect to the subject that occupies him at first.

Regarding movement of rotation, one definition is common since the time Bradwardine proposed it: The speed of the body that turns is the speed of the point that moves quickest. Our author knows and explains this opinion, which he calls the opinion of William Heytesbury[46]. Is worthy to remark that he rejects it, as John Majoris did at the same time, for resuming the theory sustained in the treatise *De proportionalitate motuum et magnitudinum* which we met at the origin of the Kinematics itself[47]: When the radius of a circle or a part of this radius rotates around the center of the circle, the movement of this line segment is *uniformiter difformis quoad subjectum*;

> the speed[48] of this uniformly difform motion relative to the subject must be regarded as equivalent in measure (*commensurari*) to the average degree of the total latitude of this uniformly difform motion.

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[39] Alvari Thomæ *Op. laud.*, first fol. after yii, col. *d*; fol. sign. C 1, col. *a*.

[40] Alvari Thomæ *Op. laud.*, first fol. after the fol. sign. z 3, col. *d*.

[41] Alvari Thomæ *Op. laud.*, *ibid.*, and the first fol. after the fol. sign. A i, col. *b*.

[42] Alvari Thomæ *Op. laud.*, fol. sign. x 2, col. *d*.

[43] Alvari Thomæ *Op. laud.*, fol. sign. B 1, col. *a*.

[44] Alvari Thomæ *Op. laud.*, fol. sign. gii, col. *a*.

[45] Alvari Thomæ *Op. laud.*, first fol. after the fol. sign. diii, col. *d*.

[46] Alvari Thomæ *Op. laud.*, fol. following the fol. sign. n 2, col. *c*.

[47] See § VIII [section 19].

[48] Alvari Thomæ *Op. laud.*, fol. sign. o 3, col. *c*.

This finding lets us see in what sense Alvaro Thomas, approaching the study of difform motion with respect to time, answers the following questions[49]:

> Should all uniformly difform movement over time be measured by the average degree? Should all difformly difform movement with respect to time be measured by reduction to uniformity?

If we believe our author, the discussion of these issues was, at the University of Paris, extensive and at the same time extremely complicated.

> We will examine,

he said[50],

> on what basis one should measure the speed of difform movement over time, as well as of uniformly difform motion and difformly difform motion; we will discuss this question within the limits of our feeble intelligence. In this region, in fact, a deep chasm opens; the labyrinth that ensnares this subject is inextricable and incomprehensible for a finite reason; among the various cases which will be posed, we will see what monstrosities and difformities one can imagine in difformly difform movements.

Indeed, the arguments of those who want to reject the opinion that "Uniformly difform motion is measured by its average degree" arise in a long series of *sed contra*s; it is a nice list of *sophismata*, able to exercise the sagacity of the dialecticians eager to solve them; it sufficed to compare this thorny discussion to the simple and such clear chapter where Oresme addressed the same topic, to understand all the evil that the Oxford Logic has done to the Parisian Logic.

However, Alvaro Thomas found the latter clear when he came to rejecting the multitude of these *sed contra*s and came to a conclusion:

> In contrast to these objections,

he said[51],

> is the common opinion of the philosophers; and, in this part, this opinion has a great vigor and strength. Moreover, in the total duration of such a difform motion, whatever it is, some space is crossed. This same space can, at the same time, be crossed with a certain uniform speed. This uniform velocity thus has a value as much as the speed of this difform movement, since using these two speeds, the same space is crossed in the same time; this obviously follows from the definition of movements equal in speed. Therefore, all difform motion corresponds to a uniform motion to which it is equivalent.

This passage defines in a very clear way what the reduction to uniformity of any difform movement will be.

How will this reduction be performed in the case of uniformly difform motion?

> The uniformly difform motion can terminate at zero in one of its termini[52], or it can be terminated, on either side, at a certain degree. Of each of these uniformly difform movements,

[49] Alvari Thomæ *Op. Laud.*, fol. sign. o 3, col. *d*.

[50] Alvari Thomæ *Op. laud.*, first fol. after the fol. sign. n 2, col. *d*; following fol., col. *a*.

[51] Alvari Thomæ *Op. Laud.*, third fol. after fol. sign. o 3, col. *b*.

[52] Alvari Thomæ *Op. laud.*, fol. cit., col. *c*.

> one says that it corresponds to its average degree, i.e., to the degree of movement that it has at the middle of its duration. Indeed, in the most intense half of the movement, the mobile moved with uniformly difform motion moves more quickly [than the average degree]; and in the less intense half, it moves less quickly by an equal amount; it therefore moves with the same speed as if it moved with the average degree.

This, one can easily see, is a sort of summary of the reasoning of Nicole Oresme, very similar to what John Majoris gave to his students.

The Portuguese Master goes on to list, regarding uniformly difform movement, various properties whose statements and demonstrations he borrows from the *Tractatus de motu locali* and *Probationes conclusionum* of William Heytesbury. In particular, Heytesbury and his Italian commentators suggest to him the following remark[53]:

> It is one thing, for the latitude of movement, to grow or decrease uniformly in intensity, and it is, for the mobile, another thing to move uniformly. When, in effect, the latitude of movement uniformly grows in intensity from zero or from a certain degree to some other degree, the mobile always moves with a uniformly difform movement. And likewise, when the latitude of movement uniformly relaxes from a certain degree until zero or to some other degree, the mobile moves with a uniformly difform movement. It remains that any movement gained or lost in a uniform manner is a uniformly difform movement. You can study this matter more fully using the first chapter of the *Treatise on Local Motion* of Hentisber, and the commentaries of the same Hentisber, which are found adjoined to the end of this treatise[54].

Guided by the *Probationes conclusionum* of Heytesbury and the *Calculationes* of Pseudo-Suiseth, Alvaro Thomas formulates and establishes the following propositions[55]:

1. In all movement where the intensity increases or decreases in a uniform manner, the speed corresponds to the average degree, because such a movement is uniformly difform.
2. All movement whose intensity grows faster and faster corresponds, in speed, to a degree less intense than the average degree between the two intensity extremes.
3. Any movement whose intensity becomes slower and slower corresponds, regarding the distance traveled, to a more intense degree than the average between the two extremal intensities.

Having thus developed the teachings of Hentisberus and the Calculator, the Regent of the College of Coqueret will draw from the lessons of Oresme; it is from this author, in particular, that he takes four lemmas, the subject of which he expresses in these terms[56]:

> So as not to appear triumphant in bringing spoils that are not ours, we will declare this: These four conclusions come out of the factory and from the insightful intelligence of the very learned Master Nicole Horen; you will find them in the fourth chapter of his *Treatise of Proportions*, with all of their support and mathematical proofs.

53 Alvari Thomæ *Op. laud.*, fol. sign. p 2, col. *c*.

54 Namely, in the *Probationes conclusionum*.

55 Alvari Thomæ *Op. laud.*, fol. sign. p 2, coll. *c* and *d*; fol. seq., coll. *a*, *b*, and *c*.

56 Alvari Thomæ *Op. laud.*, second fol. after fol. sign. p 2; col *d*.

These lemmas, moreover, will serve to solve problems of which Oresme gave the prototype[57]: One hour was divided into successive *proportional parts* whose durations decrease in geometrical progression of ratio $1/2$; during each of these periods, a mobile moves with uniform motion; the speeds of these successive uniform movements are as the successive integers; what is the distance traveled by the mobile in this hour?

To this problem Oresme joined another of the same kind, where the uniformly varied movements alternated with uniform movements; Bernard Torni had treated some problems of the same kind and John Dullaert had added others. Alvaro Thomas proposes to resolve some much broader questions than were studied before him; either he is undecided as to the ratio of the geometric progression according to which the proportional parts of the hour decrease, or he imposes various laws on the increase of successive speeds, he no longer seeks to solve numerically particularized problems, but to establish algebraic theorems which include an infinite number of such numerical solutions.

The problems examined by the Portuguese Master frequently reduced to a very simple series of summations related to the geometric progression; he then knows how to complete the solution, to demonstrate that the space crossed is infinite or, if it is finite, to give its value.

In other cases he meets some series that he cannot sum, for example, this one contained in his twelfth conclusion[58]:

$$\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2 \cdot 2^3} + \frac{4}{3 \cdot 2^4} + \frac{5}{4 \cdot 2^5} + \cdots. \tag{32.1}$$

But he remarkes that the sum is greater than that of the geometric progression

$$\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \frac{1}{2^5} + \cdots = 1 \tag{32.2}$$

and smaller than that of the series

$$\frac{1}{2} + \frac{2}{2^2} + \frac{3}{2^3} + \frac{4}{2^4} + \frac{5}{2^5} + \cdots = 2, \tag{32.3}$$

which Oresme evaluated.

Other problems are less easy to solve, and our author thinks that one could compose those which exceed the scope of a natural intelligence of finite capacity. We must not hasten, however, to declare that such a particular case is insoluble.

> Here, indeed, it should be noted[59] that, sometimes, a man will think that there is no sequence nor order of proportions in a case which is proposed to him; however, if he develops the question further, it may be that this order becomes obvious to him.

[57] See § XVIII [chapter 26].

[58] Alvari Thomæ *Op. laud.*, second fol. after the fol. sign. q 3, col. *d*, and fol. seq., coll. *a* and *b*.

[59] Alvari Thomæ *Op. laud.*, third fol. after fol. sign. 3 q, col. *d*.

These summations of more or less complicated series and their use in the problems of Kinematics were by no means, when the Regent of Coqueret wrote, exercises reserved for a very few mathematicians; such problems were commonly proposed in these kinds of dialectical jousts which found so great a favor near the University of Paris; we read the proof in this advice that Alvaro Thomas gives[60] to those whom such a question would embarrass:

> But, you say to me, what should we respond to the calculator who proposes such cases, in a public literary tournament, before a large audience?
>
> To answer, I admit a certain proposition that the very learned author who studied the proportions, Master Nicole Oresme, admitted: When one is confronted with a large number of magnitudes and the values of the ratios of these quantities are not readily apparent, one must think that many of these variables are incommensurable between themselves[61]. Thus, the spaces traversed are generally incommensurable between themselves. When, therefore, one proposes to you a similar case, you must respond that the space traversed in the entire hour is incommensurable with the space traversed in the first proportional part.

In affirming that the sum of a series of commensurable numbers will be, in general, a incommensurable number, our Regent of the College of Coqueret demonstrated by a divination that is quite insightful. He foresees, however, the case where the response he just dictated would not satisfy the calculator to whom it would be given.

