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Sir *Isaac Newton*'s MATHEMATICK PHILOSOPHY More easily DEMONSTRATED:

WITH

Dr. *Halley*'s ACCOUNT of COMETS Illustrated.

Being Forty LECTURES Read in the Publick Schools at *Cambridge*.

By WILLIAM WHISTON, M. A.
Mr. LUCAS's, Professor of the Mathematicks in that University.

For the Use of the Young Students there.

In this *English* Edition the Whole is Corrected and Improved by the AUTHOR.

corrected by Cann see Whiston's Life I. 150

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Mathematical Philosophy.

LECTURE I.

AFTER having dispatch'd the Matters of pure Astronomy, we proceed unto the other Part of our Work, the Philosophy of the Famous Sir *Isaac Newton*. For we are purpos'd to trace the Steps of that Great Man, and to set forth his principal and most noble Philosophical Inventions in a more easy Method; that so we may bring that (as I may say) Divine Philosophy within the Reach and Comprehension of those, who are but indifferently perhaps exercis'd in the Mathematicks, and communicate the Knowledge thereof as far as may be. But since that it is necessary for any one that would undertake this Philosophy, that besides some Knowledge of Geometry, Arithmetic, and Astronomy, he should also be furnish'd with the Knowledge of the true Laws of Motions; and especially should understand something of the Nature and Properties of those Curve Lines, which are called the Conic Sections; the Nature of our Purpose, which, as was said, was to direct our selves chiefly to Mathematicians of the lower Form, and who only understand the first Elements of Geometry and Astronomy; therefore 'tis requir'd of us, that we should in the Beginning touch upon, and in some measure explicate, as well the Conic Sections, as the of late demonstrated Laws of Motion; that no one through his being ignorant of these Things, may lose his Labour in his Study of that Philosophy which we have now in hand. For indeed, as to the first Laws of Motions and Collisions, *Des Cartes* was so miserably mistaken about them, when he went about to establish them, and hath so boldly impos'd upon the World false Rules concerning Collision and Reflection of Bodies, that it is worth the while to endeavour to root out of the Minds of Men the Prejudices which have sprung from thence.

We shall therefore begin with the Conic Sections; and before we go about any thing else, give some Knowledge and Understanding of those Lines which are interwoven with all the Philosophy of the Famous *Newton*, who shews that all the Paths, whether of the Planets or Comets of our System, are according to some or other of the said Sections. But we shall not spend so much Time about this Matter, as to deliver the Conic Elements in any other than a summary Way, or otherwise than by bringing into View out of the Writers of Conics, and especially the Famous *De La Hire*; the Natures, and chief Properties, and Affections of these Curve Lines without their Demonstrations, assuming them for demonstrated. And forasmuch as although the said Lines may be set forth by mere Delineations in a Plane, as will be done afterwards; yet the Geometricians, as well the Moderns as those of old, have for the most part expounded them by the Sections of a Cone;

Cone; and because also these Curves can in no other Way be shew'd all together, and at once, and consequently, the mutual Relation and Cognation which is betwixt them, cannot in any other way of explicating them be so clearly made known: For these Reasons, I shall, in the first place, open the Natures of these Lines, and set them forth by the Sections of a Cone, proposing to explicate them by Delineations on a Plane afterwards.

If you take any immoveable Point without a Plane in which a Circle is describ'd, and imagine a right Line drawn through that Point, and produc'd infinitely both ways, to be mov'd about the Circumference of the Circle, the Superficies which will arise from this Motion, is called a Conical Surface; and the Surfaces on both Sides the immoveable Point taken conjunctly, are termed Surfaces vertically opposite: The immoveable Point, common to both, is nam'd the Vertex; the Circle, the Base; and the Solid comprehended under the Conical Surface and the Base, and which may be suppos'd to be infinitely produc'd, is called a Cone; to which Solid, that generated beyond the Vertex is both equal and like. The Right Line, which is drawn from the Vertex to the Center of the Circle which is the Base, is the Axis of the Cone: Which Right Line, if it be perpendicular to the Plane of the Base, the Cone is called a Right Cone; but if not, an Oblique or Scalene one. Further, a Plane howsoever posited, so that it passeth not through the Vertex it self, doth cut the Conic Superficies, and is called, A Secant or Cutting Plane; and another Plane which doth pass through the Vertex, and is every where Parallel to the Secant, goes by the Name of the Vertical Plane; and that Curve Line which the Conic Superficies describes in the Cutting Plane, is called a Conic Section; which Section varies according to the different Inclinations of the Cutting Plane to the Cone.

Hence will arise three Cases: (1.) When the Vertical Plane toucheth the Conical Surface or Surfaces, and then the Section in the Cutting Plane is called a *Parabola.* (2.) When the Vertical Plane neither touches nor cuts either of the Surfaces, then the Section is called an Ellipsis. (3.) When the Vertical Plane cuts one of the Surfaces, and consequently the other, then the Secant Plane also cuts both Surfaces, (since it is Parallel to the Vertical one) and the Sections are called *Hyperbolæ, Oppositæ,* or opposite Sections.

If therefore the Secant, and the Vertical Plane, be so mov'd round in a Parallelism each to other, that the Vertical Plane doth sometimes cut the Base, sometimes touch the Conic Superficies, sometimes is placed wholly without the Cone; It is manifest, that by those Conical Superficies, divers Species of Hyberbola's, divers Parabola's, and divers Species of Ellipsis, will be delineated in the Secant Plane. And moreover we plainly see, what a near Affinity there is betwixt all these Lines. For if the Section be parallel to the Base, or even in a Scalene Cone, if it be subcontrarily posited, it will be a Circle; which therefore is deservedly reckon'd amongst the Conic Sections, as being one of the Extremes of the Ellipsis; from which then, if you proceed by a gradual Change of the Inclination of the Cutting Plane, there will be produc'd infinite Species of Ellipses; until at length the Inclination becoming Parallel to the Side of the Cone, the other Extreme of the Ellipsis passeth into a Parabola. But then the Inclination of the Cutting Plane being never so

little

little changed further, there will arise an Hyperbola; of which there are infinite Species, according to the divers Inclination of the Vertical Plane within the Cone. So that the Ellipses do on this Side end in a Circle, and on that in a Parabola; the Parabola on this Side in an Ellipsis, and on that in an Hyperbola; the Hyperbolæ on one part in a Parabola, and on the other in a strait Line. But because the Conical Delineation of the Regular Curves may seem too difficult to many, I shall pursue it no farther, but proceed to that Exposition of these Lines, which is us'd by *Cartes* and others, and is perform'd by an easy Delineation of them upon a Plane.

For a right Conception therefore of the Production and Nature of an Ellipsis, let (*Plate* 1. *Fig.* 1.) H and I be two Points, or two Nails or little Pegs, about which let there be put a Thread BHI; and then putting your Finger, or a Pin, to the Thread, and keeping the same always in an equal Tension, move your Finger round from the Point B, until you return to the same Point B again. By this Revolution of the Point B, is describ'd the Curve Line, called the Ellipsis, which differs from the Delineation of a Circle only in this, that a Circle hath only one Center, but the Ellipsis hath, as it were, two Centers; which indeed, if the said Points H and I, their Distance vanishing away, should come together into one, the Elliptic Curve would become perfectly Circular. But by how much the greater the Distance is betwixt those Points, the same Length of the Thread still remaining; so much the farther is this Figure remov'd from the Circular. So that according to the divers Proportion of the Distance HI to the Thread BHI, or to the Line DK, which is equal to the same Thread made less by the

Diſtance HI, divers Species of Ellipſes will be deſcribed. But then, if the Length of the Thread ſhall be increas'd or diminiſh'd, in the ſame Proportion as the Diſtance of the Points H and I ſhall be increas'd or diminiſh'd, there will be deſcrib'd indeed divers Ellipſes, but which are all of the ſame Species, or like to one another. From whence it appears, that Ellipſes are not only innumerable in Magnitude but in Species alſo, and reach from a Circle to a Right Line: For like as when the Points H and I meet together, the Ellipſis becomes a Circle; ſo when they are remov'd from each other half the Length of the Thread, it becomes a Right Line, both Sides meeting together. From whence alſo it is manifeſt, that every Species of Ellipſes is no leſs different from any other, than the Extremes of them are different on this Side from a Circle, and on that from a Right Line. It alſo appears from this Delineation, that if from a Point taken at Pleaſure in the Elliptick Periphery, as the Point B, you draw two Lines to the two Central Points; theſe two Lines BH and BI taken together, will be equal to the greateſt Diameter DK; and conſequently that the Sum of them is always given: Which thing the Conſtruction it ſelf ſhews. For that Part of the Thread, which is extended from I to B, and from thence back to H, is the ſame with that which returneth from I to F, and from thence back to H; and again, that Part of the Thread which reaches from D to H, is the ſame with that which reacheth from K to I, or DH is equal to IK; therefore IB + BH, which by the former is equal to ID + DH, is equal to ID + IK, that is, to KD.

And thus much for the Production of the Figure in a Plane; we ſhall now ſubjoin the Names of of the chief Lines in it, and the moſt notable Properties thereof; ſo as to give ſome Sort of Knowledge at leaſt of this moſt Noble Curve, for the more right underſtanding true Aſtronomy, and the Courſes of the Planets.

In *Fig.* 2. *Plate* 1. DFKR is an Ellipſis; C the Center; the Points H and I, the Foci thereof; DK the greater Axis, or the Tranſvers Axis, or the principal Diameter, or *Latus tranſverſum*, the Tranſvers Side; FR is the leſſer Axis: All the Right Lines paſſing through the Center C are Diameters: All Right Lines terminated at the Periphery, and which are divided into two equal Parts by any Diameter whatever, are called Ordinates, or Lines orderly applied, to wit, with reſpect to that Diameter. Thus MG paſſing through the Center, is a Diameter; and PK which is divided into two equal Parts by the ſame, is an Ordinate thereof, or a Line orderly applied thereto. That Part of every Diameter, which is intercepted betwixt the Vertex thereof and the Ordinate, as Mμ, is call'd the Abſciſſa or Abſciſs thereof, (as being cut off from the ſame Diameter:) A Line drawn from the Vertex of the Diameter, parallel to the Ordinates thereof, as $n\theta$, is a Tangent to the Ellipſis in that Vertex. A Diameter parallel to the Ordinates of another Diameter, and which conſequently hath its Ordinates parallel to the former Diameter, is term'd a Conjugate Diameter. Thus GM and VT are conjugate each to other, and the Ordinate PK is parallel to the Diameter VT, and the Ordinate KE to the Diameter GM. The Ordinate to the greater Axis, which paſſeth through either of the Foci, as MA in the firſt Fig. is term'd the principal *Latus rectum*, or the Parameter of the greater Axis.

Now the most notable Properties of this Ellipsis are these: (1.) The Ordinates of every Diameter, which by the foregoing Definitions are bisected by the Diameter, are parallel each to other.

(2.) The Ordinates of the Axes are perpendicular to the Axes themselves: But the Ordinates of the rest of the Diameters are oblique to their Diameters; and in Ellipses of divers Species, so much the more oblique, at equal Distances from the Axis, by how much the Proportion of the greater Axis to the lesser is the greater; but in the same Ellipsis, so much the more oblique, by how much the more remote the Diameters are from the Axes.

(3.) There be only two Conjugate Diameters, which are equal each to other; those, to wit, whose Vertices are at equal Distances from the Vertices of the Axes. Thus the Diameter V T is conjugate and equal to that other GM, where, to wit, VF is equal to MF, and VD equal to MK.

(4.) The obtuse Angle VCM of these two Diameters, which are conjugate and equal, is greater, and the acute Angle VCG is less than every other Angle contain'd by the rest of the Diameters that are conjugate to each other.

(5.) If the Lines μP and νB be Semi-ordinates to any Diameter, as MG; the Square of the Semi-ordinate μP is to the Square of the Semi-ordinate νB, as is the Rectangle Mμ x μG to the Rectangle Mν x νG; that is, μPq is to the Rectangle comprehended under the two Parts, into which the Diameter is divided by the Ordinate KP, as νBq is to the Rectangle under the Parts of the Diameter made by the Ordinate AB.

(6.) The

(6.) The Parameter, or *Latus rectum* of any Diameter is a third Proportional to that Diameter, and its Conjugate. That is (in Figure 1.) if the Diameter D K, is to its conjugate Diameter E F, as E F to Y, then Y is the Parameter or *Latus rectum* of the Diameter D K. Whence A M, an Ordinate to the Axis thro' the Focus, is as above, equal to the principal Parameter, and is a third Proportional after the greater and lesser Axis. For the Axes are the principal Pair of conjugate Diameters.

(7.) The Square of every Semi-ordinate, (as M I in the first Figure) is less than the Rectangle made of any Abscis whatever, (as I K) drawn into the *Latus rectum* of its own Diameter, (or than I K × Y.) And in the other Figure, P μ q is less than the Rectangle made of the Abscis M μ, and the *Latus rectum* of M G. From which Defect, or ἔλλειψις this Section hath its Name.

(8.) If from any Point, as B in the first Figure, you draw the right Lines B H and B I to the Foci, the Sum of them will be equal to the greater Axis, as was shew'd above. And if the Angle I B H comprehended by those Lines be bisected by the right Line b a; the Line a is perpendicular to the Tangent V B in the Point B, that is, to the Curve in the Point of Contact.

(9.) The Curvature, with respect to the Center of the Ellipsis, is at divers Distances from that Center in the Quadruplicate Proportion of those Distances directly: As, if C K be double of C F, the Curvature in the greater distance K, shall be to the Curvature in the lesser distance F, as 16 is to 1; and if C K be Treble, C F, the Curvature in K, will be to that in F as 81 to 1. And so of the rest.

(10.) The

10. The Curvature of the Elliptic Arches, with respect to the Focus, is in divers Distances from that Focus, in the simple Proportion of the Distance directly. Thus, if H D be half of H K, the Curvature at D, if you respect the Focus; XH will be half of that at K, respecting the same Focus, and so of the rest. And the Thing is the same in a Parabola and Hyperbola.

(11.) The Distance of a Body turn'd round in an Ellipsis, about the Focus H, from the same Focus, is the greatest of all in the Point K, least of all in the Point D, and mean in the Points E and F; and that mean distance H F is equal to the greater Half-Axis D C or CK; as is manifest from the Production of the Ellipsis.

(12.) The vanishing Subtense of the Angle of Contact, parallel to the Distance from the Focus, at an equal perpendicular Interval from that distance, always remains given and unvaried in the same Ellipsis, yea and in the same Parabola and Hyperbola too. Thus if d Z be always given, g d also will always remain given in a distance infinitely small.

(13.) The Area of the Ellipsis is to the Area of the Circle circumscrib'd, as the lesser Axis is to the greater; and so are all correspondent Parts whatever amongst themselves, as M I K, m I K; and the Ordinates to the greater Axis, as M I are divided by the Elliptic Periphery always in the same Proportion; so that M I is to m I always in the same Proportion; to wit that of the lesser Axis to the greater. And we are to reason in the same manner concerning a Circle inscrib'd in the Ellipsis.

(14.) All Parallelograms describ'd about the conjugate Diameters of the Ellipsis, and comprehending the Ellipsis, are equal. Thus the Parallelogram

logram $\alpha\beta\gamma\delta$ is equal to that other $\epsilon\zeta\eta\vartheta$; and thus it is every where.

(15.) If a right Line always passing through one of the Foci be so mov'd, that the Elliptic Area describ'd by the same is proportional to the time; the Angular Motion of a right Light drawn from the other Focus to the former Line, will be almost equable. Thus in the former Figure, if the Angular Motion of the Line H B be so attempered, that the same being according to the reciprocal Proportion of the Distance accelerated or retarded, doth describe the Area DHB proportional to the time, the Angular Motion K I B, about the other Focus I will be almost proportional to the time, and consequently without any notable Acceleration or Retardation, and nearly equable; that is to say, where the Ellipses doth not differ much from a Circle.

Feb. 7. $170\frac{3}{4}$.

LECT. II.

TO pass now from the Ellipsis to the Parabola, let D I be an Infinite right Line, and I L another perpendicular to it. Then there being taken in the Line D I any Point F, let the Line F I be bisected in the Point T. And let there be taken Two Threads joined together in the Point T, one T I, the other T F. And let a Pin fixed to the Threads in the Point T be moved to the Right and Left, in such a manner, that when the Pin is in any other Position as in P, the Thread

T I which here becomes P L be always perpendicular to I L, or, which is the ſame Thing parallel to D I, but equal to the Thread T F, which in this Caſe becomes P F, ever paſſing thro' the Point F. And the Curve thus generated by the Pin, infinitely produced both ways, is called a Parabola. In which g P i T s R o is the Periphery; I D the Axis or principal Diameter; F the Focus. The Point T the principal Vertex; an Ordinate to the Axis through the Focus is equal to the principal *Latus Rectum*. All right Lines n i, or R Z Parallel to the Axis, are Diameters, as dividing the Lines i h and K T which are Parallel to the Tangents at their Verttices into Two equal Parts; and they are called Diameters belonging to the Vertices in which they terminate, as T, i.

Now the principal Properties of a Parabola are theſe.

(1.) Every Diameter or right Line parallel to the Axis, biſects all the Lines within the Figure which are parallel to the Tangent of the vertical Point. Which biſected Lines are as hath been ſaid called Ordinates.

(2.) The Ordinates of the Axis are perpendicular thereto: But the Ordinates of the reſt of the Diameters are oblique to their Diameters; and ſo much the more oblique, by how much the Vertex of the Diameter is further remov'd from the principal Vertex of the Parabola.

(3.) The *Latus rectum*, or Parameter to every Diameter, is a third Geometrical Proportional after any abſciſſe and its ſemi-ordinate; that is the *Latus rectum* of the Diameter (i n), (or that of the Vertex (i)) is y; if it be thus; as the Abſciſs (iq) is to the Semi-ordinate (q k); ſo is that Semi-ordinate (q k), to y.

(4.) The

(4.) The principal *Latus rectum*, or that belonging to the Axis, is equal to the Ordinate (h i) passing through the Focus; and fourfold of F T, the least distance of the Focus from the principal Vertex.

(5.) The *Latus rectum* belonging to any Vertex or Diameter, is also fourfold of the distance of that Vertex from the Focus. Thus the *Latus rectum* of the Vertex s is fourfold F s, and so it is every where.

(6.) The distance of any Vertex or Point in the Parabola whatsoever from the Focus, is equal to the least distance of the same from the Line L L, which is perpendicular to the Axis, and is distant from the principal Vertex by a Quarter of the *principal Latus rectum*. For by the Construction, the Line F P is equal to P L.

(7.) The Square of every Semi-ordinate, as (q k) is equal to a Rectangle made of the *Latus rectum*, of the same Vertex as Y, and (i q) the Absciſſe of the Diameter of the Vertex. And from the Equality of the παραβολη, or Compariſon in the Figure, betwixt the Rectangle and the Square of the Semi-ordinate, without any Exceſs or Defect, the Name of the Section is derived.

(8.) When therefore, the *Latus rectum* in any Diameter is given, the Abſciſſes are as the Squares, or in the duplicate Proportion of the Semi-ordinate. Thus T F is to T G as i F q is to g G q; and so likewiſe is i q to i r, as the Square of q T is to the Square of r l; and thus every where. From whence alſo, when the Abſciſs of the Axis is equal to the principal *Latus rectum*, or fourfold of the distance from the Vertex, it will be equal to its ſemi-ordinate.

(9.) The

(9.) The Angle comprehended by any Tangent whatever, and a Line from the Focus, is equal to an Angle comprehended by the same Tangent, and any Diameter or the Axis. Thus the Angles I i F and p i n are equal. From whence indeed, (which thing is to be noted by the way) all the Rays of Light which fall upon the Concave part of the Surface produced by the Convolution of the Parabola about the Axis, which fall, I say, upon the same Parallel to the Axis, will be reflected from a Concave Paraboloid Figure to the Focus F, and will beget there a most vehement burning; from which Property indeed the Point F hath the Name of Focus; and hath communicated the same Name to the like Points in an Hyperbola and Ellipsis.

(10.) A Parabola, like as an Hyperbola, doth not enclose a Space, but is stretched forth *in infinitum.*

(11.) A Parabolic Curve always tends more and more *in infinitum* to a Parallelism with its Diameters, but can never reach thereto.

(12.) If two Parabolæ's be described with the same Axis and Vertex, the Ordinates to the common Axis will be cut off by the Parabolæ in a given Proportion; and the Area's comprehended by the same Axis and Ordinate, and the respective Curves will be in the same given Proportion to one another.

(13.) Every Parabolic Space, comprehended betwixt the Curve and the Ordinate, is to the Parallellogram made of the same Base and Altitude in a Subsesquialteral Proprortion, that is, as 2 is to 3, and to the external Space in a double Proportion, or as 2 is to 1. So q i T is to q i I as 2 is to 3, and to i I T as 2 is to 1. From whence it becomes easy to square the Parabola.

(14.) The

(14.) The distance betwixt the Vertex of the Axis, and the Point where any Tangent whatever intersects it, as I, is equal to the Abscis of the Axis which belongs to the Ordinate apply'd from the Point of Contact. So T I is equal to T F; and thus it is every where.

(15.) All Parabolæ's are like, or of the same Species; as are also all Circles.

(16.) If a Diameter be continued through the Point of meeting of Two Tangents, this Diameter will bisect the Line that joins the Contacts. Which Property of the Parabola is likewise to be applyed to the Ellipsis and Hyperbola.

And thus much for the Parabola. We now come to the *Hyperbola*. (*Fig.* 4. *Plate* 1.) Take a Staff or Rule of a sufficient Length as I B, let I and H be two Central Points answering to the Foci of an Ellipsis, in which let Nails be fastned; then there being tied to one end of the Stick a Rope or Thread, twofold shorter than the Stick, let the other end thereof be bor'd through, and so fixed upon the Nail I; but as for the other end of the Rope let it be fixed by a Knot upon the other Nail H; which done, laying your Finger upon the Point B, where the Rope and Staff are tyed together, let your Finger descend so long that you have thereby now applyed and joyn'd the whole Rope to the Staff or Rule, the Staff having been in the mean while, as it needs must, wheel'd about the Centre I. And thus you have describ'd by the Point B, the Vertex of the Angle H B I, a Curve Line, X B D which is part of an Hyperbola; the whole consisting of that Curve wich will result from the Curve X B D, which hath added to it the Curve Y D, the Product of the Rule and Work as turn'd to the other Side.

Side. Furthermore, if you transfer the Hole or Knot of your Rope to the Nail I, and fasten the end of the Staff upon the Nail H, you will describe another Hyperbola vertically opposite to the former, which is altogether like and equal thereto. But then, if without changing any thing in the Rule and Nails, you shall only apply a longer Rope, you will describe an Hyperbola of a different Species from the former; and if you shall still lengthen the Rope somewhat, you will have another Sort of Hyperbola; until at length making the Rope double in length of the Rule, you will have the Hyperbola chang'd into a right Line. But if you alter the Distance of the Nails in the very same Proportion, in which you change the difference betwixt the Length of the Rope and that of the Stick; in this Case you will have Hyperbolæ mark'd out, which are altogether of the same Species, but have their similar Parts differing in Magnitude. And lastly, if the Length of the Rope and Rule be equally increas'd, their Difference in the mean while, and the Interval of the Nails remaining the same; not a different Hyperbola either as to Species or Magnitude will be describ'd, nor any other than a greater Part of the same Hyperbola. And this for the Mechanical Construction of an Hyperbola in a Plane.

But it is to be acknowledg'd, that many Properties of an Hyperbola are better known from another manner of generating the Figure; which Way is this: (See *Fig.* 5. *Plate* 1.) Let LL and MM be infinite Right Lines interfecting each other in any Angle whatever in the Point C: From any Point whatever, as D or e, let Dc, Dd, be drawn parallel to the first Lines, or (ec, ed;) which with the Lines first drawn make the Parel-

Parallelograms as D c C d, or e c C d: Now conceive two Sides of the Parallelogram as D c, D d, or e c, e d, to be so mov'd this way and that way, that they always keep the same Parallelism, and that at the same time the Area's always remain equal: That is to say, that Dc and e c remain always Parallel to M M, and D d or e d always Parallel to L l; and that the Area of every Parallelogram be equal to every other; one Side being increas'd in the same Proportion wherein the other is diminish'd. By this means the Point D or e will describe a Curve-Line within the Angle comprehended by the first Lines; which is altogether the same as was describ'd above, both by the Section of a Cone, and *Cartes's* Delineation. And in like manner, in the Angle vertically opposite will be describ'd a like and equal Hyperbola, if so be the Parallelogram C c K d, equal to the former, be suppos'd to be mov'd in the same manner as before: Which Hyperbola's are, as was said before, called opposite Sections, or opposite Hyperbolæ. Now in either of the two Figures, D K is the Transvers Axis, or Transvers Diameter of the Hyperbola, or the Opposite Sections: The Point C is the Center: The Points H and I the Foci. And in the 2d Figure, all the Lines passing through the Center C, as i h are Diameters. But if Hyperbolæ be describ'd in the following Angles, as L C M, M C L, those Sections will be called the Following Sections; and if the Distance of the primary Vertex of those Hyperbolæ from the common Center C, as C β, or C γ, be equal to the Semi-tangent K ν, or K ω, at the primary Vertex of these, those Sections shall be called Conjugate Sections: And all the Figures together will be to be named the Hyperbolic System.

G Fur-

Furthermore, (i h) the Ordinate to the Axis through the Focus, is equal to the principal *Latus rectum*, or the Parameter of the Axis; and an indeterminate Diameter, whether of the following Sections, or of the former, which is parallel to the Ordinates of any determinate Diameter, is called the Conjugate Diameter of the same: and hath its Ordinates parallel to the former Diameter.

And now we come to the principal Properties of the Hyperbola, and the opposite Sections, which are as follows:

(1.) Any Diameter or right Line passing thro' the Center, bisects all its Ordinates; that is, all the Right Lines which are terminated on both Sides by the Hyperbolic Periphery, and those parallel Lines that are bisected by any Diameter whatever, are called the Ordinates of that Diameter.

(2.) The Ordinates of the Axis are perpendicular to the same: But the Ordinates of the rest of the Diameters are oblique to their Diameters; and so much the more in divers Species, at equal Distances from the Axis, by how much the Difference of the Angles including the Hyperbolæ is the greater; and in the same Hyperbola, so much the more oblique, by how much the Diameters are remov'd from the Axis.

(3.) If any Lines, as H h and Q s, be Semi-ordinates to any Diameter whatever, as K D; the Square of the Semi-ordinate H h is to the Square of the Semi-ordinate Q S, as the Rectangle K H D H is to the Rectangle K Q, D Q: And so the Square (b n) is to the Square (a K,) as the Rectangle (i b h b) is to the Rectangle (i a h a); and thus every where.

(4.) The *Latus rectum*, or Parameter of every Diameter, is a third Geometrical Proportional after

after the Diameter, and the Conjugate thereof, (or its Tangent, which is equal to it:) That is, the *Latus rectum* of any Diameter, as D K is Y, if it be thus; as the Diameter D K is to its Conjugate $\beta\nu$, or its equall ($\omega\nu$); so that Conjugate $\beta\gamma$, or that Tangent ($\omega\nu$) is to y. And as the Ordinate to the Axis through the Focus is the principal *Latus rectum*, so it is more than Quadruple of the least Distance of the Focus from the Vertex.

(5.) The Square of any Semi-ordinate whatever, as (Q r,) is greater than a Rectangle made of the Abscifs D Q, drawn into the *Latus rectum* of its own Diameter, as y: And in like manner, the Square of the Semi-ordinate (b n) is greater than the Rectangle of the Abscifs (i b) into the *Latus rectum* of the Diameter (h i.) From which ὑπερβολὴ, or Excefs, this Section hath its Name.

(6.) If from any Point of the Hyperbola, as (B) in the former Figure, there be drawn Right Lines to both the Foci, as BH, B I, the Difference of these Lines will be equal to the Axis D K; as will easily appear from the Delineation it self.

(7.) If the Angle H B I, comprehended by Lines drawn to the Foci, be bisected by the Right Line E B, that Right Line will be a Tangent to the Hyperbola in the Point B.

(8.) The Right Lines L L, and M M, which enclose the Hyperbolæ, are Asymptots of the Hyperbolæ, *i. e.* they are such unto which on both Sides the Curve approacheth nearer and nearer, but is never able to touch or coincide with the same.

(9.) The Species of Hyperbolæ are various, according to the different Magnitude of the Angle L C M comprehended by the Asymptots: But that Angle remaining the same, the Species of the Hyperbola remains unchang'd; but according to the different Magnitudes of the Parallelograms, by which the Hyperbolæ are describ'd, Hyperbolæ of divers Magnitudes do arise: But if the Angle contain'd by the Asymptots be a right Angle, the Hyperbola is called Equilateral or Rectangular, and the *Latus rectum* of all the Diameters will (as it is in a Circle) be equal to their Diameters. And lastly, if Hyperbolæ be describ'd about the same Axis, in divers Angles of the Asymptots, the Right Lines perpendicular to the Axis will be cut off in a given Proportion by them all; and the Spaces likewise enclos'd by the Right Lines or Ordinates, the produced Axis, and the Curves, will be in the same given Proportion.

(10.) If the Distances from the Center of the Hyperbola be taken in a Geometrical Proportion in one of the Asymptots, so that CI, CII, CIII, CIV, CV, CVI, be continuedly proportional geometrically; and if from those Points there be drawn parallel to the other Asymptot, the Lines, I 1, II 2, III 3, IV 4, V 5, VI 6; the Spaces I 2, II 3, III 4, IV 5, V 6, will be equal amongst themselves. And consequently, if that Asymptot C M be suppos'd to be divided, according to the Proportion of Numbers exceeding one another in a natural Series, those Spaces will be proportional to the Logarithms of all those Numbers.

Feb. 14. 170¾.

LECT.

Lect. III.

HAVING now expounded ſeverally the Curve Lines, called the Conic Sections; let us now compare them together, and briefly conſider, what Affinity there is betwixt them, what mutual Reſpect they bear to one another, and what Difference there is among them.

Let the Point A therefore (ſee *Fig.* 5. *Plate* 1.) be the Center of the Circle F X B Y, and the common Focus of all the Sections: [And it is indeed a certain Center, as it were, of all the Sections: And the Ordinate to the Axis through the Focus, or *Latus rectum*, doth in moſt of them more agree with the Diameter of the Circle paſſing through the Center thereof, than the Axis it ſelf of the Section doth:] Then let the Point F be the principal Vertex of all the Sections; and F X B Y a Circle, the Center of which Figure, as being only an Extream Ellipſis, falls in with the Foci, (where X Y will be, if I may ſo ſpeak, the *Latus rectum* of the Circle paſſing through the common Focus or Center, and equal to the reſt of the Diameters.) Let F G H I be an Ellipſis leſs Curve on the Vertex than the Circle is; the remoter Focus of which Ellipſis is the Point C; F H the principal Diameter or greater Axis; G I the leſſer Axis; e f the principal *Latus rectum*, which is more than double to A F, the Diſtance of the Vertex F from that Focus A, but leſs than Quadruple thereof. But it is to be noted, that another Ellipsis also may be drawn more Curve in F than the Circle; but then it is describ'd about the Point A, as the remoter of the Foci. But then after the greater Ellipsis, the Center thereof departing further *in infinitum*, there ariseth the Conic Section L d F c K, which we call a Parabola; which indeed is half of an Ellipsis infinitely long; the Axis whereof is the Infinite FH, and (c d) the *Latus rectum*: Which same is Quadruple of the Distance of the Vertex from the Focus A F. As for the Curvature of the Parabola in the Vertex F, it is less than that of the Ellipsis, as is easy to be seen. Then lastly, the Hyperbola M i F l N follows, whose Parameter, or principal *Latus rectum* (i l) is more than Quadruple to A F, the Distance from the Focus: and the Curvature thereof in the Vertex F, is less than that of the Parabola, and will infinitely be diminish'd, the Angle T E V, contain'd by the Asymptots, being increas'd *in infinitum*, until at length the Asymptots falling into one Right Line, the Hyperbola it self with its Asymptots, end in the Right Line O P perpendicular to the Axis. From whence it is to be noted, (1.) That the Conic Sections are in themselves a System of Regular Curves allied to each other; and that one is chang'd into another perpetually, when it is either increas'd or diminish'd *in infinitum*. Thus the Circle, the Curvature thereof being never so little increas'd or diminish'd, passeth into an Ellipsis; and the Ellipsis, its Center going away infinitely, and the Curvature being by that means diminish'd, is turn'd into a Parabola: And when the Curvature of the Parabola is never so little chang'd, there ariseth the first of the Hyperbolæ; the Species whereof, which are innumerable, will all of them arise orderly by a gradual Dimi-

nution

nution of the Curvature, until the Curvature vanishing away, the last Hyperbola ends in a Right Line perpendicular to the Axis. From whence it is manifest, that every Regular Curvature like to that of a Circle, from the Circle it self unto a Right Line, is a Conical Curvature, and is distinguish'd with its peculiar Name, according to the divers Degree of that Curvature. (2.) That the *Latus rectum* of a Circle is double to the Distance from the Vertex; that all the *Latera recta* of the Ellipses are in all Proportions to that Distance betwixt the Double and Quadruple, according to their different Species: That the *Latus rectum* of the Parabola, is just Quadruple to that Distance; and lastly, that the *Latera recta* of Hyperbolæ are in all Proportions beyond the Quadruple, according to their various Kinds. (3.) That all the Diameters in a Circle and Ellipsis intersect one another in the Center of the Figure within the Section: That in the Parabola they are all parallel amongst themselves, and to the Axis; but that in the Hyperbola they intersect one another, but this without the Section, in the common Center of the opposite Sections. (4.) That the Curvature, with respect to the Focus in all these Figures, is increas'd or diminish'd proportionably to the Increase or Diminution of the Distance from the Focus. For although by reason of the Obliquity of the Tangents, the Curvature for the most part seems greater in a lesser Distance from the Focus, and less in a greater; yet the true Curvature, which is to be defin'd by the Subtense of the Angle of Contact, is on the contrary greater in a greater Distance, and lesser in a less, and greater or less in proportion to the Increase or Diminution of the Distance; as was above noted, and will be more fully opened in the Sequel. And thus much for the Conic Sections.

And forasmuch as we shall make some sort of Use of the Line called the Cycloid, we shall briefly describe it.

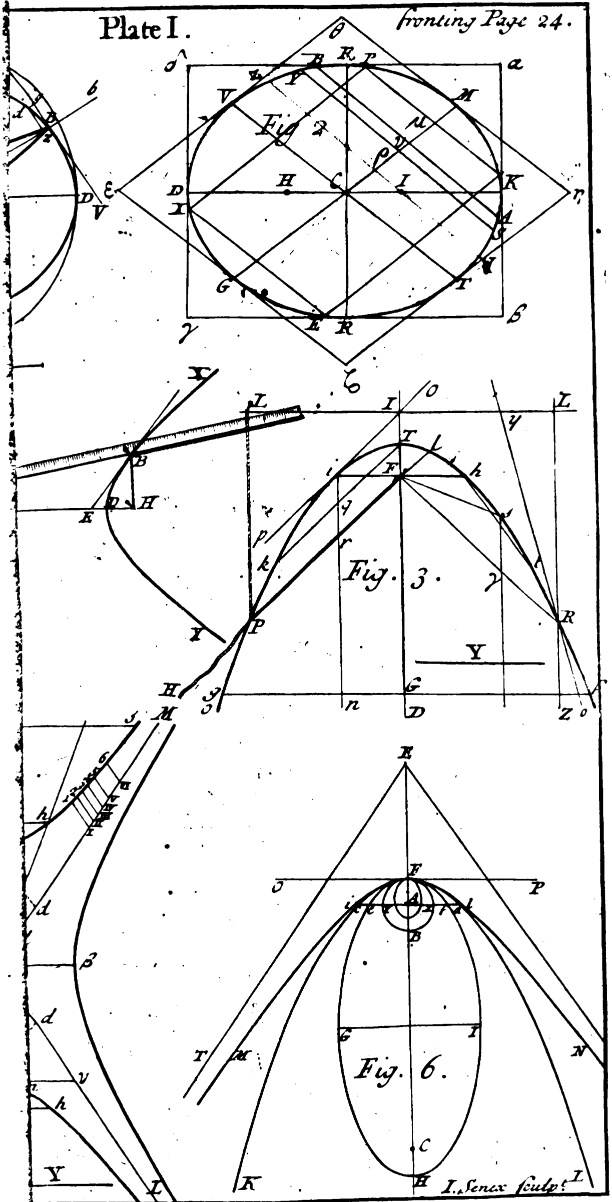
If upon the Right Line A E (see *Fig.* 1. *Plate* 2.) the Wheel or Circle A B C D be roll'd along, until the Point A, in which it at first touched the said Line, doth again after an entire Revolution meet and touch the same in E; the generating Circle A B C D will describe the Line A E equal to its own Periphery, and the Point A by its Compound Motion will describe the Curve Line A F E, which is called a Trochoid or Cycloid: The Length of which Line is Quadruple to the Diameter of the generating Circle; and the Cycloidal Space comprehended by this Curve, and the Subtense A E, is triple the Area of the generating Circle. Moreover, any part whatever estimated from the Vertex, as F I is every where double the Chord of the Circle F b, and the Tangent thereof G I H is perpetually parallel to the same Chord F b. And thus much for the Cycloid.

Now after this preparatory short Explication of the Conic Sections, we come to our proper Work: Intending to proceed next to the true Laws of Motion, both those commonly known, and those which were lately found out and establish'd by the Famous Sir *Isaac Newton.*

In the setting forth of whose Noble Inventions, we shall generally make use of the very Words of that great Man; but yet so, that every where we shall endeavour to explicate, demonstrate, and to make clear and plain to all, what either *Words or Things seem more obscure and difficult.*

DE-

Plate I. fronting Page 24.



DEFINITIONS.

(1.) BODY or Matter is an extended Substance, Solid, or Impenetrable, of it self merely Passive, and indifferent to Motion or Rest; but capable of any sort of Motion whatever, and of all Figures and Forms. I call it a Substance extended, because that it possesseth some part of extended Space; but Solid and Impenetrable: not because it cannot be penetrated by Space, or perchance by other incorporeal Substances, but because it is impenetrable by all other Matter; and upon that account it doth eminently claim the Name of Solid. I put in the Definition [its being indifferent to Motion and Rest] not that I reckon Motion, as well as Rest, a Thing plainly Negative or Privative, but because the Conception of a Body, as in Motion, is as easy and familiar as of a Body at Rest. I call it in it self Passive, because we perceive nothing of Action or Energy, or of a Power of moving it self, either in its Nature or Affections; but on the contrary from all the Phænomena of Motions we every where meet with its meer Inactivity. But I say that it is capable of any sort of Motion and of all Figures and Forms; since the daily Appearances in the World, and infinite Experiments, doe shew this to be the Nature of it. Time, Space, Place, and Motion, as being things so well known to all, scarce need to be defined. But however, for the taking away some Prejudices out of Mens Minds, it is very expedient, that with the famous *Newton* we should distinguish these Quantities into Absolute and Relative, True and Apparent, Mathematical and Vulgar, and so in a sort

ſort deſcribe them; which for Order's ſake ſhall be done in the following Definitions.