> But, you might tell me, the calculator will insist with all his might, with bitterness and brutality; his distended mouth will make the words rumble to great effect; the eyebrow raised, forehead wrinkled, and face tragic, he will loudly affirm that his argument is unsolvable; by his repeated cries, he will seek to prove vulgarly that his opponent is conquered and defeated.
>
> In such a circumstance, I will answer, I believe that you must use two kinds of tricks.
>
> First trick: You have to turn the argument of the opponent into ridicule and derision, treat it as a useless and unintelligible matter; ask that you are given a pen and an inkpot, so that with large amounts of multiplications and algorithms of all kinds it is possible to calculate the intensity of the speed in the case that you proposed.
>
> Second trick: Answer briefly to him who argues that this speed cannot be calculated in an infallible and precisely accurate manner; it is the same for many other difform speeds that one cannot, in a natural way, reduce it to uniformity. Maybe he will, loudly asserting the contrary, seek to incapacitate the one who answers him thus. The respondent, in turn, proposes to him another analogous case and tells him to evaluate the space traversed by a mobile moved with such a difform speed. If he says it is not possible, in this case, to find in a natural way the equivalent speed, the respondent will immediately rejoin that it is the same, and for the same reason, in the case that the calculator proposed. If the latter declares, on the contrary, that this space is naturally assignable, but that he does not wish to assign it, the same must be said to him.

Thanks to Master Alvaro Thomas, we just, so to speak, witnessed one of these scholarly disputes for which the humanists have found neither enough contempt nor anger. Looking only at the staging, they were, admittedly, of ultimate ridicule; these two masters of arts who defy summing a series, with attitudes the heroes of Homer

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[60] Alvari Thomæ *Op. laud., ibid.*

[61] This proposition is, in fact, the basis of the *Tractatus de proportionalitate motuum cœlestium* composed by Nicole Oresme.

assumed to provoke themselves for combat, are perfectly made for providing characters to a comedy. But how the impression changes if one so passionately considers the questions discussed and not the manner of debating them! The problems that these masters and regents are struggling to solve, whose solution they sometimes glimpse, despite their rudimentary knowledge in Mathematics, are the two major problems of the integration of functions and the summation of series. And then one wonders what results these men would not have obtained, what advancement they would not have imprinted on Mathematics if they had read Archimedes.

The Spanish masters. Juan de Celaya. Luis Coronel.

At the University of Paris, the Spaniards and Portuguese were part of the same nation, the nation of Berry; the relationship between them was to become intimate and frequent.

Thus the Spaniard Juan de Celaya, from the Kingdom of Valencia, is regent at St. Barbara; his most faithful disciple is a Portuguese, Juan Ribeyro, of Lisbon.

At the end of the *Explanation of the Physics* of Juan de Celaya[62], there is a letter that Juan Ribeyro addresses, from Paris, to his brother Gonzalo. After having sailed on the coast of Ethiopia in the hope of making their fortune, after having very poorly managed his business, Juan Ribeyro headed to Paris to get back into the good graces of the belles-lettres. There, he was attached to the teachings of Juan de Celaya for whom he professes such a great admiration that he regretted not seeing his brother among the auditors of such a master; the praise that he gave him indeed reached the highest peaks of the dithyramb.

Later, Juan Ribeyro was to mark his devotion to Juan de Celaya by publishing and annotating the *Dialectical Introductions* that he composed[63].

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[62] *Expositio magistri* ioannis de Celaya Valentini *in octo libros phisicorum Aristotelis: cum questionibus eiusdem, secundum triplicem viam beati Thome, realium et nominalium.* Venundatur Parrhisiis ab Hemundo le Feure in vico sancti Jacobi prope edem sancti Benedicti sub intersignio crescentis lune commorantis. Cum gratia et Privilegio regis amplissimo. — Colophon:

> Explicit in libros phisicorum Aristotelis expositio a magistro Joanne de Celaya Hyspano de regno Valentie edita: dum regeret Parisius in famatissimo dive Barbare gymnasio pro cursu secundo anno a virgineo partu decimoseptimo supra millesimum et quingentesimum VII idus Decembris. diligenter impressa arte Johannis de prato et Jacobi le messier in vico puretarum prope collegium cluniacense commorantium: Sumptibus vero honesti viri Hemundi le feure in vico sancti Jacobi prope edem sancti benedicti Sub intersignio crescentis lune moram trahentis. Laus deo.

[63] *Dialectice introductiones sive termini* Magistri Joannis de celaya Valentini: *cum nonnullis* (Magistri Johannis ribeyro Ulyxbonensis *sui discipuli*) *additionibus recenter impresse: et per eundem sue integritati restitute*. Colophon:

> Imprime a Caen pour Michel et Girard dietz augier, et Jacquet berthelot libraires Demeurans audict lieu a lenseigne du mont-Sainct Michel Près les Cordeliers. Et a este acheue le. *xxviij*. jour de juillet MDXXVIJ.

The attachment of Juan Ribeyro for Juan de Celaya shows us what intimate relationships were sometimes established in Paris between the Portuguese and Spanish masters. It is possible to believe that the Spanish regent of the College of St. Barbara, Juan de Celaya, knew the Portuguese regent of the College of Coqueret, Alvaro Thomas; the parallels that we have to make between the writings of these two masters will, therefore, be very natural.

In his explanations on the *Physics*, *De Cælo et Mundo*, and *De generatione et corruptione*, Juan de Celaya generally follows this order: He gives the text of Aristotle, to explain the literal commentary; then, under the heading *Sequitur glosa*, he discusses the various opinions and formulates what is his own. It is quite different in the third book of the *Physics*, after he commentated on what Aristotle, in the first three chapters of this book, says of movement. The title *Sequitur tractatus proportionum* announces[64], between the third and fourth chapter of Aristotle, the insertion of a writing that has nothing more than a commentary on the work of Stagirite, and which fills seventy-four sheets[65].

"As we plan to treat three forms of motion (*motus triplicitatem rimaturi*)…" It is in these terms that the treatise on proportions by Juan de Celaya begins. These words evoke at once, to our mind, the title of *Liber de triplici motu* composed by Alvaro Thomas. And indeed the treatise that the Spanish Regent inserts into his *Expositio in libros Physicorum* follows exactly the same plan as the treatise published a few years before by his Portuguese colleague; that one is no different from this one except that it is conciser.

The documentation of John of Celaya is the same as that of Alvaro Thomas. The most quoted name in his treatise is that of the Calculator; it is pronounced a dozen times. That of William Heytesbury is pronounced almost as often. Jacopo da Forlì is mentioned twice; in one of these citations[66], we recall that he wanted to characterize a uniformly difform latitude not by its average degree but by its most intense degree.

The Regent of St. Barbara read the Italian commentators of Heytesbury; here, regarding a fallacy related to acceleration, he quotes[67] the reply of "*Angelus Forsempionensis, commentator Entisberi*"; there, he recalls[68] how Cajetan of Tiene demonstrates a conclusion of Heytesbury.

The name of Cajetan of Tiene was quoted by Alvaro Thomas; the Angel of Fossombrone had not been; the Portuguese Regent had not pronounced the name of Bernard Torni; we will find it in the writings of Juan de Celaya, in some circumstances that merit our attention.

A chapter[69] of the treatise of Juan de Celaya has this title: *Sequuntur conclusiones Nicolai Orem*. It begins with these words:

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[64] Joannis de Celaya Expositio in libros Physicorum, fol. lxiij, col. *d*.

[65] Joannis de Celaya *Op. laud.*, fol. lxiij, col. *d*, to fol. cxvij, col. *c*.

[66] Joannis de Celaya *Op. laud.*, fol. lxxxiij, col. *d*.

[67] Joannis de Celaya *Op. laud.*, fol. lxxxv, col. *a*.

[68] Joannis de Celaya *Op. laud.*, fol. xcv, col. *a*.

[69] Joannis de Celaya *Op. laud.*, fol. lxxxviij, col. *b*.

> These preliminaries laid out, we will draw some conclusions that Bernard Torni of Florence, commentator on Hentisberus, attributes to Nicole Oresme.

Juan de Celaya cannot declare more clearly that he did not check the correctness of the attribution that Bernard Torni formulated, and, therefore, that he did not read the *De difformitate qualitatam* of Nicole Oresme.

Alvaro Thomas gave the solutions of Oresme and Torni without mentioning any author by name, despite the fact that he had carefully mentioned the name of Oresme whenever he borrowed a proposition from the *Tractatus proportionum*.

As for John Dullaert, he had attributed to Oresme four conclusions, two of which were of this author and two of Bernard Torni; obviously, he knew the work of the Norman master through the treatise of the Florentine master.

Similarly, Luis Coronel de Segovia, in his *Perscrutationes physicæ*[70] which we will study later, gives a demonstration of the first proposition of Nicole Oresme; these reflections follow it[71]:

> In his commentary on the treatise of local motion of Heytesbury, Bernard Torni proves this conclusion; Nicole Horent has also given, in his *Sophismata*, a proof that Bernard says is admirable; "it is a beautiful conclusion," he said, "and the demonstration is extremely beautiful…" Suiset the Calculator, too, in his treatise *De defformibus* formulates that conclusion, and he uses another demonstration which is the following…

The various remarks we just produced necessarily lead to this result: In Paris at the beginning of 16th century, all the masters read fluently the *Tractatus de motu locali* of Bernard Torni; none of them read the *Tractatus de figuratione potentiarum et mensurarum difformitatum* of Nicole Oresme; of this latter work, we know only what the former has repeated.

Therefore, what explanation can one give? This and, it seems, this one only: The treatise of Bernard Torni was printed; that of Oresme remained in manuscript.

If one peruses, in fact, the list of works cited by John Dullaert, Alvaro Thomas, Juan de Celaya, and Luis Coronel, we see that these are all books that the nascent printing press had reproduced. The Calculator, whose treatise already has several editions, is the most consistently read author. The collection printed in Venice in 1191 made Heytesbury and his commentators known. The treatises on proportions of Thomas Bradwardine, Albert of Saxony, and Nicole Oresme are cited, because they have all been printed. However, no one reads the *De difformitate qualitatum* of Oresme which no printer has edited; the same oblivion affected the *De primo motore* of Swineshead and the *Summa* of John Dumbleton.

During the half-century that followed his birth, the printing press will ensure the popularity and life of several writings composed in the Middle Age; but, at the same

70 *Physice perscrutationes* magistri Ludovici Coronel Hispani Segoviensis. Prostant in edibus Joannis Barbier librarii jurati Parrhisiensis academie sub signo ensis in via regia ad divum Jacobum. On the back of the first page, after the title, is a letter from Simon Agobert to John Agobert dated: Parrhisiis, MDXI. — Another edition of this book was published in 1530 Lugduni, in edibus J. Giunti; it is entitled: *Physice perscrutationes egregii interpretis magistri* Ludovici Coronel. Our quotes are all from the first edition.

71 Ludovici Coronel *Op. laud.*, lib. III, *De difformibus*; edit. 151, fol. LXIX, col. *d*.

time, it habituated the learned to read only the pages the press transcribed. All which during this half century did not have the fortune of being printed fell into oblivion, from where many were never again released.

Now chance, rather than a reasoned choice, determined the writings that the first printers were to publish. It also happened that the invention of printing was the occasion of great injustices. By reproducing in droves some second-hand books, the press gave them an undeserved reputation, whereas it forsook the work of the inventor, whose rare exemplary manuscripts, forgotten by readers, would fall prey to mold and worms. The *Opus aureum calculationum*—a boring jumble without originality, without idea—was avidly read, studied deeply, and fervently discussed in the same University where Nicole Oresme had taught; and no one, for centuries, was ever notified that the *Tractatus de difformitate qualitatum* might abound in brilliant views.

Let us return to Master Juan de Celaya and the problems he borrows from Oresme via Bernard Torni. He generalized these problems so that each of the formulated theorems contains an infinite number of particular cases; these theorems are, indeed, almost literally borrowed from Alvaro Thomas, whose very recognizable influence is apparent in many passages.

When he announced these problems, Celaya, to emphasize their importance, has this curious language[72]:

> These findings can be applied not only to Medicine but also to sacred Theology; it is sufficient, in fact, to replace the terms "to move", "movement", with some of these: "to have a fever", "fever", or: "to merit", "merit".

There we have an example of this strange confidence in the scope of the mathematical method that we already mentioned in studying the School of Oxford. The Scholastics of Paris, strong on this confidence, at the beginning of the 16th century, did not hesitate to consider only the intensities of fever, but also the degrees of moral merit that proceeded according to a series of convergences or divergences; not content to create mathematical Mechanics and Physics, they dreamed of a mathematical Medicine, a mathematical Morality, a mathematical Theology; amazed at the power of the instrument they were trying to handle, they did not think that there existed any work for which this instrument was improper. The Humanists laughed at this enthusiasm, and the laughs were on the side of the Humanists; the laughers will always scoff at the inventor, because between the truth he sees and the illusion whose seductive mirage extends this truth to infinity, the inventor never discerns the border.

The echo of the quodlibets that were the object of Parisian Scholasticism certainly reaches the ears of Celaya. But, in this Scholasticism, everything seemed a good bargain for the mockers, easy to rejoice at little cost. That two mobiles should go with different movements, that two men have unequally high fevers, that one Christian sins worse than another, these two mobiles, these two men, both Christians invariably called Socrates and Plato or, rather, *Sortes* and *Plato*; in all the sophismata, in all the *calculationes* that encumber Physics, Medicine, and Theology, we saw the

[72] Joannis de Celaya *Op. laud.*, fol. lxxxviij, col. *b*.

inevitable *Sortes* reappear; also the Parisian *calculatores* received from their opponents nicknames that Nifo invented: *captiunculatores*, *Sorticolæ*.

Celaya suffered, no doubt, to hear himself called Sorticole; he apologizes for imposing so often on *Sortes* the movements of varied difformity.

> Do not be surprised,

he said[73],

> if, to establish these findings, I have used names such as *Sortes* and *Plato* and not letters of the alphabet; these letters put a great deal of fog into the minds of a large number of scholars; also, in what follows, I shall only use them rarely.

The extreme analogy that can be recognized between the *Liber de triplici motu* of Alvaro Thomas and the treatise inserted by Juan de Celaya into his *Expositio in octo libros Physicorum* commits us to not analyzing the latter treatise; we indicate only briefly what he says about the uniformly difform latitude.

William Heytesbury, Albert of Saxony, and Paul of Venice thought that the speed of a spinning wheel was the speed of the point that moves the quickest[74]; against this view, we can raise a host of objections, such that one is led to bring in a second opinion, supported by other *Nominales*[75]; according to this opinion, the speed of a uniformly difform motion relative to the subject must be evaluated by the speed of the midpoint; if the movement is difformly difform, this evaluation should be done by reduction to uniformity.

By analogy with the first of those opinions, Jacopo da Forlì wanted[76] that the speed of a difform motion is the speed reached at the moment when the movement is most intense.

> Another opinion is that of William Heytesbury, the Calculator, and almost all other philosophers; they hold that in such a difform motion with respect to time, the difformities must be reduced to uniformity, and the speed should be assessed by the degree to which this reduction leads.
>
> From this opinion they derive some corollaries. The first is this: All uniformly difform motion starting at zero and ending at a certain degree, or starting at a certain degree and ending at some degree, corresponds to the average degree between zero and the extremal degree, or between the extremal degrees…

This opinion gives rise to a lengthy argument where the names of Heytesbury and the Calculator keep coming back, and justly, because in this theory their influence is incessant; but the influence of Alvaro Thomas is no less constant and no less recognizable, although the name of the Portuguese Master is not pronounced.

The rule that reduces a uniformly difform motion to uniformity is frequently applied in the course of this argumentation; it is not demonstrated there. To obtain a demonstration of it, we will look to where Celaya deals, in general matter, with difform qualities.

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[73] Joannis de Celaya *Op. laud.*, fol. lxxviij, col. *a*.

[74] Joannis de Celaya *Op. laud.*, fol. lxxxj, col. *c*.

[75] Joannis de Celaya *Op. laud.*, fol. lxxxij, col. *c*.

[76] Joannis de Celaya *Op. laud.*, fol. lxxxiij, col. *d*.

In the general case of any difform quality, contrary to what Jacopo da Forlì will support,

> the Calculator[77] defends an opinion that is commonly held to be the most likely. The intensity of a difform form should not be evaluated by the most intense part of the form, but by reducing the difformities to uniformity.

In particular,

> a uniformly difform quality between zero and a certain degree is as intense as the average degree between zero and this extreme degree. If, for example, a quality is uniformly difform between 0 and 8, it is as intense as the degree 4, which is the average degree between 0 and 8. I thus demonstrate: Take the excess by which the most intense half surpasses 4; place this excess on the other half so that the most intense extremity of this excess is placed on the extremity where the weaker half reaches the zeroth degree, and position the lesser intense extremity of this excess on the side facing the most intense half. The quality obtained will be uniform and of degree 4. Or, as much as it has lost in one of its halves, it has acquired in the other. Previously, therefore, it also corresponded to degree 4.
>
> And if you ask what this excess is, I would say that it is a [uniformly difform] quality starting at 0 and ending at degree 4…
>
> A second conclusion is this: If a uniformly difform quality starts at a certain degree and ends at another degree, it corresponds to the average degree between the two extreme degrees… This conclusion can be proved in the same way as before.

None of the English, Italian, or Parisian masters we have cited so far has given this demonstration a form closer to that which Oresme adopted; indeed, it is the demonstration of Oresme itself; it lacks only the figure, which would have given it more clarity.

On closer inspection, it also lacks the definition of the quantity of a form, a definition that only Oresme explicitly gave.

The excerpts of the book of Celaya, given above, are sufficient to show that the Regent of St. Barbara was more versed in the science of difform latitudes and their reduction to uniformity; his interest in this study is noticeable even in works other than the *Expositio in libros Physicorum*. Thus, in the *Expositio in libros de Cælo et Mundo* that he gave a year later, we hear him[78] rectify an illegitimate application of the rule of Oresme.

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77 Joannis de Celaya *Op. laud.*, fol. ciij, coll. *c* and *d*.

78 *Expositio magistri* ioannis de Celaya Valentini *in quator libros de celo et mundo Aristotelis: cum questionibus eiusdem*. Venundantur in edibus Hemundi le Feure in via divi Jacobi prope edem sancti Benedicti sub signo crescentis Lune moram trahentis. Cum Gratia et Privilegio regis amplissimo. Colophon:

> Explicit expositio Magistri Joannis de Celaya Valentini in quatuor Libros Aristotelis de Celo et Mundo, cum questionibus eiusdem, novissime et cum maxima vigilantia in lucem redacta: ac impressa arte ac artificio Joannis du pre et Jacobi le messier. Anno a partu virgineo Millesimo, Quingentesimo decimooctavo die vicesimaprima Mensis Junii Sumptibus vero Hedmundi le feure: in vico sancti Jacobi prope edem sancti Benedecti, sub intersignio crescentis Lune moram trahentis;

fol. XIX, col. *c*.

The writings of John Dullaert of Ghent, Alvaro Thomas of Lisbon, and Juan de Celaya of Valencia showed us what development the mathematical study of triple movement—of local movement, augmentation, and alteration—took in Paris at the beginning of 16th century.

The *Quæstiones in libros Physicorum* of Dullaert were printed in 1506; the *Liber de triplici motu* of Alvaro Thomas is dated 1509; the *Expositio in libros Physicorum* of Celaya appeared in 1517; it is thus between the last two writings that chronological order places the *Perscrutationes physicæ* composed by a Spanish regent of the College of Montaigu, Luis Coronel of Segovia; the first edition[79] of these *Perscrutationes* bears, in fact, the date of 1511.

Like the *Questions* of Dullaert and the *Explanation* of Celaya, it is the third book of the *Physicæ perscrutationes* that will teach us what we should think of the three movements and their speeds. Luis Coronel divided this book into four parts. The first part, devoted to local movement, discusses the nature of this movement and, in particular, the movement of projectiles and *impetus*. The second part relates to the movement of alteration; it includes not only the discussion of the various doctrines on the intensity of the forms, but also, under the title: *de difformibus*, most of the considerations on uniform and difform latitudes which we will discuss here. The third part, which is very short, studies the movement of augmentation. Finally, the fourth researches how the speed in each of the three movements should be assessed. The analogy of this fourth part with the *Treatise of proportions* of Albert of Saxony is visible and, moreover, acknowledged by the author.

> The shortness of time,

he wrote when ending it[80],

> presses me to move forward with speed; I shall thus not dwell any longer on the study of speed, that those who want to be informed more plainly about this material may see what Hentisberus and the Calculator wrote about local movement, and what Albert of Saxony said in the booklet *On Proportions*.

This passage teaches us both what authors inspired Luis Coronel and what summary form he has given to the chapters they suggested.