(2.) Time Abſolute, True, and Mathematical, is an eternal and equable Duration, compounded of Parts, ſucceeding each other in an immutable Order. For in it ſelf, and its own Nature it flows equably. Nor doth it depend on the Motion of Things, much leſs on their Reſt, nor indeed upon their Exiſtence. For whether any thing were mov'd or not, whether any Thing did exiſt, or nothing at all were in Being, it would be all one in this Caſe. Time flows equably, whatſoever relation any other Things have to one another.

(3.) Relative Time, or that which is Apparent and Vulgar, is ſome ſenſible and external Meaſure of Duration (whether it be by Motion, or ſome other way; whether it be Accurate, and even or uneven;) which is vulgarly uſed inſtead of the true time, as an Hour, Day, Month, Year, the Duration of the World, or any Syſtem from the beginning to the End, *&c.* In Aſtronomy, Abſolute Time is diſtinguiſh'd from Relative, by the Equation of the Vulgar Time: For the Natural Days are unequal, which are nevertheleſs commonly taken for equal in the meaſuring of time. This Inequality Aſtronomers correct, that they may meaſure the Heavenly Motions by a truer Time. It is poſſible, that there may be no even Motion at all by which Time may be accurately meaſured. All Motions may be accelerated and retarded, but the flowing of abſolute Time cannot be chang'd. The Duration or Perſeverance of the Exiſtence of Things is the ſame, whether their Motions be ſwift, ſlow, or none at all. Conſequently this Duration is juſtly diſtinguiſh'd from its ſenſible Meaſures, and collected

from

from them by Aſtronomic Equation. For this is that which Aſtronomers have labour'd after; namely, that from the unequal Motions of the Heavenly Bodies, they might find an equable Motion about ſome Center; from whence they may more eaſily and accurately meaſure Duration, that flows equably.

(4.) True, Abſolute, Mathematical Space, is an Extenſion Penetrable, Indiſcerpible, Immoveable, Infinite, Eternal, and every where like to it ſelf. Whether or no ſuch an Extenſion doth really exiſt diſtinct from Matter, is another Queſtion. But that this is the common Notion of Space with all, muſt be allowed by every reaſonable Man; and therefore, is to be taken as a Definition; for ſo Geometricians do at firſt define a Circle, a Square, a Triangle, *&c.* not troubling themſelves with the Queſtion, whether ſuch Figures do really exiſt or no. We ought therefore to lay down a Deſcription of Space ſhould be laid down aforehand, leaſt afterwards there ſhould ariſe Strife about Words; as we may afterwards enquire whether it be the Idea of a Thing really exiſtent.

(5.) Relative Space (which alſo as I ſuppoſe is commonly called Place) is the Meaſure of Abſolute Space, or any moveable Dimenſion, which is defin'd and determin'd by our Senſes, from its Poſition with reſpect to certain Bodies, and is commonly us'd by the Vulgar for immoveable Space. As the Dimenſion of an Aereal, Celeſtial, or Subterraneous Space, is defined by its Poſition in reſpect of the Earth. So Space, Abſolute and Relative, are the ſame in Species and Magnitude, but do not always remain the ſame in Number: That is, if we conſider the Space or Cavity contain'd in any Veſſel, whitherſoever the Veſſel is mov'd, mov'd, the Space or Cavity included within the Sides thereof will always be of the same Nature, by reason that the nature of Space is every where similar to it self; and will remain likewise of the same Magnitude, because of the given Magnitude of the containing Vessel. But it doth not remain the same Space numerically, for that is changed perpetually by the Motion of the Vessel. In like manner, if the Earth be mov'd with an annual Motion about the Sun, the Space of our Air which relatively, and in respect of the Earth remains still the same, that is of the same Nature and Quantity, will sometimes be one part of Absolute Space, sometimes another, and so will absolutely and really be changed perpetually. For, indeed, as the Order of the Parts of Time is unchangeable, so likewise is the Order of the Parts of Space; although the Things which are in them are continually mov'd and chang'd. For Times and Spaces are, as it were, the Places of themselves, and of all other Things; which are placed in Time as to order of Succession, and in Space as to order of Situation. They are Places by their Essence, and it is absurd to say that the primary Places can be mov'd. These therefore are the Absolute Places; and the Translations which are from these Places, are the only Absolute Motions. But then, because the Parts of Space cannot be seen in themselves, or distinguish'd from each other by our Senses, instead of them therefore we use sensible Measures; defining all Places from the Positions of Things, with respect to some Body which we look upon as unmov'd, and their Distances from the same; and estimating all Motions with respect to the said Places, and so far as we conceive Bodies to be transferr'd from them. And thus instead of absolute Places

and

and Motions, we make uſe of Relative; and this indeed not unfitly in common Affairs: But in Philoſophical Matters we ought to abſtract from the Senſes: for it is poſſible, that no Body is really quieſcent, to which Places and Motions may not in this manner be referr'd to.

(6.) An abſolute Place is that part of the abſolute Space which the Body poſſeſſeth.

(7.) A Relative Place is that part of Relative Space which a Body poſſeſſeth. I ſay that Place is a part of Space, not the Situation of a Body, or the ambient Surface as ſome have defin'd it. For the Place of equal Solids are equal; and the ſame quantity of Matter always poſſeſſeth the ſame Quantity of Space, of whatſoever Figure or Denſity it is. As for Example, The Places of a Sphere, and of a Cube of the ſame abſolute Magnitude will be equal, or they will fill and be adequate to equal Places; although the ambient Surfaces, by reaſon of the diſſimilitude of the Figures will be unequal; and ſo in all other Figures. Further, the Motion of the whole is the ſame with the Sum of the Motions of all the Parts, that is, the Tranſlation of the whole from its Place is the ſame with the Sum or Aggregate of the Tranſlations of all the Parts from their Places; and conſequently, the Place of the whole is the ſame with the Sum of the Places of the Parts, and therefore is Internal, and in the whole Body. But Situations properly ſpeaking have no Quantity, and cannot be ſaid to be greater or leſſer, neither are ſo much Places as Affections of Places.

(8.) Abſolute Motion is a Tranſlation of any Body or Subſtance from one abſolute Place, or immoveable Space, into another abſolute Place or immoveable Space.

(9.) Re-

(9.) Relative Motion is a Translation of a Body from a Relative Place or some moveable Space, into some other Relative Place, or moveable Space; or a transferring of a Body from the Neighbourhood of some ambient Bodies into the Neighbourhood of others; or lastly, a Translation of a Body from its Situation amongst some certain Bodies into another Situation.

Thus in a Ship which is under Sail, the Relative Place of a Body is that part of the Ship in which it is; or that part of the whole Cavity which such a Body fills; and which consequently is mov'd with the Ship: And the Relative Rest of that Body, is the abiding thereof in the same part of the Ship, or Cavity. But the true Rest thereof is its continuance in the same part of the immoveable Space. From whence if the Earth did truly rest, the Body which relatively rests in the Ship, would be mov'd truly and absolutely with the same Velocity wherewith the Ship is mov'd on the Earth.

But if the Earth be also mov'd; the true and absolute Motion of the Body will arise, partly from the Motion of the Earth in the unmov'd Space; partly from the Relative Motions, both of the the Earth and of the Body in the Ship; and from these Relative Motions will arise a Relative Motion of the Body on the Earth. So if that Part of the Earth in which the Ship is, be really mov'd towards the *East* with a Velocity of 10010 Parts, and the Ship be carried towards the *West* by the Wind with a Velocity of Ten Parts; and the Mariner walk in the Ship towards the *East* with one Part of Velocity: The Mariner will be mov'd truly and absolutely in the unmov'd Space towards the *East* with 10001 Parts of Velocity, and Relatively on the Earth towards the *West* with Nine Parts of Velocity.

Feb. 28. 170$\frac{3}{4}$.

LEC-

Lect. IV.

WE have already laid down ſome Definitions by way of Preparation to the *Newtonian* Philoſophy. We will now ſuper-add a General Scholium appertaining to the two laſt Definitions.

A General Scholium.] Reſt and Motion Abſolute and Relative, are diſtinguiſh'd one from another by their Properties, Cauſes, and Effects.

It is plain by what hath been ſaid, that altho' any two Bodies, each of which doth truly reſt, do alſo reſt betwixt themſelves; yet it doth in no wiſe follow from their reſting betwixt themſelves, that they do truly reſt: For there may be ſome truly Quieſcent Body in or far beyond the Region of the Fixed Stars, with reſpect to which both the ſaid Bodies do change their Poſition.

But from the Situation of Bodies in our Regions, in reſpect of one another, we cannot diſcover whether any of them keep a given Poſition in reſpect of that remote one; and ſo true Reſt cannot be defined by their Situation between themſelves. The Property of Abſolute Motion is, that thoſe Parts which keep given Poſitions to the Wholes, participate of the Motions of thoſe Wholes: For all the Parts of Revolving Bodies endeavour to recede from the Axis of Motion.

And the Impetus of moving Bodies ariſes from the conjoint Impetus of each of their Parts: Therefore in Ambient Bodies, thoſe move which are relatively at reſt. And therefore true and abſolute

solute Motion cannot be defined by a Translation from a Vicinity of Ambient Bodies, consider'd as at Rest. Those Ambient Bodies ought not only to be look'd upon as Quiescent, but also to be truly so: But all included Bodies, besides their Translation from the Neighbourhood of Ambient Bodies, also participate of the true Motion of those Bodies; and that Translation being taken away, they are not truly, but only seem to be at Rest. For Ambient Bodies are to the included ones, as the outward Part of the Whole is to the inward one, or as the Shell to the Kernel. But if the Shell be mov'd, the Kernel or part of the Whole is also mov'd together, without a Translation from the Shell.

In like manner, if a Relative Place be moved, a Body therein plac'd is also mov'd; and a Body which is moved from a moved Place partakes of the Motion of its Place. So the Motion of any one walking backwards and forwards in a Ship whilst it is under Sail, is greater or lesser in respect of the Earth, or Shore, according as it tends towards the same or contrary Part with the Ship. But if he stand still in any certain Part of the Ship, he partakes of its Motion, and moves with the same Celerity: And if it tends towards the same part of the Ship, in respect of the Earth it will be moved swifter than the Ship, if to the contrary slower: And so we ought to reason concerning the Motion of the Earth if it doth move. Therefore, all the Motions which are made from moved Places, are only Parts of Whole and Absolute Motions; and every entire Motion is compounded of the Motion of the Body from its first Place, and of the Motion of this Place from its Place, and so on; till we come to an unmoved Place; as appears in the above-mentioned Example. Whence

Motions

Motions entire and abſolute can be defined by unmoved Places only: And therefore abſolute Motions are referred to unmoved Places, and relative Motions to moveable Places. But Places are not unmoved, unleſs they all keep the ſame Poſitions to one another from Infinity to Infinity; and therefore unmoved Places always abide and conſtitute the Space which we call immoveable.

The Cauſes by which true and relative Motions are diſtinguiſhed from one another, are the Forces impreſſed on Bodies to generate Motion. True Motion is neither generated nor changed, unleſs by a Force impreſſed on the Body it ſelf. For ſince any Part of Matter whatſoever is inactive and merely paſſive, it cannot be moved without ſome Force impreſſed from ſome other Place, nor thruſt from its State without ſome Force which may change its State. But relative Motions (ſuch only as *Cartes* owns) may be generated and changed without Forces impreſſed on the Bodies themſelves. For it is ſufficient, if Forces be impreſſed on other Bodies, to which the Relation is made to alter that Relation in which the relative Reſt or Motion of theſe conſiſt, if thoſe other Bodies give way. So indeed, according to *Cartes*, it is ſufficient that the Earth only be revolved, in order to the relative Motion of the Fixed Stars; and that the Earth will be at Reſt, while it is carried round the Sun in the Solar Vortex, if it is in the ſame Ambient Parts of ſubtil Matter, altho' together with thoſe Parts it annually performs a whole Revolution in the Eclipticks, and is abſolutely moved about the Sun. Again, true Motion is always changed by Forces impreſſed on the moved Body. But relative Motion is not neceſſarily changed by theſe Forces: For if the ſame Forces are ſo impreſſed on other Bodies alſo to which there is a Relation, that the relative Situation be preſerved, that Relation will alſo be preſerv'd, in which the relative Motion conſiſts. As if a Syſtem of Bodies be moved among themſelves after what manner ſoever, and an equal Force act upon equal Parts of the Syſtem, according to parallel Lines, altho' that Force really changes the true Motion of every Part, nevertheleſs it will not change the relative one: For the Poſitions and relative Motions of the Parts acting equally and by parallel Lines, will remain among themſelves as they were before. Therefore every relative Motion may be changed, tho' the true be preſerved, (*viz.* by the Mutation of the Motions of other Bodies,) and preſerved where the true is changed; as appears from the laſt Example: And therefore true Motion doth not conſiſt in Relations of any Kind.

The principal Effect whereby Abſolute and Relative Motions are diſtinguiſh'd one from the other, is a Force whereby a Body departs, or endeavours to depart from the Axis of the Circular Motion. For in a Circular Motion barely relative, this Force is none at all; but in a true and abſolute one, it is greater or leſs according to the Quantity of the Motion. If a Bucket, which hangs upon a long Rope, be turn'd round perpetually, ſo that the Bottom of it always remains parallel to the Horizon, and the Axis of the Motion perpendicular thereto, until the Rope by twiſting is become very ſtiff: Then fill it with Water, and let both Bucket and Water be at reſt; then by a ſudden Force will the Bucket turn about with a Motion contrary to the former; and by the Strings untwiſting it ſelf, it will continue in this Motion. The Surface of the Water will at firſt be plain, and parallel to

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the Horizon, as before the Motion of the Veſſel: But after the Veſſel by a Force impreſs'd on the Water by little and little, hath at length caus'd that the Water ſhould begin to be turn'd about ſenſibly in the Form of a Whirl-pool, as it were; it will depart by degrees from the Middle, and will aſcend to the Sides of the Veſſel, putting on a Concave Form; and will aſcend with a ſwifter Motion, more and more, until at length performing its Revolutions in equal Times with the Veſſel it ſelf, it comes to reſt relatively in the ſame. Now this Aſcent ſhews an Endeavour of departing from the Axis of the Motion. For although the Receſſion from the Axis of the Motion be in it ſelf perpendicular to the Axis, yet ſeeing the Veſſel doth in that Place hinder the actual Receſſion, the Force will be impreſs'd upon the next Particles, and become ſenſible where it hath room; and becauſe the true Circular Motion will be greater in Particles which are moſt remov'd from the Center, foraſmuch as it is communicated to them from the Veſſel firſt, and chiefly by reaſon of the greater Circles and greater Celerity which is towards the Circumference, the Parts more remote from the Center will recede the more from that Center: And thus that Aſcent of the Water ariſeth from its true Circular Motion, and is meaſured by its Endeavour of receding from the Center. And it is to be obſerv'd, that the true Circular Motion is in this Place altogether contrary to the relative Motion. For at firſt, when the relative Motion of the Water, with reſpect to the Veſſel, was the greateſt of all, foraſmuch as the Veſſel was whirl'd about, the Water remaining almoſt unmov'd; and conſequently the Water it ſelf, which is contain'd, was moſt ſwiftly mov'd to the contrary Part, in reſpect of the Vessel, without any true Motion in it self; then, I say, that relative Motion excited no Endeavour of receding from the Axis; the Water as yet remain'd plain and Level: But after that the Relative Circular Motion of the Water decreas'd, and the True sensibly begun, the Ascent to the Sides of the Vessel shew'd an Endeavour of receding from the Axis; which Endeavour shew'd the true Circular Motion, more and more increasing, until it became the greatest, which was when the Water came to rest in the Vessel relatively. It is plain therefore, that the said Endeavour depends not upon a Translation of the Water, in respect of the Ambient Vessel. (Where the Vessel alone is moved, and from thence a relative Motion only is given to the Water.) And therefore true Circular Motion is not to be defined by such Translations. There is only one true Circular Motion of every revolving Body, to which one single Endeavour answers, as its proper and adequate Effect: But relative Motions, according to divers Relations to divers Bodies, and divers Situations, according as this or that Body is respected, are innumerable, and tend towards all Parts at once; and as it is with Relations in general, are destitute of all true Effects, any farther than they participate of true Motions. From whence also in their System, who would have the Heavens below the Sphere of the Fixed Stars to be turn'd round, and to carry the Planets along with them; the Planets which relatively rest in their Heavens, are notwithstanding truly mov'd, as well as the Heavens themselves: For they change their Positions according to their different Periods of Revolutions, which is the Case of Bodies really moved. Accordingly the Stars themselves, as Parts of the Revolving Spheres, partake of their Motion,

Motion, and endeavour to recede from the Axis.

Therefore the Relative Quantities, which we have now diſtinguiſh'd from the true, are not thoſe very Quantities which they are reckon'd to be, as the Space contain'd betwixt the Walls of a Chamber, the Diurnal Motion of the Stars, *&c.* but they are their ſenſible Meaſures, (whether true or falſe) which are vulgarly made uſe of inſtead of the true meaſured Quantities themſelves. Wherefore, if the Significations of Words are to be defin'd from their Uſe, by the Names of Time, Space, Place, and Motion, theſe Meaſures are properly to be underſtood; and the Expreſſion will be unuſual and purely Mathematical, if the abſolute Quantities themſelves be underſtood. And therefore as they do Violence to the Holy Scripture, who there interpret theſe Words, as intending the abſolute Quantities; ſo alſo do thoſe who from the Reſt aſſign'd to the Earth, and Motion to the Sun, in the Words of the Scripture, are wont to diſpute concerning the true Frame of the World, contrary to evident Reaſons of Aſtronomy and Philoſophy; as they do likewiſe, if ſuch there be, who from the Words wherein it is predicted, That *Time ſhall be no more,* do from thence collect, that Eternal Duration, or Abſolute Time, ſhall be annihilated. Nor do thoſe any whit leſs defile Mathematicks and Philoſophy, who confound the true Quantities with their Relations and vulgar Meaſures.

Now to know the true Motions of Bodies, and actually to diſtinguiſh them from the apparent, is indeed difficult; becauſe the Parts of the unmoveable Space in which the Bodies are truly mov'd, do not encounter the Senſes. Yet notwithſtanding, the Caſe is not altogether deſperate; for we have certain Tokens and Arguments of the ſame, partly from the apparent Motions which are the Differences of the true, partly from that Force which is the Cauſe and Effect of the true Motions. As if two Globes, tied together by a Cord at an even Diſtance from each other, ſhould be rolled-round about a Center of Gravity common to both, the Endeavour in the Globes of departing from the Axis of the Motion, would ſhew forth it ſelf in the ſtretching of the Cord; and from thence the Quantity of the Circular Motion might be computed. Then, if any equal Force whatever ſhould be at the ſame time impreſſed upon the Alternate, that is, the diametrically oppoſite Faces of the Globes, to increaſe or diminiſh the Circular Motion; that is, if one ſhould be impreſs'd on one Part, and the other on the contrary Part at the ſame time, the Increaſe or Decreaſe of the Circular Motion would be ſeen from the increas'd or diminiſh'd Tenſion of the Cord. And from thence, at length, would be found, Which are the Faces of the Globes on which the Force ought to be impreſs'd, for the augmenting the Motion moſt of all; to wit, the hinder Faces, or thoſe which in the Circular Motion do follow. But the Faces which follow being known, and by conſequence the oppoſite ones, or thoſe which go before, the Determination of the Motion will be known. After this manner, both the Quantity and Determination of this Circular Motion might be found in any immenſe Vacuum, where there is nothing ſenſible and external with which the Globes might be compar'd. If now there ſhould be placed in that Space ſome far diſtant Bodies, keeping a given Poſition one with reſpect to another, ſuch as are the Fixed Stars in our Regions; it could not be known from the relative

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Translation of the Globes amongst Bodies, whether the Motion were to be attributed to these or those; like as we upon the Earth cannot by any apparent Motion of the Fixed Stars, determine whether it be the Earth or they that is indeed mov'd: But if the Cord be minded, and it be found that the Tension thereof is the very same which the Motion of the Globes requir'd, we might conclude that the Motion is in the Globes; and then at length from the Translation of the Globes amongst the other, collect the Determination of the Motion. For seeing that from the Tension of the Cord, it would be manifest, that the Motion is truly in the Globes, and not in the remote Bodies; the Motion of the Globes, as well in respect of Velocity as Direction, will easily be determin'd by those Bodies, which are deservedly now to be look'd upon as unmov'd. And in this way we collect the annual Motion of the Earth, as being exactly proportional to the Centri-petal Force towards the Sun; and likewise easily gather the Stability of the Fixed Stars from the annual Motion of the Earth. Then the Motion of the Earth, and Stability of the Fixed Stars being known, it is as easy to deduce from thence the Velocity and Direction of the annual Motion. But in what manner true Motions are to be collected from their Causes, Effects, and different Appearances; and on the other hand, in what manner, from Motions either true or apparent, their Causes and Effects are to be gathered, will be taught more largely in the Process.

(10.) The Quantity of Matter is the Measure of the same, arising from the Density and the Magnitude thereof conjunctly.

Air, which is as denſe again as ſome other Air, and poſſeſſes double the Space, is Fourfold of the other. And if a Cubic Veſſel contain Air, which by compreſſion is reduced into a leſſer Cube, the Denſity in the leſſer Cube will be to that in the greater, as the greater Cube is to the leſs; or in the Triplicate Proportion of the Sides reciprocally; and the Diſtances of the Particles of Air which are like, and in like manner poſited, will be in the Proportion of the Cubic Sides. The ſame Thing is to be underſtood of Snow and Powders condens'd by Compreſſion or Liquefaction; and there is the like reaſon of all Bodies in whatſoever manner condens'd. We have no regard in this Place to a Medium pervading the Inſterſtices of the Parts, if there be any ſuch. But we ſhall call this Quantity of Matter, which is to be reckon'd from the Denſity and Magnitude conjointly, every where hereafter Body or Maſs. And the ſame is known by the Weight of every Body; for an equal Quantity of Matter, of what ſort ſoever it is, doth equally gravitate; as is manifeſt by Experiments of Pendulums which have been moſt accurately made, as will be taught in the Sequel. And from hence indeed, that we may note this by the way, It is certain, that either there is no Æthereal Medium pervading the Pores of Bodies; or if there be any, ſeeing it doth in no wiſe gravitate nor hinder the Motion of Bodies, it ought to be reckon'd Matter differing from that of all other Bodies; yea, in ſpeaking properly, it deſerves not the Name of Body or Matter at all. But we ſhall have occaſion to ſay more of this hereafter.

(II.) The Quantity of Motion is the Meaſure of the ſame, ariſing from the Velocity, and from the Quantity of the Matter conjunctly.

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The Motion of the Whole is the Sum of the Motions of all the Parts; and consequently in a Body double of some other, and mov'd with equal Velocity, the Motion is Twofold of the Motion which is in the other Body; and in the Double Velocity of the greater Body Fourfold. The Quantity of Matter therefore is equal to the Rectangle of the Density drawn into the Magnitude; and the Quantity of the Motion equal to the Rectangle of the Velocity drawn into the Quantity of the Matter. From which Principle the Forces of Machines are easily deduced. For wheresoever in the Equilibrium of Machines a Body is greater, there the Celerity of that Body will be so much the lesser; and where the Body is the less, the Celerity will be so much the greater; so that the Quantity of Motion resulting from the Body, as drawn into its own Velocity, is equal on both Sides; as will be more largely set forth afterwards.

(12.) The innate Force of Matter is a Power of resisting, whereby every Body, as much as it can, perseveres in its own State of Resting, or Moving uniformly strait forwards in a right Line.

This Force is proportional to the Body, and differs nothing from the Inactivity of the Body, but in the manner of conceiving it, by which it comes to pass, that a Body is not without difficulty put out of its State whether of Rest or Motion. From whence, by a very significant Name it may be called the Force of Inactivity. But a Body exerciseth this Force only when it is acted upon by some Force from without; under which exercise of its Innate Force it is considered in a Double Respect; to wit, as Resistance and Impulse. Resistance, as far as it struggles with the impressed Force,

Force, in order to preserve its own State; Impulse, as the same Body by not easily giving way to the Force of the resisting Obstacle, endeavours to change its State. Indeed, it seems most proper to attribute Resistance to quiescent, and Impulse to moving Bodies; and I should assign any Impetus whatsoever, where one of the Bodies is at Rest, to the positive Force of the moved Body, rather than to the Negative Force of the quiescent one.

(13.) The impress'd Force is an Action exercis'd on a Body for the changing its State, whether of Rest or uniform direct Motion.

Thus this Force consists in Action alone, and remains not in the Body at all after the Action. For the Body perseveres in every new State by its sole Force of Inactivity. Now Force impress'd is from divers Causes, as from a Blow, a Pressure, or Tendency to a Center.

(14.) The Centripetal Force is that, whereby a Body is drawn, impell'd, or in some way or other tends to a Center.

Of this Sort is Gravity, whereby a Body tends to the Center of the Earth; the Force Magnetic, or that whereby Iron tends to the Center of the Loadstone; the Attraction or stretching of the Cord to retain the Stone that is whirl'd round in a Sling. Hither also is to be referr'd that Force, whatsoever it is, whereby the Planets are continually held back from rectilineal Motions, and compell'd to revolve in Curvilinear Ones. The Quantity of this Centripetal Force is of Three Sorts, Absolute, Accelerating, and Moving.

(15.) The Absolute Quantity of Centripetal Force is the Measure of the same, greater or lesser, according to the Efficacy of the Central Cause, which propagates it from the Center all round about.

about. Thus the Strength of Magnets is different, and greater, *Cæteris Paribus*, in the greater Magnet; and lesser in the less. The Attraction or Tension of the Cord greater in the Circumvolution of a greater Stone than in that of a less; and in the swifter Circumrotation of the same Stone than in a Slower. And thus we may conceive, that the Gravitation of Bodies to the Sun, which is a Body so much greater than the Earth, is greater at an equal distance than the same is towards the Earth.

(16.) The Accelerating Quantity of this Centripetal Force is the Measure of it in divers Distances from the same Center; which is proportional to the Velocity which it produceth in a given time.

As the Virtue of the one and the same Magnet (in which consequently the Absolute Quantity remains the same) is greater in a less distance than in a greater; the gravitating Force in the Surface of the Earth is something greater about the Poles than about the Equator; it is greater also near the Surface of the Earth than at a greater Distance from the Center. But this Accelerating Force, which is distinctly to be noted, is at equal Distances from the Center every where the same, and this in all Bodies whether they be Heavy or Light, Great or Small, Solid or Fluid; that is to say, if you do here abstract from the resistance of the Air. Which Thing is prov'd by the equally swift Descent of all falling Bodies in Tubes emptied of Air; and from the Motion of all Pendulums, what Matter or Magnitude soever, vibrating together in like Circles or Cycloids.

(17.) The Moving Quantity of the Centripetal Force, is the measure of the same Proporti-

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onal to the Motion which it generates in a given time.

This Force is the propenſion of the whole Body towards the Center, which is eſtimated by the Quantity of the Force contrary thereto, which is requir'd to the hindring its Deſcent, which is called the Weight of a Body, and is greater in a Body which is greater; and greater in the ſame Body by how much it is nearer to the Earth. The Abſolute Quantity therefore of this Force we are treating of, is defin'd from Magnitude, or at leaſt from the Strength and Efficacy of the Central Body. The Accelerating is that Force as perpetually decreaſing in the Increaſe of the Diſtance, and on the contrary. The Moving Force is the Weight it ſelf; which ariſeth from the Body or Maſs drawn into the Accelerating Force. From whence the Abſolute Force being given, the moving Force in a given Body will be as the Accelerating; and the Accelerating being given it will be as the Body. Theſe three Forces therefore are referr'd to three Things, to Bodies, to the Places of Bodies, and to the Center of Force. The Motive Force reſpects the Body and the Endeavour and Propenſion thereof to the Center, as compounded of the Endeavours and Propenſions of all the Parts. The Accelerating refers to the Place of the Body in the Medium as the Efficacy of the ſame Abſolute Force according to divers Diſtances from the Center; and the Abſolute Force reſpects the Center or Central Body it ſelf, as endowed with ſome Power, without which the moving Forces are not propagated round about; whether that Power or Cauſe be the Central Body (as the Magnet in the Center of the Magnetick Force,) or the Earth in the Center of the gravitating Force, or be ſome other Thing which

which doth not appear. At least, this is a Mathematical Conception, and sufficient for our present Purpose; for we do not yet consider the Physical Causes of those Forces. The Accelerating Force therefore is to the Motive as Swiftness is to Motion; for from the same, as multiplied into the Quantity of the same Matter, the Moving Force arises, like as the Quantity of Motion ariseth from the Celerity multiplied into the Body. For the Sum of the Actions of the Accelerating Force upon each Particle of the Body, is the Moving Force of the Whole; from whence, where the Accelerating Gravity is the same, the Moving Gravity or Weight is as the Body. And in the same Body where the Acceleration is diminish'd, as in the upper Regions, the Weight is likewise diminished. Thus where the Accelerative Force is Twofold less, the Weight of a Body Twofold or Threefold less, will become Fourfold or Sixfold less. Furthermore, we call Impulses and Attractions Accelerative and Motive in the same Sense. And we use the Words [Attraction, Impulse, and Propension to a Center] indifferently and promiscuously; We at present considering these Forces not Physically but Mathematically, as was said before, and now say again, to Caution our Reader from understanding us, as Meaning and Defining Physical Causes or Reasons of Motions, or attributing to Centres, which are Mathematical Points, true and proper Physical Force; when at any time we say that the Centers draw, or have Force in them. And so far we have given you the Definitions requisite to be premis'd to Sir *Isaac Newton*'s Philosophy.

Feb. 28. 170$\frac{3}{4}$.

LECT.

LECT. V.

Axioms or Laws of Motions.

1. EVERY Body perſeveres in its own preſent State, whether it be that of Reſt, or uniform direct Motion; unleſs it be compelled by ſome Force impreſs'd, to change that State.

Projectiles hold on their Motion, ſo far as they are not hinder'd by the Reſiſtance of the Air, or their own Gravity. A Top, whoſe Parts by cohering continually draw themſelves from the Rectilineal Motion, ceaſeth not to be whirl'd about, ſo far as is not retarded by the Air, or the unevenneſs of the Surface, on which it turns. But the greater Bodies of Planets and Comets maintain their Motions, whether Progreſſive or Circular, much longer in Spaces leſs reſiſting. This Law of Motion is indeed the fundamental Law of all, and is moſt evident from the merely Paſſive Nature of Matter; which makes it naturally as impoſſible for a Body of it ſelf to ſtop its own Motion once begun, as it is for it to move it ſelf originally.

(2.) All Motion is of it ſelf Rectilinear.

For Motion cannot be conceived, but it muſt be directed and determin'd towards ſome Place or other, and it will by the Law foregoing keep the ſame Direction which it firſt had, until it be hinder'd or put out of its way by ſome Extrinſic Cauſe.

And

And consequently, whenever any Body is mov'd in a Curve, that Curvature must needs proceed from External Force; and therefore must cease when that Force ceaseth. Which, when it doth, then by this and the foregoing Law, the Motion will be continued in a Right Line, which is the Tangent of the Curve, at the very Point of the said ceasing Force, or in the last Rectilineal Direction. Thus it is in a Stone wheel'd about in a Sling, which slipping out of the Sling is not now carried forward in its former Circle or any Circle at all, but in a Tangent of the former Circle; where indeed, by reason of the Force of Gravity, compounded with the projectile Force, it describes a Parabolic Line: But of this afterwards.

(3.) All Bodies carried about, endeavour to recede from the Center of their Motion; and by how much the Motion is the swifter, this Endeavour is the greater.

For seeing Bodies do of themselves tend unto a Rectilineal Motion, or that which is according to Tangents of Curves; and seeing all the Parts of the Tangents are further distant from the Center of Motion, than the Parts of the Curves, unto which the Bodies are drawn by the Centripetal Force: It is manifest, that that Endeavour of going off according to Tangents, doth as much draw back the Bodies from the Center, as the Centripetal Force draws them to it, and is exactly equal to the contrary Endeavour of the Centripetal Force.

(4.) The Mutation of Motion is proportional to the moving Force impress'd; and is according to the right Line in which that Force is impress'd. If any Force generates any Motion, a Double Force will generate a Double one, a Treble a Treble

Treble one; and this whether the Force be impreſs'd all at once, or ſucceſſively.

And then this Motion impreſs'd, (for as much as it is always determin'd to the ſame Part, with the generating Force) if the Body on which the Impreſſion is, was before in Motion, either is added to the Motion thereof, as conſpiring together with it; or ſubducted therefrom, as being contrary thereto; and thus it either increaſeth or diminiſheth the former Velocity. But if the Impulſe be oblique, it is added obliquely, and compounded with the former Motion, according to the Directions of both. So that if it were at right Angles, the Velocity as conſidered in the firſt Line will neither be increas'd nor diminiſh'd.

(5.) Re-action is always contrary and equal to Action. That is, the Actions of Two Bodies acting upon each other, whether they be Impulſes or Attractions, are always directed each to the contrary Part, and are alſo equal.

Whatſoever preſſeth or draweth another thing, is equally preſſed or drawn thereby. If you preſs a Stone with your Finger, your Finger is equally preſs'd by the Stone. When a Horſe draws a Stone tied to a Rope, the Horſe will equally be drawn back to the Stone: For the Rope, which is diſtended on both Sides, will, with the ſame Endeavour of relaxing it ſelf, draw the Horſe to the Stone, as it doth the Stone to the Horſe; and will ſo much hinder the Progreſs of one, as it forwards the Progreſs of the other. If one Body daſhing upon another, ſhall by its Force in any ſort change the other's Motion, it will alſo reciprocally undergo the like Change in its Motion, to the contrary Part, by the Force of the other, and this by reaſon of mutual Preſſure. But then by theſe Actions are made equal Changes, not

not of Velocities but Motions; to wit, in Bodies not otherwise impeded. For Mutations of Velocities made to the contrary Parts, since the Motions are equally changed, are reciprocally proportional to the Bodies: We may shew the Matter briefly thus in Attractions. Let some Obstacle be suppos'd to be interpos'd betwixt the Bodies A and B, which attract each other, to keep them from meeting together. If either of the Bodies, as A, be more drawn towards B, than B is towards A; the Obstacle will be press'd more with the Action of A upon it, than with that of B, and consequently will not remain in an Æquilibrium. The stronger Pressure therefore prevailing, will make that the System of the three Bodies will be mov'd directly unto that Part which is from A to B; and so in a free Space will be mov'd *in infinitum* in that Direction, with a Motion continually accelerated: Which is absurd and contrary to the first Law of Motion. For by that, the System ought to persevere in its State, whether of Resting or moving right forwards; and consequently the Bodies will equally press the Obstacle, and so will equally be attracted to each other. And the Thing is the same if there be no Obstacle; for the stronger Motion will in the Meeting overcome the Weaker, and carry both Bodies to the same Part, and according to its own Direction. Wherefore, either there is no Attraction in a System of Bodies where the first Law hath Place, such as the Solar one is, as we shall hereafter clearly demonstrate it to be hereafter; or the Attraction is mutual and equal. The Famous Sir *Isaac Newton* hath made an Experiment of the Matter in the Magnet and Iron. If these be separately put in two proper Vessels, swimming close to one another in a standing Water; neither will prevail over the other,

 but

but with an Equality of Attraction on both sides, they will sustain their mutual Endeavours on one another, and at length being in Equilibrio will be at rest. So likewise, the Gravity betwixt the Earth and its Parts is mutual and equal. If the Globe of the Earth H E F G K I (see *Fig.* 2. *Plate* 2.) be divided into Two unequal Parts by a Plane G E, the Gravity of the Part E G F, towards the rest of the Earth, will be equal to the Gravity of the rest of the Earth towards this Part: which is prov'd thus. Imagine the Earth to be divided by Parallel Planes into three Parts, E G F, H K I, E G K H; of which E G F and H K I, are equal to each other, and lye upon the middle Part E G K H. Here it will be manifest, that the middle Part E G K H, doth by its own Weight incline to neither of the Parts, but hangs, as we may say, in Æquilibrio betwixt both, and so rests. But the Extream Part H K I lies with its whole Weight upon the middle Part, and urgeth that towards the other Extream Part E G F. And therefore, the Force wherewith the Sum of the Parts H K I, and E G K H, tends towards the Third Part E G F, which is equal to the Weight of the same Parts, is equal to the Third Part E G F. And therefore if the Earth be divided by any Plane whatever, as E G into Two Parts E G F and E G I, the Force wherewith the greater Part E G I tends to the lesser E G F is equal to the Force wherewith the lesser Part tends to the greater; and unless those Weights were equal, the whole Earth would give place to the greater Weight, and in yielding to it would fly away, and there would no Place be found for it; which, as before, is absurd, and contrary to the first Law.

(6.) If

(6.) If of two equal Bodies, void of Elasticity, one of them which is in Motion meets the other at rest, upon the meeting they will both be carried forwards together, to the same part, with half the Velocity of the Body which was moved. For the Body put in Motion, will in the Shock communicate its Motion so long to the other quiescent Body, till that moves with the same Velocity with it self. For whilst the Velocity of the mov'd Body is greater than the Velocity of that which was before quiescent, the former Body will still force the other, and accelerate its Motion; but assoon as the Body that was quiescent hath gotten a Velocity equal to what the moving Body moves with it can force it no further, but follows it closely. And thus the Motion of the former being now equally divided betwixt them, it appears that they are both carried with half the Velocity which the former had before.

(7.) If two equal Bodies, void of Elasticity, do directly meet each other with the same Velocity, they upon the Collision will both of them rest.

For so much as either of them tends to go forward it is repelled by the other: And so these Two equal Forces, or Quantities of Motion, tending to the contrary Parts, will destroy one another; whereupon, there being no new Cause of Motion, they must needs both of them rest. In which Case, the Motion wholly perisheth, contrary to the Opinion of *Cartes*, who would have the same Quantity of Motion always to remain in the World.

(8.) If two unequal Bodies, destitute of Elasticity, meet one another with such Velocity, that by how much the greater exceeds the other in Magnitude, by so much it is exceeded by the lesſer in Swiftneſs, ſo that the Velocities are reciprocal to the Bodies; they will both reſt after the meeting.

For the Motions which are directly oppoſite to each other, being as to the Quantities of them equal, they will deſtroy one another, as before.

(9.) If a moving Body ſtrike another at reſt, (but both void of Elaſticity) how unequal ſoever they be in Bulk and Quantity of Matter, they will both move after the ſhock with the ſame Velocity towards the ſame Parts, as in the Sixth Law: And the common Velocity will be ſo much leſs than the firſt, as both the Bodies together are greater than the Body firſt moved. For ſince all the Motion of the former Body is now divided between the two Bodies, the Velocity will be ſo much diminiſhed, as the Quantity of Matter to be moved is increaſed.

Corollary, Therefore, when the Bodies are given, there will be alſo given the Proportion of the Velocity of the moved Body before the Shock, to the common Velocity of the Bodies after the Shock. For as both Bodies together are to the moved Body, ſo will the Velocity of the moved Body before the Shock, be to the common Velocity of both after the Shock.

(10.) If two unequal Bodies, void of Elaſticity, which are carried with equal Velocity to oppoſite Parts, hit againſt one another, the Quantity of Motion in both, taken together after the Colliſion will be the difference only of the former Motions; for the leſſer Quantity of Motion on either Part will be equivalent to an equal Quantity of Motion on the other Part, and as above will deſtroy it; wherefore, there remains only the Exceſs of the Motion, as the ſole Cauſe of it after the Shock. And the Caſe will be juſt the

ſame;

ſame, as if the Body that had the greater Quantity of Motion, ſtruck another at reſt with that difference of Motions, and after the Shock ought to be calculated in the ſame manner.

(11.) If two equal Bodies, void of Elaſticity, be mov'd with unequal Velocity towards the ſame Part, upon their Colliſion there will remain the ſame Quantity or Sum of their Motion, but the common Velocity will be the half of both the former Velocities put together.

For the Exceſs of Velocity will now be divided equally betwixt both Bodies, and ſo they will go away together with a mean Velocity.

(12.) If in two unequal Bodies, void of Elaſticity, the Greater overtakes the Leſſer, the common Velocity, after the Shock, will be greater than half the Sum of the former Velocities. And on the contrary, it will be leſs when the leſſer Body overtakes the greater. For if the Bodies were equal, it would, by the foregoing, be juſt half that Sum. Wherefore it will be more or leſs than half, in proportion to the Greatneſs or Smallneſs of the hindmoſt Body.

Corollary, Therefore the Velocities and Magnitudes of Bodies before the Shock being given, it will be eaſy to compute the common Velocity of the Bodies after the Shock. For the Sum of the Motions, divided by the Sum of the Bodies, gives the common Velocity, when the Motions are made towards the ſame Parts; or the difference of the Motions divided by the Sum of the Bodies gives the common Velocity, when the Motion is towards the contrary Parts.

Scholium, Theſe are the true Laws of Motion in Bodies, which yield ſomewhat, but do not reſtore themſelves, or are endued with no elaſtic Force; and the ſame Laws may perhaps hold alſo in Bodies perfectly hard, so that they be not Elastic. But the Rules of Motion in Bodies perfectly Elastic, or which restore themselves with the same Force wherewith they are compress'd, are altogether different from the former; and therefore require a distinct and separate Consideration. And forasmuch, as the Collisions of these Bodies do afford many both difficult and notable Phænomena; and the famous Monsieur *Hugens* hath undertaken in a posthumous Work to explicate and demonstrate them; but this indeed not without much going about, and a long Pomp of Arguments and Figures according to the manner of the old Geometricians, we shall deliver the Laws of Motion of Elastic Bodies according to his Order, but in a briefer Method and one that is more natural; that so Beginners may be able in some measure to comprehend the Certainty and Physical Origin of these Laws; the first and fundamental one of which is this,

(13.) If a Body perfectly Elastic dasheth upon another Body of the same sort which is Quiescent and Equal; after the Shock the Motion will be wholly transferr'd into that which was quiescent before, and with the same Celerity, but the Body which was mov'd before, will now rest. For the impelling Body, whether it were Elastic or no, will by the Sixth Law communicate half of its Motion to the other, and begin to go along with the other with the same Pace; and by its Elasticity, the Force of which is equal to the Force of the direct Impulse, it will communicate the other half of its Motion; from whence it comes, that the Motion of the Body before quiescent, will now be equal to that which the Impellent had before; and consequently, that seeing so much of Motion as the Impellent transfers to the other, so much it loseth of its own

own Motion, the Motion upon the whole will be convey'd into the Quiescent, the Impellent having lost its Motion.

Corollary (1.) If a greater Body dasheth upon a lesser, the former will not rest, but only be mov'd more slowly; and the other which before rested will, in its being mov'd, gain a greater Velocity indeed than was in the Impellent, but a less Quantity of Motion.

Coroll. (2.) If a lesser Body dasheth upon a greater, it will not rest but go back; and the Quiescent will gain a less Velocity indeed, but a greater Quantity of Motion than was in the Impellent.

Coroll. (3.) If a Body, put into Motion, hit upon divers Bodies contiguous to one another, and quiescent, they will all rest but the last or furthest of them; and this will be mov'd wth a Celerity, equal to, or greater or less, than that of the Impellent, according as the Impellent Body is equal to, or greater or less than the last Body. These *Corollaries* follow naturally from the present Law of Motion, and therefore seem to require no special Demonstration.

(14.) If two Bodies perfectly Elastic, which are equal, but mov'd with an unequal Celerity, dash one upon another, they, whether they were before carried to the same part or to the contrary, will after the Contact be mov'd each with that Celerity which the other had before.

For if they tend towards the same Parts, the common Velocity on both Sides being taken away, there will only remain the difference of the Velocities as the sole Cause of the Change in the Shock; and since by the foregoing Law that Velocity will be communicated to the slower Body, it follows, that the striking Body shall lose that Excess of Motion, and the slower Body get it; that is, in other Words but the same Sense, they will move with each others Velocities. And in like manner, we may demonstrate the same in the second Case, where the Bodies carried towards different Parts are supposed to strike one another. For the common Velocity being taken away on both Sides, the difference of Velocity, which after the Shock tends the contrary way, and will not at all change the former common Velocity, will remain as before, the sole Cause of changing the Velocity; which by the foregoing Law will be transferred from the swifter to the slower Body: Whence as before, it will follow, that elastic equal Bodies after the Shock will move with each others Velocities.

(15.) Any Body how great soever, may be moved by any Body how small soever, with any Velocity whatsoever. This Law of Motion is indeed an Axiom, manifest in it self, and wants no Demonstration.

(16.) When two Bodies, perfectly Elastical, are dash'd one upon the other, they depart from one another with the same Celerity wherewith they approach'd one to the other; that is, not with the same absolute Celerity perhaps, but with the same relative Celerity. This Law indeed is the Foundation of all the following Laws of Motion. The Thing was before prov'd concerning two Elastical Bodies which are equal, when it was demonstrated, that in their departure from each other, there are the same True and Absolute Celerities on both Sides, the Seats of them only being chang'd. And therefore it is necessary, that the Relative Velocity of departing from one another be the same with that of coming towards one another. Now concerning Bodies unequal it is thus shew'd. If a greater Body ſtrikes a leſſer, which is either quieſcent, or at leaſt moved ſlower, it will communicate ſome of its Motion to the quieſcent or ſlower Body; and the Elaſticity being laid aſide, it will not reſt; and by ſuch Communication together with the quieſcent or ſlower Body it will continue to go on with a direct Impulſe, and alſo by the Elaſtick Re-action accelerate that quieſcent or ſlow Body, until it recede from it ſelf with the ſame Velocity, by which it had withſtood its Motion, and compreſſed its Elaſticity; that is, by which it ſelf approached the other. Indeed, the greater Body muſt neceſſarily impreſs this Velocity on the leſſer; but it cannot impreſs a greater; (altho' the leſſer Body of it ſelf is capable of a greater:) for as ſoon as the quieſcent or ſlower moved Body hath gotten a Degree of Velocity equal to the Impulſe or former Relative Velocity, it will fly thence, and will ſuſtain no farther Impulſe whatſoever. But if a leſſer Body ſtrike a greater, either quieſcent or moved ſlower, it is impoſſible the leſſer Body ſhould impreſs the whole Exceſs of its Velocity on the quieſcent or ſlower Body, (for that will be in that Caſe only where the Bodies were equal, as we juſt now ſaw in the 13th and 14th Caſes.) But in the Communication of the Motion, the Exceſs of the ſwifter is loſt, even when the Elaſticity is not conſidered. And while the Bodies go on together in that manner, the hindmoſt will react on the foremoſt until they are ſeparated with the ſame Relative Velocity, with which at firſt they came together; for in this, and only in this Caſe can the Elaſtic Force be equal to the Impulſe; or rather ſo far, and no farther can the leſſer Body ſuffer the Re-action as in the former Caſe. But in thoſe Bodies which mutually ſtrike one another

ther with unequal Velocities, we muſt take away the common Velocity from both, ſo that it will generate the ſame Velocity after the Shock but the Seats changed; but then there will be left only the difference of the Velocities,as the ſole Cauſe to change the Velocity; which Cauſe indeed will not ceaſe, but in Acting and Re-acting, the Bodies will depart from one another with the ſame Relative Velocity, with which they came together. And the Matter will every where depend on this, that the Elaſtick Forces, every where equal to the impreſſed, produce their Whole and Pure Effects only; which cannot be done any otherwiſe, than if the Relative Velocity of Receding, exactly anſwer to the Relative Velocity of approaching.

(17.) If two Bodies perfectly Elaſtical, do each return to the Impulſe with the ſame Celerity wherewith they rebounded from it; they will each of them, after the Second Impulſe, require the ſame Celerity as they had before the firſt Meeting. For by reaſon of the given Quantity of the Stroke in the Colliſion, there will be given therewithal a certain Rectangle, whoſe two Factors are the Diſtances from the Point of Concourſe, both the primary diſtance, and that to which they return on both Sides after the 1ſt Conflict; if therefore we divide the Rectangle by the firſt Diſtance, there will come forth the ſecond Diſtance as the Quotient; and if we divide it by the ſecond Diſtance we ſhall obtain the firſt Diſtance for a Quotient; and ſo perpetually. From whence it follows, that thoſe Diſtances as deſcrib'd in a given time, or the Velocities of coming to, and receding from one another, do anſwer to each other mutually, and follow one upon the other.

(18.) In

(18.) In two Bodies which meet one another, whether they be Elastic or not Elastic, there doth not always remain the same Quantity of Motion as was before, but it may be greater or less. This Proposition, which directly contradicts *Cartes*, we deduced out of the Seventh Law, as to Bodies not Elastic; and it follows out of the last Law save one, concerning Bodies Elastic. For seeing the Quantity of Motion is estimated from the Celerity drawn into the Matter; and seeing in Bodies howsoever unequal, and unequally mov'd, the thing is indeed thus, that the Sum of their Velocities, or the Relative Velocity remains given, the Quantity of Motion will be very unequal, as the greater or lesser Body gains a greater or lesser Part of the entire respective Velocity; as will more clearly appear from that computation of Motions which will presently follow.

(19.) If a Body perfectly Elastical, which is greater, meets a lesser one which is quiescent, it will give a Velocity to it less than the double of its own. For seeing, after the Impulse, the Bodies ought to be separated from each other with the same respective Velocity, with which they came to one another, that is, in the present Case with the Velocity with which the greater was mov'd before the Impulse; if the Velocity of the quiescent Body were double to the Velocity of the Body incurring, then after the Motion communicated to the Quiescent, the Impellent ought to go forward with the same Celerity, which it had before, without any diminution of it: which is absurd.

(20.) If two Bodies perfectly Elastic, the Celerities whereof are in reciprocal Proportion to their Magnitudes, meet one another directly and oppositely, they will both rebound with the same

Celerity

Celerity with which they came to each other. For ſeeing the Force which ariſeth from the mere Impulſe of Bodies without any conſideration of the Elaſticity, is on both Sides equal, they by the Eighth Law will mutually ſuſtain and deſtroy each other; ſo that there will remain no Cauſe of Motion but the Elaſtic Force; which ſeeing it is on both Sides equal will beget equal Motions on both Sides; and conſequently both the Bodies will rebound with the ſame Celerity which they had before.

Scholium, A Problem. There being given two unequal Bodies perfectly Elaſtic meeting one another directly, both of which are mov'd or one only, and the Celerity of both, or of the one, if only one be mov'd, being alſo given, to find the Celerities with which both are mov'd after the Meeting. Let it be made thus, as the Sum of the Bodies, is to the Double of the Second Body, ſo is the given reſpective Celerity of Approach to the other Celerity. The Difference betwixt this laſt found Celerity, and the Celerity of the firſt Body before the Impulſe (or in one Caſe the Sum of them, to wit, where the firſt Body in the Motion goes before) will give the Celerity of the firſt Body after the Meeting; which Celerity being ſubducted out of the whole reſpective Celerity given, the remainder will be the Celerity of the ſecond Body after the Meeting. Which Rule is thus demonſtrated. The Velocity of the firſt Body after the Meeting will be the Difference betwixt the Velocity of the firſt before the Meeting and the whole Velocity, where the Bodies are put to be equal, ſo that the Sum of the Bodies is equal to the Double of the ſecond Body, as appears from the 14th Law: It is therefore manifeſt, that all the Difference, that is, the Motion of of the first Body after the Meeting, doth arise from the Difference of the Sum of the Bodies, and the double of the second Body; and consequently is proportional to the same. Which is the very Thing that the present Analogy supposeth.

For Example: Let the first Body be moved towards the right Hand with the Celerity of Six Parts; and the second to the contrary Part with the Celerity of Four Parts: Let the first Body also be quadruple of the second Body. The respective Velocity therefore of approach will be of 10 Parts, $6+4=10$. And the Sum of the Bodies will be of Five Parts. It will therefore be thus; as the Sum of the Bodies $=5$ is to the Difference of them $=2$; so is the whole respective Velocity $=10$ to $\frac{2\times10}{5}=4$; the Difference of which Velocity, and of the Velocity of the first before the meeting $=2$, will give the Velocity of the first after the meeting. From whence the Velocity of the second after the Meeting will be found to be of Twelve Parts, Q E I.

But if the other Body doth rest, the Celerity thereof after the Meeting will easily and immediately become known by the former Analogy. To wit, if the greater Body in the former Example be put to be unmov'd, the Motion thereof will immediately be found thus. For as the Sum of the Bodies $=5$, is to the Double of the second Body $=2$; so is the whole respective Velocity $=4$, to the Velocity of the Second after the Meeting $=\frac{2\times4}{5}=\frac{8}{5}$ or $1\frac{3}{5}$. For the Difference betwixt the Celerity of the first Body before the Meeting, which was none at all, and this Celerity, will be the very Celerity of the first after the Meeting, and consequently the Velocity of the second will be Parts $1\frac{2}{5}$ or $2\frac{2}{5}$.

(21.) The

(21.) The Celerity which a greater Body perfectly Elastic, gives to a lesser perfectly quiescent which is perfectly Elastic, hath that Proportion to that Velocity, which the lesser moved with the like Celerity gives to the greater which is quiescent, which the Magnitude of the greater hath to the Magnitude of the less. For by reason of the given respective Velocity in both, and the Sum of the Bodies also given, the Computation will be alike in both Cases; to wit, as the given Sum of the Bodies is to the given respective Velocity; so is the double of the greater Body or the double of the lesser to the sought Velocity. The Velocities therefore are as the Bodies, Q E D.

Scholium, We shall in this Place, by way of *Corollary*, annex the three remaining Theorems of *Hugens* hitherto belonging, (albeit the Demonstration of them is longer than agrees to this Place) both because they are in themselves most noble Theorems,, and because, they may sufficiently appear manifest from a Calculation taught under the foregoing Problem.

(1.) Two Bodies perfectly Elastic meeting one another, that which is produc'd from drawing the Magnitudes of each into the Squares of their respective Velocities, being added together, both before and after the meeting of the Bodies, will be found equal on both Sides; if to wit, the Proportions both of the Magnitudes, and of the Velocities, be express'd in Numbers or Lines.

(2.) If any Body perfectly Elastic, meets another Body which is quiescent, whether greater or less; it will give a greater Celerity thereto, by an interpos'd Elastic Body of a mean Magnitude, which is likewise quiescent, than if it hit upon it without the Interposition of the other Body: And

And it will confer upon it the greateſt Celerity of all, if the interpos'd Body be a mean proportional betwixt the Extreams.

(3.) By how much the greater Number of Bodies perfectly Elaſtic, is interpos'd betwixt two unequal Bodies perfectly Elaſtic, whereof the one reſts and the other is mov'd, by ſo much the greater quantity of Motion will be produced in the quieſcent Body: But the greateſt Motion of all will be convey'd through the Multitude of the interpos'd Bodies, if the interpos'd ones, together with the Extremes, do conſtitute one continued Series of Geometrical Proportionals.

And it is to be noted, that it appears from the two laſt Theorems, according to the Author's Computation; that if there be given an 100 Bodies placed in a Line, which are in a double Proportion, and the Motion begins at the greateſt, the Celerity of the leaſt will be to the Celerity of the greateſt, as 14,760,000,000 to 1 or thereabouts. But if the Motion begins with the leaſt, the Quantity of the Motion will be increas'd in the End, in about that Proportion which 1 bears to 4,677,000,000,000. Where in the former Caſe is ſeen a moſt prodigious increaſe of Celerity; and in the latter, a more ſtupendious Augmentation of the Quantity of Motion.

But to conclude, what things *Hugens* aſſerted (that I may advertiſe this at length) concerning all Bodies, or at leaſt concerning all Bodies perfectly hard, we have all along with our Famous Mathematicians *Wallis* and *Newton*, demonſtrated of Bodies perfectly Elaſtic only. Nor certainly, ought they to be otherwiſe underſtood or affirm'd. For the Laws of Motion which agree to Bodies not Elaſtic, are for the moſt part altogether different from theſe, as is abundantly manifeſt from

from what hath been ſaid, and therefore ought in no wiſe to be mingled together with the Laws of Elaſtics. But as to what concerns Bodies imperfectly Elaſtic, we ſhall deliver their Laws out of the famous *Newton* in the following Lecture.

May 8. 1704.

LECT. VI.

Law. (22.) EVERY Body will in the ſame Time deſcribe the Diagonal of a Parallelogram with Forces conjunct, that it would do the Sides with thoſe Forces ſeparate.

(In *Fig.* 3. *Plate* 2.) Let the Body A be carried in a given time by the ſingle Force A B, impreſs'd according to the Direction of the Line A B, from A to B; and by the ſingle Force A C impreſs'd according to the Line A C let it be carried in the ſame time from A to C; and let the Parallelogram A B D C be compleated; I ſay, that by both the Forces impreſs'd together, it will be carried in the given time from A along the Diagonal unto D. For becauſe theſe Forces impreſs'd together are not oppoſite one to the other, they can in no wiſe deſtroy one another, but will beget a certain Motion which is in the middle betwixt both. For ſeeing the latter Force A C, acts according to the Line A C, which is Parallel and Equal to B D, this Force ought not at all to change the Velocity of coming to the Line B D, which is produc'd by the former Force. Therefore

the

the Body will come in the same time to the Line B D, whether the latter Force be impressed or no; and consequently the Body, in the End of the given time will be found somewhere in that Line B D. And by the same Argument, since the former Force A B, acts according to the Line A B, which is Parallel and Equal to C D, this Force ought not at all to change the Velocity of coming to the Line C D, which was generated by the latter Force. Therefore the Body would come in the same time to the Line C D, whether the former Force were impress'd or no; and consequently in the End of the given Time will be found somewhere in that Line C D. And therefore it is necessary, that in the End of that time the Body should be found in D, the Concourse of the two Lines B D and C D. Furthermore, seeing the same Thing may in altogether the same manner be demonstrated of innumerable Points d, d, d, &c. in the same Diagonal Line; it is manifest, that the Body with these Forces conjoin'd, ought always to describe this Diagonal Right Line. Q. E. D.

Coroll. (1.) The Forces being given, the Velocity arising from the Conjunction of them will be so much the greater, by how much the Directions of the first Forces do the more conspire together, or by how much the Angle B A C is the less; and so much the less as the Directions of those Forces are the more opposite to one another, or the Angle B A C is the greater: And the Velocity of both Directions which tend to go according to the Parallel Lines A C, B D, and A B, C D, parallel to the Lines B D and C D, or any others whatsoever, is in no wise chang'd by the conjunction of these Forces, but always remains the ſame, as is manifeſt from what has been demonſtrated in this Propoſition.

Coroll. (2) The ſame Diagonal Line A D, may be deſcrib'd from the conjunction of innumerable Pairs of Forces. Thus, if inſtead of the former Force A B you ſuppoſe another, to wit A E; and for A C put A F, and then compleat the Parallelogram A E D F, the Line A D being the common Diagonal: The Body A from the conjunction of theſe Forces will deſcribe the ſame Line A D which it did before, as is manifeſt from the Propoſition. And there is the ſame Reaſon for any other Pair of Forces whatever, by which the Sides of a Parallelogram, whoſe Diagonal is A D, ought to be deſcrib'd.

Coroll. (3.) Therefore the Forces being given both in Magnitude and Direction, there is alſo given one right Line to be deſcrib'd, to wit the Diagonal of a Parallelogram: but the deſcrib'd Line or Diagonal being given, the Forces are not thence given, nor the Directions by which that Parallelogram was deſcrib'd. For the Sides of a Parallelogram being given, and the Angle included, there is given therewithal the Parallelogram it ſelf, and conſequently the Diagonal of that Parallelogram; but a Line being given in Length and Direction for a certain Diagonal, the Parallelogram it ſelf is not from thence given; to wit, becauſe the Line may be the Diagonal of innumerable Parallelograms. For as the Sides of the Parallelogram, without the included Angle, do determine no certain Diagonal; ſo neither doth the Diagonal without the adjacent Angles determine any certain Sides.

Coroll. (4.) Where the primary Forces B A, B D (ſee *Fig.* 4. *Plate* 2.) are equal, and comprehend the Angle A B D of 120 Degrees, the Velocity reſulting from the conjunct Forces, will be the ſame

same as that of either of the separate Forces; and the Directions of the Force only will be chang'd; for in this Case the Triangles A B C and B C D will be Equilateral, and compose a Rhombus; and the Diagonal therefore B C will be equal to either of the Sides.

Coroll. (5.) Where the primary Forces are equal, and the Angle included by the Sides is a right Angle, the Velocity arising from the Forces conjoin'd will be incommensurable to either of them separate; to wit, because the Diagonal of a Square is incommensurable to the Side.

Scholium. What has been spoken in this Proposition, and its Corollaries, concerning real Motions and Velocities, is to be applied to any Endeavours or Tendencies to Motion whatever. Thus, if the Body A in the former Figure be impell'd by two Forces, which have that Proportion amongst themselves, which the Lines A C and A B have, and also are impell'd according to the Directions of the same Lines, or be press'd, or drawn in those Directions, or any other way tend according to the same, although actual Motion should not presently follow by reason of some Obstacles, yet notwithstanding the Impulse or Force arising from the conjoin'd Forces, tends according to the Direction of the Diagonal A D; and the Velocity to be produced is to be express'd or represented by the Line A D; as will more easily be understood from what follows.

(23.) All Forces and Motions whatever may be resolv'd into innumerable Forces and Motions; and on the contrary, direct Forces, and rectilinear Motions, may be compounded of innumerable oblique Motions and Forces.

Thus in the former Figure, the Line and Direction of the Motion is the ſame, whether it be compounded of the Forces A B, A C, or of the Forces A E, A F, or ariſe from one ſingle Motion impreſs'd according to the Line A D. And on the other Hand, any Motion whatever along the Line A D, although it may ariſe perhaps from one ſingle Force impelling right forward, yet may be conſidered as compounded of A B, A C, or A E, A F, and innumerable other the like; foraſmuch, as the very ſame Motion would ariſe from all thoſe combin'd Forces. And in the ſame manner are we to reaſon of Motions more compounded. For in the firſt Place, having conſidered a Diagonal Line as reſulting from two Forces combin'd; we may then reduce thoſe two unto one ſingle Force, and conceive a third Force as ſuperadded; which being join'd to the former, will produce a Motion along another Diagonal of ſome ſecond Parallelogram, and then may we in our Conception ſuperinduce a fourth Force, and after that a fifth, and ſo on infinitely. Nor can indeed any direct Force, where there is occaſion to reſolve it into more, be otherwiſe reſolv'd than thus. Now this Compoſition and Reſolution of Forces occurs very frequently, and is abundantly confirm'd from *Mechanics*, as we ſhall now ſhew with our Author.

If unequal Rays O M, O N (ſee *Fig.* 5. *Plate* 2.) going forth from O, the Center of ſome Wheel, do by the Threads M A, N P, ſuſtain Weights in Equilibrium, and the Forces of the Weights unto the moving of the Wheel be requir'd: Through the Center O let the Line K O L be drawn, meeting the Threads which ſuſtain the Weights perpendicularly; and from the Center O with O L, the greater Interval of the two

O K

O K, OL, let there be describ'd a Circle meeting the Thread M A in D; then through O and D let there be drawn the right Line O D; to which let D C be perpendicular, and A C Parallel; and let the Parallelogram D C A be compleated. Now because it nothing matters, whether the Points of the Threads K L D be fastned or not fastned to the Plane of the Wheel; the Weights will be of the same Force, whether they be hanged on the Points K and L, or those D and L; for (setting aside the Weight of the Thread) the Gravity of the same Body is the same wheresoever the Thread is fixed, so it be in a Line perpendicular to the Horizon: Let therefore the whole gravitating Force at A be represented by the Line A D as the Diagonal of a Parallelogram, that from the Proportion of the Diagonal to the Side of the Parallelogram, we may come to know where it is, that one of the Forces is none at all. Now that whole Force which A D designs may be resolv'd into innumerable Pairs of Forces, but seeing others are foreign to our Purpose, let it be resolv'd into D c (or A C) and D C; the one to wit according to the direction of the protracted Radius D O, the other Perpendicular to the same Radius. One of these Forces A C or c D, by reason that it draws the Radius O D directly from the Center (for it tends from D to c in the Protracted Radius) is of no force at all for the moving the Wheel; but the other Force D C which draws the Radius D O perpendicularly, is of the same Force as if it drew the Radius O L, equal to O D perpendicularly: But seeing the Wheel doth by the Hypothesis rest in Equilibrio, the Weight P will be to the Weight A, as the Force D C is to the Force D A. For the whole Force of the Weight P draws the Radius O L perpendicularly, and so confers its whole Force to the moving of the Wheel; but only that Part of the entire Weight A represented by the Line A D, I say, only that Part of this Weight which is expounded by D C, draws the Radius O D, which is equal to O L perpendicularly, the Force of the other Part which tends according to the Radius O being wholly lost; that Part therefore D C only avails to the moving of the Wheel. Since therefore, because of the Equilibrium suppos'd on both Sides, the entire Force of the Weight P is equivalent only to a certain Part of the Weight A, to wit, to D C; it is manifest, that the Weight A ought to be so much greater than the Weight P, by how much the Diagonal D A is greater than the Side D C; and that because of the Declination of the Body A from the perpendicular D C. As therefore the Weight A is to the Weight P, so is D A to D C; that is, because of like Triangles A D C, D O K, as O D or O L is to O K, therefore the Weights O and P, which are reciprocally as the Rays O L and O K which are placed in a straight Line, will be of the same Force on both Sides, and consequently stand in an Equilibrium. And this indeed is the most known, and the fundamental Property of the Balance, Leaver, and Axis in Peritrochio, which from the resolution of Forces is easily demonstrated. But if either of the Weights be greater than in this Proportion, its stronger Force will prevail, and suffice to move the Wheel. But if the Weight π, equal to the Weight P, be partly hang'd upon the Thread N π, and doth partly lye upon the Oblique Plane π G; let N H and π H be drawn, the former Perpendicular to the Horizon, the latter to the Plane π G; and let the Parallelogram

rallelogram π N R H be compleated. And if the entire Force of the Weight π tending downwards, be represented by the Line N H, it may be resolv'd into the Forces π N, R N. Now if to the Thread π N, some Plane as π Q were perpendicular, cutting the other Plane π G in a Line parallel to the Horizon, and the Weight π lay wholly upon these two π Q, π G; the Weight π would press these Planes perpendicularly, to wit π Q by the Force π N, and the Plane π G by the Force R N. And therefore if the Plane π Q should be taken away, that the Weight should stretch the Thread, because the Thread doth now by sustaining the Weight supply the Place of the Plane which is taken away, it will be stretch'd with the same Force π N wherewith the Plane was before press'd. From whence the Tension of this Oblique Thread will be to the Tension of the other perpendicular Thread P N, as π N is to N H; and therefore if the Weight π be increas'd in the Proportion of N H to N π, it will sustain the Weight A, and the Wheel will not be mov'd. From whence, if the Weight π be to the Weight A in the reciprocal Proportion of the least Distances of their Threads A M P N from the Center of the Wheel, or as K O to O L, and also in the direct Proportion of N H to π N; that is, joining both Proportions together, as the Rectangle K O $\times$ N H to the Rectangle O L $\times$ π N, the Weights will be of equal Force to the moving of the Wheel; and consequently they will sustain each other in an Equilibrium, as any one may easily find upon Tryal.

Corollary (1.) From hence we may discover a new way for measuring all lesser Weights from one given Weight. For if the Plane π G, perfectly polish'd, be placed gradually at divers Degrees of

Inclination, the same Weight π or P will be equivalent to divers Weights whatsoever less than it self; to wit, in the Proportion of the Line π N to H N. And consequently, if a Table should be made, of the Proportions of the Lines π N and H N in the divers Degrees of Inclination; it will be easy from the Inclination of the Plane π G, and one only given Weight π or P to examine and determine the Weights of all Bodies less than π or P.

Coroll. (2.) Hence likewise we may estimate the Velocities or Weights of Bodies, descending or declining in any Plane whatsoever: Let A B be the inclining Plane, and (see *Fig.* 6. *Plate* 2.) of the Body descending along that Plane, or leaning upon it; let the entire Force of its Gravity be represented by the Line d f perpendicular to the Horizon; and let that whole Force be resolv'd into two Forces f c, and f g, of which let the one be perpendicular to the inclined Plane, for the bearing of which therefore that Plane adequately sufficeth; and let the other be put Parallelwise with respect to the inclined Plane, which therefore is fitted for exciting a Motion, or at least for procuring an Endeavour towards Motion without any Impediment. The Motion therefore or Weight in the inclined Plane, is to the Motion or Weight in the Plane perpendicular to the Horizon, as the Side f g is to the Diagonal Line f d; that is, because of the likeness of the Triangles f g d and A B C as A C is to A B, or as the Radius is to the Secant of the Angle BAC; which is a Proposition very well known in *Mechanics*.

Coroll. (3.) From hence also the Force of the Wedge appears. Let (*Fig.* 7. *Plate* 2.) C C A be a Wedge struck by a Mallet with a direct Blow; let the

the whole Force of the Stroke be expressed by the Line D A; and let it be resolv'd into two Forces D Q and D R; the one of which let be perpendicular to the Face of the Wood C A, and consequently directly set to remove the same Face, and the other D R parallel to the same Face, and consequently posited to go forward directly; and let the same be understood of the other half of the Wedge D A C. The removal then of the lateral Obstacle, according to the Line D Q, is to the Progress of the Force downwards, according to the Line D R, as D Q is to D R; that is, because of the Triangles D Q A, D C A, which are like, as D C is to D A; or the force of the other Part being computed, as C C is to D A; which also is a most known Property of a Wedge, and universally receiv'd in *Mechanics*. Or, if we be minded to dispatch the Matter with Sir *Isaac Newton* out of what has been before demonstrated; The Weight π, in the last Figure save one, lying upon the two oblique Planes π Q, π G, will have the Nature of a Wedge betwixt the interval Faces of the cloven Body; and from thence the Force of the Wedge and Mallet will be known. for the Force wherewith the Weight π presseth upon the Plane π Q, is to the Force wherewith the same is impell'd, either by its own Gravity, or by the Stroke of the Mallet, according to the Line perpendicular to the Horizon, as π N is to N H; and is to the Force wherewith it presseth upon the other Plane π G, as π N is to N R. Nay the Force of the Screw likewise may be collected by the like division of Forces, forasmuch as the same, in our *Author*'s Opinion is a Wedge forced by a Leaver.

Scholium,

Scholium, The use therefore of this Composition and Resolution of Motion, appears of very wide Extent, and from its Clearness demonstrates its own Truth, since all the Mechanic Science, which is diversly demonstrated by Authors, depends upon the Things which have been now said: For from these are deriv'd the Forces of the Machines which are wont to be compos'd of Wheels, Screws, Leavers, and Weights, ascending directly or obliquely, and the rest of the Mechanic Powers; as also the Force of Muscles for moving the Bones of living Creatures.

Octob. 23, 1704.

Lect. VII.

(24.) THE Quantity of Motion which is collected, by taking the Sum of Motions made to the same Part, and the difference of those made to the contrary Parts, is not chang'd by the Actions of Bodies one upon another.

For Action, and the contrary to it Re-action, are equal by the Fifth Law; and consequently by the Fourth, they make equal Mutations of Bodies towards the contrary Parts. Therefore, if the Motions be made to the same Part, whatsoever is added to the Motion of the Body, which flies away, will be subducted from the Motion of that which follows, so that the Sum shall remain

main as before. But if Bodies meet one another in the ſame Line, there will be an equal ſubduction from the Motion of both, and conſequently the difference of the Motions made to contrary Parts will remain the ſame. As if the Spherical Body A be Threefold greater than the Spherical Body B, and have two Parts of Velocity; and B follows in the ſame right Line with 10 Parts of Velocity; and conſequently the Motion of the Body A, reſulting from its Velocity and Magnitude together, is to the Motion of the Body B, eſtimated in the ſame manner as 6 is to 10; therefore the Sum of the Motions to the ſame Part is of 16 Parts. In the meeting together therefore of the Bodies A and B, if the Body A, according to the Quantity of its Elaſticity, doth gain three Parts of Motion, or four or five, the Body B ſhall loſe ſo many; and conſequently the Body will go forward after Reflection, with Nine Parts, or Ten, or Eleven, and B with Seven, or Six, or Five, the Sum of Sixteen Parts, remaining always as before; as the thing will always happen in Bodies not at all, or at moſt in a leſs degree Elaſtic. But if the Body ſhall gain 9, 10, or 11, or 12 Parts, and ſo go forwards, after the meeting with Fifteen Parts in all, or Sixteen, or Seventeen or Eighteen; the Body B, whilſt it loſeth ſo many Parts as A gains, either will go forward with one Part having loſt Nine; or it will reſt, its progreſſive Motion of Ten Parts being wholly loſt; or it will go back with one Part, having loſt its Motion, and (as I may ſay) one Part more, or it will retrocede with two Parts, becauſe of the progreſſive Motion of twelve Parts which was taken away. And ſo the Sums of the conſpiring Motions 15+1, or 16+0; and alſo the Differences of the contrary Motions 17--1, or 18--2, will always be of Sixteen Parts, as it was before the Meeting or Reflection; which

will

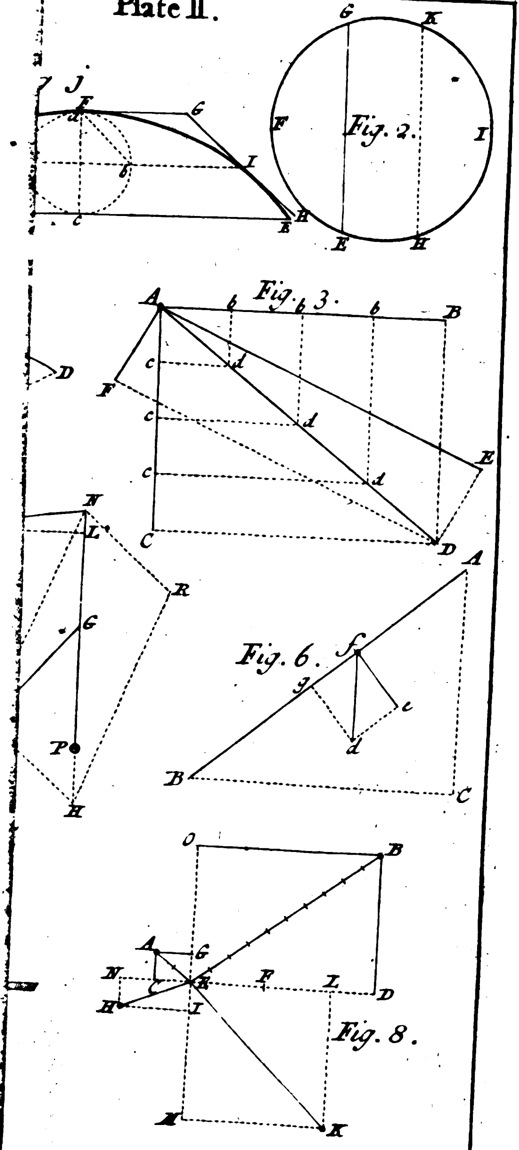
will happen alſo in Bodies imperfectly Elaſtic, as may ſufficiently be underſtood from the Laws of Motion before delivered, and is afterwards to be ſaid concerning Bodies imperfectly Elaſtic. But the Motion wherewith Bodies go forwards after Reflection being known, there will be found the Velocity of the ſame after the Reflection, by ſaying it is to the Velocity which was before the Reflection, as the Motion after is to that which was before. As in the laſt Caſe, where the Motion of the Body A was of Six Parts before the Reflection, and of Eighteen afterwards, and its Velocity was of two Parts before the Reflexion; its Velocity will be found to be of Six Parts after the Reflexion, by ſaying according to the Golden Rule; as Six Parts of Motion before the Reflexion is to Eighteen Parts afterwards, ſo is the Velocity of two Parts before the Reflexion to Six Parts of Velocity after. For ſeeing the Quantity of Motion doth ariſe from the Velocity and Magnitude conjunctly, in a given Body the Quantity of Motion will be eſtimated from the Velocity alone, and conſequently the Quantity of Motion and Velocity will be directly proportional to each other. But if Bodies not Spherical, or which move in divers right Lines, fall one upon another obliquely, and their Motions after the Reflexion be required; the ſituation of the Plane, by which the concurring Bodies are touched in the Point of Concourſe, is to be conſidered and known. Thus the Motion of both Bodies is to be diſtinguiſh'd into two, one perpendicular to the Plane, the other parallel to the ſame; but the parallel Motions, by reaſon that they are in no wiſe oppoſite to each other, the Bodies acting upon one another, according to a Line perpendicular to this Plane, the ſame are to be retain'd after

after the Reflexion as well as before; and equal Mutations to the contrary Parts are to be attributed to the perpendicular Motions, in such sort, that the Sum of the conspiring Motions, and the difference of the contrary ones remain always as before. As for Example: Let the Body A (see *Fig.* 8. *Plate* 2.) which is Spherical, and perfectly Elastic, be threefold of the Body B, which is also Spherical and perfectly Elastic, and let A have two Parts of Velocity, represented by the Line A E, divided into two equal Parts; and let the Body B meet it obliquely according to the right Line B E, in the Angle A E B, with ten Parts of Velocity, represented by the Division of the Line B E into Ten Parts equal amongst themselves, and to the former; let the Angle A E B be bisected by the right Line O E M; Let A G and B O be let down perpendicular to the Line E O; and the Parallelograms A C E G, B O E D be compleated. The Plane then which passeth through O M, will be that by which the Spherical Bodies will be touched in the Point of Concourse; and the oblique Motions along the Diagonals A E and B E will be distinguish'd on both Sides into two, to wit A E into A G and A C, and B E into B O and B D; one of which Motions A G and B O, or C F and E D are perpendicular to the Plane of Concourse; to which alone therefore, as being directly opposite to each other, and tending to the contrary Parts E C and E D, all the change of the Motions in the Concourse is to be referred, in the mean while that the other A C and B D, or G E and O E which are parallel each to the other, and in the Point of Concourse tend wholly unto the same Part, are so far from being contrary to one another, that they are rather to be reckon'd to conspire together directly,

rectly, and consequently are to be retain'd after the Reflexion as well as before. Wherefore let E I be equal to E G, and E M equal to E O; and that we may estimate the Mutations of Motions made to the contrary Parts, and to be directed according to the Line C D; let us make the computation according to the Twentieth Law of Motion, borrowed from *Hugens.* Let it be made then, as the Sum of the Bodies A and B = 4 is to the double of the Body B = 2; so is C D the respective Celerity of the approach, which is of Twelve Parts, (for because the Triangles A G E, B O E are like, A G or C E is to B O or E D as A E = 2 is to E B = 10; and consequently, A E + E B = 12) to the half of C D = C F = 6. And the Difference betwixt the Celerity of Six Parts, and the Celerity of the Body A before the Impulse which was of Two Parts, which is equal to 4, will give the Celerity wherewith the Body A will be mov'd after the Concourse; which Celerity being taken away out of the whole respective Celerity which was before the Impulse, to wit, 12 − 4 = 8, there remains the Celerity of the Body B after the meeting. Let therefore E N be of Four Parts, and E L of 8, and the Parallelograms E N H I, and E L K M being compleated, and the Diagonals E H and E K being drawn, the Bodies A and B in the same time in which they hastened to the Meeting before, according to the Diagonals A E and B E will come after the Meeting, to the Points H and K, being reflected into the Diagonals E H, and E K; and the Motion of the Body A will be of 4 × 3 = 12 Parts; and the Motion of the Body B = 8 × 1 = 8 Parts, the difference of which Motions is Four Parts, which was also the difference of the Motions before the Meeting.