The main sources from which he draws are, indeed, those he named: The *Tractatus proportionum* of Albert of Saxony, the *Tractatus de tribus prædicamentis* of William Heytesbury, and finally the treatise of the Calculator. He also read and willingly cites the *Summa philosophiæ* of Paul of Venice and the *De intensione et remissione formarum* of Jacopo da Forlì. Finally, he probably studied the Italian commentators of Heytesbury; he cites[81] an opinion issued "by Cajetan in his commentary on the treatise of Hentisberus on the maximum and the minimum"; and we saw that he borrows from Bernard Torni a theorem of Nicole Oresme.

The documentation of Luis Coronel is identical to that of Alvaro Thomas and Juan de Celaya; the entire doctrine that he extracts is also similar to what they had

[79] We described this edition above (p. 546 [395]), from which all of our quotes will be drawn.

[80] Ludovici Coronel *Op. laud.*, lib. III, pars IV; ed. 1511, fol. LXXX, col. *b*

[81] Ludovici Coronel *Op. laud.*, lib. II, pars III; ed. 1511, fol. xl, col. *at.*

deduced; but he does not give it the full development that his colleagues of Coqueret and St. Barbara had given it. Of this doctrine the Regent of Montaigu merely formulates the propositions that seem the most important.

On some problems of Nicole Oresme and Bernard Torni, Alvaro Thomas had grafted a fairly extensive mathematical theory, an outline of the theory of series; Juan de Celaya would reproduce the theory in full. Luis Coronel takes up neither the four problems that Bernard Torni explained nor even the first two, which are of Oresme; he merely solves the first of these problems.

In treating *de difformibus*, Coronel states[82] the rule by which a uniformly difform quality corresponds to its average degree; he produces no demonstration of this rule; he merely destroys a misinterpretation that the Calculator gave it.

He again invokes this rule to reduce to uniformity a velocity distributed in a uniformly difform way, whether in the subject or over time; what he says of this reduction ends with these words[83]:

> If one of these two mobiles or both of them move in a uniformly difform way, or even if the speed is difformly difform, the difformity should be reduced to uniformity according to its average degree, and we say that the mobile moves in a difform way with this degree of movement. Almost everything that has been said of difform qualities can be applied to difform movement, so I no longer insist on these considerations. One may consult the rules that Heytesbury gives in the *Tractatus de motu locali*; they are quite good and easy. As for him who wants to use his time in vain, let him see the rules of Suiset; because for me I find it unnecessary to dwell any longer on these questions.

The desire to be brief has not only, it seems, dictated this subject; one can guess there was a great weariness for these minute quibbles so cherished to the Calculator. We know[84] the first symptoms of this weariness, that the Humanists carried to the point of the most profound disgust, were felt until the circle of John Majoris; at the discretion of the disciples of the Scottish Master, and of this master himself, it was time to impose an end to the dialectical excesses that the influence of Oxford made fashionable; there was an urgent need to simplify Logic and Physics. The *Perscrutationes physicæ* of Luis Coronel visibly strove toward this simplification. Unfortunately, the departure between the unnecessary and cumbersome straw that it was necessary to abandon and the fertile grain that was good to keep is not, in these *Perscrutationes*, always done with full discernment; many of the "trivialities *à la* Suiseth" have been preserved, while the author rejects some theories that will prove fertile in the future; so that Luis Coronel might avoid any mistake of this kind, it would have been necessary for a prophetic intuition to reveal to him all the future progress of the Science.

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[82] Ludovici Coronel *Op. laud.*, lib. III, pars II; ed. 151. fol. lxix, col. *at*.

[83] Ludovici Coronel *Op. laud.*, lib. III, pars IV; ed. 1511, fol. lxxix. col. *b*.

[84] *The tradition of Buridan and the Italian Science in 16th century*, IV: The decadence of Parisian Scholasticism after the death of Leonardo da Vinci. The attacks of Humanism; Desiderius Erasmus and Luis Vives [chapter 9].

Domingo Soto and the laws of falling bodies

It is difficult to read the writings of John Dullaert, Alvaro Thomas, Luis Coronel, and Juan de Celaya without making a remark, nor to make this remark without being surprised. All these authors, after Heytesbury, the Calculator, and their Italian commentators, treat at length uniformly difform movement; none of them takes care to show by example that such a movement occurs or may occur in nature. The example, however, appeared to be at the immediate disposal of our regents of Montaigu, Coqueret, and St. Barbara. Albert of Saxony had stated the hypothesis of uniformly accelerated movement as one of two suppositions that one could make on the fall of heavy bodies; this opinion was reproduced, then printed, in the various editions of the *Quæstiones in libros de cælo et mundo*; only the editions published in Paris, in 1516 and 1518, omitted it. Our scholastics, who read and quoted Albert of Saxony so willingly, could hardly have assumed nor encountered this hypothesis; he would have let it pass unnoticed that they had found it in the manual of Philosophy of Pierre Tataret, so often printed in their time, where it was copied. Surprising though the fact may seem, it is, however, certain and easy to recognize; no Parisian master at the beginning of the 16th century had thought to mention falling bodies as an example of uniformly difform motion.

About the same time, Leonardo da Vinci, guided no doubt by reading Albert of Saxony, was strongly attached to proclaiming this truth: The fall of bodies is uniformly accelerated motion. But, although he had studied the writings of Heytesbury, the Calculator, and Angel of Fossombrone, he does not seem to have taken advantage of what these writings teach about uniformly difform motion; he has failed to recognize exactly the law which links the elapsed time to the path traversed in a uniformly accelerated motion.

By the early 16th century, therefore, the two propositions that govern falling bodies have been formulated for one hundred and fifty years; since that time, each was repeated a very large number of times; but, still, those who formulate the first of these propositions seem to ignore the second, and those who teach only the second do not breathe a word of the first; no one yet seems to have thought to unite them and, in uniting them, to create the theory of motion of heavy bodies.

So, who first had the idea to weld these two propositions together? We cannot say; but reading the *Questions* of Soto, we find that the weld is made; the learned Dominican, moreover, does not seem to present it as something new and of which he is the author.

We know that Francisco Soto, when he came to study in Paris, was received by his compatriot Luis Coronel of Segovia; so we will not be surprised that Soto teaches, regarding the difformity of latitudes, a doctrine similar to what Coronel professed; and indeed, if the explanation that the professor of Salamanca gives to this question differs from that given by the regent of Montaigu, it is only by a greater brevity and by a more complete abandonment of mathematical subtleties.

It is in his *Questions* on the seventh book of the *Physics* of Aristotle that Soto developed his opinion regarding the speed of local movement; to conform himself to a custom almost constantly followed since Bradwardine and Albert of Saxony—

and which Dullaert, Alvaro Thomas, and John Celaya have been careful to avoid—he prefaced this development with an arithmetic introduction[85] which he entitled: *Digressio de proportionibus*. Immediately after this mathematical digression comes a question formulated in these terms:

> Is the speed of a movement, considered in its effect, evaluated by the amount of space traversed[86]?

The difformity of the movement may depend either on its distribution within the mobile or on its succession in time; the difformity relative to the moving subject first captures the attention of the Professor of Salamanca.

In a rotational movement, the speed is that of the most rapidly moving point. Soto sides[87] with this "conclusion of Hentisberus, which the philosophers quite rightly admit." But, for this, it was necessary to reject this objection[88]:

> In any kind of movement, one must adopt the same measure. However, in the movement of alteration, when the heat is distributed in a uniformly difform manner in any subject, from degree zero, for example, to degree 8, we denominate this heat not by its highest degree, but by its average degree, i.e., the degree 4. And therefore when in a wheel, driven with a rotational movement, the speed of movement extends with uniform difformity from the center to the circumference, the speed of the entire movement of the wheel should be evaluated by the speed of the midpoint of the radius.

We come to what the Professor of Salamanca teaches[89] on difform movement in time.

> Uniformly difform motion with respect to time is that whose difformity is as follows: If we divide it according to time, viz., according to successive parts in time, in each point, the movement of the midpoint exceeds the weakest extremal movement of this same part by an amount equal to that which the more intense extremal movement exceeded it.
>
> This kind of movement is specific to moving bodies that move with a natural movement and to projectiles.
>
> Whenever, in fact, a mass falls from a certain height within a homogeneous medium, it moves faster at the end than at the beginning. On the contrary, the movement of bodies projected [upwards] is lower at the end than at the beginning. As the former is accelerated uniformly, the latter is delayed uniformly.

We give the Latin text in full of this such clear and important passage:

> *Motus uniformiter difformis quoad tempus est motus ita difformis ut, si dividatur secundum tempus (scilicet secundum prius et posterius), cujuscunque partis punctum medium illa proportione excedit remissimum extremum illius partis qua exceditur ab intensissimo.*
>
> *Hæc motus species proprie accidit naturaliter motis et projectis.*

---

[85] *Reverendi Patris* Dominici Soto Segobiensis *Theologi ordinis prædicatorum super octo libros Physicorum Aristotelis Qusestiones*. Cum Privilegie Salmanticæ. In ædibus Dominici a Portonariis, Cath. M. Typographi. MDLXXII. Fol. 90, col. *a* in fol. 92 col. *b*.

[86] Dominici Soto *Op. laud.*, lib. VIII, quæst. III; ed. cit., fol. 92, col. *b*.

[87] Dominici Soto *Op. laud.*, quæst. cit.; ed. cit., fol. 93, col. *b*.

[88] Dominici Soto *Op. laud.*, quæst. cit.; ed. cit., fol. 92, col. *c*.

[89] Dominici Soto *Op. laud.*, quæst. cit.; ed. cit., fol. 92, col. *d*.

> *Ubi enim moles ab alto cadit per medium uniforme, velocius movetur in fine quam in principio. Projectorum vero motus remissior is in fine quam in principio. Atque adeo primus uniformiter difformiter intenditur, secundus vero uniformiter difformiter remittitur.*

An obvious inadvertence is introduced twice, in the last sentence; the word *difformiter* should not be included there. Soto wants that the fall of the weight and the ascent of the projectile are two *uniformiter difformes* movements; therefore, as Heytesbury and a host of writers after him constantly did, he should have said *uniformiter intenditur* of the former *uniformiter remittitur* of the latter. We have seen in § XXIV [chapter 29] that Cajetan of Tiene, Messino, and Angel of Fossombrone had, all three, insisted on the synonymy of the expressions with the qualification *uniformiter difformis*.

We have also translated these expressions as: the movement accelerates uniformly, retards uniformly. To justify the correctness of this translation, we could use the authority of Messino; we will invoke an even more convincing one; Juan de Celaya will tell us that this sense is the one attributed to such expressions among the Spanish Masters of the University of Paris when Soto collected their teachings.

> It is one thing,

Celaya said[90],

> you have to be warned about; properly speaking, no one should say that the movement is intense (*intensus*) or weak (*remissus*), or that it is fast (*velox*) or slow (*tardus*); but the common way of speaking decided on the contrary; now it is the opinion of the Philosopher that one must speak like the many and think like the few; we will thus constantly use these terms: "intense movement", "weak movement", instead of these: "fast movement", "slow movement"; we will use the expression: "grows in intensity" (*intenditur*) instead of the words: "is accelerated" (*velocitatur*), the words: "is weakened" (*remittitur*) instead of the words: "is delayed" (*retardetur*).

These various explanations do not seem to leave any room for doubt; we can confidently attribute these two propositions to Domingo Soto:

1. The fall of a weight is a uniformly accelerated motion.
2. The ascent of a projectile is a uniformly retarded movement.

In such a movement, which law will make known the path described by the mobile in a given time? Soto will now tell us[91]:

> Uniformly difform movement over time follows almost the same rule as uniform movement. If two mobiles, in fact, travel equal lengths in the same time, although one moves uniformly and the other in any difform manner—describing, for example, a foot during the first half hour and two feet during the second—as long as the latter, in the entire hour, traverses just as many feet as the first, which moves uniformly, these two mobiles will move equally.
>
> But here a doubt arises: Must the speed of a mobile moved with uniformly difform motion be denoted by its most intense degree? If, for example, the speed of a weight that falls for an hour increases from degree 0 to degree 8, should we say this weight has a movement of degree 8? It seems the affirmative answer is true because this is the law which the uniformly

[90] Magistri Johannis de Celaya *Expositio in libros Physicorum*; fol. LXXXV, col. *d*.

[91] Dominici Soto, *Op. laud.*, quæst. cit.; ed. cit., fol. 93, col. d et fol. 94, col. *a*.

difform movement of the moving subject appears to follow. We nevertheless reply that the speed of the uniformly difform motion with respect to time is measured by the average degree and must take its name from this degree. One must not reason with respect to it as with respect to uniformly difform movement relative to the subject. In the latter case, in fact, the reason for the rule adopted was as follows: The entire mobile describes with itself the line that describes the quickest moving point, so that everything moves as fast as this point. While a mobile moving with uniformly difform motion with respect to time does not describe as great a path as if it moved uniformly over the same period with the speed it reaches at its highest degree; this is self-evident. We therefore believe that the uniformly difform motion must be referred denoted by its average degree. Example: If mobile A moves for one hour by continuously accelerating its movement from degree zero to degree 8, it will travel just as far as the mobile B, which, during the same time, would uniformly move with degree 4.

It follows that, whenever the mobiles are moved with difform movement, it is necessary to reduce these movements to uniformity.

Oresme gave some examples of this reduction, which are of a somewhat elevated mathematical style, and these examples were repeatedly multiplied and generalized by Bernard Torni, John Dullaert, and Alvaro Thomas; Juan de Celaya had reproduced the theory of Thomas, but Luis Coronel was forced to borrow from Oresme one of his problems, the first and the most simple. Into this mathematical study, Soto penetrates even less; he merely shows, treating two special cases, how to reduce to uniformity a movement of continuous speed, formed by the succession of two uniformly accelerated movements.

During the reading of the passage just quoted, two points can be made:

First, the fall of a weight is taken as an example of uniformly difform motion; there, the proposition that such a fall is uniformly accelerated is affirmed again.

In the second place, Soto discusses if the average degree of movement must serve to denominate a uniformly difform movement; but regarding the rule for measuring the distance traveled by a similar movement, he feels no hesitation; he said at the outset that this path is equal to what the mobile describes, in the same time, by a uniform motion where the speed is the average between the largest and smallest speed of the uniformly difform motion.

Of this rule Soto sketches no demonstration; obviously, he regards it as a truth of common usage; reading Jean de Celaya has also shown us that those who wanted to justify it knew the need, at that time, of resuming the considerations that Nicole Oresme developed.

Here is what the testimony of Soto tells us:

Before the mid-16th century, the Parisian Scholastics and their followers regarded these truths as trivial:

1. The free fall of a weight is a uniformly accelerated motion; the vertical ascent of a projectile is a uniformly retarded motion.
2. In a uniformly varied motion, the path traversed is the same as a uniform movement, of the same duration, whose speed is the average between the two extremes of the first movement.

The immense labor, the history of which the previous pages have briefly traced, had borne fruit; two of the essential laws of falling bodies were known; in favor of these laws, Galileo may well bring new arguments from either reason or experience, but at least he will not have to invent them.

Chapter 33
Conclusion. The Parisian tradition and Galileo.

It is customary to attribute the invention of these two propositions to Galileo. Is this attribution legitimate? Let us examine the titles which adduce it[1].

On 16 October 1604, Galileo wrote to his friend Fra Paolo Sarpi a well-known letter[2]. Galileo said that, to account for the various characteristics he observed in the fall of heavy bodies, he lacks until now "a totally indubitable principle" that can be given as an "axiom".

> I contented myself,

he continues,

> with a natural and evident proposition; this proposition admitted, one can prove all the rest, namely: that the spaces traversed by the natural movement are a squared ratio of the durations of the fall; therefore, that the spaces crossed in equal times are to each other as the successive odd numbers starting with one, etc. The principle in question is this one: Bodies that move naturally ıncrease speed in the same ratio that they move away from the start of their movement (*Ed il principio è questo, che il mobile naturale vadia crescendo di velocità, con quella proporzione, che si discosta dal principio del suo moto*).

So that no doubt remains in the mind of his correspondent, Galileo explains his thought with a figure; departing from A, the weight falls vertically to B, then to C; the degree of speed (*grado di velocità*) at C is the degree of speed it has at B, as the distance CA is to the distance BA.

Galileo added that, if a projectile is launched vertically upwards, the speed it takes, successively, will be exactly reproduced in the reverse order when it will fall.

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[1] We will not examine here the ensemble of ideas of Galileo on Dynamics and, in particular, on the cause of the accelerated fall of weights. We refer the reader wanting to know these ideas to our study entitled: *De l'accélération produite par une force constante. Notes pour servir à l'histoire de la Dynamique.* This study was published in the *Comptes rendus du II° Congrès international de Philosophie*, Geneva, September 1904, pp. 859-915.

[2] This letter is the first of those reproduced in the edition of the *Opere di* Galileo Galilei published in Padua in 1744 (vol. III, p. 342). It has since been reproduced in the edition of Alberi, Firenze, 1847 (t. VI, pp. 24-25) and in the national edition (vol. X, p. 115).

So, in 1604 the illustrious Pisan knows the law which connects the time of fall with the path described by a falling weight; but he admits, to connect to the same time the speed that animates this weight, a law that is false and from which the first could not be deduced. Galileo claimed to Sarpi that this deduction was possible, without saying, however, how he would perform it.

We found, in the 19th century, many fragments and some essays composed by Galileo; written by the hand of Galileo or recopied by some of his friends, they had never been printed. These fragments were carefully published in the national edition of the works of Galileo; unfortunately, it is, in general, impossible to assign a specific date to them or even to order them chronologically.

Among these fragments there is one, written in Italian by the hand of Galileo, which develops the thoughts stated in the letter to Paolo Sarpi, retaining the same order and almost exactly the same form; it is conceivable that the fragment is roughly contemporary with the letter.

This fragment will show us the demonstration that Galileo used. We give the translation of key passages[3].

> I suppose (and perhaps I can demonstrate) that a weight that falls constantly increases its speed in proportion to its increasing distance from its starting point. If, for example, the weight starts from point A (Figure 33.1) and falls along the line AB, I assume that the degree

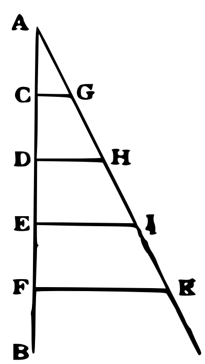

Figure 33.1 [Galileo's demonstration of average speed]

> of speed at point D surpasses the degree of speed at point C in the proportion where the distance DA is greater than the distance CA; that, similarly, the degree of speed at E is the degree of speed at D as EA is to DA; thus, the weight at any point of the line AB has a speed proportional to the distance of this same point to the origin A. This principle seems to me very natural; it answers all the experiments that we verified on machinery and instruments whose job is to strike; in these machines, in fact, the piece that strikes produces an even

[3] *Le Opere di* Galileo Galilei. Edizione Nazionale sotto gli auspicii di sua Maestà il Re d'Italia. Vol. VIII, Firenze, 1908. Frammenti attenenti ai *Discorsi e Dimostrazioni matematiche intorno a due Nuove Scienze*, pp. 373-374.

> greater effect the greater the height from which it falls. This principle accepted, I will show the rest.
>
> Let the line AK make any angle with the line AF, and through the points C, D, E, F, draw parallels CG, DH, EI, FK; since the lines FK, EI, DH, CG are to each other as the lines FA, EA, DA, CA, the speeds at the points F, E, D, C are to each other as the lines FK, EI, DH, CG. The degrees of speed in all the points of the line AF will therefore constantly increase according to the increase of the parallels drawn from these same points.
>
> Also, as the speed of the mobile from A to D is comprised of all the degrees of speed acquired in all of points of the line AD, and the speed with which it crosses the line AC is composed of all the degrees of speed it acquired in all the points of the line AC, the speed with which it traveled the line AD has, to the speed with which it traveled the line AC, a ratio equal to that which all the parallels drawn from all points of the line AD to the line AH have to all the parallels drawn from all the points of the line AC to AG; and this latter ratio is that of the triangle ADH to the triangle ACG, viz., the square of AD to the square of AC. So the ratio of the speed with which the mobile has traversed the line AD to the speed with which it traversed the line AC is the square of the ratio of DA to CA.
>
> But the ratio of the speed to the speed is the inverse of the ratio of the time to the time, because time decreases in the same time that the speed increases; the duration of the movement along AD therefore has to the duration of the movement along AC a ratio that is the square root of the ratio of the distance AD to the distance AC. The distances to the starting point are also as the square of the time; hence, the spaces traversed in equal times are to each other as the successive odd numbers starting from one; this answers what I have always said, and the experiments observed; all the truths are thus found to agree.

Galileo continues to demonstrate that his principle leads to a corollary: A projectile that rises vertically takes successively all the speeds that it will retake in reverse order when it will fall along the same line.

Let us analyze the passage we have just reproduced.

To draw a right conclusion from his false principle, Galileo successively committed two serious fallacies.