 Wherefore

Plate II.



I. Senex sculp.t

Wherefore in this Case the Quantity of Motion which is collected by taking the difference of the Motions made to the contrary Parts, is not changed by the Action of the two Bodies upon one another; and consequently in Bodies dashing one upon another obliquely, this Rule holds good, as well as in Bodies which directly meet one another. Now from these Reflexions, there are wont to arise circular Motions of Bodies about their own Centres: But we shall have no occasion to consider these Cases in what follows; and it would be too long to demonstrate all the Things hereto appertaining.

A Lemma to the 25th Law.

If two right Lines given in Position, A C and B D, (*Fig.* 1. *Plate* 3.) be terminated at the given Points A and B, and have a given Proportion to one another; and the right Line C D, wherewith the indeterminate Points C and D are join'd, be divided in the given Proportion in K; I say, that the Point K will be placed in a right Line given in Position

For let the right Lines A C and B D (if they be not Parallel) meet together in the Point E; and in B E let B G be taken in the same Proportion to A E as is B D to A C: And let F D be equal to E G. Here E C will be to G D, that is, to E F, equal by Hypothesis to G D, as A C is to B D, and consequently will be in the Proportion given; therefore the Triangle E F C will be given in Species (to wit as having the Angle C E F, and the Proportion of the Sides about the same Angle given) let C F be cut in L in that given Proportion, and so there will be given in Species the Triangle E F L (by reason of of the given Proportion of the Sides about the given Angle E F C ;) and therefore the Point L will always be plac'd in the Line E L given in Position. Join L K ; and because of F D which is given, as being equal to E G given, and the proportion of L K to F D which is given, the same to wit as that of C K to C D, L K will be given. Let E H be taken equal to L K, and E L K H will be a Parallelogram, for L K is parallel to F D, and consequently to E H the protracted part of the same Line, to which it is by the Hypothesis equal. Therefore the Point K is placed in H K a side of a Parallelogram which is given in Position. Q. E. D. But then, if the right Lines A C, B D, be parallel each to other, the Point of Concourse will be infinitely distant, that is none at all ; and all the Lines E C, E L, H K, E D, will be parallel to one another.(see *Fig.* 2. *Plate* 3.) In which Case the Lemma is thus demonstrated. Let the Points terminating the Lines A C, B D, which have a given Proportion, be join'd by the Lines A B, C D, and let these joining Lines be protracted to meet together in Q ; through the Point K which divides the Line C D in the given Proportion, let H K be drawn parallel to A C and B D : I say, that the Point K is placed in the right Line H K given in Position. For wheresoever the Points C and D are taken in the right Lines A C, and B D, the Line joining these Points will tend to the Point Q, as in the Points c and d, and the joining Line c d will be divided in that given Proportion by the Line H K : For according to the Hypothesis, and in this Figure A c is to B d as A C is to B D ; as also according to the Hypothesis and in this Figure c k is to c d as C K is to C D. It is manifest therefore

fore in this Caſe, that the Point K is always placed in the right Line given in Poſition.

Coroll. (1.) If two Points go forwards together with an uniform Motion in right Lines, and their diſtance be divided in a given Proportion, the dividing Point will be placed in a right Line given in Poſition; and that Point as K will be mov'd uniformly in that right Line. For, becauſe of the Uniform and Even Motion of both Points, the Lines of Motion which they deſcribe at the ſame time will always be in a given Proportion, to wit, in the Proportion of the Celerities which are on both Sides equable: From whence it is manifeſt by the things already demonſtrated, that the Point K will always be carried in the right Line HK. But that it is carried uniformly, and with an equable Motion, will be thus demonſtrated: HK is always equal to EL, and EL increaſeth in the ſame Proportion as the Lines EC and EF proportional to it, which Lines are alſo proportional according to what hath been already ſaid, to AC and BD, along which the Bodies are mov'd at the ſame time. EC therefore is to EF, as AC to BD; from whence, ſince thoſe Lines by the Uniformity of the Motion do increaſe equally; EL alſo, and HK, which is proportional to the ſame, will alſo increaſe equally; or, which is the ſame thing, the Point K will be carried with an uniform and equable Motion along the Line HK. Q. E. D. And we may argue in like manner in the ſecond Caſe where the Lines were ſuppos'd parallel. Nor is there need of more Words in ſo plain a Matter. The truth of the Lemma will likewiſe be concluded concerning a ſolid Place by almoſt the like Demonſtration, *viz.* if you conceive a Plane cutting the leaſt diſtance of the

Lines in the ſame Proportion and perpendicular to the ſame diſtance; and if you imagine Lines let fall perpendicular to the ſaid Plane, the Demonſtration will be as in this Propoſition, if inſtead of the Lines of Motion, ſuppos'd in different Planes, we uſe the Lines joining the Perpendiculars, which will be in the ſame Plane.

Coroll. (2.) If both Points go forwards unto the ſame Part, the dividing Point will alſo go forwards unto that Part: From whence in every Caſe, that dividing Point K will either reſt or be mov'd uniformly in a right Line. If one of the Points be mov'd unto this Part, another to the contrary, the Point K will be mov'd more ſlowly unto one Part or the other, according as the Proportions of the greater Celerity, or of the diſtance from the Point K ſhall require: Or laſtly, if thoſe ſhall be Proportions of Equality, and prevail on neither ſide, the dividing Point will be mov'd to neither Side, but wholly reſt.

(25.) The common Center of Gravity of a Syſtem of Bodies doth not change its State either of Motion or Reſt, from the Actions of the Bodies amongſt themſelves, (whether they be Attractions or Impulſes;) and therefore the common Center of Gravity of all Bodies acting upon one another (Actions and Impediments, whether External or otherwiſe gotten being excluded) doth either reſt, or is mov'd uniformly ſtraight forwards. For if two Bodies or Points, as C D go forwards with an uniform Motion in the right Lines A C, B D, and their (ſee *Fig.* 1, 2. *Plate* 3.) diſtance C D be divided in a given Proportion; (as the Line always paſſing through the Centers of the mov'd Bodies is divided by K the common Center of Gravity of both in a given Proportion, to wit, that which is reciprocal to the Bodies) their common

mon Center of Gravity K, will either reſt or be mov'd uniformly in theLine KH. Therefore if any Number of Bodies be mov'd uniformly in right Lines,the common Center of any two of them either reſteth or goeth forwards uniformly in a right Line; becauſe that the Line connecting the Centers of thoſe Bodies which go forward uniformly in right Lines, is divided by the common Center of Gravity in a given Proportion; in like manner, the common Center of theſe two, and any third Body whatever, either reſts or goes forwards uniformly in a right Line; becauſe that the diſtance of the Center of Gravity of theſe two, and of the Center of the third, is divided thereby in a given Proportion, to wit, a reciprocal Proportion to the third Body, and the Syſtem of the two: For the common Center of Gravity of the two goes forwards uniformly in a right Line, and conſequently is to be reckon'd as if it were the Center of a ſingle Body. In the ſame manner, the common Center of Gravity of theſe Three and any Fourth,either reſts or goes forwards uniformly in a right Line; becauſe the diſtance betwixt the Center of Gravity of the Three, and of a Fourth, is divided in a given Proportion; and ſo on *in infinitum.* Therefore in a Syſtem of Bodies which are altogether free from Actions upon one another, and others alſo impreſs'd from without; and which conſequently do either reſt or are mov'd uniformly in ſeveral right Lines, the common Center of Gravity of all either reſts or is mov'd uniformly ſtraight forwards. Moreover, in a Syſtem of two Bodies acting upon one another, ſince the Diſtances of the Centers of both from the common Center of Gravity is reciprocally as the Bodies, the Relative Motions of the ſame Bodies, of coming to that Center or departing from the ſame, (whether the one be from Attraction, or a Centripetal Force, or the other be from an Impulſe or a Centrifugal Force) are equal betwixt themſelves, and the Velocities of Acceſs or Receſs are reciprocally proportional to the Bodies, that is, directly proportional to the Diſtances from the Center of Gravity of both. From whence, by thoſe Actions the diſtance from the Center will be proportionally increas'd or diminiſh'd; and therefore that Center is neither advanced forwards nor retarded, nor ſuffers any change in its own State as to Motion or Reſt, from the equal Changes made to the contrary Parts, and conſequently the Actions of theſe Bodies amongſt themſelves, whether they repel one another or attract, do not any ways alter the State of the common Center of Gravity. Now in a Syſtem of more Bodies, becauſe the common Center of Gravity of any two acting mutually upon each other, doth not in any wiſe by reaſon of that Action change its State; and the common Center of Gravity of the reſt ſuffers nothing therefrom; but the diſtance of the Centers of theſe two is divided by the common Center of all the Bodies into Parts reciprocally proportional to the Sums total of the Bodies whoſe Center of Gravity they are, and conſequently thoſe two Centers keeping their State of Motion or Reſt, the common Center of Gravity of all will alſo keep its State; from hence it is manifeſt, that that common Center of all never changeth its State as to Motion or Reſt, becauſe of the Actions of two Bodies betwixt themſelves. But in ſuch a Syſtem of all, the Actions of all amongſt themſelves are either of two Bodies; in which Caſe the common Center of Gravity of the whole Syſtem is nothing

thing chang'd, as we have already ſhewn; or compounded of the Actions which are betwixt Couples of Bodies; and therefore they will ſuperinduce no change in the State of Motion or Reſt to the common Center of Gravity. For if by the Action of A upon B, the State of the Center of Gravity is nothing chang'd, nor by the Action of C upon B; neither will the ſame be diſturb'd by the conjunct Forces of C and A upon B. Wherefore, ſeeing that the common Center of Gravity, when Bodies do not act upon one another, either reſts or goes forwards uniformly in ſome ſtraight Line; the ſame will continue notwithſtanding the Actions of Bodies one upon another, either always to reſt or to go forwards in a right Line uniformly, unleſs it be moved from this State by ſome extrinſic Force impreſs'd upon the Syſtem. There is the ſame Law therefore of a Syſtem of Bodies as to perſeverance in the State of Motion or Reſt, that there is of one ſingle Body. For the progreſſive Motion, whether of a ſolitary Body or of a Syſtem of Bodies, ought always to be eſtimated from the Motion of the Center of Gravity.

Octob. 30, 1704.

LECT. VIII.

(26.) THE Motions of two Bodies included in a given Space, and partaking of the Motion thereof, are the ſame amongſt themſelves, whether that Space reſteth, or that the ſame is mov'd uniformly ſtraight forward without a Circular Motion.

For the difference of the Motions tending to the ſame Part, and the Sums of thoſe which tend to the contrary Parts, are the ſame at the beginning in both Caſes (by the Hypotheſis) ; and from theſe Sums or Differences ariſe the Congreſſes or Shocks whereby the Bodies encounter one another, [to wit, of the Sums of the Motions tending to the contrary Parts, and the Differences of the ſame when tending to the ſame Part.] Therefore by Law 4th, the Effects of the Congreſſes will be equal in both Caſes, and therefore the Motions amongſt themſelves in one Caſe will remain equal to the Motions amongſt themſelves in another. For the common and uniform Motion of the Space, and included Bodies which tends to the ſame Part, will either by equally accelerating all, as in caſe they all tend to the ſame Part with the Space it ſelf, or by adding ſo much to one as it takes away from another, as in thoſe which tend to the contrary Parts, make that the Forces at the Meetings of the Bodies will be in no wiſe changed. This ſame thing is prov'd by a manifeſt Experiment ; for all Mo-

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tions are in the ſame manner in a Ship, whether it reſteth, or be carried uniformly ſtraight forwards.

(27.) If Bodies be mov'd in any wiſe amongſt themſelves, and be preſſed with equal accelerative Forces according to parallel Lines, they will all continue to be mov'd in the ſame manner amongſt themſelves, as if they were not preſſed with thoſe Forces. For thoſe Forces will move all the Bodies equally as to Velocity, whilſt they act according to the Quantities of the Bodies to be mov'd, and in parallel Lines; and therefore they will not change their Poſitions and Motions in reſpect of one another.

A Lemma to the Experiments following:

The Velocity of a pendulous Body in the loweſt Point of any Circle, is always as the Chord of the Arch which is deſcrib'd in the falling. (See *Fig.* 3. *Plate* 3.)

Let C A B be a right Angle, C or H a moveable Body hanging by the Thread C A or H A upon the Center A, which will fall down in the Arch C B or H B. I ſay that the Velocity of the Body C, in the loweſt Point B, is to the Velocity of the Body H in the ſame Point, or rather the Velocity of the ſame Body falling firſt along the Arch C B, and afterwards the Arch H B, is as the Chord C B is to the Chord H B. For the Velocity of the Body, falling through the Arch C B, as we ſhall demonſtrate by and by (*Coroll.* 5. *Prop.* 6. following) is in the loweſt Point B (that Velocity, to wit, wherewith the Body would go on to be mov'd in a right Line, which toucheth the Circle in B, if it ſhould leave the Thread in B) is, I ſay, the ſame, as that which the Body would have in the Point F, if it had fallen perpendicularly along C F. And by the ſame, the Velocity of the Body which falls from H according to the Arch H B, is the ſame as that which it would have in the Point K, if it ſhould fall perpendicularly along H K: [the ſame Celerity, to wit, being impreſs'd in Spaces betwixt parallel Planes, whether the Tranſit through thoſe Planes be perpendicular, as in Bodies falling perpendicular to the Horizon; or whether it be oblique, as in pendulous Bodies deſcribing circular Arches, as will be more fully made appear afterwards.] Therefore the Velocity of a Body deſcending along the Arch H C B, is to the Velocity of the ſame deſcending along the Arch H B, as the Velocity of a Body falling along C F, is to the Velocity of that which falls down H K. But (by *Coroll. Prop.* 4. beneath) The Velocity of a Body falling down in C F, is to the Velocity of a Body falling down in H K, in the ſubduplicate Proportion of the Line C F to the Line H K; and (as will appear from *Prop.* 2. following) the Chord C B is to the Chord H B in the ſame ſubduplicate Proportion of C F to H K. From whence it follows, that the Velocity of the Body deſcending along the Arch C B, is to the Velocity of a Body falling down the Arch H B in the loweſt Point B, as the Chord C B is to the Chord H B. Q. E. D.

Corollary. From hence is to be corrected the Error of *Hugens*, or rather of his Editors, who ſuppoſe that the Proportion of Velocity in the Point B, is the ſame as that of the Lines themſelves C F, H K, when it is only the ſubduplicate Proportion of the ſame, as we have already

* De vi Centrifugâ. p. 426, 427.

ready demonstrated from the Principles of *Hugens* himself.

A general *Scholium.* The truth of these Laws hath heretofore been prov'd by Sir *Christopher Wren* before the Royal Society by the Experiment of Pendulums; which thing the famous *Mariotte* hath taken upon him to set forth and declare in an entire Book. But that this kind of Experiments may agree with the Theories, regard is to be had not only to the Elasticity of the Pendulous Bodies, but also to the Resistance of the Air. Let the Bodies A and B (see *Fig.* 4. *Plate* 3.) hang by the parallel Threads A C and B D upon the Centers C and D: From these Centers, and at equal Intervals, let there be describ'd the Semi-Circles E A F, G B H, bisected respectively by the Radii C A and D B. Let the Body A be drawn unto any Point of the Arch E A F as R, and the Body B being withdrawn, let it be let down from thence, and return after one Vibration, compounded of going and returning, to the Point V. Here R V is the Retardation from the resistance of the Air. Of this R V let S T be made a fourth part, placed in the middle; and let R Q be equal to Q V; and thus S T will represent the retardatio nvery near which is in the Descent from S to A. For if in the Double both Ascent and Descent, the Retardation be R V, the Retardation in one Ascent or one Descent will be one fourth Part thereof. And seeing two Arches be greater, and two be less than the Arch Q A, the resistance of the Air is to be taken neither in the greatest Arches, nor in the least, but in a mean betwixt them. From whence the fourth Part S T is neither to be placed at the highest Part R, nor at the lowest V, but in a middle one which is betwixt both. Now let the Body B be restor'd into

into its Place: Let the Body A fall from the Point S; and the Velocity thereof in the Point of Reflection A, will, without sensible Error, be the same, as if it had fallen in a Vacuum from the Point T; the Body A by falling something higher, compensating the Resistance of the Air; and therefore, according to the just now demonstrated *Lemma*, let this Velocity of the Body in the Point A be represented by the Chord of the Arch T A. After the Reflexion, let the Body A come unto the Place s, and the Body B to the Place k, whether the Bodies be Elastic or not. Let the Body B be taken away, and the Place (u) be found, from which if the Body A was let down, and should after one entire Vibration return to the Point (r); (s t) may be a fourth Part of (r u) which (s t) is situate also as before in the middle: And by the Chord of the Arch (t A) let the Velocity which the Body A hath after its Reflexion in the Point A be represented; for (t) will be the true and correct Place unto which the Body A, the resistance of the Air being taken away, ought to have ascended; and by the like Method will the Place (k) be to be corrected, that to wit, to which the Body B ascends, and the Place (l) to be found, that to wit, unto which the Body ought to have ascended in a Vacuum. By this means we may try all this sort of Experiments in like manner as if we were placed in a Vacuum. Then at last the Body A is to be drawn into the Chord T A, which represents its Velocity, that the Motion of it in the Point A just before the Reflexion may be had; and afterwards into the Chord (t A), that the Motion of the same in the Point A presently after the Reflexion may also be had: As in like manner the Body B is to be drawn into the Chord (B l) that the Motion of it presently after

after Reflection may be had; and by the like Method where two Bodies are let down together from divers Places, there are to be found the Motions of both, as well before as after the Reflection, and then those Motions are to be compar'd at length betwixt themselves, and the Effects of the Reflexion to be collected. By experiencing the thing in this manner in Pendulums of ten Feet, and this in Bodies both equal and unequal, and by making that the Bodies should meet together from very great Intervals, as of Eight, Twelve, or Sixteen Feet, the Famous Sir *Isaac Newton* always found, without an Error of Three Inches in the measures, when the Bodies did directly meet each other, that the change of the Motion to the contrary Parts was equally in both Bodies, and consequently that Action and Re-action, according to the fifth Law were always equal. As if the Body A should fall upon the Body B, which is at rest, with Nine Parts of Motion, and seven Parts being lost in the Conflict, should go forward after the Reflexion with two Parts only, the Body B would rebound with those Seven which A lost. If Bodies meet one another, A with Twelve Parts of Motion, and B with Six; and A returns with Two, B would return with Eight; there being made to wit, a subduction of Fourteen Parts on both Sides. Let Twelve Parts be subducted from the Motion of A, and there will remain nothing; let there be substracted other two Parts, and then there will be a Motion of Two Parts unto the contrary Part. And so as to the Motion of the Body B of six Parts, by subducting 14 Parts, there will arise Eight Parts of Motion the contrary way. But if the Bodies should be carried to the same Part, A more swiftly with fourteen Parts, and B more slowly with

five

five Parts, and after the Reflexion A ſhould go forward with five Parts, B would go forwards with fourteen, there being made a Tranſlation of Nine Parts from the Body A to the Body B, and ſo in all other Caſes. The Quantity of Motion which was collected from the Sums of the conſpiring Motions, and the difference of the contrary ones, was never found in the Tryals of the foreſaid great Man to be changed by the Congreſs or Colliſion of Bodies. For the Error of one Inch or two in the Meaſures is to be attributed to the difficulty of performing every thing with due exactneſs. For it was not an eaſy thing to let down the Pendulums juſt at one time, ſo that the Bodies ſhould daſh one upon another in the loweſt Place A B; then to note the Places s and k, to which the Bodies aſcended after the Congreſs, was difficult; and in the Balls themſelves which were to be us'd, the unequal Denſity of the Parts, and the irregular Texture ariſing from other Cauſes muſt needs bring in ſome ſort of Errors. But further, leſt any ſhould object, that the Rule, for the proving of which this Experiment was invented, doth preſuppoſe either that the Bodies are abſolutely hard, or at leaſt perfectly Elaſtic, of which ſort there are none perhaps to be found in natural Compoſitions; we add, that the Experiments now deſcrib'd do ſucceed as well in ſoft Bodies as in Bodies Hard and Elaſtic; theſe Experiments not depending at all upon the Condition of the Hardneſs or Elaſticity of Bodies. For if the thing were to be tryed in Bodies not perfectly Hard or Elaſtic, the Reflexion ought only to be diminiſh'd in a certain Proportion according to the Quantity of the Elaſtic Force which is diminiſh'd. In the Theory of *Wren* and *Hugens*, Bodies abſolutely hard return from one another with the Relative Velocity of

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the Congreſs; but with the famous *Wallis* it is altogether to be ſaid, that this holds in Bodies per-perfectly Elaſtic only; and it is to be aſſerted, that other Laws have place in Bodies not Elaſtic, whether ſoft or hard, that obtain not in Elaſtic ones; as is abundantly manifeſt from what has been above opened. And particularly thoſe Bodies only which are perfectly Elaſtic, do after mutual Colliſions return from one another with the Velocity of the Congreſs, according to the 16th Law of Motion thereto belonging. In thoſe which are imperfectly Elaſtic, the Velocity of the Return is to be diminiſh'd together with the Elaſtic Force,and in the proportion of the diminution of the ſame: becauſe that that Elaſtic Force (unleſs where the Parts of the Bodies are hurt by the Meeting, or ſuffer ſome kind of Extenſion as under a Mallet) ſeems to be in it ſelf certain and determinate, and makes Bodies to return from one another with that Relative Velocity, which is in a given Proportion to the Relative Velocity before the Concourſe. Which thing Sir *Iſaac Newton* did thus experiment in Balls made of Wool, moſt ſtraitly wound up and preſſed together; firſt by letting down Pendulums, and meaſuring the Reflexion, he found the Quantity of the Elaſtic Force; then by this Force he computed the Reflexions which were to be expected in other Caſes of Congreſs, and the Experiments anſwer'd. The Balls always return'd from one another with a relative Velocity, which was to the Relative Velocity of the Concourſe, as the Number Five is to Nine. Balls of Steel were almoſt perfectly Elaſtic, for they return'd almoſt with the very Velocity of the Concourſe; thoſe of Cork with ſomething leſs; but in Glaſs Balls the Proportion was of about Fifteen to Sixteen.

And

And by this means, the Fifth Law of Motion as to Stroaks and Reflexions was prov'd by Dr. *Wallis*'s Theory, which plainly agrees with Experience. Sir *Isaac Newton* doth alſo ſhew in this Place briefly that this Rule holds alſo in Attractions; to wit, that the Quantity of Motion, collected by gathering the Sum of the Motions made to the ſame Part, and the difference of thoſe which are made to the contrary Parts, is not changed by the Actions of Bodies amongſt themſelves; whoſe reaſoning in this Matter we conſidered above under the Fifth Law, and conſequently ſhall at preſent omit it, and come to the reſt of his Obſervations belonging to the preſent Place. As therefore Bodies in Concourſe and Reflexion are of the ſame Force, whoſe Velocities are reciprocally as the implanted Force or the Bodies themſelves, as may be underſtood from the Eight and Seventeenth Laws, and *Hugens*'s Eighth Propoſition; ſo in moving Mechanic Inſtruments, Agents are of the ſame Force, and ſuſtain each other by contrary Endeavours, whoſe Velocities, eſtimated according to the determination of their Forces, be reciprocally as their Forces. Thus Weights are of equal Force to the moving the Beam of a Pair of Scales, which in the Vibration of the Balance are reciprocally as their Velocities up and down; that is, the Weights if they aſcend and deſcend perpendicularly, are of equal Force betwixt themſelves when they are reciprocally, as the Diſtances of the Points on which they are hanged from the Axis of the Balance. But if being hinder'd by oblique Planes, or ſome other Obſtacles, they aſcend and deſcend obliquely, thoſe Weights are Equipollent, which are as the perpendicular Aſcent and Deſcent, and this becauſe of the determination of Gravity downwards. Thus in a

Wheel or Pulley, the Force of the Hand which directly draws the Rope, is to the Weight ascending either directly or obliquely, as is the Velocity of the perpendicular Ascent to the Velocity of the Hand drawing the Rope, the Hand will sustain the Weight in Equilibrio. In Watches, and the like Instruments which are fram'd of little Wheels put together, if the contrary Forces for the furthering and hindring the Motion of the Wheels be reciprocally, as the Velocities of the Parts of the Wheels on which they are impress'd, they will sustain one another. The Force of a Screw to press a Body, is to the Force of the Hand turning round the Handle, as the circular Velocity of the Handle in that Part where it is press'd by the Hand to the progressive Velocity of the Screw towards the pressed Body. The Force with which the Wedge lies upon the Two Parts of the cloven Wood, is to the Force of the Mallet upon the Wedge, as the Progress of the Wedge according to the determination of the Force impress'd upon it by the Mallet, is to the Velocity wherewith the Parts of the Wood give Place to the Wedge according to Lines perpendicular to the Forces of the Wedge. And the Reason is the same in all Machines. Their Efficacy and Use consisting only in this, that by diminishing the Velocity we may increase the Force, and on the contrary. From whence in all Kinds of fit Instruments that so much talk'd-of Problem is solv'd, of moving a given Weight by any given force, or of overcoming any other given Resistance by any given Force how small soever. For if Machines be so form'd, that the Velocities of the Agent and Resistents are reciprocally as the Force, the Agent will sustain the Resistent, and overcome the same with a disparity of

Velocities:

Velocities: Certainly, if the disparity of Velocities be so great, that it will overcome even all Resistance, which is wont to arise as well from the Attrition of Bodies contiguous, sliding one upon another, as from the Cohesion of Bodies continuous, and which are to be separated one from another, and the Weights of Bodies to be lifted up; all that Resistance being overcome, the Force redounding will produce an Acceleration of Motion proportional to it self, partly in the Parts of the Machine, partly in the residing Body. But to treat of, and handle Mechanics as it ought to be done, belongs not to our Purpose; we by these Things only shew how far and wide this Rule extends, and how certain the Fifth Law of Motion above deliver'd is. For if the Action of the Agent be estimated from its Force and Velocity conjunctly, and the Re-action of the Resistent from the Velocities of each of its Parts, and their Force in resisting, arising from their Attrition, Cohesion, Weight and Acceleration; Action and Re-action will be in all Kinds of Instruments equal one to the other; and so far forth as the Action is propagated by the Instrument, and at length impress'd upon every resisting Body, its last Determination will always be contrary to the determination of the Re-action.

Corollary. From these two Laws of Motion now sufficiently explain'd and prov'd, the gross Errors of *Des Cartes* about the same do manifestly appear; whose Laws of Motions, are so far from agreeing every where with the true Laws of the same, that they are rather found every where to disagree with them. And consequently it is no wonder, if he in like manner err'd in the rest of the Phenomena of Nature. The Laws of Motion being now dispatch'd, we come unto the Propositions.

November 6th, 1704. LECT.

Lect. IX.

Propositions.

I. THE last Proportion of the Tangent Subtense or Chord to the Curve Arch which belongeth to them, is the proportion of Equality; that is, the Tangent, Arch, and Chord, where the least Arch of all, or that which vanisheth away is taken, do at length end in one and the same Line. And the same thing is to be understood of the Sine. Let A b be the Arch of a Circle or some other Curve, which is as little as may be; let A f be the Tangent thereof, and A b the Subtense: I would know what is the Proportion of these Lines one to another if they be taken as near as may be to the Point A, or if the Point b doth as it were coincide with the Point A; and I say, that the Proportion of the Arch, whether to the Tangent above, or to the Subtense below, is the Proportion of Equality. For from the Nature of Curves it is manifest, that all the difference betwixt the Tangent and Subtense of any Arch whatever doth arise from the length of the intermediate Arch, and that the difference is always so much the greater, as the Arch which is taken is the greater, and so much the less by how much the Arch which is taken is the less; from whence it follows, that in an Arch the least that may be the difference will be the least that may be, and in an Arch infinitely small, such as we now intend, the Difference will be infinitely small, or none at all. And if the Difference betwixt the Tangent and the Subtense be none at all, much more shall the Difference betwixt the Tangent and the intermediate Arch be none at all, or the Difference betwixt the Subtense and that Arch; since that Arch is every where of an intermediate Length betwixt the Tangent and the Subtense. And this Equality of the least Tangents, Arches and Subtenses, and of Sines also, is a Thing which Geometricians have always supposed and acknowledged; whilst they have considered the Perimeters of Curves, as being innumerable Sides of Polygons, and as arising from the Coalescence of the inscrib'd and circumscrib'd Figures, the Difference betwixt them vanishing away.

Corollary, If therefore it can be demonstrated, that d b D b (*Fig.* 5. *Plate* 3.) the Subtenses of the Angles of Contact, are always betwixt themselves in the duplicate Proportion of the Subtenses A b, A B, as will presently be done, it will from thence follow, that the same vanishing Subtenses are also in the duplicate Proportion of their conterminous Arches A b, A B, or of the Sines c b, C B; because the Subtense A b doth in the foregoing Case fall in, and become the same with the Arch A b or its Sine c b, and so doth at length the subtense A B with the Arch A B, or its Sine C B.

II. The Subtenses of the Angles of Contact in Circles, are always in the duplicate Proportion of the Subtenses of the conterminous Arches.

Let there be in the same Figure any two Arches as A B and A b; and D B, d b (equal to A C and A c, the versed Sines of the same Arches) the Subtenses of the Angle of Contact. To these Subtenses (by Book the 3d, Prop. 31. of the Elements)

Elements) G B and G b drawn from the Point G will be perpendicular; let the Rectangles ADBC, and A d b c be compleated. This done, the Square of A B (by VI. 8. of the Elements with VI. 17. of the same) is equal to the Rectangle of A G into A C or D B; and in like manner, the Square of A b is equal to the Rectangle under A G and A c or d b. And consequently, the Proportion of the Square of A B to the Square of A b, is the same as that of the Rectangle under A G and D B, so that under A G and d b; that is, (by VI. 1. Elements) the same as of the Line D B to the Line d b. Q. E. D.

Corollary. Therefore any Subtense whatever of the Angle of Contact, as D B or d b is equal to the Square of the Chord applied to the Diameter of the Circle. For as A G is to A B, to is A B to A C or D B: From whence by the Golden Rule, $BD = \frac{AB \times AB}{AG}$ or $= \frac{ABq.}{AG}$ And in like manner, A G is to A b as A b to A c or d b; from whence $d\,b = \frac{A\,b\,q}{AG.}$ Q. E. D.

Coroll. (2.) In the least Segments of perspective Glasses, the Altitudes or Axes of the Segments A C, and A c, are to be reckon'd to have the same Proportion betwixt themselves, which the Squares of the Bases or Apertures E b and R B &c. have. (see *Fig.* 5. *Plate* 3.) For we have shew'd that A C and A c have the same Proportion as the Squares of the Subtenses; and seeing in very small Arches the Subtenses or Sines, or their Doubles R B and E B are almost in the same Proportion amongst themselves; it follows, that the Altitudes also A C and A c have almost the same Proportion as the Squares of the double Sines R B and E b, that is, of the Apertures. Q. E. D.

Coroll. (3.) In very ſmall Angles, the Exceſs of the Secants above the Radius, are alſo very near as the Squares of the Subtenſes, or Sines, or Tangents, or even of the Arches. For thoſe Exceſſes (ſee *Fig.* 5. *Plate* 3.) b f and BF, do in that Caſe coincide with the Subtenſes of the Angle of Contact b d and B D, and conſequently have the ſame proportion amongſt themſelves as they. Thus you may ſee in the Tables of Secants, that the Radius of the Circle being put to be of 10000000 of equal Parts, the Exceſs of the Secant of two firſt Minutes is of two Parts, and that of the Secant of Four firſt Minutes is of Eight Parts; from whence the difference of the former Secant, and the Radius, is fourfold of the difference of the latter Secant to the double Arch, and the Radius; that is, thoſe Differences are betwixt themſelves as the Squares of the Arches; and ſo of the reſt.

Coroll. (4.) The Naſcent or Evaneſcent Subtenſes of the Angle of Contact, are in the duplicate proportion of the conterminous Arches. For they are every where, by what hath been demonſtrated before, in the duplicate Proportion of the Chords. But ſeeing the Chords do at laſt end in the Arches, that is, in Diſtances infinitely ſmall do coincide with them, and are equal to them, as we demonſtrated above, thoſe Subtenſes in like manner will in the preſent Caſe be in the duplicate Proportion of the Arches.

Coroll. (5.) From whence alſo in the ſame Caſe, according to the firſt Corollary of this Propoſition, the vaniſhing Subtenſe of the Angle of Contact, will be equal to the Square of the Arch it ſelf, applied to the Diameter of the Circle.

Coroll. (6.) Hence is gathered that Noble and Fundamental Theorem of Sir *Iſaac Newton*, and of of Mr. *Hugens* alſo; to wit, that in the circular Motion of a Body, the Centripetal Forces, or the Gravities toward the Center are every where, as the Squares of the Velocities of the Arches, deſcrib'd at the ſame time, applied to the Diameters or Radii of the Circles. For (in *Fig.* 6. *Plate* 3.) let the revolving Bodies B and b deſcribe the Circumference of the Circles B D and b d, and in the ſame given Time let them deſcribe the infinitely ſmall Arches B D and b d; becauſe by their Innate Force alone they would deſcribe the Tangents B C, b c equal to theſe Arches; by the firſt Law of Motion it is manifeſt, that there is ſome centripetal Force which perpetually draws back the Bodies from the Tangents to the Circumferences of the Circles; and conſequently, theſe are one to another in the firſt Proportion of the Naſcent Lines C D and c d; that is, as $\frac{BDq}{BG}$ is to $\frac{bdq}{bg}$; or, by taking half the Diviſors, as $\frac{BDq}{BS}$ to $\frac{bdq}{bs}$ and becauſe the Velocities are in the Proportion of the Arches deſcrib'd at the ſame time, thoſe Forces will be as the Squares of thoſe Velocities applyed to the Radii of the Circles. But if the Circles be equal betwixt themſelves, then by reaſon of the given Diameters thoſe Forces will be as the Squares themſelves of the Arches deſcrib'd at the ſame time, or of the Velocities; as we will ſhew more fully afterwards.

Coroll. (7.) By means of the foregoing Corollary, we gather the Proportion of the Centripetal Force to any known Force, as that of Gravity. For ſince that Force in the time that the Body deſcribes the Line B C, or an Arch equal to it,

 impells

impells the ſame along the Line C D; which in the beginning of Motion is equal to the Square of that Arch B D, applied to the Diameter of the Circle. And ſince every Body by an uniformly accelerated Motion, or the ſame Force continued towards the ſame Part always, doth deſcribe Spaces in the duplicate Proportion of the Times, as will preſently be ſhewed (under *Prop.* 4.) that Force in which time the revolving Body doth deſcribe any given Arch, will make the ſame Body, going right forwards, deſcribe a Space equal to the Square of that Arch applied to the Diameter; and conſequently, is to the Force of Gravity as that Space is to the Space which an heavy Body in falling doth deſcribe in the ſame time. As for Example, from the Experiments of Pendulums, and other ways it is manifeſt, that all Bodies whatever deſcribe in a Vacuum 16,14 *Engliſh* Feet in one Second of Time whilſt they are falling by the Force of Gravity: I therefore would know what Proportion the Centripetal Force, whereby the Moon is held in its Orbit bears to the Force of Gravity with us? For this purpoſe the Square of the Arch of the Moon's Orbit, which is deſcribed in one Second of Time, is to be divided by the Diameter of the Orbit; and ſo we ſhall find the Line which the Moon (if the circular Motion thereof were deſtroyed, and it deſcended as an heavy Body towards the Earth) would deſcribe in the ſame time. The mean Diſtance of the Moon from the Center of the Earth is about Sixty-Times the Earth's ſemi-Diameter, or of about *Engliſh* Feet in Number 1257696000. The Circumference therefore of its Orbit, if reduced to a Circular, will be of about 7897834380 Feet; which Periphery, ſince the Moon deſcribes it in the Space of a periodical Month, or 27 Days, 7 Hours, 43 Minutes, that is, in 2360580 Seconds;

let

let the Circumference 789783438o be divided by 236058o the Seconds which belong to the same, and the Quotient 3346 will give the Length of the Arch described by the Moon in a second of Time in *English* Feet, the Square whereof 11128976 being divided by the Diameter 2515392000 will give .00443 Parts of an *English* Foot to be described in one Second by the Moon falling, and in one Minute 16.14 Feet, or thereabouts; therefore the Centripetal Force, or Gravity of the Moon, is to the Centripetal Force of Bodies with us upon the Surface of the Earth, as 443 Hundred Thousandth Parts of one Foot is to 16.14 Feet, that is almost as 1 to 3600. And consequently, the Force of Gravitation towards the Earth at the distance of the Moon is only the 3600th Part of the Force of Gravity with us.

III. The Velocities of a Body accelerated by any uniform urging Force whatever, are betwixt themselves, as the Times are wherein that uniform Force is impress'd; that is, in Double the Time Double, in Treble the Time Treble of it self, and in Four times the Time a Quadruple Velocity. For if the Accelerating Force be Equable and Uniform, which is here suppos'd; and the Body consequently, whether it rested at the First, or was mov'd with any Celerity, doth receive equal Degrees of Velocity, and an equal Increase in equal Time; it is manifest, that the Velocity of the Body is exactly proportional to the Time. For if in every given Particle of Time that Force doth generate a certain Velocity, it will be able in the next equal small Portion of Time to generate a Velocity like and equal to the former; and so likewise in the Third, and Fourth, and Fifth, *&c.* Particle of Time, and so *in infinitum.* From whence the entire Velocity will every where be as the Space of Time, in which that Accelerating Force is impress'd on the Body. Q. E. D.

Corollary. Seeing therefore it is manifest by Experiments, that all Bodies whatever being accelerated by the Force of Gravity, do receive increases of Velocity every where proportional to the Times; it is manifest, that the Force of Gravity doth act uniformly, and doth affect Bodies most swiftly descending, as well as quiescent. From whence it appears, that the Gravity of Bodies is to be ascrib'd to no Pressure of the Air, or Impulse of either, nor to the Mechanical Endeavour of any Mater towards Motion. For all these Impulses or Endeavours would most of all affect a Body at Rest, and by how much the more swiftly the Body should be mov'd, they would so much the less continually urge it, until at length, the Celerity produc'd becoming equal to the Force of the Cause which generates it, all the Impulse would cease, and no Acceleration of Motion follow thence forward.

Some Lemmata *to the 4th* Prop.