Firstly, by this vague proposition:

> The [average] speed with which the mobile traveled the line AD is composed of speeds taken from all the points of AD,

he was led to regard this average speed as measured by the area of triangle ADH; this is what allowed him to say that the ratio of the two average speeds with which the mobile has successively crossed the distances AC, AD was equal to the ratio of the areas of the two triangles ACG, ADH.

Secondly, Galileo invoked this principle: The times are in inverse ratio of the speeds (*La velocità alla velocità ha contraria proporzione di quella che ha il tempo al tempo*). He forgot to add that this principle compares the times and speeds with which a single path was traversed in different circumstances. He was quick to apply it to a case where the two paths traversed, AC, AD, are different. It is surprising to see such a genius make mistakes that one would condemn in a beginner of Geometry. We will find these same errors, at least in part, from the pen of another genius, Descartes.

On 13 November 1629, Descartes replies[4] to a question that Mersenne posed for him regarding the time employed by a weight descending from various heights. In

[4] Descartes, *Œuvres* published by Ch. Adam and Paul Tannery, *Correspondance*, piece n° XIX, t. I, pp. 69-73.

response, he inserts, into his letter written in French, a fragment written in Latin. According to Mr. Adam and Mr. Tannery[5], this fragment must have been composed during the first sojourn of Descartes in Holland, i.e., between 1617 and July 1619.

Descartes starts from this principle, so dear to the Terminist School of Paris: The body falling from A to B, then from B to C

> describes much faster the space BC than the space AB, because, while it travels this space BC, it retains all the *impetus* by which it is moved along the path AB and, in addition, the effect of gravity that presses it again at each moment increases its *impetus*.

The power of the speed thus imprinted by this *impetus* (*vis celeritatis impressa*) therefore increases from one moment to another. Descartes continues in these terms:

> The proportion in which this speed increases is demonstrated by the triangle ABCDE (Figure 33.2). The first line, in fact, denotes the power of speed impressed at the first moment;

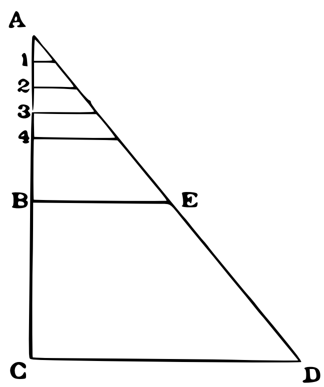

Figure 33.2 [Descartes's demonstration of the proportion in which this speed increases]

> the second, the power imparted at the second moment; the third, the third power communicated (*vis indita*) and so on. One thus forms the triangle ACD representing the increase of the speed of movement while the weight descends from A to C; let triangle ABE represent the increase of speed in the first half of the space that the weight travels; finally, let the trapezoid BCDE represent the increase of speed in the second half of the space that the weight travels. As the trapezoid BCDE is three times greater than the triangle ABE, as is evident, it results that the weight descends three times more quickly from B to C than from A to B; i.e., that if it descends in three moments from A to B, it will descend in one moment from B to C; thus, in four moments it will go twice the distance than in three; therefore, in 12 moments, it will make twice more than in 9; in 16 moments, four times more than 9; and so on.

This fragment of Descartes is clear if one is careful to draw the figure as we did; it is absolutely incomprehensible if one uses the figure drawn in the letter to Mersenne; the lines designated by the numbers 1, 2, 3, 4, are not parallel to CD there; they are parallel to AC and move away from AC as their order increases. It

[5] Note des éditeurs, *ibid.*, p. 75.

does not seems doubtful that the latter line is due to an inadvertence, committed perhaps when Descartes copied this fragment to insert it in the letter to Mersenne. We therefore assume that the figure we drew is the one that Descartes had in mind when constructing his reasoning.

Therefore, the passage quoted above gives rise to various remarks.

1. Like Galileo in 1604, Descartes clearly admits in this passage that the speed of a falling weight is not proportional to the duration of the fall, but to the path traveled by the mobile.
2. This speed is thus a uniformly difform latitude whose path traveled is the longitude. Descartes will represent this uniformly varying latitude as Nicole Oresme taught him to do, like how most printed books in the late 15$^{\text{th}}$ and early 16$^{\text{th}}$ centuries and Galileo in his Paduan notes did it; he will, in our modern language, make the longitudes or the paths traversed the x-coordinates and the latitudes or speeds the y-coordinates, so that the uniformly difform latitude is represented by a triangle.
   But the fragment we have just quoted would be, if we accept the approximate date that Mr. Adam and Mr. Tannery assigned to it, the oldest product of the genius of Descartes to have survived; it would be older than the time when Descartes created his Geometry. If it is so, before Descartes applied himself to Geometry, he knew about the use of coordinates under the form that Nicole Oresme had proposed it, and he used the coordinates for problems similar to those which Oresme treated.
3. The uniformly difform latitude thus drawn corresponds to something that Nicole Oresme named the quantity or measure of the latitude; this something is measured by the area of the representative figure; Descartes called this something "increase of speed" (*augmentum velocitatis*). Imitating a proposition that Heytesbury and all his commentators formulated regarding uniformly difform speed with respect to time, he can state this theorem: While the mobile traverses the second half of the path, the increase in speed is three times what it was during the course of the first half of the path.
4. As long as one does not state the meaning of the words *augmentum velocitatis* differently, this proposition can be accepted as absolutely correct; but it is evident that in the mind of Descartes, the meaning of this word is stated by an error analogous to that which is found in the mind of Galileo; Descartes identifies the *augmentum velocitatis* relative to the path AB with the *augmentum velocitatis* relative to the path BC, with the average speed along each of these two paths.
   Since both paths AB and BC are equal, Descartes can then declare that the times the weight takes to traverse them are inversely proportional to the corresponding average speeds and, therefore, that the duration of the fall along BC is a third of the duration of the fall along AB.

Of the two fallacies Galileo committed, Descartes kept the first and avoided the second; also, departing from the same principle as the Pisan, he came to a different conclusion.

Obtained from a false principle by a severe fault in reasoning, this conclusion is erroneous; it is an unfortunate reversal of the exact classical theorem since Heytesbury: The distance a weight travels during the second half of the duration of the fall is three times the distance covered during the first half of this same period.

From the first essays of Galileo and Descartes on the laws of falling bodies, an overall impression emerges, which can be formulated as follows:

These two authors start from this false principle: The speed of the movement of the weight is proportional to the duration of the fall. On the other hand, they are haunted by the considerations that the Scholasticism of Paris and Oxford developed regarding the movements whose speed increases in proportion to time. So they strive to adapt, to the principle which they use, considerations similar to those which they cannot make without committing serious fallacies.

In the unpublished papers of Descartes, Leibniz copied various fragments composed from the years 1619 to 1621, fragments that Foucher of Careil published under the title *Cogitationes privatæ*. One of these fragments[6] relates to the fall of a weight in a vacuum. Less detailed than the fragment Descartes sent to Mersenne, it contains the same errors and the same fallacies. After what we wrote regarding the letter to Mersenne, the analysis of this fragment would be a repetition.

This fragment begins with these words:

> Contingit mihi ante paucos dies familiaritate uti ingeniosissimi viri, qui talem mihi quæstionem proposuit:
>
> > *Lapis*, aiebat, *descendit ab A ad B unâ horâ; attrahitur autem a terra perpetua eadem vi, nec quid deperdit ab illa celeritate quæ illi impressa est priori attractione. Quod enim in vacuo movetur, semper moveri* existimabat. Quæritur: *quo tempore tale spatium percurrat.*

Who is this *vir ingeniosissimus* with whom Descartes was familiar and who posed this problem? The recent discovery from the journal of Isaac Beeckman makes it known to us.

The first fragment[7] of this newspaper has the title: Why does a stone falling in the vacuum fall faster and faster? The answer to this question is the following:

> Here is how things move towards the center of the earth, when the medium is empty.
>
> During the first moment, the mobile travels as much space as it can traverse by the effect of the attraction of the earth. During the second moment, while the mobile persists in this movement, a new pulling movement is superimposed, such that a double space is traversed in this second movement. For the third moment, this double movement[8] perseveres and, by the pulling effect of the earth, a third is superimposed, such that in one moment, a space triple that of the first is traversed.

The proportionality of the speed to the duration of the fall is, in this passage, formally admitted and explained.

[6] Foucher of Careil, *Op. laud.*, p. 18. — *Œuvres de Descartes*, published by Ch. Adam and P. Tannery, t. X, pp. 219-220.

[7] Descartes and Beeckman, *Varia*, n° XI. — *Œuvres de* Descartes, ed. Ch. Adam and P. Tannery, t. X, p. 58.

[8] The text says: *duplex spacium*.

Following this first fragment is another[9] entitled: “Calculation of the fall time of a stone: *Lapidis cadentis tempus supputatum.*”

> As the moments he just mentioned are indivisible,

Beeckman wrote,

> ADE (Figure 33.3) will be the value of the space the thing traveled in one hour. The space

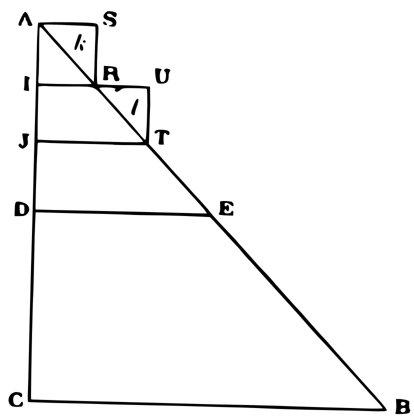

Figure 33.3 [Beeckman’s demonstration]

> in which the rock falls in two hours is the squared ratio of the time.
>
> [These two spaces] are thus to each other as ADE is to ACB, which is the squared ratio of AD to AC. — *Cum autem momenta hæc sint individua, habebit spatium per quod res unâ horâ cadit,* ADE. *Spatium per quod duabus horis cadit duplicat proportionem temporis, id est* ADE *ad* ACB, *quæ est duplicata proportio* AD *ad* AC.

He goes further; he happens to correctly infer the second truth from the first. If, he said, during the first moment of time the body traversed a “moment of space” AIRS, then during the first two moments of time, AJ, it will describe three moments of space, represented by the figure AJTURS. The space traversed in any time will be represented by the corresponding triangle, augmented by the small triangles *k*, *l*, … equal to each other.

> But these equal triangles thus added are as much less as the moments of space are less; these added areas are therefore of zero magnitude when it is posited that the moment is of zero magnitude. But this moment is the moment of space in which the thing falls. It remains that the space in which the thing falls in one hour is to the space it falls in two hours as the triangle ADE is to the triangle ACB. *Cumque hæ æqualia adjecta semper eo minora fiant, quo momenta spatii minora sunt: sequitur hæc adjecta nullius quantitatis fore, quando momentum nullius quantitatis statuitur. Tale autem momentum est spatii per quod res cadit. Restât igitur spatium per quod res cadit unâ horâ, se habere ad spatium per quod cadit duabus horis, ut triangulum* ADE *ad triangulum* ACB.

[9] Descartes and Beeckman, *Varia*, n° XI *bis*. — *Œuvres de* Descartes, ed. cit., t. X, pp. 58-61.

Beeckman did not therefore merely reproduce two key propositions that the Parisian Scholasticism certainly possessed in the 16th century, as Domingo Soto testifies. He again links one of these propositions to the other by a link that the method of indivisibles, the infinitesimal method, allows him to establish. Is all this his invention? Certainly not, because the passage just quoted is immediately followed by this:

> Hæc ita demonstravit Mr. Peron, cùm ei ansam præbuissem, rogando an possit quis scire quantum spatium res cadendo conficeret unicâ horâ, cum scitur quantum conficiat duabus horis, secundum mea fundamenta, videlicet *quod semel movetur semper movetur, in vacuo*, et supponendo inter terram et lapidem cadentem esse vacuum.

Beeckman thus did not give this doctrine as his; it is the answer that René Descartes, Lord Du Perron, made to the problem that he posed.

Now if we compare this response that Beeckman reported to that which is preserved in the papers of Descartes or transmitted to Mersenne, we find deep differences which are, moreover, in favor of the first. Beeckman admits the proportionality of the speed to the duration of the fall, whereas Descartes takes this speed as proportional to the path traversed. Beeckman employs exactly the rule of Oresme, whereas Descartes substitutes this rule for an entirely false formula.

What explanation should we give for these differences? Of the problem stated by Beeckman, has Descartes, for his interlocutor, given a correct solution, which he then distorted when he drafted it to keep it in his papers? Or has "Mr. Peron" not suggested to Beeckman some mistakes, errors that Beeckman would have turned into truths without even noticing the happy modification he made them undergo? It seems difficult to choose between these two assumptions.

This choice will not become easier when we read another passage[10] that Beeckman devoted to the same problem and which he called:

> *Lapis in vacuo versus terræ centrum cadens quantum singulis momentis motu crescat, ratio Des Cartes.*
>
> In the proposed question,

he said,

> we imagine that at each instant (*singulis temporibus*), a new force is added whereby the weight tends downward; I say that this force accrues in the same way as the transverse lines IR, JT, DE, and as the other transversals, infinite in number, that can be imagined between these.

Our author seeks to establish that parallels to the base CD of the triangle represent the successive instantaneous speeds. His whole argument supposes that the various lengths on the height AC measure the durations of fall; he explicitly says, moreover, that the divisions marked by him on this height are of "*minima temporis*". What we read at the beginning of his note is thus consistent with what he previously explained in the summarized passage.

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[10] Descartes and Beeckman, *Physico-Mathematica*, II. — *Œuvres de* Descartes, ed. cit., t. X, pp. 70-78.

But here, when concluding the demonstration, an inadvertence slips in; the lengths along AC no longer represent the durations of fall, but the paths that the mobile traveled; this is clearly evident in these lines:

> Ex quibus patet, si imaginetur, verbi gratiâ, lapis ex A ad C trahi a terra in vacuo per vim quæ æqualiter ab illâ semper fluat, priori remanente, motum primum in A se habere ad ultimum qui est in C ut punctum A se habet ad lineam CD; mediam vero partem DC triplo celerius pertransiri a lapide, quam alia media pars AD, quia triplo majori vi a terra trahitur; spatium enim LDCB triplum est spatii ALD, ut facile probatur.

Departing therefore from an accurate assumption, of the proportionality between the speed of movement to the duration of the fall, Beeckman swaps it, along the way, for the false law that takes the speed proportional to the path traveled; moreover, for the rule that correctly assesses the progress made in a given time, he substitutes the erroneous rule that we read in the papers of Descartes, which claims to evaluate the duration employed for traversing a given path.

Thus, having known—either by having received it from Descartes or by having conceived it himself—the true theory of falling bodies, Isaac Beeckman is quick to forget it for resuming the errors which the great philosopher seemed to have stopped. He, too, after having reasoned rightly, tries to use fallacies to compete with the first works of Galileo.

Galileo would get rid of these paralogisms in admitting that the fall of weights is a uniformly accelerated movement; it would then be possible for him to keep, without committing any contradiction, everything that Scholasticism said about uniformly accelerated motion.

In the second day of the *Dialogho delle dui massimi sistemi del mondo*, Galileo admits that in the fall of a weight, the speed increases in proportion to the time, without giving any indication of the reasons which led him to adopt this principle in preference to the one which seduced him first. The reason, it seems, can easily be guessed. As early as 1604, the letter to Sarpi witnesses, Galileo was assured of the law which connects the path traversed to the duration of the fall; if he admitted the proportionality of the speed to path traveled, it is only as a postulate to demonstrate that this law; a more careful reflection made him recognize that this premise, employed without any fault of reasoning, was absolutely unsuitable for what was required of him; to obtain the law he was to prove, it was enough, as the Scholastics had shown since the middle of 14th century, to assume uniformly accelerated motion.

> As in accelerated motion,

Galileo said,

> the increase is continuous, one cannot divide the speed, which unceasingly grows, into any given number of degrees because, changing from moment to moment, they are infinite in number. Therefore, we can better represent our intention by imagining a triangle such as ABC, (Figure 33.4) in taking on the side AC as many equal parts AD, DE, EF, FG as we like and drawing through the points D, E, F, G straight lines parallel to the base BC; then, if the parts marked on the line AC are equal, we will assume that the parallels drawn through the points D, E, F, G represent the degrees of the accelerated speed, degrees that also grow in equal times…

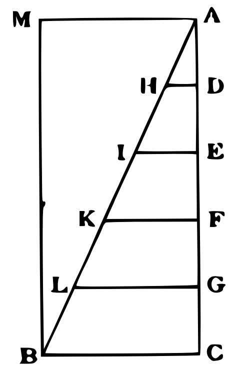

Figure 33.4 [Galileo on uniformly accelerated motion]

> But because the acceleration occurs continuously from moment to moment, not in a manner interrupted with such and such a duration…, before the mobile has reached the degree of speed DH acquired at the end of time AD, it has passed through an infinite number of smaller and smaller degrees, gained at instants in infinite number which the time DA contains, instants corresponding to the infinity of points that are in the line DA; therefore, to represent the infinite degrees of speed that precede the degree DH, it is necessary to imagine an infinite number of lines, always smaller and smaller, that are drawn, parallel to DH, from the various points of infinite number of the line DA; in the limit (*in ultimo*), this infinity of lines represents the area of the triangle AHD.
>
> Let us complete the entire parallelogram AMBC and extend up to its side BM not only the parallels that have been drawn in the triangle, but also the parallels of infinite number that one conceives issuing from all the points of the side AC. The line BC, which is the largest of the parallels drawn in the triangle, represents the highest degree of the speed acquired by the mobile in its accelerated movement; the total area of the triangle is the totality and the sum of all the speed (*la massa e la somma di tutta la velocità*) with which the mobile, in the time AC, has traversed such a space. Similarly, the parallelogram comes to be the totality and the aggregate (*la massa e aggregato*) of all degrees of speed, each of which is equal to the maximum degree BC. This totality of speeds comes to be twice the totality of increasing speeds of the triangle, just as the parallelogram is double the triangle. Therefore, if the mobile which, in falling, used the degrees of an accelerated speed consistent with the triangle ABC, has crossed in such time such a space, it is very reasonable and probable that by using uniform speeds which correspond to the parallelogram, it would have in the same time, with a uniform motion, crossed a space double that which it traversed with accelerated movement.

To obtain this proposition, which is equivalent to the classic one since the time of Nicole Oresme, Galileo, in summary, reasoned as follows:

The area of the figure with free fall durations as abscissas and speeds as ordinates represents something that one agrees to name totality (*masse*) or sum of speeds.

It is postulated that this totality or sum is identical to the space traversed during the time to which it relates.

It is postulated, we say, not demonstrated, for is it possible to assign the name of demonstration to that discourse where an area is supposed to be formed by joining an infinite number of lines? Certainly not, and the demonstration of Galileo, like that of Oresme, ultimately rests on an implicit assumption, on the same implicit assumption as that of Oresme. If it differs from that of Oresme, it is, by these considerations, illogical that an area is treated as a sum of juxtaposed lines. For the logician, therefore, it is more vicious than that of Oresme; but for the historian, it is superior to it, and for that very reason which depreciates it in the eyes of the logician; it is, in fact, by such paralogisms that the human mind was oriented in the direction where it was to discover the calculus.

In this direction, moreover, Galileo might have been able, without much effort, to make more progress. What Beeckman said on this same subject was of a different accuracy and perfection than the reasonings of the Mechanician of Pisa. Beeckman therefore—or Descartes, of whom he declares himself the interpreter—is the true inventor of the deduction appropriate for justifying the rule which determines the path traveled in a uniformly varied motion. But this discovery, which Descartes and Beeckman themselves misunderstood, did not have any direct influence on the approach to Dynamics; it was necessary that Gassendi reworked it.

Let us return to the works of Galileo.

From 1604 to 1630, Galileo transformed into an exact theory his erroneous ideas about the accelerated fall of weights, and this transformation had the effect of reconciling the thinking of the Pisan with that of the Scholastics of Paris and Oxford; from 1630 to 1638, this rapprochement will become closer at the same time that the doctrine of Galileo will become clearer.

In the third day of the *Dialoghi delle scienze nuove*, a treatise *De motu naturaliler accelerato* is inserted. From the beginning of this treatise, Galileo admits that the fall of bodies is a uniformly accelerated motion, and he gives no other reason than the simplicity of this hypothesis:

> We are led as if by the hand to the study of uniformly accelerated motion when we observe what the purpose and rule is that nature follows in all its other operations; to accomplish this, it usually uses primitive means, the simplest, easiest; a person, I think, will not believe that we could swim or fly with a simpler and easier process than the instinctive, natural way that fish or birds use. Therefore, when I see a stone descend from the high place where it stood at rest, and reacquire increases of speed, how can I believe that these increases do not follow the simplest and most obvious law? And on the other hand, when I think about it carefully, I do not see any method of addition and increase simpler than that of always adding in the same way.