(1.) Odd Numbers being added to themselves continually do make all the Square Numbers. Thus One is the first of odd Numbers, and the first also of Square Numbers. But if the Number 3, which is the Second odd Number be added to One, there is made the Number Four, the Second of Square Numbers; and if the Number Five be added to Four there is made the Number Nine, the Third Square Number, and so on *in infinitum.* We shall bring Two Demonstrations of this *Lemma*, one out of *Tacquet*, the other out of our own Store. *Tacquet* shews the Thing thus. In the natural

Pract. Arithm. Book V. Chap. 1. Theor. 7.

tural Progression, saith he of odd Numbers, 1, 3, 5, 7, *&c.* the total Sum is equal to the Square of the Number of the Terms. For according to the Nature of Arithmetical Progression, the Sum of all the Terms is equal to the Product of Half the Sum of the Extremes, drawn into the Number of the Terms: But half the Sum of the Extremes of an Arithmetical Progression of odd Numbers, beginning with Unity, is equal to the Number of the Terms, (for it goeth on from Unity by Two's superadded, whilst the Number of the Terms increaseth only by Ones) and consequently, that Product is equal to the Square of the Number of the Terms, and therefore the Sum Total of odd Numbers, beginning with Unity, is equal to the Square of the Number of the Terms, Q. E. D. We demonstrate it thus: Let (a c) or (a b) (see *Fig.* 7. *Plate* 3.) be Unity, and (a d) the Square of Unity; I say, that the Addition of the odd Numbers, 3, 5, 7, 9, *&c.* is necessary to make the Squares (a h, a n, a s, a z, a A) of all Numbers proceeding from Unity; for to the making the Second Square, or the Square of the Number Two, there are Three other Squares of Unity to be added to (a d) to wit, the Two lateral Squares (c d and d f) and the Diagonal Vertical Square (i g.) And then through all the rest of the Terms, the Number of the Squares to be added is always to be increas'd by Two for the making up of the rest of the Squares; to wit, three Squares (k i, l h, and g g) corresponding to the Three which were added before are first to be added, then another Square (h p) because that the Square, added by the Side of the Diagonal, doth always require a Pair of corresponding Squares to be super-added, to which at length is to be added another Diagonal Square (m o). And thus it is every where (the

Number

Number of the Squares to be added always, exceeding the former by Two,) that the Squares (a d, a h, a n, a s, *&c.*) from Unity may be compleated. From whence it plainly appears, that the continual Addition of uneven Numbers begets all Square Numbers, Q. E. D. But he that will be content with an Induction, carried on without End, may safely enough pass By this way of demonstrating the Thing; howbeit it is indeed so easy, that it will not require much Attention of Mind to comprehend it.

Lemma (2.) If a Body doth in a given time depart gradually and uniformly from Rest, and by that means describe a certain Line; the same Body in the same given time will, from the last Celerity acquired, if it be uniformly continued, describe a Line double to the former. For since the Body in departing from Rest acquired a certain Degree of Velocity by equal increases, the Line describ'd by the same, will be to be distinguish'd into innumerable Lines greater each to the former gradually: And if those little Lines gradually increasing, were dispos'd not Length-wise but orderly at the Sides, they would compose a certain Triangle, (a b c) or at least, according to *Cavallier*'s Method of Indivisibles, are to be reckon'd to compose one; where the Vertical Point of the Triangle, to wit (a) is the Point of Rest, and the Base (c b) designs the last (see *Fig.* 8. *Plate* 3.) Line of Motion, and the rest of the Parallel little Lines, the Lines of the diverse Velocity which the Body had passed through. Now if we had put the greatest Line (1 b) to have been measured in the same full time, or had dispos'd from the Point (a) to the Base (1 b) so many Times equal to the greatest, as we had before dispos'd Lines gradually increasing, we had compos'd pos'd a Parallelogram, double to the former Triangle, (I. 41. *of the Elements.*) And consequently, the uniform Motion, gradually increas'd from the Point of Rest, is in the given time double to the former, Q. E. D.

IV. The Lines which Bodies by any urging uniform Force do describe, are in the duplicate proportion of the Times, *i. e.* if the Times be Seconds, One, Two, Three, Four, Five, *&c.* the whole Lines describ'd will be amongst themselves, as One, Four, Nine, Sixteen, Twenty-five, *&c.* which are the Squares of the former. For if any Body whatever, by any urging uniform Force whatever, shall as it falls describe in the least Portion of Time, as one Second, some Line; in the second equal Portion of Time it will describe a Line equal to the former, by reason of the continuation of a Force equal to the former; and because of the gradual Acquisition, and Increase of the Velocity of Motion, it shall by *Lemma* the Second describe a Line double to the former; therefore from both Causes conjoin'd it will now describe a Line treble to the former. But in the Third Particle of Time, by reason of the Force of Gravity still acting, a Line equal to the former will be describ'd; and because of the Velocity of the former, which was double to B, continued in the equal Time a Line will be describ'd double to the former, that is, quadruple of the first, and so from the Forces conjoin'd a Line will be describ'd five-fold of the First; and so forwards from the continual Impression of Gravity a Line equal to the first will always be to be added, and then another Line equal to the first by reason that the Velocity is continually increas'd by one Part; and consequently, two Parts or Lines equal to the first are every time to be added; and conseqently, the

the entire Lines describ'd in every successive Particle of Time will be to be design'd perpetually by odd Numbers. Seeing therefore (by *Lemma* 1.) the odd Numbers added one to another do orderly make all the Square Numbers, the Lines of these Moments added together will necessarily make the entire Lines of the Moments to exceed the latter the foregoing in a duplicate Proportion, or in the Proportion of a Square Number to a Square Number. Thus if in one Second of Time Bodies be carried downwards by the Force of Gravity about Sixteen *English* Feet, as is manifest from Experience; they will in two Seconds be carried Sixty-four Feet, and in three one Hundred Forty-four Feet, or thereabouts.

Or according to *Galilæus* in his *Systema Cosmicum*, we may demonstrate the Proposition thus. (*Fig.* 8. *Plate* 3.) Let equal Times be represented by equal Lines a b, b c, c d, d e, and the Velocity in the end of the first time b g, by the Line (b 1). Seeing then, that Velocity which the falling Body hath in that Place, was acquir'd not together and at once, but gradually, and in a certain Space of time, represented by the whole Line(a b) from the continual and uniform accelerating Force; therefore it is necessary, that it should have had all the lesser Degrees of Velocity before it got that Velocity; from whence those former Degrees of Velocity will be represented by lesser Lines drawn from the Parts of the Time a b parallel to the Line; and seeing the Velocity doth increase uniformly with the Times, those Lines according to the Method of indivisibles will constitute and compose the Triangle (a b 1). Therefore the whole Line which shall be described from all those Velocities join'd together,

Prop. 3. *foregoing.*

ther, will be proportional to the Aggregate of all those Lines, that is, to the Triangle (a b 1); and will rightly be represented by that Triangle. But in the second time, when the Body shall now have acquir'd a Velocity proportional to the Line (b 1) and represented by the same; with that only Velocity continued, it will describe a Line double to the former, and consequently to be represented by the Parallelogram (a b 1 h) or (b 1 k c) double to the Triangle (a b 1); and over and above, by the new Velocity, arising as before, from the perpetual and uniform Incitation of the same Force, a Line will be described equal to the first Line; therefore if you add both Forces together, in the second Time, the Line describ'd will be treble to the former, and to be represented by the Trapezium; and the Sum of the Lines describ'd in the first and second Time, will be to the Line described in the first time alone as the Triangle (a c 2) is to the Triangle (a b 1); that is, in the duplicate Proportion of the homologous Sides (a c) and (a b) which represent the Times; or as the Squares of the Times themselves. In like manner, in the third Time the Body with the Celerity hitherto acquir'd, or the mere permanence of the Motion now got, will describe a Line to be represented by the Parallelogram (c 2 n d); and by the new added Force arising from Gravity still, and continually uniformly inciting, will describe a Line to be represented by the Triangle (2 n 3.) From whence the Line describ'd in the third Time will be fivefold of the First, and to be represented by the Trapezium (2 c d 3); and the Sum of the Lines in the first, second, and third Times, will be to the Line described in the first time only, as the Triangle (a d 3) is to the Triangle (a b 1) or as the

the Squares of the Times (a d); and (a b) and so *in infinitum*.

Corollary, Since according to what hath been before demonstrated, Celerity is every where proportional to the time, and seeing the Lines described by Bodies falling down be in the duplicate Proportion of the Times, the same Lines will also be in the duplicate Proportion of the Celerities, or as the Squares of the Velocities. As for Example, if the Velocities last acquir'd of two Bodies falling to the Earth, be one to the other as the Number Two is to One, the Heights of the Fall shall be betwixt themselves as Four is to One. If the Velocity of one Body be treble to the Velocity of the other, the Height of the Descent of the same shall be Ninefold of the Height of the Descent of the other. And so on *ad infinitum*.

Nov. 13, 1704.

Lect. X.

V. If a Body shall begin to tend upwards with that same Velocity which it had acquir'd in the End of its Descent, it will ascend to the same Altitude in the same Time, from whence it before descended, and shall equally lose its Velocity in equal Times.

For by Force of what was demonstrated in the last Proposition it appears, that the Velocity once acquir'd, as (3 d) will always describe

Plate III

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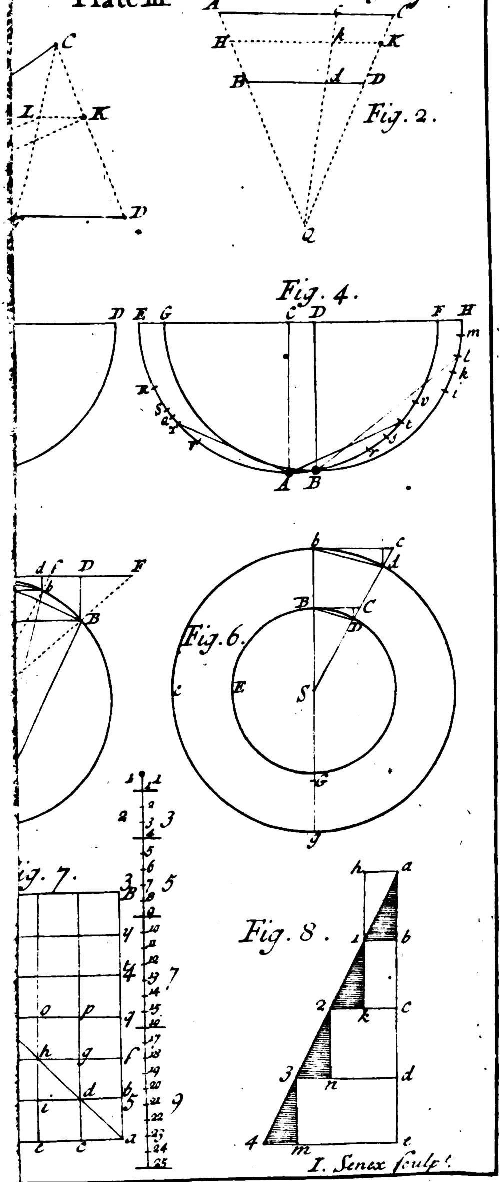


scribe an equal Parallelogram, whether the Body ascend or descend: But seeing the new Force of Gravity in the Descent increases by the Triangle (3 m 4 3) and in the Ascent diminisheth the same by an equal Triangle; it is manifest that the Trapezium now to be described in the Ascent will be equal to the Parallelogram before describ'd in the Descent, to wit the Trapezium 3 2 c d; and so on. From whence the Lines describ'd which are proportional to these Trapeziums, and the Velocities proportional to the Bases of the Trapeziums, will every where be the same in the Ascent that they were in the Descent; until at length the Body reacheth to the last Point of the Ascent in the same time that it had descended from it.

VI. The Celerities of heavy Bodies acquir'd by descending upon divers Inclinations of Planes will be equal, if the Elevations of the Planes or their perpendicular Altitudes be equal.

Let E G be a Line perpendicular to the Horizon, and E F a Line inclin'd to the Horizon (*Fig.* 1. *Plate* 4.) in any Angle whatever, and let G A be perpendicular to E F. I say, that any heavy Body whatever will acquire the same Velocity in descending along the inclined Line EF, which it would acquire in the Line EG by the perpendicular Fall. For from what was demonstrated before, the Force of Gravity in the oblique Plane E F, is to the Force of Gravity in the perpendicular E G, as A B is to A C, or, on account of the likeness of the Triangles A C B, E F G, as E G to E F; or because the Triangle E G A is like to them, as E A is to E G. From whence, by reason of the divers Forces, the Motion and Velocity of the Body descending along

Corol. 2. *Law of Mot.* 23.

E A

E A in the inclined Plane, will be to the Motion and Velocity of the Body deſcending along E G, the Time of the perpendicular Deſcent being given, as E A is to E G, or as E G is to E F; and the Velocity of the Body deſcending along E A, unto the Velocity of the ſame deſcending along E F, will be in the ſubduplicate Proportion of E A to E F, that is, in the Proportion of E A unto E G. Therefore the Velocity of the Body in the Point A, is to the Velocity of the Body in the Point F, and to the Velocity of the ſame deſcending perpendicularly in the Point G, in the ſame Proportion, to wit, that of the Line E A to the Line E G, or that of the Line E G to the Line E F. From whence it appears, that thoſe Velocities are equal one to the other. Q. E. D.

VI. 8. *Elements.*

Coroll. (1.) Whilſt a Body falling perpendicularly deſcribes the Line E G, another falling obliquely, will deſcribe the Line E A, determined by the Perpendicular G A.

Coroll. (2.) The Time of the perpendicular Fall is to the time of the oblique Fall, in the ſubduplicate Proportion of the Line E A to the Line E F; or as the Line E A is to the Line E G, that is, in the proportion of the perpendicular Altitude E G to the oblique Line E F. From whence, by how much the Velocity is diminiſh'd on account of the diminution of the Force, it is increas'd, by reaſon of the increaſe of the time; ſo that in the ſame perpendicular Altitude there always remain the ſame Velocity, whatſoever may be the obliquity of the inclined Plane to the Horizon.

Coroll. (3.) The Times of Deſcents upon Planes diverſly inclin'd to the Horizon, but whoſe Elevation or perpendicular Altitude is the ſame,

ſame; and betwixt themſelves as the Lengths of the Planes. For the time of Deſcent by E F is to the time of Deſcent by E G, (*Fig.* 1. *Plate* 4.) according to what hath been already demonſtrated, as E F is to E G; and the time of Deſcent thro' E G is to the time of Deſcent thro' E H as E G to E H; from whence by equality of Proportion, the time of Deſcent through E F will be to the Time of Deſcent thro' E G, as E F is to E H, Q. E. D.

Coroll. (4.) If a Body deſcend from the ſame perpendicular Altitude, with a continued Motion through how many ſoever and whatſoever contiguous Planes, as E I, I K, K L, howſoever inclin'd, it will always acquire the ſame Velocity in the End; that, to wit, which it would have acquired by falling perpendicularly from the ſame Heighth. For by the Dermination of Mr. *Hugens*, the ſame will be the Velocity, according to what hath been already demonſtrated, of the Body (*Fig.* 1. *Plate* 4.) falling to the Point I, whether it fall along E I or M I; from whence the Velocity will alſo be the ſame in its going along I K, as in falling along N K; from whence the Velocity will be the ſame at the Point K, whether the Fall were through E I or I K or along M K, or even N K; from whence will follow that there will be the ſame Velocity in falling along K L at the Point L, which would be the Deſcent were according to one ſingle Plane N L, or two Planes M K and K L, or even three Planes, E I, I K, K L; the ſame to wit, according to what hath been already demonſtrated, which the Body falling perpendicularly could acquire at the Point G. Q. E. D.

Coroll. (5.) Hence it is alſo manifeſt, according to the Determination of the ſame *Hugens*, that a Body deſcending along the Circumference of a Circle, or a Cycloid, or any Curve Line whatever, the ſame Velocity will always be acquir'd, if it deſcend from an equal Altitude; and that that Velocity will be ſo great as the Body ought to acquire by a perpendicular Fall from the ſame Altitude. For Curve Lines are as it were, compounded of many innumerable right ones; and ſince the Propoſition is true in any rectilinear Perimeters whatever, and how many ſoever, it will be true likewiſe where they are in Number infinite, that is, where they end in Curve Lines. Q. E. D.

Coroll. (6.) Hence it alſo appears, that if an heavy Body's Motion be turned from a Deſcent upwards, it will aſcend unto the ſame Heighth from whence it came, along whatſoever Plane contiguous Surfaces, and in what ſort ſoever inclin'd it ſhall be carried. For as before, (*Prop.* 5. *Fig.* 1, Plate 4.) the ſame will be the Velocity in any Point K and I, whether the heavy Body deſcend or whether it aſcend; from whence certainly the ſame will be the Limit of the aſcending or deſcending Velocity, the ſame the Term of it at the Point E. From whence alſo, if there be an infinite Multitude of Planes; that is, if the Surface be Curve, the Body will ariſe along the Curve Line alſo unto the Height from whence it came, and no higher.

Coroll. (7.) If a Body falls perpendicularly, or deſcends along any Surface whatever; and then ſhall, from the Force acquir'd by the Deſcent be carried upwards along any other Surface; it will have in Aſcending and Deſcending the ſame Velocity in Points of equal Heighth. And if the Plane or Surface of the Aſcent be like and equal to the Surface of the Deſcent, it will aſcend in equal time in which it deſcended. Theſe Things do ſo clearly follow from the Things already demonſtrated, that there needs no more Words about them.

Lemma's

Lemmata to the 7th Proposition.

(1.) If a Curve Line be of that Nature that it will every where sustain the Force of Gravity in proportion to its length; so that by how much the Part of the Line to be describ'd is the greater, by so much will the accelerating Force be the greater, and altogether in the same Proportion; and by how much the part of the Line which is to be describ'd is the less, by so much the less will the accelerating Forces be, and this in the same Proportion; the Times of Descent along such a Curve, whether the Arches describ'd be greater or less, will always be equal to one another. For the Velocity in the given Time is as the moving Force; if therefore the Line to be describ'd be also as the same moving Force, it will be likewise as the Velocity; but if the Velocity of the Motion be every where as the Line to be described, it is manifest, that any Line whatever, whether great or small, ought to be describ'd in the same time. But that this is the Nature and Property of the Cycloid is what comes to be demonstrated in what follows.

Lemma (2.) Let D A C (*Fig.* 2. Plate 4.) be a Semi-cycloid, D F A half of that Circle which produced the Cycloid; and from any Point in the Cycloid as B, let there be drawn the Line B E parallel to the Base D C, meeting the Semi-Circle D F A in E; then let the Chord A E be drawn, and from the Point B in the Cycloid the Line B L, parallel to the Chord A E; the Line B L will be a Tangent to the Cycloid in the Point B.

Lemma (3.) And the Arch of the Cycloid A B will be equal to the double Chord A E. These two Lemmata are manifest from the Elements of the Cycloid; and we have them demonstrated by the famous Sir *Christopher Wren* and others.

See Wallis's *Works* Vol. I. P. 533, *&c.*

VII. In a Cycloid inverted, whose Axis is erected perpendicular, the Times of the Descent wherein a Body let down from any Point whatever in it, comes to the lowest Point A, always equal betwixt themselves.

Let B A and O A be any Arches whatever in the Cycloid, and B L and O N the Tangents in the Points B and O; and let E A and F A be Chords of the Semicircle D F A or parallel to the said Tangents, by *Lemma* the second: let A F be produced to the Point K. There are therefore by the third *Lemma*, Lines to be described by the Body placed in one Case at B, in the other at O, which are betwixt themselves as the Chord E A is to the Chord F A; but the Force in the direction of the Tangent B L, or the Chord F A parallel thereto, is in the same Proportion to the Force in the direction of the Tangent O N, or A F which is parallel thereto. For (*Fig.* 2. *Plate* 4.) as the Square of E A is to the Square of F A, so is the versed Sine E P to the versed Sine F P; or so is K M to F P; or so is K A to F A. Therefore the Chord A E is a Geometrical Mean proportional betwixt the Chord A F and the Line A K; and consequently A F, A E, A K ∺. But the Force of Gravity, according to what hath been demonstrated, which is in the Plane A E, is to the Force of Gravity in the Plane A F, as A K is to A E; that is, as the Chord A E is to the Chord A F; and thus every where. But the Line to be described is as the same A E is to the same A F; and consequently the accelerating

Corol. 2. *to Law* 23.

lerating Force is every where in the same Proportion as the Line which is to be described, and therefore the Times of the Descent are every where equal. Q. E. D.

Coroll. (1.) If therefore we form other entire Semi-Cycloids Q R T, Q S C like and equal to the former A T and A C, the Vertices whereof touch the Base of the other at the Points T and C; and the heavy Body V hangs from the Center Q upon the Thread Q R V, which is equal to Q D A, or the double of D A; and be mov'd betwixt those Semi-Cycloids Q R T, Q S C, the pendulous heavy Body will, from the Evolution of the Thread describe the entire primary Cycloid, as is manifest from the Properties of the Cycloid, and will perform the Vibrations of what Amplitude soever, even to the greatest of them T A C, in the same times exactly, and so that the Center of the Oscillation shall always be in the Curve Line it self T A C.

Coroll. (2.) Seeing all Vibrations whatever in a Cycloid are always in equal Times, and seeing the least Vibrations in the least Arch of the Circle, the Radius whereof is Q A, and in the least Arch of the Cycloid T A C, by reason of the manifest Coincidence in this Case of the Arch of the Circle and of the Cycloid in the lowest Point are the same; it is manifest that the time of every Vibration in the Cycloid is equal to the time of the least Vibration in a Circle, the Radius whereof is double to the Diameter of the Circle which produced the Cycloid.

Coroll. (3.) By reason also of the same Coincidence of the least Arches of the Circle and Cycloid in the lowest Point, the Vibrations in the Circle will be so much the more in equal Times, by how much the describ'd Arches are the less;

so that in very small Arches they may very well be reckon'd to be in equal times.

Coroll.(4.)Therefore in Pendulum Clocks which have longer Strings or Wires for the pendulous Bodies to swing by; the Times of the Vibrations of the lesser Arches which are described, come nearer to an Equality, than they do where the Strings or Wires are shorter; and consequently, the former Clocks are to be preferr'd far before the latter.

Coroll. (5.) The Times of Vibrations in divers Cycloids are in the subduplicate Proportion of the Cycloids or Radii Q A; or the Lengths of the Pendulums are in the duplicate Proportion of the Times; this will easily appear from what was demonstrated (*Prop.* 4.) before, as applied to the present Case. But it is to be noted, that the same Thing is also to be understood of the Times of Vibrations in Circles as well as in Cycloids. Thus, because a Pendulum of 39L25 Inches perform any Vibrations whatever in a Cycloid, and also the least Vibrations in a Circle in the time of one Second, a Pendulum of 157 Inches would make the like Vibrations in the time of two Seconds, and one of 353L25 Inches in the Space of 3 Seconds.

Coroll. (6.) Since the Times of all Vibrations whatever are equal in a Cycloid only; and upon this Account only are to be reckon'd equal in the least circular Arches; to wit, because the Arches of the Cycloid and Circle coincide no where else but in the lowest Points, they being in every other Place sufficiently different from one another: It is manifest, that the Vibrations in Arches of a Circle are so much the less Isochronal, by how much the greater they are; and consequently, that in larger Arches they are far enough remov'd from Isochronism. And according to *Hugens*, the time

Horolog. Oscillat. P. 9.

time of Descent in a Quadrant of a Circle, is to that which is in the least Arch, as 34 to 29, supposing that the Vibration is made in a Vacuum. Which therefore will arise to a very sensible Difference when we compare the greatest Vibrations and the least together.

Coroll. (7.) Because it appears from Experiments, and the Computation made thereupon, that each single Vibration to and fro, where the Pendulum is 96⌊85 Inches long (each one I mean in a Cycloid, and the least in a Circle) is perform'd in 94⌊25 Thirds of Time, or 1″. 34‴⌊25. And because, by what *Hugens* hath demonstrated, the Time of Vibration is to the time of the perpendicular Fall along the Quadruple of the Diameter of the generating Circle; or along double the Length of the Pendulum = 193⌊76 Inches, or 16⌊1 *English* Feet, as the Circumference of the Circle is to the Diameter doubled; or as 94⌊25 Thirds of Time to 60 or one Second, [for 355 : 226 : 94‴⌊25 : 60‴ = 1″ ;] thence it follows, that an heavy Body will descend by the Force of its Weight 16⌊1 *English*, or 15 $\frac{1}{12}$ Feet of *Paris*, in the space of one Second. Which Velocity of Descent, deduced from the pendulary Experiments, agrees notably with the said Author's Experiments about falling Bodies; and therefore is to be accounted for certainly true.

Horol. Osc. *P.* 57, 58. & de vi Centrifug. *Prop.* 12.

Coroll. (8.) Therefore the perpendicular Line of a falling Body being given for any space of time, the Line of Descent, whether Perpendicular or Oblique, is given for any other Space of Time; as being always in the duplicate Proportion of the Time. Thus in a direct Fall, as ten Seconds Square = 100 is to the Square of one Second = 1; so is 1610 *English* Feet describ'd in 10″, to 16⌊1 Feet describ'd in one Second. And in oblique Descents

scents it is not much otherwise. For the Lines of Descent in an oblique Plane are by the same Reason one to another as the Squares of the Times; all the Difference is, that the Force of Gravity which is the Cause of the Descent, is to be diminish'd in this Case in the Proportion of the Perpendicular Line to the oblique. (See *Fig.* 1. *Plate* 4. *Coroll.* 1. *Pr.* 6.) For since the oblique heavy Body, as we shew'd before, descends in the same time through the Line E A, that the Perpendicular doth through the Line E G; it is manifest, that the moving Force is every where in the same Proportion. Therefore if we put the Case, that the heavy Body descends along a Plain so very oblique, that E G is only a third Part of E F; what we have to do is only to diminish the Force of the Gravity in the same Proportion, so that the Body be suppos'd to descend along a Line of 5L37 Feet only in the space of one Second, and the Calculation will be the same as in the direct Fall.

November 27, 1704.

Lect. XI.

VIII. ALL Projectiles, not perpendicular to the Horizon, describe Parabola's, so far as they are not hindred by the resistance of the Air.

(*Fig.* 3. *Plate* 4.) Let any Body be supposed at T, and let it in any given time tend by the Force of the Horizontal Projection according to the Tangent T E, from T to a, so far as it is not hinder'd by some other Force: Then let the Force

Force of Gravity surpervene, which acts in the Line T K perpendicular to the Horizon, or any of the Lines parallel to it, (a l, b m, c n, d o, e p) for by reason of the great distance of the Center of the Earth, those Lines are to be accounted for Parallel. Since therefore, the Force of the Projection begets an uniform equable Motion, according to the Direction T e or F l, G m, H n, L o, K p, which are parallel to the same T E, and the Velocity of this Motion, according to the primary Direction, suffers nothing from the Force of Gravity that supervenes; the Body will in the end of the first time be found somewhere in the Line (a l); of the second Time somewhere in the Line b m, of the third somewhere in c n, of the fourth somewhere in d o, as being Lines parallel and equi-distant. Then let us consider the Force of Gravity as supervening, from which alone in the mean while, that the Body would by the projectile Force alone describe the Line T a, it is carried downwards according to any little Line T F or a l, so that if there were no other moving Force but this of Gravity, the Body would in the End of the first time come to the Line F l. Since therefore, the Velocity of this Force suffers nothing from the combination of the projectile Force therewith, any more than the Velocity of this latter suffers from it, the Body, notwithstanding the projectile Force will be found still in the end of the first time in the Line F 1. From whence it appears, that in the end of every time, it must by the Conjunction of these Forces be found in the Intersections of those Lines a l and F l, b m and G m, *&c.* to wit, in the end of the first time in the Point L, of the second in the Point M, of the third in n, of the fourth in o, and so on. Since therefore, if a l be of one Part,

Corol. 1. *Law* 22.

b m

b m is 4, c n is 9, d o is 16, e p is 25, and so on, they being amongst themselves by *Prop.* 4. as the Squares of the Times or Distances T a, T B, T c, T d, T e; and from the Nature of the compound Motion, T F is to T G, as F l squared is to G m squared, and so in the rest. And since, according to the primary Property of the Parabola, the Abscisses of any Diameter whatever T F and T G, are also betwixt themselves as the Squares of the *S*emi-ordinates F l and G m, it is manifest, that the Points l, m, n, o, p, are in the Curve of a Parabola, the principal Diameter whereof is T K; and T F, T G, T H, T I, T K, are the Abscisses, and F l, G m, H n, I o, K p, are the Semi-ordinates. And seeing all the Things here demonstrated do alike belong to any Diameter whatever, how oblique soever the Tangent of the same may be to it, as well as to the Axis; it is manifest, that the Trajectories of all Projectiles are universally truly parabolic; that is, so far as they are not hindered by the Resistance of the Medium. Q. E. D.

Coroll. (1.) Hence we may learn the Foundations of the Art of Gunnery. For since all Projectiles carried to any Inclination whatever, do describe Parabola's greater or lesser, or at least a greater or lesser Part of the same Parabola, so far as they are not hinder'd by the resistance of the Air, and seeing the Air is of small or no moment for the retarding the Motion of these Projectiles, by reason of their Solidity, and the Velocity of the Motion; it is plain that the principles of that Art are to be taken from the Nature of the Parabola. The Use of this *Corollary* extends it self a great way, and will be illustrated in the Sequel by many Examples taken out of the said Art.

Coroll.

Coroll. (2.) The Velocity of a Projectile being given, whatsoever the Angle of Inclination may be, there will also be given the distance of the Focus of the Parabola, from the Point where the Projection begun. Let s (See *Fig.* 4. Plate 4.) be the Point of the Projection, where the Projectile being thrown along the Tangent s v, begins to move in a parabolic Curve, and let s v be the Line to be describ'd by the Body in any given time by the projectile Force alone; let also v c, or s r be a little Line to be describ'd by the force of Gravity alone in the same given time. In the end therefore of that time, the Projectile will be found in the Point of the Parabola; and by reason of the given Force of Gravity as well as of Projection, there will be given also, whatsoever may be the Inclination of the Tangent to the Horizon, the Lines v c or s r and s v or c v, that is, the Absciss of the Diameter s o, and the semi-ordinate of the same; the third Proportional of which two is the *Latus rectum* belonging to the Vertex s; which therefore will necessarily be given when the former things are given. From whence also, the fourth Part of that *Latus rectum*, which is the distance of the Vertex s from the Focus of the Parabola, will also be given. Although, therefore, from the same Velocity of Projection, divers Parabola's will be describ'd in divers Elevations, yet the Foci of them all will be equidistant from the Vertex or Point where the Motion began, and consequently will be placed in a Circumference, whose Center is in that Point, Q. E. D.

Coroll. (3.) The Horizontal Range is then the longest, when it is directed according to a Line which is in the middle betwixt the Horizontal and perpendicular Lines, or in an Angle of

45 Degrees above the Horizon. For, seeing the principal Vertex of any Parabola whatever, describ'd by Projectiles, is in the greatest Height of the Projectile, under which in the Axis it self the Focus D F is placed; seeing the Distance of the same Focus from the Vertex is given: Seeing lastly, the Horizontal Range will be the longest where s F, the distance of the Vertex s, from the Focus, is measured by e g an Ordinate to the Axis passing through the Vertex s; since these things are so, the Horizontal Range will certainly be the longest, where s F the distance of the Vertex s from the Focus, coincides with s g the Ordinate to the Axis; for otherwise, by reason of the given distance of the Focus, 3 F s g will be greater than s F doubled: But where s F coincides with s g, s g will be double to s F, and consequently, the Horizontal Range will be the longest, where s F coincides with s g; that is, where the Angle v s h is half-right. For the Angle v s f comprehended by the Tangent v s and s F, the distance of the Vertex s from the Focus is always equal to the Angle b s o, comprehended by the same Tangent, and s o the Diameter of the Parabola. If therefore, the Angle b s o be an half right one, v s F will also be an half-right one, and consequently the Angle o s F will be a right one, and the Line s F will become s h, and will coincide with the Ordinage s g, and the Ordinate s g will be the longest Horizontal Range of all.

Coroll. (4.) Since therefore, the Tangent of the Parabola doth only in that Case comprehend with the Diameter an half right Angle, where it toucheth the same at the Extremity of the principal *Latus rectum*, which passeth through the Focus; it is manifest, that every longest Horizontal Cast will be comprehended betwixt that

Part

Part of the parabolic Curve that is placed above the *latus rectum*, the Focus it self being in the Horizontal Line; and that the highest Altitude from the Horizon in this Case is T F, a Quarter of the principal *Latus rectum*.

Coroll. (5.) If the Angle of Elevation differs equally from an half-right one, whether it be greater or less, the greatest Horizontal Range will equally be diminish'd. For because of the right Angle h s o, and the Angles v s F, o s b, which are equal one to the other, their Excess or Defect with reference to a right Angle, will be equal to the Angle F s h, whether the Focus be above the Horizontal Line, as in the greater Elevation, or beneath it, as in the less Elevation. But the acute Angle F s h, and the right one F h s, and the Side F s being given, there is given withal the Side s h the semi-ordinate of the Axis, and s g the ordinate determining the Horizontal Range. Thus in two Projections of equal Velocity, where the Angles of Elevation are one of 40, the other of 50 Degrees, the Horizontal Range will be equal on both Sides, and in like manner in the Degrees 30 and 60, 20 and 70, *&c.* as is well known to them that practise this Art.

Coroll. (6.) The Horizontal Distances produced in a given Velocity, in divers Angles of Elevation, are as the right Sines of the double Angles of Elevation. To wit, as g s is every where, so is h s the half thereof; but in the right Angled Triangle F h s, because of the given Radius F s, and the Angle h F s, double to b s o, the Angle of the Tangent and perpendicular s h will be every where the right Sine of that Angle; and consequently, the Horizontal Distances will always be betwixt themselves as those Sines.

Coroll.

Coroll. (7.) The Times of every Horizontal Range from a given Velocity in divers Degrees of Elevation, are betwixt themselves as the right Sines of the Angles of Elevation. Let one Body be thrown (see *Fig.* 4. Plate 4.) according to the Angle of Elevation l c d, and another according to the Angle L A D. I say, that the time in which the first Body reacheth through the Parabolic Arch c T l to the Point l, situate in the same Horizontal Plane with the Point c, will be to the time in which the latter Body reacheth along the Arch A t L (*Fig.* 5. Plate 4.) to the Point L, situate in the same Horizontal Plane with the Point A, as the Sine of the Angle d c l is to the Sine of the Angle D A L. For let there be in these Figures, as taken together, △ A equal to d c: △ e also (because of the Equality of the Time, in which the Bodies together would describe the equal Lines d c and △ a by the projectile Force alone) will be equal to d l. But by the Nature of a Parabola before declared, D L is to △ e as D A squared is to △ A squared; or as D L squared is to D I squared. Therefore D L, △ i, △ e, are three Lines continually proportional. And since (*Prop.* 4.) the Lines D L, △ e are in the duplicate Proportion of the Times, △ i, and △ e will themselves be in the proportion of the Times; therefore, the former Time will be to the latter, as △ e, or d l is to △ i; that is, as the Sines of the Angle of Elevation d c l and D A L. Q. E. D.

Coroll. (8.) The greatest Heights of Projectiles in a given Velocity in divers Angles of Elevation, are one to another as the Squares of the right Sines of the Angles of Elevation. To wit, as d l or △ e squared is to △ I squared, so are the greatest Altitudes d l or △ e to D L. Q. E. D.

Coroll. (9.) The greatest Altitude of Projectiles in a given Velocity, is where the Projection is perpendicular to the Horizon, and is the 4th Part of the *Latus rectum*, which in a given Velocity is always given; as in this Case the Parabola ending in a right Line, the Vertex T (*Fig.* 4. Plate 4.) coincides with the Focus F, and the highest Altitude becomes equal to F s a quarter of the given *Latus rectum*; and consequently, which is to be noted by the way, half of the longest Horizontal Range; as we shall presently demonstrate.

Coroll. (10.) The Angle of Projection being given, but the Velocity being chang'd, both the highest Altitude, (*Fig.* 4. Plate 4.) that is, the principal Vertex of the Parabola, and the longest Horizontal Range, or the Ordinate s g will be changed in the duplicate Proportion of the Velocity. The former Part is manifest from what has been before demonstrated, seeing the Altitudes of the Lines, or their Ascents, or Descents, are always in the duplicate Proportion of the Velocities by *Prop.* 4. foregoing. And from this Part of the Proposition the other also follows; for, because of the likeness of all Parabola's, and of the likeness of the Parts of like Figures on both Sides, if the Altitude T h be changed in the duplicate Proportion of the Velocity, the rest of the Lines also, as s h and s g will be changed in the same Proportion. But the latter part of the *Corollary* may also be easily deduc'd otherwise, and from of the Nature of the Parabola it self; for let us put the Velocity to be twofold greater than it was before, in this Case, in what time the Projectile would describe before the Line s v, it will now describe the double of that Line; but because of the Uniformity of the Force of Gravity, the

Line

Line vc or sr will not be changed. Therefore, as vc or sr given is Double to the Line sv; so is that double Line to another Line, to wit, the *Latus Rectum* of the Vertex s, which is Fourfold of the *Latus Rectum* that belonged to that Vertex before. From whence the Fourth Part of this *Latus Rectum*, or sF, will be Fourfold also of SF the fourth Part of the former *Latus Rectum*; and because of the Likeness of the Triangles sFh, sFh in both Cases, the Lines sh and sg will be Fourfold also of themselves, sh and sg; and so in the rest. The longest Horizontal Range therefore in a Velocity Twofold greater, is Fourfold greater, in a treble Velocity Ninefold, and so *in infinitum*. Nay, indeed it is generally to be affirmed, that all the Lines in a Parabola which are like, and in the like manner placed, are always increas'd and diminish'd, in the duplicate Proportion of the Increase and Diminution of the Velocity; as may be gathered from what has been already said.

Coroll. (11.) The longest Horizontal Range of every Parabola, is equal to half the *Latus Rectum* belonging to the Vertex that terminates the principal *Latus Rectum*. For in that Case, Fs is equal to sh; but Fs is the fourth Part of the *Latus Rectum* belonging to the said Vertex; and sg is double to sh; from whence sg is half the longest Horizontal Range of that *Latus Rectum*.

Coroll. (12.) Hence we may determine the longest Horizontal Range belonging to every Degree of Velocity. For let it be made thus, as the Line sr, describ'd in one Second of Time by the Force of Gravity = 16[1/12] *English* Feet, is to the Velocity of the Projectile sv, or rc, to be computed in the same Time; so is that Velocity to a fourth Number, which will give the *Latus*

Rectum

Rectum of the Vertex s in the same Feet: But the half of this Number will give the longest Horizontal Range, as is abundantly manifest from what hath been said. Thus, if the Projectile Velocity be so great, that it would carry the Ball One Thousand *English* Feet in one Second, say 16$\lfloor$1 : 1000 : 1000 : 62112 the *Latus Rectum* of the Vertex s in *English* Feet. The longest Horizontal Range therefore is 31056 Feet; beyond which Distance, as nothing can be reach'd, so within the same it may reach any assign'd Place whatever, as will be shewn in the next.

Coroll. (13.) Probl. (1.) To reach by a Projectile, in a Given Velocity, any Place whatever assign'd in an Horizontal Plane, not distant above half of the *Latus Rectum* of the Vertex s. Let the Place be at the Distance of 20,000 *English* Feet, and the Velocity be the same which was suppos'd in the foregoing Corollary. Because therefore the Velocity is given, there is given the *Latus Rectum* of the Vertex where the Projectile will begin its Motion in a Curve, and consequently the 4th Part thereof, or the Line F s, to wit, of 15528 Feet. But according to what was said before, s h is 10000 Feet. From these things therefore, let there be found the Angle h s F by this Analogy, as s h is to s F, or as 10000 is to 15528; so will the Radius be to the Secant of the Angle F s h, to be found by the Table of Secants; to wit, 49°. 47′. Which Angle being taken out of a right one, or superadded thereto, will give the Sum of the equal Angles v s F and o s b; the half of which v s F, or o s b, will determine the Angle which the Tangent v b ought to comprehend with the Perpendicular s o; to wit, of 90°.—49°. 47′.=40°. 13′. (or 90°. × 49°. 47′.=139°. 47′.) the half of which is 20°.

K 6′.

6′. 30″. or 69°. 53′. 30″. to wit, according as we would have the Elevation greater or lesser than the Mean. If therefore a Leaden Ball be thrown with the given Velocity in the said Angles, it will describe the Parabola required, and consequently reach the Place assign'd, without any other Declination from the Mark, than what the Resistance of the Air makes, which indeed is so small, that it ought not to be regarded. Thus we have solv'd the Problem, and taught how, with a given Vecity, to hit any Mark in any Horizontal Plane; which is not too far distant.

Coroll. (14). Problem (2.) To reach by a Projectile Motion, in a given Elevation, any Place assign'd in an Horizontal Plane; that is, from the Distance of the Place given s g, and the Angle h s v given to determine the Velocity s v. Here the Quadruple of s F will give the *Latus Rectum* belonging to the Vertex s: That s v therefore may be found, v c or s r is to be drawn into the Quadruple of s f; and from thence will arise a Rectangle equal to the Square of v s or c r; the Quadratick Root therefore being extracted out of that Rectangle, there will be found v s or c r, that Semi-ordinate which the Projectile ought to describe in one Second of Time. As for Example: Let sg, the Distance of the Object, be as before, to wit, 20000 *English* Feet, and the given Angle h s v 69°. 53′. 30″. The Angle F s v, or o s b, will be of 20°. 6′. 30″. and the Angle F s h of 49°. 47′. Whence, from the Tables of Sines, the Proportion of the Line s h to F s will be that of 10,000 to 15,528. From whence F s will be given, and the *Latus Rectum* of the Vertex s of 62112 Feet; which Number being drawn into v c or s r = 16$\frac{1}{12}$ Feet, there will arise the Rectangular Number 1000,000, the square

ſquare Root whereof is 1000, which affords the Number of Feet in the Line s v. If therefore in the given Angle, the primary Velocity of the Projectile be ſuch, as to carry it One Thouſand Feet in one Second of Time, it will reach the Mark g placed in the Parabolick Curve s T g, if it is not a little retarded by a very ſmall Reſiſtance of the Air. And the Computation is altogether the ſame, if the Angle F s v, or o s b, were ſuppos'd to be 69°. 53'. 30". as will eaſily appear from what was ſaid in the laſt Corollary foregoing.

Coroll. (15.) Hence alſo, from the Elevation given, or from the Velocity given, any Place whatever, as l may be reached that is placed out of the Horizontal Plane; that is, if in the ſame Parabola produc'd, if there be Occaſion, we note ſome other Point, as g, placed in the Horizontal Plane. For the ſame Caſt which reacheth to the Place g, will alſo reach unto l, or any other Place ſituate in the ſame Parabola.

Coroll. (16.) The Velocity of a Body which deſcribes a Parabola, is every where as a right Line drawn from T, the Vertex of the Parabola unto the Middle of the Semi-ordinate; or as part of the Tangent drawn betwixt the Point of Contact m, and the Axis; that is, as the Secant of the Angle of Elevation above the Horizon. (See *Fig.* 3. *Plate* 4.) For the Line to be deſcrib'd in the given Time, is as the Diagonal of the Parallelogram m Q R P; the Side whereof m Q is always given, and in P is equal to b m doubled, or to S G. Therefore the Velocity in the Point m, is to the original Velocity in the Point T, as m R is to P R, or as S m is to G M; and thus it is every where. Therefore the Velocity in m, any Point of the Parabola, is to the Velocity o n, which is any other Point of the Parabola, as the

Part of the Tangent S m is to Part of the Tangent X 4, both being taken betwixt equi-distant Diameters b m and T G; that is, as the Secants of the Angles of Elevation. *Q. E. D.*

Coroll. (17.) The least Velocity of all therefore is in T, the Vertex of the Parabola; and the Velocity is always by so much the greater, by how much the greater the Distance is from that Vertex.

Coroll. (18.) If therefore the Velocities of Bodies, thrown in divers Angles, be in the Proportion of the Secants of the Angles of the Elevation above the Horizon, they will all of them describe the same or an equal Parabola, that is, Parts of the same or an equal Parabola; Parts greater, where the Angle of Elevation is greater, and lesser where the said Angle is less. But if the Velocities be in some other Proportion, they must needs describe divers Parabola's, or Parts of divers.

Decemb. 4. 1714.

Lect. XII.

A Lemma *to the 9th and following Propositions.*

THE Quantity of the Centripetal Force of Bodies mov'd round in Circles, is to be estimated from two things conjunctly, to wit, the Curvature of the Arches describ'd at the same time, and the Velocity of the Motions in that Curvature.

For

For ſince all Motion is in it ſelf Rectilinear, and Bodies can be mov'd in Curves only by a centripetal Force impreſs'd from without; it is reaſonable that the Velocity being given, we ſhould determine the Curvature, which is generated from an extrinſic centripetal Force only, proportional a the ſame Force. And on the other Side, ſince to greater centripetal Force is requir'd to the making the ſame Curvature, where the Velocity of the Projection, or of the equable Original Motion is the greater, and a leſs centripetal Force is requir'd where the ſaid Velocity is the leſs: It is reaſonable that the Curvature being given, we ſhould determine the centripetal Force, proportional to the ſame Velocity. As therefore in comparing Rectangles, we determine their true Proportions by thoſe of their Longitudes and Latitudes; ſo it is likewiſe to be done in the comparing of centripetal Force; by defining, to wit, their true Proportions in every given Time, from the Proportions of their Curvatures and Velocities conjunctly. Let it therefore be laid down for a certain Truth, That the Proportions of centripetal Forces are every where to be eſtimated from the Proportions of their Curvatures and Velocities conjunctly.

Scholium. For the Underſtanding the true Proportions of Curvature and Velocity, we are to obſerve, that the Curvature is every where equal in the leaſt equal Angles, if the Subtenſes of the Angles of Contact be one to another, as the Radii or Diſtances from the Center; as the Proportion of like Figures doth altogether require: And if the Curvature doth recede from that Proportion of the Diſtances, the Proportions of the Exceſs or Defect are to be taken for the true Proportions of exceeding or deficient Curvature, which are afterwards to be computed. But as for the Velocity, it is every where to be consider'd according to that Degree in which it serves to promote the Angular Motion, and consequently every where in a Line Perpendicular to the Radius, or, which comes to the same, it is to be estimated in the least Circular Arch. For wheresoever the Direction of the Motion is either upwards or downwards, by how much the Velocity is increas'd, by so much is the Curvature always diminish'd; and so on the contrary: The Quantity which ariseth from the conjunct Forces of the same, being yet in no wise chang'd; which is to be observ'd every where.

IX. If two Bodies do in equal (*Fig.* 5. *Plate* 3.) Times run over Two whole unequal Circumferences b d g e, and B D G E, with an equable Motion, the centripetal Force in the greater Circumference will be to that which is in the less, as the Circumferences are one to another directly; or, which is the same, as their Diameters or Radii.

For because of the Curvature given on both Sides, to wit, that of a whole Circle; the centripetal Force in the greater Circumference will be to that which is in the Less, as the Velocities of the Bodies; that is, as the Circumferences of the Circles, directly. Q. E. D.

Corollary: If the periodic Times of Bodies revolving in Circles be Equal, then both the Velocities, and the centripetal Forces proportional to the same, will be one to another, as the Circumferences or Diameters of the Circles directly; and on the contrary, if the centripetal Forces of Bodies revolving in Circles, be one to the other, as the Circumferences or Radii of the Circles directly;

rectly; then their Velocities will also be in the same Proportion, and the periodic Times will be equal.

Corol. (21.) If the Force of some central attractive Body be directly as the Distances from the same Center; the periodic Times of all Bodies revolving in Circles about the central Body will be equal. And the same Thing is to be asserted of Ellipses, since their entire Curvatures be equal to the entire Curvature of any Circle whatever; and their Circumference an intermediate one, as it were betwixt the Circumferences of Circles, taken on this Side and on that. From whence from the Equality of periodical Times in Circles, whether greater or lesser than Ellipses, it will be obvious enough to ascribe the same Equality of periodic Times, to the intermediate Ellipses also about their Centers.

X. If two Bodies revolve in the same, or equal Circles with unequal Celerities, but both with an equable Motion, the centripetal Force of the Swifter will be to that of the Slower, in the Proportion of the Celerities duplicated; or as the Squares of the Arches described together. For because of the given Curvature of equal Circles in equal Arches; increasing together with the Velocity, the Curvature also will increase in the same Proportion; therefore the centripetal Force, which is to be estimated from the Curvature, and the Velocity conjunct, will be in a given Time in Proportion of the Arch, to the Arch described at the same Time, upon Account of the Velocity; and in Proportion of the same Arch, describ'd at the same Time; consider'd in respect of the Curvature; from whence by the Conjunction of both Proportions, the centripetal Force will be (the Rectangle being reduc'd to a

Square,) in the duplicate Proportion of the Arches describ'd at the same Time; or as the Squares of those Arches. Q. E. D.

Coroll. (1.) Since the Periodic Times in equal Circles are reciprocally Proportional to the Velocities, the centripetal Forces will be reciprocally in the duplicate Proportion of the periodic Times, or as the Squares of the periodic Times reciprocally; so that by how much the Square of the periodic Time is the greater, the centripetal Force is so much the less; and by how much that Square is the less, so much the greater is the centripetal Force, and in the same Proportion.

Coroll. (2.) If many Bodies be mov'd in Circles about many Centers, each about their own, and this at the same Distance every one from the Centers; the attractive Force of the central Bodies will easily appear, since they are amongst themselves as the Squares of the Times reciprocally; and the Velocities also will easily appear, since they be in the reciprocal Proportion of the periodical Times.

XI. If two Bodies be carried in unequal Circles with equal Velocity, their centripetal Force will be in the reciprocal Proportion of their Circumference or Diameters; so that in the lesser Circumference there will be the greater centripetal Force, and in the greater the less.

For because of the given Velocity, the centripetal Force in the given Time, will be as the Curvature of equal Arches, that is as the Circumferences, the Diameters or Radii directly. Q. E. D.

Coroll. (1.) Since the periodic Times in Bodies equally swift, are betwixt themselves in the same Proportion, as the Circumferences to be described; if the periodic Times of Bodies

mov'd

mov'd in divers Circles, be directly as the Circumferences of the Circles, the centripetal Forces will be as those Circumferences or Radii reciprocally; and on the other Hand, if the centripetal Forces be as the Radii or Distances reciprocally, the periodic Times will be as the Radii directly.

Coroll. (2.) If the Force of any attractive central Body be reciprocally as the Distances of Bodies from the Center; so that by how much the nearer Bodies approach thereto, so much the greater the central Force is; and by how much they are more remov'd, so much the less is that Force: The peroidic Times of Bodies, plac'd at divers Distances, will be as those Distances directly, and the Velocities of those Bodies will be equal.

XII. If two Bodies be mov'd in unequal Circles, with an unequal Velocity, in the sub-duplicate Proportion of the Circumferences, Diameters, or Radii, the centripetal Forces will be equal every where, and neither increas'd in the Access or Recess.

For because of the greater Velocity in the greater Circle, and this in the sub-duplicate Proportion of the Circumferences, the centripetal Force is to be increas'd in the greater Circle in the same Proportion. And because of the greater Curvature in the lesser Circle, and this in the sub-duplicate Proportion of the Circumferences, the centripetal Force is to be increas'd reciprocally in the lesser Circle. It is therefore manifest, That the centripetal Forces are to be increas'd on both Sides in an equal Proportion, and consequently that they are still equal on both Sides. Q. E. D.

As

As for Example, Let the Radius of the greater Circle be Fourfold of the Radius of the lesser Circle, or as 4 to 1; and let the Velocity in the greater be to the Velocity in the lesser, in a subduplicate Proportion of the Radii, or as 2 to 1. Seeing now the Curvature of the greater is equal to the Curvature of the lesser, in like Arches, and in Arches equal, is to the same reciprocally as the Radii; it is necessary that in a double Arch, which the double Velocity in the greater, will describe in the given Time, the Curvature should be half of the other. The Velocity therefore of the former Body is to that of the latter, as 2 to 1; and the Curvature of the latter to the Curvature of the former, as 2 to 1. From whence the Quantity of the centripetal Force in the former, will be to the Quantity of the same in the latter, as a Rectangle made of the Velocity of the former, drawn into the Curvature thereof, or as 2 x 1, is to a Rectangle of the Velocity of the latter, and the Curvature of the same conjunctly, or as 1 x 2; that is, in the Proportion of Equality: And thus every where.

Coroll. (2.) Since the periodic Times be in this Case one to another, in the sub-duplicate Proportion of the Circumferences, Diameters, or Radii, the Squares of the periodic Times will be betwixt themselves as the Circumferences, *&c.* If therefore the Squares of the periodic Times be one to another as the Circumferences, *&c.* the centripetal Force will be equal in all Distances, and the Velocities in the sub-duplicate Proportion of the same Circumferences, Diameters, *&c.* And on the other Hand, if the centripetal Forces be equall in all Distances, the Squares of the periodic Times, will be as the Distances or Radii; and

the

the Velocities still in the sub-duplicate Proportion of the same.

Coroll. (2.) If the centripetal Force of any attractive central Body be altogether the same in all Distances, the Velocities will be in the sub-duplicate Proportion of the Distances; and the Squares of the periodic Times will be to one another, as those Distances or Diameters, or Circumferences.

XIII. If two Bodies be mov'd in unequal Circles, with an unequal Velocity, in the sub-duplicate Proportion of the Circumferences, *&c.* reciprocally; so that in the greater Circle the Velocity be the lesser, and in the lesser Circle the greater, and this in the said sub-duplicate reciprocal Proportion, the centripetal Force will be reciprocally as the Squares of the Radii or Distances.

For because of the lesser Curvature in the greater Circle, and this in the sesquialteral reciprocal Proportion of the Radii; and because of the lesser Celerity also in the greater Circle, and this in the sub-duplicate Proportion of the Radii, the centripetal Forces to be deriv'd from these conjunct Proportions will be in the duplicate reciprocal Proportion of the Radii, or reciprocally as the Squares of the Radii, Q. E. D.

For Example: Let the Radius of the greater Circle be Ninefold of the Radius of the lesser, or as 9 to 1; and let the Velocity in the greater be to the Velocity in the lesser in the Subduplicate Proportion of the Radii reciprocally, or as 1 to 3: Seeing the Curvature of the greater is to that of the lesser as before, that is, in like Arches equal, and in equal Arches reciprocally as the Radii: It must needs be, that in an Arch equal to the Third Part only of the other, which the Third Part of the

the other's Velocity will describe in the Given Time, should be in the Greater only one 27th Part of the other, or as 1 to 27. Therefore the Velocity in the greater Circle is to that in the lesser as 1 to 3, and the Curvature in the greater to that in the less, as 1 to 27. From whence the Quantity of the Centripetal Force in the greater, will be to the Quantity of the same in the lesser, as a Rectangle of the Velocity and Curvature in the greater conjunctly, or as $1 \times 1 = 1$ is to the Rectangle of the Velocity and Curvature in the less conjunctly, or as $3 \times 27 = 81$; that is, as the Square of the Radius of the less $10 = 1$ is to the Square of the greater $= 81$. And so every where. For the Periodic Times will be to one another, as 27 is to 1; that is, in the Sesquialteral Proportion; for 27 is a Geometrical Mean Proportional betwixt 9 and 81; and consequently the Proportion of 27 to 1, contains the Proportion of 9 to 1; and the halved Proportion of 81 to 9, or the Subduplicate of 81 to 27, [1:3:9:27:81: ∺.] and thus every-where.

Coroll. (1.) Seeing the Periodical Times in this Case are one to another in the Sesqui-plicate Proportion of the Radii, the Squares of the Periodical Times will be betwixt themselves as the Cubes of the Radii. If therefore the Squares of the Periodic Times be betwixt themselves as the Cubes of the Radii, the Centri-petal Forces will be betwixt themselves as the Squares of the Radii reciprocally; and the Velocities still in the Subduplicate Proportion of the Radii reciprocally. And on the other hand, if the Centri-petal Forces be inversly as the Squares of the Distances or Radii, the Squares of the Periodical Times will be betwixt themselves as are the Cubes of the Radii, and

and the Velocities ſtill in the Subduplicate Proportion of the Radii reciprocally.

Coroll. (2.) If the Centri-petal Forces of any Central attractive Body whatever be in divers Diſtances from their Center, as the Squares of thoſe Diſtances reciprocally, the Velocities of Bodies revolved in the divers Diſtances will be in the Subduplicate Proportion of thoſe Diſtances reciprocally; and the Proportion of the Periodical Times duplicated, will be equal to the Proportion of the Diſtances triplicated, or the Squares of the Periodical Times will be betwixt themſelves as the Cubes of the Radii.

Coroll. (3.) If the Motion be in an Ellipſis, then let the Middle Diſtance betwixt the greateſt and the leaſt be taken; and then alſo the Squares of the Periodical Times will be as the Cubes of the Radii as well as in Circles.

XIV. If two Bodies be carried in unequal Circles with an unequal Celerity; ſo that by how much greater the Radius, Diameter or Circumference is, ſo much the leſs the Velocity is; and by how much the leſs the Radius is, ſo much the greater is the Velocity, and this in the Reciprocal Proportion of the Radii, the Centri-petal Forces will be as the Cubes of the Radii reciprocally.

For becauſe of the leſſer Celerity in the greater Circle, and this in the Reciprocal Proportion of the Radii; and by reaſon alſo of the leſſer Curvature in that Circle, and this in the Duplicate Reciprocal Proportion of the Radii, the Centripetal Forces to be deriv'd from thoſe Conjunct Proportions will be in the Reciprocal Triplicated Proportion of the Radii, or as the Cubes of the ſame reciprocally.

As

As for Example: Let the Radius of the greater Circle be Twofold of the Radius of the less, or as 2 to 1. And let the Velocity in the greater be to the Velocity in the less Reciprocally as the Radii, or as 1 to 2: In this Case the Curvature of the greater will be to the Curvature of the less in the Given Time as 1 to 4: Therefore the Velocity in the lesser, is to the Velocity in the greater, as 2 to 1; and the Curvature in the less, to the Curvature in the greater, as 4 to 1. From whence the Quantity of the Centri-petal Force in the less, will be to the Quantity of the same in the greater; as the Rectangle $2 \times 4 = 8$ is to the Rectangle $1 \times 1 = 1$, or as the Cubes of the Radii reciprocally; and so every-where.

Coroll. (1.) Since the Periodic Times be in this Case in the Duplicate Proportion of the Radii, if the Squares of the Periodic Times be betwixt themselves, as the Biquadrate of the Radii; or which is the same, if the Periodic Times themselves be one to another as the Squares of the Radii; the Centri-petal Forces will be betwixt themselves as the Cubes of the Distances or Radii inversely, and the Velocities inversely as the Radii. And on the other hand, if the Centri-petal Forces be inversely as the Cubes of the Distances, the Periodic Times will be betwixt themselves as the Squares of the Radii; and the Velocities still as the Radii themselves inversely.

Coroll. (2.) If the Centri-petal Forces of any Central attractive Body whatever be in divers Distances from their Center, as the Cubes of those Distances reciprocally, the Velocities of Bodies revolved in divers Distances will be in the Reciprocal Proportion of the Distances; and the Periodic

dic Times in the Duplicate Proportion of thoſe Diſtances.

Coroll. (3.) All the ſame Things concerning Times, Velocities, and Centri-petal Forces, whereby Bodies deſcribe like Parts of any Curves whatever which are like, and have their Centers in the like manner poſited, do follow from the Demonſtrations of the foregoing Matters which were applied to Circles in particular, as applied to the other Caſes.

Scholium. Since the Caſe of Propoſition 13th hath place in the Celeſtial Bodies; to wit, that the Squares of the Periodic Times are every-where one to another as the Cubes of the Diſtances; and that conſequently the Centri-petal Forces are as the Squares of the Diſtances reciprocally, and the Velocities in the Sub-duplicate Proportion of thoſe Diſtances reciprocally: Since, I ſay, this Caſe hath place in the Syſtem of the World, and this alone, as our Countrymen, Sir *Chriſtopher Wren*, Dr. *Hook*, and Dr. *Halley* have ſeverally Collected; and that the ſame is now generally received amongſt Aſtronomers; This moſt noble Caſe requires to be more largely and diligently conſider'd in what follows; while the Conſequences of the reſt are but more lightly and curſorily touch'd upon.

Dec. 11. 1704.

LECT.

Lect. XIII.

XV. THE Area's, which revolving Bodies do describe by Radii, drawn unto an unmovable Center of Force, do both lie in immovable Planes, and are proportional to the Times, and so in any given Time everywhere equal; the Velocity of Motion in the lesser Distance, and the Slowness thereof in the greater so tempering the Description of the Area's, that from those various Distances no difference of the Spaces run over in the given Time doth ever arise.

For let the Time be divided into equal Parts, and in the first small Time let the Body by its innate Force, or by a projectile Motion describe in the first Part of Time, any right Line, as A B. (See *Fig.* 6. *Plate* 4.) The same Body in the second equal Part of Time, if nothing hinder'd, and no other Force was impress'd on it, would go straight forward to B c, describing the Line B c, equal to A B, so that the Area's A S B, B S c, made by the Radii drawn to the Center S, would be equal. But when the Body comes at the Point B, let a centripetal Force, whether it be an Attraction, or some Pressure tending to the Center S, act upon the same Body by one single Impulse, which is to the projectile Motion as any Right Line, as B g to B c; this new Impulse will make that the Body should decline from the Right Line B c, and go forwards in another Right Line, to wit, B C the Diagonal of the Parellogram B g C c; so that the 2d equal Part of Time being compleated, the Body will be to be found at the Point C, in the same Plane with the Triangle A S B. Join S C, and the Area describ'd by the Radius drawn from the Center to the Body, that is, the Triangle S B C will be equal to the Area of the former; that is, to the Triangle S B c, (by I. 37. of the *Elements*,) and consequently to the first Triangle SAB, to which, by what was said before, S B c is equal. By the like Argument in the 3d equal Part of Time, the Body would reach by the Projectile Force (which being once impress'd doth still endure) from C to d; so that the Line C d, which is to be described, would be equal to C B that was last describ'd. But if any Centripetal Force whatsoever, whether greater or less than the former, should again act upon the Body in the Point C, the Body in the End of the 3d Time will be found somewhere in the Line D d parallel to S C, and would be carried along C D the Diagonal of a certain Parellogram h D d C to a certain Point D; so that the Triangle S D C would be equal to the Triangle S d C, and consequently to the Triangles S C B, S B A, which are equal one to the other; and by the same Reason, if the Centripetal Force acts successively in D, E, F, making that the Body should in each equal little Portion of Time describe a several Diagonal, all these right Lines will lie in the same Plane, and the Triangles S E D, S F E will be described equal to the former. Now let the Number of the Triangles be increas'd, and their Latitude decreas'd infinitely, their ultimate Perimeter A D F will become a Curve Line, the Sides of a Polygon ending in a

Law of Mot. 2. foregoing.

Curve, and by reason of the continued and never-ceasing Action of the Centripetal Force, the Body will perpetually be drawn back from the Tangents of the Curve, and the Areas likewise by the same Reason as before, will still be describ'd in an unmoveable Plane, and be proportional to the Times. *Q. E. D.*

Coroll. (1.) Therefore the Velocity of a Body revolved about a Center, which is estimated according to a Line perpendicular to the Radius, will be in the reciprocal Proportion of the Distances; for otherwise the Equality of the Areas could in no wise be kept.

Coroll. (2.) The Angular Velocity of a Body about the Center of Force, will likewise be in the reciprocal Duplicate Proportion of the Distances. For since the true Velocity is in the simple reciprocal Proportion of the Distances, as we have seen already, and the Distance of the Center is the greater by how much the Motion is the slower, and in the same Proportion; it is manifest, that that angular Velocity with respect to the Center of Force, is in the duplicate Proportion of the Distance reciprocally.

Coroll. (3.) Where the Position of the Tangent is perpendicular to the Radius, or Distance from the Center, and the Velocity of the projectile Motion makes the Centrifugal Force exactly proportional or correspondent to the Centripetal Force of the Central Body; the Body will neither approach to the Center, nor recede from it, but will be carried perpetually with a circular Motion about that Center.

Coroll. (4.) But where the Position of the Tangent is oblique to the Radius, although the Velocity of the Projectile Motion be proportionate and correspondent to the Centripetal Force, that Centripetripetal Force will somewhat increase even the least descending Motion by conspiring together with it, and something diminish even the least ascending Motion by opposing it; until at length the increas'd Motion exceeds the Centripetal Force, and the Body which before descended comes now to ascend; and the diminish'd Motion at length yields to the Centripetal Force, and the Body which ascended before doth again descend.

Coroll. (5.) From such like Circumstances ought to arise the Motions of Bodies revolved in Ellipses about any Center whatever. For although the Body be suppos'd to be now situated in the Course of its Revolution, at the lesser Axis of the Ellipsis, the Central Body possessing the Focus, or at the mean Distance, and the Velocity of the Projectile Motion be supposed also in that Place to correspond exactly to the Centripetal Force; yet notwithstanding, because of the oblique Position of the Tangent in the same Place, the Motion will become not Circular but Elliptic; whilst the Body, as it is in descending, doth by little and little acquire a new Force, by which it may afterwards ascend; and as it is in ascending, doth by degrees lose some Force by which it ascended before; until at length the Centripetal Force overcoming it, be compell'd to descend: And thus perpetually. From whence it is manifest, by what means an Elliptic Motion may arise from a Motion impress'd according to an Oblique Line; in the mean while that the very same Motion impress'd, according to a perpendicular Line, would have produc'd a Motion altogether Circular.

Corol. (6.) If in Vacuo the Area's describ'd be not proportional to the Times of the Description, the Forces do not tend unto a Concourse of the Rays. For if they tended thither, the Areas would necessarily be proportional to the Times. Which is contrary to the Hypothesis.

Coroll. (7.) In all Mediums, if the Description of the Areas be accelerated, the Forces tend not unto the meeting-together of the Rays, but conspire rather with the Projectile Motion; if the Description of the Areas be retarded, that is, more than the Resistance of the Medium requires, the Forces tend not unto the meeting-together of the Rays, but are rather opposite to the Projectile Motion.

XVI. Every Body which is mov'd in a Curve Line, and doth by a Radius drawn to some Point, either immoveable, or going forwards uniformly with a Rectilineal Motion, describes Areas about that Point proportional to the Times, is urged or impress'd by a Centripetal Force tending to the same Point.

Case (1.) For because of the Equality of the Triangles S c B, S C B (see *Fig.* 6. *Plate* 4.) describ'd upon the same Base S B, the Points C and and c will be (by I. 39. of the *Elements*) in the Line C c parallel to the Base; and consequently the Figure B c C g will be a Parallelogram, in which the Sides B c and B g represent the Forces, and B L is the Diagonal: And therefore the Body placed at B, is incited by the Force B g tending to S the Center of the Forces; and so likewise in all the Points, C, D, E, F. *Q. E. D.*

Case (2.) And it is the same thing, whether the Plane in which the Body describes the Curvilinear Figure doth rest, or whether it be mov'd together with the Body, the Figure described, and its Central Point S uniformly straight forward. Where-

Wherefore the Demonſtration of the former Caſe will hold in this alſo.

Scholium. A Body may be urged by a Centripetal Force, which is compounded of divers Forces, (as for Example, the Force of heavy Things towards the Center of the Earth, is compounded of Forces tending to all the Parts of the Earth, as will appear afterwards;) And in this Caſe the Senſe of the Propoſition is, that that Force which is compounded of all, when it is reduc'd to one, tends to the Center of that Force.

Coroll. (1.) Seeing therefore in the Syſtem of the Primary Planets the Areas deſcrib'd by Rays drawn to the Center of the Sun, are always proportional to the Times; as is well known to Aſtronomers, the Planets are perpetually urged by a Force tending to the Center of the Sun; and in the ſame manner muſt we reaſon concerning the Secondary Planets, as revolved about their Primary ones, *Saturn*, *Jupiter*, and the Earth.

Coroll. (2.) As the Velocity of divers Bodies about a Center of Force; where that Force is as the Squares of the Diſtances inverſely, is in divers Circles in the ſubduplicate Proportion of the Diſtances inverſely, as we demonſtrated before; ſo from this and the foregoing Propoſition it follows, that the Velocity of the ſame Body deſcribing any Eccentric Orbit, taken as placed in its divers Diſtances from the Center, let the Condition of the Centripetal Force be what it will, is as is the Diſtance inverſly; *i. e.* if the Velocity be eſtimated by a Circular Arch, or in a Line perpendicular to the Radius as before: The Cauſe of which divers Velocity is this, that in divers Circles the Areas in that Caſe are not equal on both Sides, but according to the Greatneſs of the Diſtance greater, and in the ſame Proportion of the Magnitude also greater; when notwithstanding, in the Revolution of the same Body, the Equality of the Areas doth altogether require a Velocity reciprocally proportional to the Distance. Thus, if two Planets are revolv'd about the Sun in divers Circles, the Radii of which do exceed one the other in the Quadruple Proportion, the remoter Planet would be carried with a Velocity which is only double to that of the other: But if the same Planet, performing its Circuits in a very Eccentric Ellipsis, be placed sometimes at a greater, sometimes at a lesser Distance; and the same, as before; exceeding and falling short by turns in the Quadruple Proportion, the Velocity will be in the reciprocal Proportion of the Distances, and in the lesser Distance exactly Quadruple of the other; and so in any Distances whatever. Which thing ought to be kept in mind, in the Contemplation of the whole Planetary System.

XVII. Every Body, which by a Ray drawn to the Center of any other Body howsoever mov'd, describes Areas about that Body proportional to the Times; is urged by a Force compounded of a Centripetal Force tending to the other Body, and of all the accelerative Force wherewith the other Body is urged. For, if first of all the Plane, and the Center of Forces in the Plane, do rest, the Areas will be proportional to the Times; and if both Bodies be accelerated with the same Celerity according to parallel Lines, the Areas will still remain proportional to the Times. From whence, since by the Hypothesis the Areas remain proportional to the Times, both the Centripetal Force, the Cause of them, will remain, and the accelerative Force will remain every where the same common Cause of Celerity.

Coroll.

Coroll. (1.) If any Body whatever doth with a Ray drawn to the Center describe Areas proportional to the Times, and there be subducted from the whole Force wherewith the former Body is urged, whether Simple or Compound, the whole accelerative Force wherewith the latter Body is urged, all the remaining Force wherewith the former Body is urged, will tend unto the other Body, as to the Center.

Coroll. (2.) And if those Areas be nearly proportional to the Times, the remaining Force will tend to the other Body very near.

Coroll. (3.) And on the other hand, if the remaining Force doth come very near to the tending to the other Body, those Areas will very nearly be proportional to the Times.

Coroll. (4.) If a Body doth with a Ray drawn to another Body describe Areas, which, when compared with the Times, are very unequal thereto; and that other Body doth either rest, or is moved uniformly straight forwards, the Action of a Centripetal Force tending to the other Body, is either none at all, or is mingled and disturbed by other Forces. And the whole Compound Force, if it be such a Force, will be directed to some other Center, whether immoveable or moveable, the Areas described about which will be proportional to the Times. The same thing holds where another Body is mov'd with any Motion whatever, if so be the Centripetal Force be taken to be that which remains after the Subduction of the whole Force, which acts upon that other Body.

Scholium (1.) Because an equable Description of Areas is a Token of a Center, which that Force wherewith the Body is affected doth respect, and the Body by this Centripetal Force is retain'd in a Curvilinear Orbit; and all Curvilineal Motion is rightly ſaid to be made about that Center, by the Force of which the Body is drawn back from the Rectilinear Motion, and perpetually retain'd in its Orbit: In what follows, we ſhall make uſe of that equable Deſcription of Areas, as the Index of a Center, about which the Motion which is in a Curve is perform'd in free Spaces.

Scholium (2.) This 17th Propoſition, with its Corollaries, appertains to the underſtanding the true Syſtem of the World. For although all Planetary Motions are to be derived from a Projectile Motion once impreſſed according to Tangents, and a Centripetal Force continually urging; yet thoſe Centers unto which the Centripetal Forces tend, are alſo mov'd themſelves, together with the Bodies that are revolved about them. Thus the Circulations of the Circumſaturnian, and Circumjovial Planets, and of the Moon, do proceed from a Projectile Motion once impreſs'd upon each, and from a Centripetal Force tending to the Centers of *Saturn*, *Jupiter*, and the Earth reſpectively; albeit, in the mean while thoſe Central Bodies, together with their whole Satellitium, be mov'd about the Sun, the common Center of all the Primary Planets.

XVIII. *A Problem.* There being given in any Three Places whatſoever, the Velocity wherewith the Body deſcribes a given Figure, by a Force tending unto ſome common Center or Point, to find that Center.

Let Three Right Lines, P T, T Q V, V R, (ſee *Fig.* 1. *Plate* 5.) touch a deſcribed Figure in ſo many Points, P, Q, R, whilſt they meet together in T and V. Let there be erected P A, Q B, R C, perpendicular to the Tangents in the

Points

Plate IV.

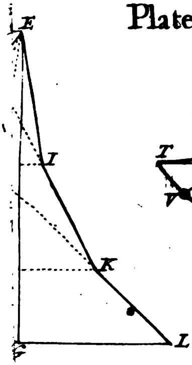


Fig. 2.

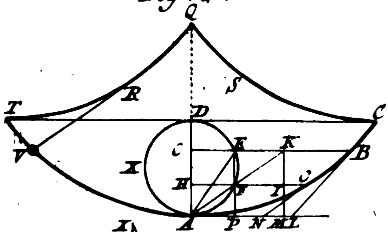
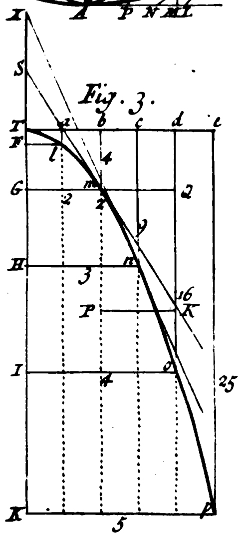
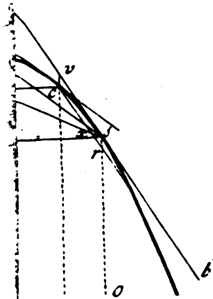
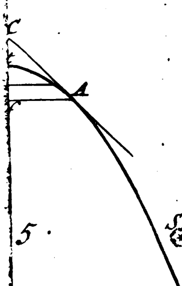


Fig. 3.

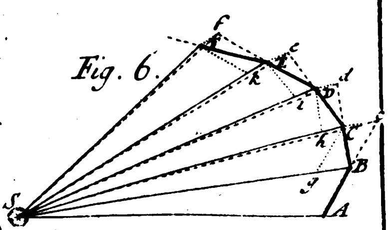






5.

Fig. 6.



I. Senex sculp.

Points of Contact, and let them be reciprocally proportional to the Velocities of the Body in those Points; that is, let P A be to Q B as the Velocity in Q is to the Velocity in P; and Q B to R C as the Velocity in R is to the Velocity in Q. At the Extremities of the Perpendiculars A, B, C, let AD, DBE, EC be drawn at right Angles to the Perpendiculars, or parallel to the Tangents, and meeting together in D and E. Let T D and V E be drawn intersecting each other in S; and from the Point E, let E r and Ev be parallel to the Perpendiculars C R and B Q respectively. And in like manner from the Point S, let Dp and Dx be parallel to the Perpendiculars A P & B Q respectively. Then lastly, from the Point S let S s, S t, S q be parallel to the same Perpendiculars respectively, or perpendicular to the Tangents: I say, that the Point S is the Center which is sought. For since the Body revolving, and placed successively in the Points P and Q, doth by Rays drawn to the Center of Force in equal Time describe equal Areas, or equal Triangles; since also those Triangles together described, are as the Velocities, or as the Lines together described in P and Q drawn into the respective Perpendiculars, let fall from the Center to the Tangents P T, Q T: Those Perpendiculars will be reciprocally as the Velocities, and consequently as the Perpendiculars D p and D x directly. But because of the Likeness of the Triangles TDx, TSt, and TDp, TSq; as is Dp to Dx, so is the Perpendicular Sq to the Perpendicular S t. And by the like Reason, as is E v to E r, so is the Perpendicular S t to the Perpendicular S s. And seeing this can be true only in this Point S the Concourse of the Lines TD & VE,

Prop. 15. *foregoing* Schol. *of* I. 41. Elements.

it

it is manifest, that S is the Center of the Centripetal Force *Q. E. D.*

Jan. 29. 170$\frac{5}{6}$.

Lect. XIV.

XIX. IF a Body be mov'd in an Ellipsis about the Center of the same, the Centripetal Force will be directly as the Distance of the Body from the same Center. For the Curvature every where in like Arches is in the quadruplicate Proportion of the Distance; but the Velocity is in the simple Proportion of the same Distance inversly. From whence the Curvature, describ'd in a given Time, will be in the duplicate Proportion of the Distance, and the Velocity in the simple Proportion of the Distance inversly, and the Centripetal Force, which is to be estimated in this Case by the Excess of the Proportion of the Curvature above that of the Velocity, will be directly as the Distance. *Q.E.D.*

Corollary. If an Ellipsis, the Center thereof passing away infinitely, be turn'd into a Parabola, the Body will be mov'd in this Parabola, and the Force now tending to a Center infinitely distant will become equable. This is the Theorem of *Galilæo* demonstrated by us above in another Method. And if a Parabolic Section of a Cone, the Inclination of the Plane to the Cone that is cut thereby being

Prop. 8.

chang'd,

chang'd, be turned into an Hyperbola; the Body will be moved about its Center in the Perimeter of the Hyperbola, the Centripetal Force being turn'd into a Centrifugal one, and that Force being greater in a lesser Distance, and lesser in a greater Distance; as the Nature of such Force doth altogether require.

Coroll. (2.) If the Centripetal Force of any attractive Body whatever be directly as the Distance, so that in a greater Distance the Attraction be in the same Proportion also greater, and in a lesser Distance less; the Body will be mov'd in the Ellipsis about a Central Body placed in the very Center of the Ellipsis, or perchance in a Circle which the Ellipsis may pass into; for, according to the Situation of the Tangent, of which before, the Body will be mov'd either in a Circle, or in an Ellipsis. *Corol.* 3, 4, 5. *of Prop.* 15.