The law which would render the speed of fall proportional to the path traveled by the weight would not be less simple, and it had seemed easier to receive when Galileo began to treat the fall of heavy bodies; but now he admitted with an admirable perspicacity, although he demonstrates it unconvincingly, the absurdity of such a law.

Let us see how, from the uniform acceleration attributed to falling bodies, Galileo will deduce this consequence, which is Theorem I of his treatise *De motu naturaliter accelerato*:

> The time that a mobile departing from rest and moving with a uniformly accelerated motion takes to traverse a certain space is equal to the time that the same mobile would take to travel the same space with a uniform motion whose degree of speed would be half the supreme and ultimate degree of the speed of the uniformly accelerated motion.
>
> Represent by the length AB (Figure 33.5) the time during which the mobile, departing from rest at C, would travel the space CD; represent by EB the largest and last of the degrees

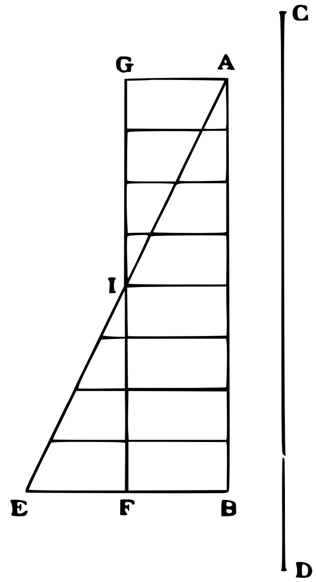

Figure 33.5 [Galileo's proof of the mean speed theorem]

> taken by the speed which grew at each instant of the time AB; raise EB perpendicularly to AB; join AE; the lines from the various points of the line AB and extended parallel to BE up to AE will represent the increasing degrees of speed from the moment A. We divide BE into two equal parts at point F and carry the parallels FG, AG to the lines BA, BF; the parallelogram AGFB thus constructed will be equivalent to the triangle AEB, and by its side GF, it will divide at I the line AE into two equal parts. Extend up to GIF the parallels drawn in the triangle AEB; the aggregate (*aggregatum*) of all the parallels contained in the quadrilateral will be equal to the aggregate of all the parallels included in the triangle; those, in fact, which are in the triangle IEF are equal to those contained in the triangle GIA; as for those in the trapezium AIFB, they are common. As the points of the line AB correspond one-to-one to the instants of time AB, and the parallels from the various points of the line AB and included in the triangle AEB represent the growing degrees of the accrued speed; as the parallels contained in the parallelogram represent as many degrees of a speed not increasing, but uniform, it appears that it had consumed as many moments of speed (*totidem velocitatis momenta absumpta esse*) in the accelerated movement that the increasing parallels in triangle AEB represent, as in the uniform motion represented by the parallels of the parallelogram GB. In fact, the moments that are missing in the first half of the accelerated movement (lacking, in fact, the movements represented by the parallels of the triangle AGI) are offset by the moments that the parallels of triangle IEF represent. It is thus evident that the spaces traversed in the same time by two mobiles—of which the one, starting from rest, would move with a uniformly accelerated motion, while the other would move with a uniform motion half that of the greatest moment of the accelerated motion—will be equal to each other; this is what we intended to demonstrate.

Let us divest the thought of Galileo from the form in which it was clothed, a form which will remain inaccurate, as we said, until, by the use of integral calculus, Gassendi, taking up the tradition of Descartes and Beeckman, will bring forth the correct idea that it hides. What remains in what we have just mentioned, if not from the considerations we repeatedly read in support of this adage: *Latitudo uniformiter difformis gradui medio correspondet?* Have we not encountered in all that Galileo has just told us the *Tractatus de figuratione potentiarum* of Nicole Oresme, the notes that a Parisian scholar put on the margins of the *Summa* of Dumbleton, the commentaries of Cajetan of Tiene on the *Regulæ* of Heytesbury, the*Expositio in libros Physicorum* of Juan de Celaya? If some prophetic insight revealed the *Dialoghi delle scienze nuove* to Nicole Oresrne, would have he been entitled to regard Galileo as his successor, while the revelation of the *Geometry* would have allowed him to claim Descartes for his disciple?

And now, one last inevitable question arises: Had Descartes and Galileo read these books, from the tradition of Paris or the tradition of Oxford, which prepared the work of Galileo and Descartes?

Concerning Descartes, we found no information that would allow us to give a confident answer to this question. But it is not the same regarding Galileo. Galileo read many of the works that introduced into Italy the theories of the School of Oxford and the Italian writings that had commentated on these theories.

The monuments left for us of the first intellectual activity of Galileo are three treatises, or rather three fragments of treatises, written in Latin, which most publishers of the great Pisan mathematician had disdained and which finally Mr. A. Favaro had the happy idea of publishing at the beginning of the national edition.

Of these treatises, the first, entitled *De Cœlo*, is a series of questions similar to those that all the Scholastics were accustomed to debate regarding the Περί Οὐρανοῦ. The second, untitled, is devoted to the degrees of forms, to action and reaction, that is to say, to problems of which the *De generatione et corruptione* supplied the text. The third, finally, is a treatise *De elementis*, designed in the style of the treatise of Achillini, which is frequently cited therein, as well as the writings of Paul of Venice.

Furthermore, we find many books quoted there. Some of these quotes are worth our attention.

There is, firstly[11], the statement of an opinion supported by "Marsilius, in the second book *De Generatione*».

A little further[12], regarding the problem of action and reaction, we read these lines:

> *Secunda dubitatio: quomodo se habént primæ qualitates in activitate et resistentia. De hac re lege* Calculatorem in tractatu De reactione, Hentisberum *in sophismate An aliquid fiat*, Marlianum *in suo introductorio De reactione*, Buccaferri *2° de generatione q° De reactione*, Thienensem *tract. De reactione*, Pomponatium *sec°. p^a. De reactione a cap. 13, et a Met. dub. 4 et 9.*

Galileo was not content to read the treatises of the Italian authors—of Marliano, Cajetan of Tiene, Buccaferri, and Pomponazzi; he tackled the abstruse writings that

[11] *Le Opere di* Galileo Galilei *ristampate fedelmente sopra la edizione nazionale*. Volume I, Firenze, 1890 [Galilei et al (1890)], p. 167 (*Tractatus de elementis*, Secunda disputatio: De primis qualitatibus. Quæstio tertia: An omnes quatuor qualitates sint activæ).

[12] Galileo, *loc. cit.*, p. 173 (Quæstio quarta: Quomodo se habeant primæ qualitates in activitate et resistentia.)

Oxford produced; he feared neither the thorny sophisms of Heytesbury nor the tedious chicaneries of the mysterious Calculator.

But maybe, in these writings, he paid no attention to the passages about uniform, difform, and uniformly difform latitudes? Let us not get hung up on this doubt. There is, in the untitled treatise, a *Quæstio ultima: De partibus sive gradibus qualitatis*; and, in this question, the following passage[13] will dispel our uncertainty:

> Note that a quality always resides in a subject endowed with magnitude; therefore, besides its own degrees, it participates in the latitude of this magnitude and can be divided according to the parts of the magnitude. Let one then compare the parts of the quality with the parts of the quantity; either in all parts of the quantity there will be equal degrees of quality, and the quality will then be called uniform; or there will be unequal degrees, and it will be called difform. Suppose that the excesses [of the degrees of quality] that these parts have over the others are equal to each other; that there is, for example, in the first part, 2 degrees, 4 in the second, 6 in the third, and so on, the excess being always equal to 2; the quality is said to be uniformly difform; if it is not so, it is said to be difformly difform. Suppose now that the unequal excesses of the quality behave such that it has, for example, 4 degrees in the first part, 6 in the second, 9 in the third, and so on; we will say that the quality is uniformly difformly difform; if the excesses are not proportional [i.e., do not form an arithmetic progression], the quality will be called difformly difformly [*sic*] difform.

When after reading this passage we hear Galileo establish, by the famous demonstration of the triangle, the law of the space traversed by a uniformly accelerated motion, will we for one moment not be able to recognize a reminiscence of the theories that Heytesbury and the Calculator taught?

Galileo knew the Kinematics of the School of Oxford, and, in the happiest way, it influenced him.

Did he know the Dynamics of Paris, the Dynamics of Jean Buridan and Albert of Saxony with which his own thoughts often offer such striking similarities?

In his youthful writings, Galileo quoted twice the Parisian Doctors, *Doctores Parisienses*.

In the treatise *De elementis*, he tells us[14] that "according to Aristotle, whom the Parisian Doctors followed", the volumes of the elements form a progression of ratio 10. This opinion is, in fact, described in detail and accepted by Themo the son of the Jew, in the sixth question of the first book of his *Meteorology*.

The second citation is more precise. In his *De Cælo*, Galileo lists the authors who think the World could have existed from all eternity.

> This opinion,

he said[15],

> is that of Saint Thomas…, Scotus…, Ockham…, and the Parisian Doctors in the first question of the eighth book of *Physics* (*Doctorum Parisiensium 8 Phys. q. p.*[a]).

[13] Galileo, *loc. cit.*, p. 120.

[14] Galileo, *loc. cit.*, p. 138 (*Tractatus de elementis*, Pars prima: De quidditate et substantia elementorum; quæstio quarta: An formæ elementorum intendantur et remittantur).

[15] Galileo, *loc. cit.*, p. 35 (*De Cælo*, tractatio prima de mundo, quæstio quarta: An mundus potuerit esse ab æterno).

Here we see that by this collective name, Parisian Doctors, Galileo does not refer, in a general and vague way, to some school, but, in a specific way, to a certain well-determined work.

Now we see that in his first question on the eighth book of the *Physics*, Albert of Saxony declares, in effect, that, apart from the teaching of the law, the World and its movement could have existed from all eternity.

So what is this book, composed by some Parisian Doctors, where, regarding a question related to the *Meteorology*, one meets the opinion that Themo admitted in his *Meteorology*—which, in the first question of the eighth book of the *Physics*, he teaches exactly what Albert of Saxony taught in the first question of the eighth book of his *Physics*? But this description leaves no room for ambiguity; we know this work; it is the collection, published in Paris, twice, in 1516 and in 1518, where George Lokert gathered the *Physics*, *De Cælo*, *De generatione et corruptione* of Albert of Saxony, the *Meteorology* of Themo, and the *De anima* and *Parva naturalia* of Jean Buridan. It is this collection that Galileo was reading when he wrote some scholastic disquisitions; it is through this collection that he was introduced to the Dynamics of Paris.

Are we not now allowed to invoke the testimony itself of the great Pisan to salute these Parisian Doctors with the title of Precursors of Galileo?

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