Coroll. (3.) And the Periodic Times of Revolutions made in all Figures about the same Center will be equal, as we also shew'd before. *Corol.* 2. *P.* 9.

XX. If a Body be mov'd in a Spiral Line, which cuts all the Radii in the same Angle, the Centripetal Force will be reciprocally as the Cube of the Distance from the Center of the Spiral. For in the divers Parts of the Spirals, the Curvature of like Arches is equal; and that of equal Arches is reciprocally as the Distance. But whilst Bodies revolve in Spirals, the Celerity will be every where reciprocally as the Distance; and from thence also the Curvature will, in the given Time, be reciprocally in the duplicate Proportion of the Distance. Therefore the Centripetal Force, which proceeds from the Proportions of the Curvature and Celerity conjunctly, will be in the triplicate Proportion of the Distance reciprocally,

cally, or reciprocally as the Cube of the Distance.

Corollary. If the Force of any attractive Body be in the triplicate Proportion of the Distances from the Center reciprocally, all Bodies, the Directions of the Projectile Motions whereof are not perpendicular to the Radii, with what Velocity soever they go forth, will be mov'd in a Spiral which cuts all the Rays in a given Angle; and if the first Body ascends, it will ascend infinitely; if it descends, it will descend to the Center in a Space of Time easily to be found from the Quantity of the Spiral Area.

Scholium. If there were any regular Curve, the Curvature whereof from any Central Point whatever were in a duplicate Proportion of the Distance directly, any Body whatever would revolve in it, if so be the Centripetal Forces were amongst themselves in the reciprocal Proportion of the Distances. For if the Curvature, in equal Angles, were according to the Hypothesis in the duplicate Proportion of the Distance directly, the Curvature would in a given Time be always equal to it self in all Distances; and since the Velocity is always reciprocally as the Distance, the Centripetal Forces, to be estimated from the Curvature and Velocity conjunctly, would be as the Distance reciprocally, and the Body would be mov'd in that Curve. *Q. E. D.*

So likewise if there were any regular Curve, the Curvature whereof from any Central Point whatever, were in the triplicate Proportion of the Distance directly, any Body whatever would revolve in it, if so be the Centripetal Force were in all Distances equal. For if the Curvature in equal Angles were, by the Hypothesis, in the triplicate Proportion of the Distance directly, the Cur-

Curvature in the given Time would always be directly as the Diſtance; and ſince the Velocity is always as the Diſtance reciprocally, the Centripetal Forces, by reaſon of the Equality of the direct and reciprocal Proportions, would always be equal, and a Body would be mov'd in that Curve.

XXI. If a Body be mov'd in an Ellipſis about its Focus, the Centripetal Force will be every where in the duplicate Proportion of the Diſtance from the ſame Focus reciprocally. For, as we noted above, the Curvature with reſpect to the Focus in divers Parts of Ellipſes, Parabolæ, and Hyperbolæ, is every where in like Arches directly as the Diſtance, and in equal Parts always equal: Now the Velocity every where is in the reciprocal Proportion of the Diſtance. Therefore in Arches deſcribed in the ſame time, the Curvature is reciprocally as the Diſtance from the Focus, and the Celerity is likewiſe in the ſame reciprocal Proportion: From whence the Centripetal Force, to be eſtimated from the conjunct Proportions of the Curvature and Celerity, will be in the duplicate Proportion of the Diſtance from the Focus reciprocally. *Q. E. D.*

Coroll. (1.) If the Force of any attractive Body whatever be in the duplicate Proportion of the Diſtances from the Center reciprocally; all Bodies, at leaſt where the Directions of the Projectile Motions are not perpendicular to the Radii, whatſoever Velocity of Motion alſo they may have, will be mov'd in Ellipſes, one of whoſe Foci will be poſſeſs'd by the Central Body: unleſs the Velocity of the Projectile Motions be ſo great, as to be able to turn the Ellipſis into Parabola's, or even Hyperbola's.

Coroll.

Coroll. (2.) If a Body, according to the Law of the Centripetal Force here assign'd, be mov'd in an Ellipsis about one of the Foci, the Periodic Time of the Body, moving in the Ellipsis, will be to the Periodic Time of a Body mov'd in a Circle, the Radius whereof is in the Middle betwixt the greatest Distance and the least, or equal to the greater Semi-axis, in the Proportion of Equality. For since the whole absolute Curvature of the Ellipsis is equal to the Curvature of the Circle, and the Sum of the absolute Velocities in equal Arches above and below the mean Distance, because of the equal Change of the Motion in an equal Arch, is always equal to the Velocity in a mean Circle; it is manifest, that the Centripetal Force is equal; and consequently that the Periodic Times are also equal one to another. Or we will rather demonstrate it thus: Let the same absolute Velocity be supposed in the mean Distance, which is in a Circle describ'd with the same Semi-diameter; the Angle then, according to the Conic Properties, or Area described in the Circle, will be to the Angle or Area describ'd at the same time in the Ellipsis, as the greater Semi-axis is to the lesser; and in the same Proportion also, according to the Conic Properties, is the entire Area of the Circle to the entire Area of the Ellipsis. From whence, because of an equable Description of Areas on both Sides, the Periodic Times also will be on both Sides equal.

Coroll. (3.) Therefore the Periodic Times in Ellipses are between themselves in the Sesquialteral Proportion of the greater Axes, as well as in Circles.

Coroll. (4.) Consequently the greater Axis being given, there is given withal the Periodic Time.

Coroll.

Coroll. (5.) Seeing the Proportion of the Curvature and Celerity in a Parabola and Hyperbola is the same, with respect to the Focus; by the same Reason as before, a Body will be mov'd in a Parabola and Hyperbola about the Focus.

Scholium. Having now dispatch'd in a more easy Method, the Demonstrations of the fundamental Propositions of Sir *Isaac Newton*; I will take Liberty for a Conclusion, to adjoin another Demonstration of the last Proposition, which is the most Noble of all, and most of all accommodated to the Mundane System; which Demonstration comes more up to Geometrical Rigor, and is that which I once transcrib'd from a Manuscript of Sir *Isaac Newton*'s himself.

A Proposition.

If any Body whatever be attracted towards the Focus of an Ellipsis, and if the Quantity and Proportion of the Attraction be such, that they make the Body to revolve in the Elliptic Perimeter; the Attraction in the least Distance will be to the Attraction in the greatest, both Distances being taken at the greater Axis, as the Squares of the Distances of the Body in those Points from the Focus of the Ellipsis reciprocally.

Let (in *Fig.* 2. *Plate* 5.) A E C D be an Ellipsis; A and C the Extremities of the greater Axis: F the Focus whither the centripetal Force tends; and A F E, C F D, those Areas which a Body doth by Radii drawn to the Focus, describe in an equal Space of Time. Now those Areas are equal one to another, as being proportional to equal Times; that is, Prop. 15.

the

Scholium *to* I. 41. Elements. VI. 14. Elements.

the Rectangles $\frac{1}{2}$ A F $\times$ A E, and $\frac{1}{2}$ F C $\times$ D C, are equal to each other; that is on the Hypothesis that the Arches AE and C D are taken so small, that they may safely enough be reckon'd for Right Lines. Therefore A E is to C D, as F C is to F A. Let us now suppose the Right Lines A M and C N to touch the Ellipsis in the Points A and C, and the little Lines E M and D N [to be supplied in the Figures] to be from the Points E and D perpendicular to those Tangents. Now because the Curvature of Ellipses (that is, if we consider it in general, and in equal Arches with respect to the Center of the same) is equal at both Extremities, these Perpendiculars E M and D N will be betwixt themselves, (*Coroll.* 4. *Prop.* 2.) as the Squares of the Arches A E and C D. E M therefore is to D N, as F C square is to F A square. But in the same time, in which the Body will from the Force of Attraction describe the Elliptick Arches A E and C D from A to E, and from C to D; the same without that Attraction would have described the Tangents A M and C N equal to those Arches. The Forces of the Attractions therefore which draw the Body back from the Tangents to the Curve, to wit, from M to E, and from N to D, are also betwixt themselves as those little Lines M E and N D subtending the Angles of Contact, which are describ'd at the same time. The Attraction therefore at the Point A is to the Attraction in the Point C, as the little Line M E to the little Line N D; that is, by the Things already demonstrated, as F C square is to F A square; or as the Squares of the Distances reciprocally. *Q. E. D.*

This Demonstration respects only the Extremities of Ellipses; those which follow will apply the same Proposition to any Parts of Ellipses whatsoever.

Lemma. If a right Line touch an Ellipsis in any Point whatsoever, and if a Line be drawn through the Centre of the Ellipsis parallel to the Tangent, which may intersect a 3d Line drawn through the Point of Contact and either of the Foci; that Part of the 3d Line which is posited betwixt the Contact, and the Intersection, will be equal to half the greater Axis.

Let (*Fig.* 2. *Plate* 5.) A P C Q be an Ellipsis: A C the greater Axis: O, the Center: F f the Foci; P the Point of Contact: O G the Line parallel to the Tangent; and P G that Part of the Line F P, which lies betwixt the Contact and the parallel to the Tangent. I say that P G is equal to C O, or to half the greater Axis.

For join the Points P F; and draw the Line f H parallel to O G. Because the Lines F f and F H are bisected in the Points O and G, A C will be equal to the Sum of the Lines P F and P f, that is, to the Sum of the two Lines P F and P H, (by the Conics) or to the double of the Line P G. And therefore the half of A C, that is C O, is equal to P G. *Q. E. D.*

Another Lemma. Any right Line whatever drawn through either of the Foci of the Ellipsis to the Periphery, is to the Diameter of the Ellipsis which is parallel to the same, as the same Diameter is to the greater Axis of the Ellipsis.

Let A P C Q (*Fig.* 2. *Plate* 5.) be an Ellipsis: A C the greater Axis: F f the Foci: O the Center: P Q any Line drawn through the Focus F:

VOS the Diameter of the Ellipsis parallel to PQ. Here PQ. VS. AC, will be ∺. For let fp be drawn parallel to QFP, cutting the Perimeter of the Ellipsis in the Point p. And join the Points Pp by the Line Pp cutting VS in the Point T. Then draw the Line PR which may touch the Ellipsis in the Point P, and cut the Diameter VS produced in the Point R. There will now be by the Conics OT: OS: OR ∺. But OT is half the Sum of FP and fp, or of FP and FQ: and consequently OT doubled is equal to PQ. OS also doubled is equal to VS, and by the *Lemma* before demonstrated, OR, or PG doubled, is equal to AC. Wherefore PQ: VS: AC are ∺. *Q. E. D.*

Coroll. $AC \times PQ = VSq = 4\,OSq.$

Lemma. (3.) If the right Line FP be drawn from either of the Foci of the Ellipsis to any Point in the Perimeter thereof: And to the Point P the Tangent of the Ellipsis Px; and if the little Line xy (See *Fig.* 3. *Plate* 5.) subtending the Angle of Contact, parallel to the Line PQ; the Rectangle of the subtending little Line, and of the same Line produc'd to the remoter Part of the Perimeter, is to the Rectangle of the greater Axis of the Ellipsis, and of the first Line which was produc'd also to the Perimeter of the Ellipsis, as the Square of the Perpendicular Distance betwixt the subtending little Line, and the first Line is to the Square of the lesser Axis.

For let AKBL be an Ellipsis: AB the greater Axis: KL the lesser: C the center: Ff the Foci: P any Point design'd in the Perimeter: FP the first Line drawn through the Focus F to P: PQ the same Line produc'd unto the Elipsis: Px the Tangent: xy the little Line subtending the Angle of Contact: xI the same subtense pro-

produc'd to the Remoter part of the Perimeter: y z the Perpendicular Diſtance of the Subtenſe and the firſt Line. Theſe things ſuppos'd, I ſay that the Rectangle y x I is to the Rectangle A B x P Q, as is y z Square to K L Square. For let V S a Diameter of the Ellipſis be parallel to the firſt Line, and G H another Diameter parallel to the Tangent S x, or the conjugate Diameter to the former Diameter. The Rectangle yxI will then, by the Conics, be to P x Square, or the Square of the Tangent, as the Rectangle S C V is to the Rectangle G C H; that is, as S V Square is to G H Square: Now, by the Conics, all Parallelograms deſcrib'd about the conjugate Diameters of every Ellipſis are alſo equal betwixt themſelves. From whence it follows, that the Rectangle of the double of P E drawn into G H will be equal to the Rectangle of the Axes A B x K L: And conſequently (by VI. 14. of the *Elements*) G H is to K L as A B, that is, by the firſt *Lemma*, as the double of P D is to the double of P E; or, becauſe of the likeneſs of the Triangles yzP and P E D (when the Point y coincides with the Point P) as P x is to yz. Therefore P x is to G H as y z to K L: And conſequently P x Square is to G H Square, as y z Square is to K L Square, (VI. 22. of the *Elements*). But by what hath been already aſſum'd, P x Square is to G H Square, as the Rectangle y x I to S V Square: And S V Square (by the *Corollary* of the 2d *Lemma*) is equal to the Rectangle of A C x P Q. Therefore the Rectangle y x I is to the Rectangle A C x P Q, as y z Square is K L q. *Q. E. D.*

Coroll. (1.) If y z be given, and conſequently y z Square, y x Square will alſo be given, and conſequently y x it ſelf: That is, if the leaſt Perpendicular Diſtance of a Point taken in the

Elliptic Perimeter from the Line drawn through the Focus, be given in divers Distances from that Focus whatsoever; there will also be given the vanishing little Line, subtending in the same Place the Angle of Contact. For by what hath been just now demonstrated, since y z is given by the Hypothesis, and K L is also given; and since, as the Rectangle y x × x I is to the Rectangle A C × P Q, so is y z square to K L square: It will also be, the Line x I ending at last in the Line P Q, as y x × P Q is to A C × P Q, so is y z square to K L q: But as y x × P Q is to A C × P Q, (VI. 1. *Elem.*) so is y x to A C. Therefore, as y x is to A C, so is y z square to K L square; and by inverting, as K L square is to y z square, so is A C to y x; since therefore the rest of the Things are given, the Subtense y x will also be given. *Q.E.D.*

Coroll. (2.) I may also in this Place infer, that the Curvature of an Ellipsis with respect to the Focus is every where in the Proportion of the Distance from the Focus directly. For since y z the vanishing Subtense of the Angle of Contact in a given Perpendicular Distance, in all Distances from the Focus is the same; y x in Distances proportional to the Radius F P at equal Angles, will be (by *Coroll.* 4. *Prop.* 2.) in the duplicate Proportion of the Radii. From which duplicated Proportion, there being taken away, as it ought to be, the Proportion of the Radius, the Proportion of the Curvature in divers Distances will be left; the same, to wit, with the direct Proportion of the Radii. Although therefore the Curvature of divers Circles in the same Angles, is with respect to the Center every where equal; yet in Ellipses, on the contrary, it is continually changed in divers Distances from the Focus, and in a greater Distance becomes greater, in a lesser Distance less;

and this in the Proportion of the Increaſe or Dimunition of the Diſtance; as we noted before.

Coroll..(3.) To conclude, we may transfer both the foregoing *Corollaries* to a Parabola and Hyperbola: For what Things have been once demonſtrated of an Ellipſis, are to be underſtood to agree to a Parabola alſo; becauſe of the Coincidence of Ellipſes infinitely long with Parabola's: And then the Affections of Ellipſes and Parabola's, becauſe of the mutual Agreement of all Conic Sections, are to be applied to Hyperbola's, changing thoſe Things which the nature of the Line requires to be changed. Wherefore I may now aſſert, that the vaniſhing Subtenſe of the Angle of Contact at all equal Perpendicular Diſtances from the Radius, is in every Conic Section always equal to it ſelf; and that the Curvature conſequently in equal Angles is in the direct Proportion of the Diſtances.

Feb. 5. 170¾.

LECT. XV.

SCholium. By almoſt the ſame Reaſoning which Sir *Iſ. Newton* made uſe of for finding out the Proportions of the vaniſhing Subtenſes with reſpect to the Focus of the Ellipſis; I may undertake to determine the Proportions of the ſame Subtenſes in Ellipſes with reſpect to the Center, by

Princ. Mathe. Book 1. Pr. 10.

by what he hath demonſtrated; *viz.* that y z ſquare, drawn into S C ſquare, (*Fig.* 4. *Plate* 5.) and then applied to the Line y x; is equal to the double of K C ſquare drawn into C B ſquare, and then applied to the Line S C; or y z q × S C cube =2 K C q × C B q × y x. If therefore z y be given, and conſequently z y ſquare, becauſe of the Solid 2 K C q × C B q which is alſo given; y x will be every where as S C cube, or in the triplicate proportion of the Diſtance directly. If therefore z y be [illegible] as it ought to be; becauſe the Subtenſe of the Angle of Contact, is in the duplicate Proportion of the Arch, the Subtenſe y x will be in the quintuplicate Proportion of the Diſtance; or the Proportion of the Diſtance being Subducted, the Curvature it ſelf will ſtill be in a quadruplicate Proportion of the Diſtance directly; or as the biquadrate of the Diſtance directly.

Another Propoſition. If a Body be drawn to one of the Foci of an Ellipſis, and from that attraction revolve in the Elliptic Perimeter, the Forces of the attraction will be every where, as the Squares of the Diſtances from the ſame Focus directly.

For (in *Fig.* 4. *Plate* 5.) Let P be the Place of a Body revolving in an Ellipſis at any Moment of Time, and P X the Tangent of the Ellipſis in that Point; along which Tangent the Body would go forward with an uniform Motion, if it were affected with no attraction: Let the Point X be the Place whither the Body would reach in a given very ſmall Space of Time; and let Y be the Place in the Perimeter of the Ellipſis, whither it doth from both Forces actually reach in the given Time. Let the Time be divided into very ſmall qual Parts, that

that they may be accounted Physical Moments or Points: Let the attraction now act not continuedly, but by Intervals, but those very little ones; once to wit, in the beginning of every Physical Moment; so that the first Force of the Attraction may Act at the Point P; the 2d at Y; and so at equal Intervals perpetually: So that the Body may be mov'd along the Chord of the Arch PY, and then along the Chord of the following Arch, and so on. Now because the attraction in the Point P is directed towards the Point F, and draws down the Body from the Tangent PX unto the Chord PY; the little Line XY produc'd by that Force of the attraction in P will be proportional to that Force, and parallel to its Direction, that is to the Line PF. Produce the Lines XY and PF unto the Elliptic Perimeter in I and Q; join the Points F, Y: and let YZ be let down Perpendicular to FP. (*Fig.* 4. *Plate* 5.) Let AB be the greater Axis of the Ellipsis, and KL the lesser. And by *Lemma* the 3d, the Rectangle YXI will be to the Rectangle $AB \times PQ$, as is YZ square to KL square. And consequently the Line YX will be equal to the Solid of $AB \times PQ \times YZq$, applied to the Solid $XI \times KLq$. In the same manner, if py be the Chord of another Elliptic Arch, which the Body describes in the given Physical Moment of Time equal to the former; and px the Tangent of the Ellipsis in the Point P; and xy the Subtense of the vanishing Angle of Contact parallel to the Line pF; and if xy and pF cut the Perimeter of the Ellipsis in q and i; let yz from the Point y be let down perpendicular to pF; the Subtense yx will by the former Reason be equal to the Solid made of AB into $pq \times yz$ squared, applied to the Solid made of

$xi \times KL$ squared; that is, because AB and KL are given and standing Quantities, as $\frac{PQ}{XI}YZ$ squar'd to $\frac{pq}{xi}yz$ squared. But because the Lines PY, py, are described by a revolving Body in equal Times, the Areas described, or Triangles PYF, pyF are equal: And the Rectangles $PF \times YZ$, and $pF \times yz$ are double those Triangles they are equal: And YZ is to yz, as pF to PF; and consequently $\frac{PQ}{XI}YZ$ squared, is to $\frac{pq}{xi}yz$ squar'd, as $\frac{PQ}{XI}pF$ squared is to $\frac{pq}{xi}pF$ squared.

Therefore, as YX is to yx, so is $\frac{PQ}{XI}pF$ squared to $\frac{pq}{xi}PF$ squared; that is, the Attraction in P is to the Attraction in p, as $\frac{PQ}{XI}pF$ square is to $\frac{pq}{xi}PF$ squared. Now, suppose the Times taken infinitely small to be equal, in which a Body describes the Subtenses PY and py; so that the Attraction may be continued, and the Body revolv'd in the Perimeter of the Ellipsis:

In this Case the Lines PQ and XI coincide; but pq and xi have been already supposed equal, therefore the Quantities $\frac{PQ}{XI}pF$ squared, and $\frac{pq}{xi}PF$ squared, become pF squared and PF squared. Therefore the Attraction in P, or the Line XY, will be to the Attraction in p, or the Line xy, as pF squared is to PF squared; or reciprocally, as the Squares of the Distances from the Focus. *Q. E. D.*

And

And the ſame Propoſition may be applied to the Parabola, as the Extreme of Ellipſes, and alſo to the Hyperbola: But ſince there are no Cœleſtial Bodies that we know of, that are carried about in Hyperbola's; I ſhall not ſearch out for a particular Demonſtration of them.

XXII. The Velocity of a Body moving in a Parabola about a Body placed in the Focus, the Force whereof is in the reciprocal duplicate Proportion of the Diſtances, is every where to the Velocity of a Body revolving in a Circle in the ſame time, in the ſubduplicate Proportion of the Number, Two to Unity; or as the Diagonal of a Square to its Side; that is, as 10 to 7 nearly.

For, ſince the Diſtance of a Body from the Central Body was ſuppoſed every where the ſame, the Force of Attraction, or Line ſubtending any Angle of Contact, will be always equal in any given Space of Time: And the Velocity in a Parabola will be to the Velocity in the Circle, as the Tangent of the Parabola to the Tangent of the Circle; to wit, where the Subtenſe is every where equal. But the leaſt Tangent in the Parabola, by the Conics, is equal to the ſquare Root of the Rectangle of the Subtenſe drawn into the Latus Rectum of that Vertex. And the leaſt Tangent in the Circle is equal to the ſquare Root of the Rectangle of the Subtenſe drawn into the Diameter of the Circle. But becauſe both Subtenſes are given, and the Latus Rectum of the Vertex of the Parabola is, by the Conics, double to the Diameter of the Circle; or as two to one: The firſt Rectangle will be double to the laſt, or as 2 to 1; from whence the Tangents or ſquare Roots will be among themſelves, as the ſquare Root of the Number two to one, or as the

III. 36. *Elem.*

Diagonal

Diagonal of a ſquare to its Side; that is as ten to ſeven nearly. *Q. E. D.*

Coroll. (1.) Since therefore the Velocity in a Parabola is to the Velocity in a Circle, at the ſame Diſtance from the Focus, in a given Proportion; to wit, as ♈. 2 to 1. And ſince the Velocity in divers Circles is in the ſubduplicate reciprocal Proportion of their Radii, the Velocity of a Body deſcribing a Parabola at divers Diſtances from the Focus will alſo be in the ſubduplicate reeiprocal Proportion of the Diſtances.

Coroll. (2.) The Velocity of a Body revolving in an Ellipſis, is leſs than in a Parabola; and in an Hyperbola, greater at the ſame Diſtance from the Focus: From whence the Velocity in an Ellipſis, will be to the Velocity in a Circle in a leſs Proportion than ♈. 2 to 1; and in an Hyperbola, in a greater at the ſame Diſtance from the Focus.

Coroll. (3.) Therefore the Velocity of a Body, at any Diſtance from the Focus, being known, the Figure of its Trajectory may be alſo known; to wit, whether it be a Circle, Ellipſis, Parabola, or Hyperbola: And from a more accurate Calculus, if it be an Ellipſis, or Hyperbola, what Species of thoſe Figures it is that a revolving Body ought to deſcribe.

Coroll. (4.) It follows from what was juſt now demonſtrated, that if any Body be moved according to any right Line whatever (unleſs it tends directly to the Focus) with any Velocity, and be acted upon at the ſame time by a Centripetal Force reciprocally proportional to the ſquare of the Diſtances from the Center; the Body will be moved in one of the Conic Sections, having the Focus in the Center of Forces: To wit, if the Line, according to which the Projectile Motion of the Body tends, be perpendicular to the Radius, and the Velo-

Velocity equipollent to the Attraction; that is, if the Velocity in any given very small Space of Time, be equal to the square Root made of the Subtense of the Angle of Contact of that Circle drawn into its Diameter, that Body will be moved in a Circle. But if the Velocity be equipollent to the Attraction, and the Line of Direction oblique to the Radius, the Body will be moved in an Ellipsis, the periodic Time whereof wou'd be equal to the periodic Time of the Circle in which it will move. But if the Velocity be greater or less than the Velocity before assigned, so nevertheless where it is greater, that it be not increased above the Proportion of the square Root of the Number Two to Unity, that Body will be moved in an Ellipsis, in the first Case greater, and in the last lesser than the Circle. But if the Velocity be to the Velocity in the Circle, as the square Root of 2 to 1, the Body will be mov'd in a Parabola. If, lastly, the Velocity be greater, the Body will be mov'd in an Hyperbola.

XXIII. *Problem.* The Centripetal Force being reciprocally proportional to the square of the Distance from the Center, to define the Times in which Bodies in falling strait down wou'd reach the Center. (See *Fig.* 5. *Plate* 5.)

Upon the same principal Axis or transverse Diameter, A B, let there be described the Extreme Ellipses; to wit, the Circle A D B, and the Right Line A B; from the Equality of these Transverse Diameters, the Periodic Times will be equal on both Sides; and consequently the Times of the half Revolutions will be equal to one another. (*Coroll.* 4. *Prop.* 21.) That is, the Time of Descent by the Diameter, is equal to the Time of Revolution along the Semi-circumference. Since therefore by what hath been de-

mon-

monstrated before, it is easy to determine that Time of the half Revolution, it will also be easy to define the Time of the direct Descent. As for Example: The Time of the Lunar Half-period contains 19671|5 Minutes; where, to wit, the Diameter of the Orbit is double to the mean Distance from the Center of the Earth. And this Time is to the Time of the Half-period for half the Distance, which is the thing we now enquire for, in the sequi-alteral Proportion of the Distances; that is, almost as 2828 is to 1000, or as 19671|5 is to 6955|5. From whence the Time of the Half-period, in half the Distance (or in case the Diameter of the Orbit was but half of what it is,) that is, the Time of the direct Fall to the Center of the Earth of a Body placed at that Distance from the Earth in which the Moon is really placed, will contain 6955|5 Minutes; or 4 Days, 19 Hours, 55′, 30″. In this Space of Time would the Moon, if the Motion thereof was stopp'd, and the Earth remain'd unmoveable, fall to the Center of the Earth. And by the same Reasoning, the Time of the Fall of any Planet may easily be determined, as is actually done in the following *Scholium.*

Scholium. Since therefore the Time of the Half-period of every Planet diminish'd in the Proportion of 1000 to 2828, is the Time of the direct Fall to the Center, the following Table, which is built upon that Foundation, will shew the Times of the Fall of all the Planets to their respective Centers.

Mer-

		Day.	Hours.
Mercury	would fall to the Sun in the Space of	15	13
Venus		39	17
The *Earth*		64	14
Mars		121	11
Jupiter		767	3
Saturn		1900	4

Of the Planets about *Jupiter*.

The inmost	would fall to *Jupiter* in the Space of	00	7
The Second		00	15
The Third		1	6
The Fourth		2	23

Of the Planets about *Saturn*.

The Inmost	would fall to *Saturn* in	0	8
The Second		0	12
The Third		0	19
The Fourth		2	20
The Fifth		14	1

The Moon, as above, would fall to the Earth in the Space of 4 Days, 20 Hours.

Feb. 19. 170$\frac{3}{4}$.

LECT.

Lect. XVI.

XXIV. A *Problem.* Suppoſing that the Centripetal Force is reciprocally proportional to the ſquare of the Diſtance, to define the Spaces which Bodies falling right downwards would deſcribe in any given Time.

If the Body doth not fall perpendicularly, it will deſcribe ſome Conic Section, the nether Focus whereof (becauſe of the Deſcent of a projectile Motion which is here ſuppos'd) will agree with the Center of Force, as is manifeſt from what goes before, *Prop.* 21. Let that Conic Section (ſee *Fig.* 5. *Plate* 5.) be the Ellipſis ARBP; where the Velocity of the Projection is to the Velocity wherewith the Body would revolve in a Circle at the ſame Diſtance, in a leſs Proportion than is the ſquare Root of the Number Two to Unity, (*Coroll.* 2. *Prop.* 22.) Let S be the nether Focus of the Ellipſis, and upon the greater Axis of the Ellipſis AB, let there be deſcrib'd a Semi-circle ADB. And the right Line DPC being ſuppos'd to paſs through the falling Body perpendicular to the Axis, and the Lines DS and PS being drawn to the Center, the Area ASD will be proportional to the Area ASP, and conſequently to the Time. For (by VI. 1. *Elements*,) as CD is to CP, ſo is the Area of the Triangle SCD to the Area of the Triangle SCP. And according to the Conics, as the ſame CD is to the ſame CP, ſo is the Circular Area CAD to the Ellip-

Elliptic Area C A P; and consequently A S D the Sum of the former Areas, will be to A S P the Sum of the latter, as C D to C P, (V. 12. of the *Elements*;) or as the greater Axis of the Ellipsis to the lesser Axis of the same; and consequently in the given Proportion, or proportional to the Time. Now A B the greater Axis of the Ellipsis, or Diameter of the Circle remaining, let the Latitude or lesser Axis of the Ellipsis be perpetually diminish'd; here, by the Force of what hath been already demonstrated, A S D will still remain proportional to the Time. Yea, let the Latitude be diminish'd infinitely, so that the Elliptic Orb A P B may at length coincide with the Axis A B: and the Focus S with B the Term of the Axis: the Body will descend in the right Line A C: and the Area A B D will become in this Case also proportional to the Time. From whence, if a right Line perpendicular to the Axis as C D, be suppos'd to be mov'd downwards always parallel to it self, so that the Area A B D should be every where proportional to the Time, the Point C will determine the Place, unto which the Body would reach that falls downwards to the Center in the same given Time.

As for Example. Let A B the mean Distance of the Moon from the Center of the Earth, be as before about 1257 696 000 feet; it is required that we should determine the Place of the Moon falling straight down, in the first Day of the Fall. It is known from what hath been already demonstrated, (*Corol.* 7. *Prop.* 2.) that if the Motion of the Moon should cease, it would fall about 16$\frac{1}{12}$ *English* Feet in the Space of one Minute. From whence (by *Coroll.* of *Prop.* 5. Select Prop. out of *Archimedes*,) the Circular Area A B D will be of about 89 483 812 704 000 square Feet, [as being equal

equal to the Rectangle of cd drawn into the half of AB. From whence, seeing there are 1440 Minutes in a whole Day, the Circular Area ABD belonging to the whole Day will be of about 128856690293760000 square Feet; but the given Time is one whole Day, or 1440′. If therefore we can define the Point D, so that the Area ABD should be of 128856690293760 000 square Feet, the Sine of the Arch AD, that is, CD will determine the Line AC which is describ'd in that Time, as being the versed Sine of the same Arch. But that Area is equal to the Rectangles $\frac{1}{2}$CD×OB, and $\frac{1}{2}$AD×OB, or to the Rectangle $\frac{1}{2}$ CD × $\frac{1}{2}$ AD × OB. If therefore the given Area be divided by the Semi-diameter OB, the Quotient will give the half of CD and OB. From the Table of Sines therefore that Arch is to be sought, the half of which being superadded to the half of its own Sine, will yield that Quotient. But that Quotient is by Calculation of 204909 120 Feet; or by reducing it to a Circle, whose Radius is of 10000000, will contain 3258484 of those Parts. And if in the Tables of Sines, we look for the Sine of 19 Degrees and almost 51 Minutes, the Sine of one Minute multiplied by 11302909 × 1130, will give 3287170 Parts, as belonging to the Arch AD, which is of 18 Degrees and 50′, the Sine of which Arch is of 3228165 Parts; so that the Sum of both will be of 6515335′; the half whereof is 3257667, which agrees with the first Number 3258484 exactly enough. The Line CD therefore is the Sine of 18° 50′, and the Line describ'd in that Space of Time is the versed Sine thereof, which is 535382 Parts long; that is, by reducing it to the Semi-diameter of the Moon's Orbit 33667390 Feet long, that is,

6376

6,376 Miles and 2,110 Feet. And in the same manner it will be defin'd, in what time the Moon would descend to the very Center of the Earth. But because we have deduced that before by another and more easy way of Computation, we shall not prosecute it any further here.

Coroll. If the Figure R P B be not an Ellipsis, but an Hyperbola or Parabola, the thing will be dispatch'd in the same manner by a Rectangular Hyperbola, or any Parabola; but by reason of the Difficulty of the Practice, and that it is not necessary, we shall pass it by here.

Coroll. (2.) The Times wherein any Bodies would fall to the Center from divers Distances, are betwixt themselves in the Sesqui-alteral Proportion of those Distances directly. For the Line A c, that in a given Time is produc'd at divers Distances, is reciprocally in the duplicate Proportion of these Distances. From whence c d the least Sine, will be in the subsesquiplicate Proportion of the Distance reciprocally; and the Area $\frac{1}{2}$ c d $\times$ A B describ'd at the same time, in the sub-duplicate Proportion of the Distance directly. From whence, since the entire semi-circular Area A D B is in the duplicate Proportion of the Distance directly, the Time proportional to the same will be in the sesquiplicate Proportion to the Distance directly. *Q. E. D.*

As for Example. Let another A B be double to this A B; then the vanishing Subtense of the Angle of Contact, or the little Line A c, will be only a 4th Part of the other A c: And the Sine c d will be sub-sesquiplicate of c d, or as the Side of the Square to the Diameter; that is, 7 to 10 almost: the Area also $\frac{1}{2}$ c d $\times$ A B will be to $\frac{1}{2}$ c d $\times$ A B well-nigh, as $2 \times 7 = 14$ is to $\times$ 10 $= 10$. From whence, the Area described in the

 greater

greater Diſtance will be to the Area in the leſs, but which is deſcribed in the ſame time, nearly as 14 to 10, or as the Diameter in a Square is to the Side. But the entire Area to be deſcribed by the greater Line AD in the Deſcent, is to the Area to be deſcribed by the leſſer Line BD in the Deſcent, as 4 to 1, or 40 to 10. Therefore the Time of the Deſcent in the greater Diſtance, will be to that in the leſſer, in that Proportion in which the Ratio of 40 to 10 exceeds that of 14 to 10. But the Proportion of that Exceſs is the ſame as that of 40 to 14, or as the Diameter of a Square is to the Quadruple of the Side. From whence the Lines are betwixt themſelves, as the Diameter of a ſquare is to the Quadruple of the Side; that is, in the ſeſqui-alteral Proportion of the Diſtances directly. *Q. E. D.*

Coroll. (3.) If therefore we ſuppoſe any one of the primary Planets, as alſo of the Secondaries of *Saturn* and *Jupiter*, to fall to that Center of its Orbit, and have the Times of that Deſcent already defin'd and computed; it will be eaſy from the known Diſtances of the reſt, to define the Times alſo of their Deſcent; which thing we have perform'd before upon another Ground, and therefore ſhall not repeat it again.

Coroll. (4.) Since therefore the Velocity in an Ellipſis in a mean Diſtance from the Focus; that is, the Velocity of a Body falling to O, the Center of an Ellipſis when it ends in a right Line, is equal to the equable Velocity of a Body revolved in a Circle, the Radius whereof is BO; it is manifeſt, that the Velocity of a Body falling in O, the very middle of the Space, is equal to the Velocity of a Body revolved in a Circle at the ſame Diſtance. From whence it alſo follows, that the Velocity of a Body falling at a remoter Diſtance,

is

is less than the circular Velocity, and at a nearer Distance greater.

XXV. *A Problem.* If the Centripetal Force be proportional to the Altitude, or the Distance of Places from the Center directly, to define the Times in which Bodies falling down will describe any given Spaces.

If the Body doth not fall perpendicularly, it will describe some Conic Section, the Center whereof will agree with the Center of Force, as appears from what hath been already said, *Prop.* 19. Let (*Fig.* 6. *Plate* 5.) the Conic Section be the Ellipsis A R P B: Let O be the Center thereof; and upon A B the greater Axis of the Ellipsis, let there de described the Semi-circle ABND, and let the right Line D P C pass through the falling Body perpendicular to the Axis. Which done, and the Line DO and P O being drawn to the Center, the Area A O D will, by the Conics, be proportional to the Area A O P, and consequently to the Time. For, as before (by VI. 1 *Elements*) as C D is to C P, so is the Area of the Triangle O C D to the Area of the Triangle O C P. And also, by the Conics, as the same C D is to the same C P, so is the circular Area C A D to the Elliptic Area C A P; and consequently, the Sum of the former Areas A O D will be (V. 12 *Elements*) to A O P the Sum of the latter Areas, as C D is to C P; or, by the Conics, as the greater Axis of the Ellipsis is to the lesser Axis of the same; and consequently in a Proportion given, proportional to the Time. Now A B the greatest Axis of the Ellipsis or Diameter of the Circle remaining, let the Breadth of the Ellipsis, or its lesser Axis, be continually diminish'd; and by the Force of what hath been already demonstrated, the Area A O D will remain proportional to the Time. And let that Breadth be diminiſh'd infinitely; ſo that the Elliptic Orb A R B P may now fall in with the Axis, the Body will deſcend in the right Line A C, and the Area A O D will in this Caſe alſo be proportional to the Time. From whence, if a right Line perpendicular to the Axis, as C D, be ſuppos'd to be mov'd always downwards parallel to it ſelf, ſo that A O D may be every where proportional to the Time, the Point C will determine the Place unto which the Body in falling down will reach in the given Time.

Coroll. (1.) Becauſe of the Equality of the circular Area that is every where to be deſcribed in equal Time about the Center of the Circle; the Motion of the Point D will always be equable, and will deſcribe equal Arches in a given Time.

Coroll. (2.) The Times therefore of Bodies falling and deſcribing what Spaces ſoever, as A C, are betwixt themſelves as the Arches themſelves A D; and the Spaces deſcribed A C, are as the verſed Sines of thoſe Arches.

Coroll. (3.) But the Velocities produced in any Places whatever, as in C, are as the right Sines of the Arches A D. For let the Line c d be drawn parallel to C D, at a Diſtance infinitely ſmall, and let d D the Tangent of the Circle be drawn. Whilſt therefore the Point D deſcribes the Tangent d D, the falling Body deſcribes the little Line c C equal to d e; and becauſe of the given Velocity of the Point D, d D will alſo be given in length for the given Time. In the Triangle therefore d e D, d D the Radius of the Circle will be given, and d e the right Sine of the Angle d D e. And becauſe of the Likeneſs of the Triangles d e D, C O D, the Radius in that

that Place will be O D, and the Line C D the right Sine of the Angle A O D; Therefore the Velocity in all Points whatever, as C, is as the right Sine of the Arch A D. *Q. E. D.*

Coroll. (4.) The Times in which Bodies fall from any Places whatever to the Center, are always equal. For since, by the Hypothesis, the accelerating Force, and consequently the Velocity arising, is as the Line to be describ'd; it is manifest, that the Times of Descent are every where equal. *Q. E. D.*

Coroll. (5.) Since, by what hath been demonstrated before, (*Corol.* 3. *Prop.* 19.) the periodic Times of all Bodies revolving about the Center of Ellipses are equal, the Quarters also of the periodic Times A B P V will be equal. And since this is true in all Ellipses whatever, it will be true also in the Extremes of Ellipses on this side, and on that; to wit, in the right Line A O, and the Quadrantal Arch A N; that is, the Times wherein one Body in falling comes from any Place whatever, as A unto O, and another in revolving describes a Quadrantal Arch, will be equal every where. *Q. E. D.*

Scholium (1.) Since therefore the periodic Time of the Moon about the Earth, is (*per Schol. Prop.* 14.) to the periodic Time of any other Body revolving about the Center of the Earth at the Distance of a Semi-diameter of the Earth, in the sesqui-alteral Proportion of the Distances; and since within the Surface of the Earth, the Centripetal Force is every where in the direct Proportion of the Distance, as will hereafter be demonstrated: It will not be unpleasant to produce an Example of the foregoing Reasoning: and to shew by Calculation, in what Space of Tine heavy Bodies would descend to the Center, down ſome empty Hole or Pit which reaches thither. For the finding therefore, according to what hath been already demonſtrated, the Quarter of the periodic Time at the Surface of the Earth, as being the Time of the Deſcent of Bodies from the Surface to the Center of the Earth: Let it be made thus; as is the Cube of the Moon's Diſtance, $60 \times 60 \times 60 = 216000$, to the Cube of the Semi-diameter of the Earth $1 \times 1 \times 1 = 1$; ſo is the ſquare of the Moon's Period $39343' \times 39343' = 1\,547\,871\,649$ to the ſquare of the Period in the Surface of the Earth $= 7\,166\llcorner 07$, the ſquare Root whereof $84\llcorner 6$ will yield the Minutes in Time, in which a Body or Planet at the Diſtance of the Semi-diameter of the Earth from the Center would perform its whole Period about it. The Quarter of which Number $21\llcorner 15$, will ſhew the Space of Time in Minutes, in which all heavy Bodies whatever would fall thro' the Semi-diameter of the Earth to the Center of the ſame. And ſince in all Diſtances whatever, the Time of falling is the ſame, as hath been already ſhew'd, (*Coroll.* 4. foregoing); it is manifeſt, that all Bodies would deſcend, and fall from any Place to the Center in 21 Minutes, and 15 Seconds.

Schol. (2.) But if the Time of the Fall through any given Space whatever be requir'd to be found without the Uſe of *Algebra*; as for Example, thro' a 4th Part of the Semi-diameter; ſeek in the Tables of Sines, what Angle that is, the verſed Sine whereof is a 4th Part of the whole right Sine; to wit, the Arch A D which is of $41^\circ\ 25'$. From whence the Time of the Fall (ſee *Fig.* 6. *Plate* 5.) along A C, the 4th Part of the Semi-diameter, will be to the Time of the whole Fall to the Center, as the Arch A D is to the Quadrantal Arch A N, (*Corol.* 2 foregoing) or as $41^\circ\ 25'$ is to 90°.

And

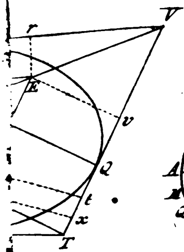
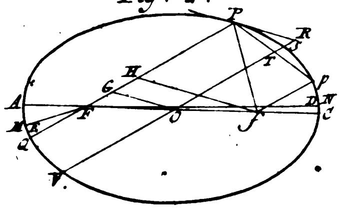


Fig. 2.



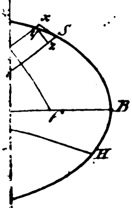
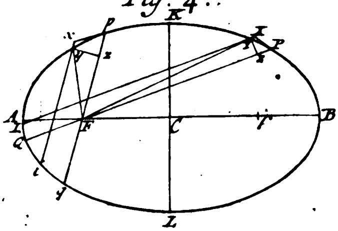


Fig. 4.



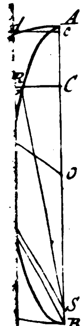
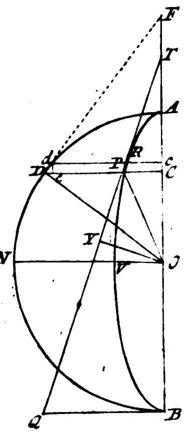


Fig. 6.



I. Senex sculp.

And since 90 : 41° 25′ : : : 21′⌊15 horary Minutes : 9′⌊97 or 9′ 58″: It is manifest, that any Body whatever would fall down a 4th Part of the Semidiameter of the Earth in 9 Minutes, and 58 Seconds. And that the Velocity in the Point C is to the greatest Velocity, or that which would be at the Center it self, in the Proportion of the Right Sine C D to the whole Sine O N, (*Corol.* 3. foregoing) or as 66153 is to 100000, as is most manifest from what hath been just now demonstrated.

April 7. 1705.

LECT. XVII.

XXVI. A *Problem.* A Body being carried about a Focus in a given parabolic Trajectory, to find the Time in which a given Arch has been or will be describ'd, whether in the Ascent or Descent.

Let F (in *Fig.* 1. *Plate* 6.) be the Focus of the Figure, T the principal Vertex, T l or T s the given Arch describ'd, or to be describ'd: T t or T q the Abscis of the same Arch, which is also given, the Arch being given: t l or q s the Semiordinate, which also is given, the Arch being given. The Time wherein the Arch T l or T s is described, is requir'd: The Parabola being given, the *Latus Rectum* of the same, and consequently T F the Fourth Part thereof is given. From the Centripetal Force of the Central Body, there is alſo given the Velocity of the Body at the principal Vertex; or that which (by *Prop.* 22. foregoing) is to the Velocity of a Body deſcribing a Circle, the Radius whereof is T F, as the ſquare Root of the Number Two is to Unity. From whence alſo there will be given the leaſt Area to be deſcrib'd by the Radius T F in any the leaſt Time which is given. But the Area FTl or FTs, is equal to two thirds of the Rectangle T t × t l, or T q × q s. To which, if there be added the Triangle F t l in the former Caſe, and in the latter the Triangle F q s be taken away from the ſame, there will alſo be given the Area F t l or F t s; which being divided by the leaſt Area deſcribed at the Vertex T, in any very ſmall Time given, will give the Time ſought.

As for Example. Let the Parabola given be that which the Comet that was ſeen in *Europe* in the Year 1680, deſcrib'd in the End of that Year, and the Beginning of the next. Let F q be equal to the Semi-diameter of the great Orb, to wit, of 10000 equal Parts, ſuch as the *Latus Rectum* contains 236|8; and conſequently F T of 59|2 Parts, and the whole Abſciſs T q of Parts 10,059|2. Let us alſo ſuppoſe the Comet to have been in the Vertex of the Parabola, or in its Perihelium T, *December* 8. Four Minutes after Noon. For the finding the Velocity of the Comet in the Vertex of the Parabola, let there be found in the firſt place the Velocity of a Planet revolving in a Circle at that Diſtance, by this Analogy; as is the ſquare Root of the Diſtance F T, of 59|2 Parts = 7|7, to the ſquare Root of the Diſtance F q, of 10,000 Parts = 100; ſo is the annual Velocity of the Earth, to the Velocity of a Planet deſcri-

See Newton, p. 494, & 498.

Prop. 13.

describing a Circle, the Radius whereof is F T. Then, as the square Root of the Number two $=1|414$ is to Unity; so will the Velocity of a Comet in the Vertex of its Parabola be, to the Velocity of a Planet in a Circle at the same Distance. But the Earth, by the mean Velocity of the same, describes $|1195$ Parts in the Space of one Minute, and $7|7 : 100 :: |1195 : 1|552$. From whence the Velocity of the Comet in its Perihelium, will be that which in the Space of one first Minute describes $\frac{1|414}{1} 1|552 = 2|19$ Parts, such as the Semi-diameter of the great Orb contains 10000, and such that $59|2$ of them are contain'd in the least Distance of the Comet. The Area therefore describ'd in that given Time by the Comet, with a Radius drawn to the Center of the Sun, is equal to the Rectangle of $\frac{1}{2}\, 59|2 \times 2|99 = 64|824$ square Parts. That therefore we may at length find the Space of Time wherein the Parabolic Arch T s, where F q is equal to the Semi-diameter of the *Orbis magnus*, is describ'd, we will compute the Area T s F, and compare it with the former Area which was describ'd in one Minute. Therefore, as T F of $59|2$ Parts is to T q of $10059|2$ Parts; so let the square of F H of $118|4$ Parts $= 14018|56$, be to $2382018|61$ square Parts; the square Root whereof $= 1543|3$ will, by the Conics, be equal to the Semi-ordinate q s: Which being drawn into half the Distance F q $1543|3 \times \frac{1}{2}\, 10000 = 7716500$, the Area of the addititious Triangle F q s will come forth. But the whole Parabolic Area T q s is equal to two 3ds of the Rectangle under T q of $10059|2$ Parts, and S q of $1543|3$ Parts, or to the square Parts of $\frac{2}{3}\, 15524363|36 = 10349575|57$. From which Number, let there be deducted the Triangle F s q of of 7716500 ſquare Parts, the remaining deſcribed Area will be of 26330755|57 ſuch Parts; which being divided by the Parts of the Area belonging to one firſt Minute, there comes forth $\frac{26330755|7}{64|824}$, the ſpace of Time ſought; or that in which the Comet would deſcribe the Arch Ts = 40611|9 = $28^d. 4^h. 59'$. Wherefore the Arch Ts will be deſcrib'd in 28 Days, and almoſt 5 Hours. And the Comet poſſeſs'd the Point s on the 5th of *January*, about 4 hours Afternoon. Which alſo exactly agrees with Sir *Iſaac Newton*'s Scheme deduc'd from Obſervations.

If therefore we, from ſuch Calculations, ſhall once have the Times rightly defin'd, wherein any Comet deſcribes any Arches whatever, as Ts of a Parabola, or rather an Ellipſis, ſo eccentric that it may ſafely be reckon'd for a Parabola, by the inverſe Method we may be able alſo to define exactly enough, the Arches belonging to any given Times; I mean the ſame way of working by which, in *Kepler*'s Hypotheſis and Tables, we are wont to find the Ceoquate Anomaly of the Planets from their mean Anomaly in a given Ellipſis.

Coroll. (1.) Seeing therefore the ablatitious Triangle Fsq vaniſheth away in the Point h, the Area to be computed at that Time will be equal to two 3ds of the Rectangle of TF drawn into Fh; or $\frac{2}{3}$ 59|2 × 118|4 = 4676|8, and conſequently the Time belonging to this Area will be equal to $\frac{4676|8}{64|824}$ = 1 h. 12′. 9″. So that Th, the Arch betwixt the principal Vertex and the Ordinate to the Axis through the Focus, was deſcribed in 1 h. 12′. 9″. And the Comet poſſeſs'd the

the Point s *December* the 8th, 17 Minutes after One in the Afternoon.

Coroll. (2.) Hence alſo the Space of Time wherein any given Arch is deſcribed, doth eaſily become known; *viz.* by computing the Time from the Perihelium to both Places, and taking away the ſhorter Time from the longer: For by that means the Interval of Time belonging to the given Arch will become known. Thus the Time agreeing to the Arch Th = 1 h. 12′. 9″. being deduc'd out of the Time agreeing to the Arch Ts = $28^d. 4^h. 59'$, the Remainder is the Interval of Time belonging to the Arch hs = $28^d. 3^h. 46'. 51''$. And ſo every where.

Coroll (3.) Hence alſo is deriv'd the Method of finding the deſcribed Arch from the given Time. For ſeeing that at the Point h the ablatitious Triangle F q s, and the addititious one F t l, doth always vaniſh away; and conſequently the Area in that Place may be eaſily computed, as being in our Example of 4667|4516 ſquare Parts: Since alſo in that Place T F is half F H; and ſince, laſtly, the Abſciſs T F doth always increaſe in the ſame Proportion, in which the ſquare of its Ordinate F H increaſeth: any Time whatever, or the Area proportional thereto, being given, the Arch belonging to the ſame will alſo be given; if that Quantity of proportional Increments or Decrements be taken, that $\frac{1}{3}$ q s × F q being taken out of $\frac{2}{3}$ q s × T q the Remainder be the Quantity of the given Area. Thus, that I may find the Arch T of $28^d. 14^h. 59''$. = 40619′, that is, that which belongeth to the Area of 2633075|57 ſquare Parts; I ſeek in the Tables of ſquare Numbers, if I would work without Algebra, where ſuch a Number is to be found, (the Line T F being taken for Unity, and the Area

F T

F T h for the square of Unity; or for $\frac{2}{3}$ T F $\times$ F h = a 563d Part of the whole Area; and F h being taken for the Number two:) That the Numbers to be added to Unity being proportional, the square of the Numbers to be added to two $\frac{1}{2}$ q s $\times$ q F being taken out of $\frac{2}{3}$ q s $\times$ T q, the Remainder may be the given Area = 563: Which Number will occur no where else, but where F q is to F T as 10000 to 59⌊2, or as 167 to 1 nearly. From whence it is manifest, that the sought Arch is no other but that of which T q of 10059⌊2 Parts is the Abscis. But since this Method consists in making Essays, and is indirect, it is not so artful. However, what hath been here delivered contains enough in it, to shew in some measure the Origin and Method of compiling Tables.

Scholium. Note, That Sir *Isaac Newton*'s Geometrical Method doth shew directly, from the given Time, the describ'd Arch; that is, if F T be made to t y, as the Time belonging to the Area T h F is to the given Time, the Point t possessing the Middle of the Line T F, and t y being drawn perpendicular to T F; the Distance from the Focus F will be equal to y s: From whence the Circle describ'd from that Radius will determine the Point. But since this Method is not so fit for Calculation, we pass it by in this Place.

See Newton, *Book I. Prop.* 30.

Scholium. Hitherto we have chiefly expounded the Motions of Bodies attracted unto an unmoveable Center, such as scarce is in the whole Compass of Nature. For Attractions are wont to be unto Bodies, and the Actions of the Bodies attracting and attracted, are always mutual and equal, as we shew'd before, (*Law of Motion* 5.); so that neither can the Attrahent rest, nor the Attracted,

tracted, if they be two Bodies; but must both, as it were, by a mutual Attraction, where the projectile Motion of both is duly impress'd upon them, be revolv'd about the common Center of Gravity. And if there be more Bodies, (which are either attracted by a single one, or attract each other mutually) these ought to be so mov'd among themselves, that the common Center of Gravity should either rest, or be mov'd uniformly in a straight Line, as we shew'd before, (*Law of Motion* 25.) For which Reason, we proceed to set forth the Motion of Bodies, as mutually drawing each other; considering the Centripetal Forces as Attractions, altho' indeed speaking physically, they may perhaps be more rightly called Impulses. For we look not here so much at the physical Causes, as at the Effect it self, considering and measuring things in a Mathematical Way, and using an easy and familiar Term, though in the strict Notion of it perhaps it may not agree.

XXVII. Two Bodies attracting one another, describe like Figures both about the Common Center of Gravity, and about one another; that is, whilst they really describe like Figures about the Common Center of Gravity, the Eye being placed in either of the two, and not perceiving its own Motion, or that of the Center of Gravity, a Figure like to the same will seem to it to be describ'd.

For the Distances from the Common Center of Gravity are reciprocally Proportional to the Bodies, and consequently in a given Proportion to each other; and by Composition in a given Proportion unto the whole Distance betwixt the Bodies. But these Distances are carried about their Terms, with a common Angular Motion,

because

because lying always in a ſtreight Line, they do not change their Inclination to one another. But Right Lines which are in a given Proportion to one another, and are carried about one another with an equal Angular Motion, deſcribe altogether like Figures about the ſame Points (in Plains which either reſt together with theſe Points, or are mov'd with any Motion which is not Angular.) And therefore the Figures which are deſcrib'd by theſe revolving Diſtances are equal. *Q. E. D.*

Scholium. Thus the Earth and the Moon are carried by a Monthly Motion about the Common Center of Gravity of both. But to us placed on the Earth, to whom neither the Motion of our own Seat, nor of the Center of Gravity, as being an Inviſible Point, is perceptible, the Moon alone ſeems to be carried about; and ſo it muſt needs happen in all the reſt of the Syſtems of the Planets.

XXVIII. If two Bodies attract one another with any Force whatever, and be in the mean while revolv'd about a common Center of Gravity; a Figure like and equal may be deſcrib'd by the ſame Force about either Body unmov'd, to the Figures which the Bodies ſo mov'd deſcribe about one another.

In *Fig.2. Plate 6.*Let S and P be revolv'd about C, the common Center of Gravity; S from S to T, and P from P to Q; from a given Point s let s p and s q be drawn, equal and parallel to S P, T Q. Here the Curve p q v, which the Point p deſcribes about the unmoved Point s, will be like and equal to the Curves, which the Bodies S and P deſcribe about each other; and conſequently, by our laſt Propoſition, like to the Curves S T and P Q V, which the ſame Bodies deſcribe about C, the

the common Center of Gravity; and this so because the Proportions of the Lines S C. C P, and S P, or s p to one another are every where given.

Case (1.) That common Center of Gravity C, either rests, or is uniformly mov'd straight forward, by the 25th Law of Motion. Let us first suppose it to rest; and in s and p let two Bodies be placed, the unmov'd one in s, the mov'd one in p; and let them be respectively like and equal to S and P. Then let the right Lines P R and p r touch the Curves P Q and p q in P and p; and let C Q and s q be drawn forth unto R and r. Here because of the likeness of the Figures C P R Q and s p r q, R Q will be to r q, as C P is to s p; and consequently in a given Proportion: Therefore if the Force wherewith the Body P is attracted to the Body S, and consequently towards the intermediate Center C, should be to that wherewith the Body p is attracted towards the Center s in that same given Proportion; these Forces would in equal Times always draw Bodies from the Tangents P R, and p r to the Arches P Q, p q by Intervals Proportional to those Forces R Q. r q; and consequently the latter Force would make that the Body p should be turn'd round in the Curve p q v, which would be like to the Curve P Q V, in which the former Force makes the Body P to be turn'd about; and the Revolutions would be compleated in the same Times. But because these Forces are not to one another in the Proportion of C P to s p; but (by reason of the Similitude and Equality of the Bodies S and s, P and p, and the Equality of the Distances S P, s p) equal to one another, the Bodies will be attracted equally from the Tangents; and therefore that the latter Body p should

should be drawn thro' the greater Interval r q, greater Time is requir'd, and this in the sub-duplicate Proportion of the Intervals; because the Spaces described are in the duplicate Proportion of the Times by Proposition 4. Therefore let the Velocity of the Body p be suppos'd to be to the Velocity of the Body P in the sub-duplicate Proportion of the Distance s p to the Distance C P; so that in Times which are in the same sub-duplicate Proportion, the Arches P Q, p q may be described, which are in the entire Proportion like to one another: In this Case the Bodies P, p which are always attracted by equal Forces, will describe about the quiescent Centers C and s, like Figures P Q V, p q v. the latter whereof p q v is like and equal to the Figure which the Body P describes about the moved Body S. *Q. E. D.*

Case (2.) Let us now suppose that the common Center of Gravity, together with the Relative Space in which the Bodies are moved amongst themselves, goes forwards uniformly in a Right Line; by the 26th Law of Motion, all the Motions will be perform'd in the Space as before; and consequently the Bodies will describe about one another the same Figures as before; which therefore will be like and equal to the Figure p q v. *Q. E. D.*

Coroll. (2.) The periodic Time about the unmov'd Body s, will be greater than the periodic Time about the moved Body S, or rather that which is about C the Center of Gravity: and that in the reciprocal Proportion of the Angles described at the same time; that is, in the subduplicate Proportion of the Radii s p and C P; that is, in the subduplicate Proportion of the Bodies S + P to the Body S. Thus, if the Moon p should

ſhould be moved about s., which is the Earth unmov'd, at the ſame Diſtance that it is: And ſince the Quantity of the Matter in the Moon is about one 40th Part of the Quantity of Matter in the Earth; the periodic Time of the Moon would be greater than that periodic Time of the ſame, which is at preſent, nearly in proportion of the Number 40 to the Number 39⌊498. For it is 40 : 39⌊498 : 39 ÷. From whence, ſince the periodic Time of that Planet is now 27d. 7^h. 43′, or 39343′; in the other Caſe it would be 39841, or 27d. 16h. 1′.

Coroll. (2.) Hence two Bodies drawing one another by Forces directly proportional to their Diſtances, deſcribe (ſee *Prop.* 19.) both about the common Center of Gravity, and about each other Ellipſes Concentrical, and which have their Centers in the Centers of the Forces. And on the contrary, if ſuch Figures be deſcribed about the Centers of Ellipſes, the Centripetal Forces are directly proportional to the Diſtances from thoſe Centers.

Coroll. (3.) Two Bodies drawing one another by Forces reciprocally proportional to the Square of their Diſtances, (ſee *Prop.* 21.) do both about the Common Center of Gravity, and about each other, deſcribe Conical Sections, which have their Foci in the Center, about which the Figures are deſcribed. And on the contrary, if ſuch Figures be deſcribed about the Focus of Conic Section, the Centripetal Forces are reciprocally proportional to the Squares of the Diſtances.

Coroll. (4.) Any Two Bodies revolving about a common Center of Gravity, (ſee *Prop.* 15.) do by their Rays drawn to that Center, and to each other, deſcribe Areas proportional to the Times, by reason of the perpetual Direction of the Rays, or Centripetal Forces to those Centers.

May 14. 1705.

Lect. XVIII.

XXIX. IF Two Bodies S and P, (see *Fig.* 2. *Plate* 6.) which are revolv'd about a common Center of Gravity, do attract each other with a Force reciprocally proportional to their Distance from the Center; the Transverse Axis of the Ellipsis, which either of them, as P describes about the other S, will be to the Transverse Axis of the Ellipsis, which the same Body P might describe in the same periodic Time about the other at Rest, as the Sum of the two Bodies S + P is to the first of two Proportionals betwixt this Sum, and that other Body S. For if the described Ellipses were equal one to the other, the periodic Times would be, by the last Proposition, in the subduplicate Proportion of the Body S, to the Sum of the Bodies S + P. Now, if the periodic Time be diminish'd in this Proportion in the latter Ellipsis, the periodic Times will become equal: And the Transverse Axis of that Ellipsis will (by *Prop.* 13.) be diminish'd in the Proportion, of which this Subduplicate is the Sesquiplicate; that is, in the Proportion of which the entire Proportion of S to S + P is triplicate; and consequently will come to be to the transverse Axis of the other Ellipsis, as the first of the two

two mean Proportionals betwixt S + P, and S is to S + P. And inversly, the transverse Axis of the Ellipsis, described about the moved Body, will be to the transverse Axis of that described about the Body unmov'd, as S + P is to the first of the two mean Proportionals betwixt S + P and S. *Q.E.D.*

Thus, if the Moon's mean Distance from the Earth; that is, half the transverse Axis of the Ellipsis described in the Supposition of the Earth's being unmoved, be of 60 Semi-diameters of the Earth, in Proportion to the given periodic Time; that Distance will be greater than 60 Semi-diameters of the Earth, on the Supposition of the Circumrotation both of the Earth and the Moon about a common Center of Gravity; and that in the Proportion of the Sum of the Earth and the Moon to the first of the two mean Proportionals betwixt the Sum of the Earth and of the Moon, to the Earth; or in the Supposition of the Moon's being a 40th Part of the Earth, as 40 is to 39⌊66. For 39 : 39⌊33 : 39⌊66 : 40. From whence, since the Distance of the Moon in the Hypothesis of the Earth's being unmoved, is put to be of 60 Semi-diameters of the Earth; it will be, in the other Hypothesis, of $60\frac{1}{2}$ Semi-diameters.

Coroll. From what was just demonstrated, it follows, that if two Bodies drawing each other by any Force whatever, and which are not moved from any thing else, nor impeded, be moved in any sort whatever; their Motions will be the same in effect, as if they did not attract each other, but were both attracted with the same Force by some 3d Body placed in the common Center of Gravity: And the Condition of the attracting Force will be the same, in respect of the Distance of the Bodies from that common Center, and in

O 2 respect

respect of their whole Distance betwixt themselves. For that Force wherewith the Bodies draw each other, because it tends to the Bodies, tends to the intermediate common Center of Gravity; and the Distances from the Center of Gravity, are every where proportional to the Distances of the Bodies; and consequently the Forces are the same, and do in the same Proportion increase or decrease, as if they proceeded from the intermediate Body in the Center of Gravity.

XXX. Many Bodies, whose Forces are proportional to the Quantity of Matter, and in the direct Proportion of the Distances, may be mov'd in divers Ellipses about their Centers, in such sort that their Motions may continue perpetually without any Perturbation, and that the common Center of Gravity of them all may in the mean while rest.

In the first place, let the two Bodies T and L (see *Fig.* 3. *Plate* 6.) be suppos'd to have D for the common Center of Gravity. If a projectile Motion be once impress'd in due Proportion according to parallel Lines situate in the same Plane, but according to Directions contrary to both, these Bodies will describe like Ellipses, having their Centers in D the common Center of Gravity, as we shewed above, *Prop.* 19.

Now let S, a 3d Body, draw the two former T and L, with the accelerating Forces S T and S L, and be reciprocally drawn by them. The Force S T may, by the 22d *Law of Motion*, be resolv'd in the Forces S D, D T; and the Force S L into the Forces S D, D L. But the Forces D T, D L, which are as T L the Sum of them: [For since the Proportions of the Parts T D and D L do always remain the same, the Proportion of the whole also T L will remain the same in all the Distances

of

of the Bodies T and L.] And the accelerating Forces of the Bodies themselves T and L, are as the Distances T L; and the additional Force arising from the Body S, and tending according to the Line T L, is, as we have already seen, as the same Distances T L. Therefore the Sum of the Forces T D and L D, respecting the Center of Gravity, are as the Distances D T and T L. But these are greater than the former Forces; and consequently will make that those Bodies should describe Ellipses, either like to the former, with a swifter Motion, if the projectile Force be accelerated in Proportion to the additional Centripetal Force; or of another Species, if that projectile Motion remain given. The remaining accelerating Forces SD and SD, whilst they draw those Bodies equally, and according to the Lines T I, L K, parallel to D S, do nothing at all change their Situations each to other, but cause that they should equally approach to the Line I K perpendicular to S D. But that Access to the Line I K will be hindred, by causing that the System of the Bodies T and L; that is, that D the Center of Gravity of the Two on one Part, and the Body S on the other, should be revolv'd with due Velocities in the given Plane about C the common Center of the Three, according to parallel Lines. The Body S by such a Motion (because that the Sums of the Motions, being on both Sides directly proportional to the Distance S D, and consequently to those C D and C S, do draw the Bodies towards the Center C:) The Body S, I say, for this Reason will (by the said Motion) describe an Ellipsis about the same Point C: and the Point D will describe an Ellipsis on the contrary Part; in the mean while that the Bodies T and L go on to describe their Ellipses, as before, about the moveable Center D.

Now, let a fourth Body, as V, be added, and it will be concluded by the like Argument, that this and the Point C may describe Ellipses about B the common Center of Gravity of them all; the Motions of the former Bodies T L and S about the Centers D and C still remaining, but something accelerated: And the thing will be the same in case of more Bodies.

Coroll. (1.) The Case of a System of Bodies revolving about other Bodies, where the Centripetal Forces are directly as the Distances, affords exact Ellipses, such as are in no wise disordered by the Addition of more Bodies. But by how much the more the Quantities of the Centripetal Forces depart from this Proportion, the Bodies must necessarily, *cæteris paribus*, the more disorder and disturb one another's Motions.

Coroll. (2.) But if the Centripetal Forces be reciprocally as the Squares of the Distances, and a System of two or more lesser Bodies revolving about a common Center of Gravity placed in the Focus of the Ellipsis, be pressed on one Side by a Body far greater than any of them, but sufficiently remote; and be pressed in such sort, that the common Center of Gravity of them all becomes not far distant from the Center of the greater Body, the common Center of Gravity of the System of the lesser Bodies will describe an Ellipsis about the greatest Body, or rather about the common Center of Gravity of them all. But divers Inequalities will arise in the Motions of the lesser Bodies, which we shall explain in what follows. Such indeed as Astronomers have noted in our Moon from most certain Observations.

Coroll. (3.) But the greatest Disorder of all will arise in the lesser System, if the greatest Bo-

dy

dy should attract divers Parts of that System unequally at equal Distances; that is, if the divers Kinds of the various Bodies should gravitate unequally, or in divers Degrees towards the greatest Body; especially if the Inequality of this Proportion should be greater than the Inequality of the Proportion of the Distances from the greatest Body. For if the accelerating Force, whilst it acts equally, and according to parallel Lines, does nothing at all disturb the Motions of Bodies amongst themselves, a Disturbance must necessarily arise from the Inequality of the Action; and must be greater or lesser, according to the greater or lesser Inequality. The Excesses of the greater Impulse, whilst they act upon some Bodies, and not upon others; or act less upon some than others, will necessarily change their Situation amongst themselves. And this Disturbance being added to the Disturbance which ariseth necessarily from the Inclination and Inequality of the Lines, will make the whole Disturbance the greater.

Coroll. (4.) From whence, if the Parts of the lesser System should be moved in Ellipses about the Focus, or in Circles about the Center, without any other Disturbance of their Motions, than what proceeds from the Inclination and Inequality of Lines drawn from the greatest Body; it is manifest, that the accelerating Forces of all the Parts of the System towards the greatest Body, are in equal Distances equal; and that all the Bodies comprehended in the lesser System, do equally gravitate towards the greatest Body at equal Distances.

Coroll. (5.) Hence it is also manifest, that the Parts of that lesser System are either urged by no accelerating Force, but what tends to the greatest Body, except it be very lightly and insensibly; or at least are urged very nearly with equal Impressions, and according to parallel Lines. All which things it were easy to apply to the Systems of the Earth and Moon, of *Jupiter* and his *Satellites*, of *Saturn* and his *Satellites* revolving about the Sun.

XXXI. If a primary Planet revolving about the Sun carry a Moon along with it, this will be so mov'd about the Primary, that it will perpetually be accelerated from the Quadrature with the Sun, unto the Conjunction or Opposition next following; but from the Conjunction to the Quadrature, it will be retarded; and consequently will be carried more swiftly about the Conjunction and Opposition, but more slowly about the Quadratures.

In *Fig.* 4. *Plate* 6. let Q be the Sun, S the primary Planet revolving in E S E its annual Orb, P or p a Moon describing its own menstrual Period A D B C about the Primary; in which Orbit, let the Points A and B design the *Syzigies* with the Sun; that is, the Conjunction and Opposition: C and D the Quadratures, that is, the Points distant on this Side and on that a Quarter of a Circle from the Conjunction & Opposition. Further, let QS, or QK, or Qk, the mean Distance of the Moon or *Satellit* from the Sun, represent the Quantity of the accelerating Attraction; that, to wit, whereby the Secondary Planet tends to the Sun, where it is placed at the same Distance from the Sun, as the Primary; and P or p be supposed to be the Place of the *Satellit* in its own Orbit. And let QL or Ql be taken in the Line PQ or pQ, produced if need be, which QL or Ql let be to QK or Qk in the duplicate Proportion of QK or Qk to QP or Qp; that is, so that it may be thus, PQ : QK : QR : QL ÷, or Qp : Qk : Qr : Ql ÷: These Things being thus, this Line QL or Ql, which

which was last found, will express the accelerative Attraction of the Moon placed at L or l towards the Sun in Q. Then let S, P or S, p be joined, and L M or L m drawn parallel to it, and meeting Q S in M or m. Here the accelerative Attraction in Q L or Q l, by the 2d *Law of Motion*, will be resolved into the Attractions L M, and L F, or M Q; or into l m, and l f, or m Q; and this with the Directions of those Lines. Of which Attractions, that which is represented by M Q or m Q is reduced to the Attraction m S; by taking away the Attraction as Q S, which is common to the Primary and its Satelles; and which consequently brings in no Anomalies. By which means the Attraction of the Satelles, tending according to the Direction S Q, which ought to be reckoned in this Place, is reduced to the Attraction M S in the Place P, so much as the Satelles is more attracted to the Sun than the Primary. From whence M S in the former Case, and m S in the latter, will design the Difference of the Attractions tending along S Q: And consequently the Satelles, by this means, is urged with a threefold Attraction, or rather with such an Attraction as may very well be resolv'd into three. The first and chief being that wherewith the Primary S draws this Secondary P or p; the second that which is proportional to L M or l m, with the Direction of the Line L M or l m; that is, with the Direction of P S or p S parallel to L M or l m: From whence the whole Force, compounded of these two Attractions, when it respects the Center of the Primary S, will make that the Body P or p, if it were impress'd with these alone, would even yet describe Areas about the same Center S proportional to the Times, by *Prop.* 15. But the Satelles is also

also acted on with a third Force, with one that is as M S or m S, and with the Direction from M or m towards S: that is, from L or l towards F or f. This, in the Position P, tends more to the Sun than its Primary; and that according to the Direction parallel to Q S by the Excess M S. And in the Position p, it tends less to the Sun than its Primary; and this according to the same Direction parallel to Q S by the Defect m S. Which will come altogether to the same, as if we should reckon the Excess M S from L towards F; and the Defect m S from f towards l; or the Excess from M towards S, and the Defect from m towards S; or as if the Satelles were disturbed on this Side and on that, by a double Sun placed opposite each to other. For when the Primary is drawn back from its Secondary towards the Sun by a true Excess of Attraction, there will be altogether, as to the Primary, all the same sensible Effects, (and those alone are what we are now searching after) as would be if the Primary being unmov'd, the Secondary were drawn away by the same Difference of Attraction unto the Part opposite to the Sun. But now, since this third Force which ariseth from the Difference of Attractions parallel to S Q, doth not tend to the Center S, neither doth the total Force compounded of these three Attractions, that, to wit, wherewith the Satelles is mov'd, tend unto the said Center. Wherefore, by *Prop.* 17. and 18. the Satelles will not describe equable Areas about the Center of the Primary, or such as are proportional to the Times. But the Force represented by M S or m S, will disturb the equable Description of Areas. In the Quadrant C A, of the Semi-circle C A D, supposing the monthly Motion to be perform'd from *West* to

East

East through A,D,B,C, the said Force accelerates the Motion of the Satelles about S, from C to A, by conspiring together with it; but after the Conjunction in A in the Quadrant AD, it retards the Motion by being opposite to it. But the Satelles being come unto the Quadrature about D, the 3d Force MS or ms vanisheth away; (because QK or Qk : QP or Qp; and consequently QL and Ql also are then equal.) And therefore the Force expressed every where by the said MS, can have no Effect in this Place. Therefore the Satelles, which about the Quadratures is urged by the rest of the Forces, and those only tending unto the Center of the Primary, will describe equable Areas by Rays drawn to the Center, or proportional to the Times. But whilst the Satelles goes over the Quadrant DB, Qm falls short of QS; and if we refer the disturbing Force to the Satelles alone, it will tend from m to S, and will again accelerate the Motion thereof by conspiring together with it: But after the Opposition in B, the Force will still tend from m towards S; but will now retard the Motion of the Satelles by being contrary thereto; untill again about the Quadrature C, mS vanisheth away, and consequently its Effects cease. Again, seeing the Force MS or mS, which disturbs the Area in the Passage of the Satelles from C to A, and from D to B, is continually increas'd, and in A and B becomes the greatest; and from these Points again is continually diminish'd, whilst the Planet is carried from A to D, and from B to C, until it at length vanisheth away in the Points D and C; it is manifest, that the Motion of the Satelles, as beheld from its Primary, is the swiftest, *cæteris paribus*, in the Conjunction and Opposition A and B, and slowest in the Quadratures C and D. *Q. E. D.*

Coroll.

Coroll. (1.) From hence we may salve that Inequality in the Motion of the Moon, named by Astronomy, *The Variation*; wherein the Moon is carried more swiftly in the Conjunction and Opposition, than in the Quadratures; and this so, that in moving from a *Syrigy* to an Octant, it gains about 35 Minutes above the middle Motion; and loseth again the same Quantity in its Motion from an Octant to a Quadrature; and so perpetually. And the like Anomaly is to be expected in the little Moons of *Jupiter* and *Saturn*; although by reason of their great Distance from the Sun and from us, and of their short menstrual Periods, it is not sensible to us.

Coroll. (2.) Hence also it follows, that the Orbit of any Moon, *cæteris paribus*, will be more curve in the Quadratures than in the Conjunction and Opposition. And consequently, if it be in it self Circular, it will become something Elliptical, in such sort that the lesser Axis will be always placed in the Conjunction and Opposition, and the greater in the Quadratures. But if the Orbit be of it self Elliptic about its Primary placed in one of the Foci, it will partake more of that Figure, than if it were not affected with this Anomaly. *Cartes* was the first that I know of, that assigned this oblong Figure to the Orb of the Moon, which he did only by way of Hypothesis and Conjecture: But in the mean while he fell into a great Error, when he determin'd that the Moon comes nearer to the Earth in all Conjunction and Opposition, and departs further off in the Quadratures; when on the contrary, by the proper Eccentricity of that Orb, the Line of the Apsides being put the Conjunction and Opposition, the Moon is more remote from the Earth in the highest Apsis, than in the Quadra-

tures;

tures; that Inequality which we have been speaking of notwithstanding. But the great Dr. *Halley* was the first who, from Observations, attributed this oblong Figure to the said Orbit; or at least the first that communicated the same to the Publick; and from thence shew'd, that the Lunar Theory was to be corrected: But as to the Demonstration of this Corollary, it is easily deduced out of the Proposition. For Bodies which are swifter, do decline less from the right Path than slower ones: And besides, the disturbing Force MS or mS in the Conjunction and Opposition, is not only the greatest in it self, but is also directly contrary to that Force wherewith the Central Body S draws the Body P or p; and consequently diminisheth that Force by being contrary thereto. But the Body p or P will decline less from the right Path, when it is less urged towards the Central Body S; and consequently will be more carried in an Oblong Elliptic Path about its Primary.

May 21. 1705.

Lect. XIX.

XXXII. If by reason of the Distance betwixt the Sun, and a primary Planet increas'd and diminish'd by turns, the Action of the Sun be alternately increas'd and diminish'd; the Radius of the Orbit of the *Satelles* will withal be increas'd and diminish'd, and the periodic Time of the *Satelles* about its Primary will be chang'd alternately; that

that is, will be increas'd when the Radius is increas'd; and on the contrary, diminish'd when it is so.

The Force wherewith the Primary draws its Moon, is increased when the Moon is in the Quadratures C and D, by the Addition of the Force S P or S p; the Force S M or S m vanishing away; and is diminish'd when the *Satelles* is in the Conjunction and Opposition, by the taking away of the Force S M or S M. And because the Force S m or S m in the Conjunction and Opposition, is twofold of S P or S p in the Quadratures, where the Point R or r falls-in almost with the Point B or A; the attractive Force of the Primary will be more increas'd than diminish'd in every Synodical Month, and consequently is to be reckon'd for absolutely increas'd. Therefore the Force of the Sun being increas'd about the Perihelion of the System, the attractive Force of the Primary will be more languid, and the Orbit will be enlarg'd; but the Force of the Sun being diminish'd about the Aphelion of the System, the attractive Force of the Primary will be more strong, and the Orbit will be contracted. But the periodic Time of the *Satelles* will be increas'd with the enlarging of the Orbit; and on the contrary, diminish'd with it: and thus every Year the middle Motion of the *Satelles* will be greater and lesser by turns; and is to be accounted truly mean, only in a mean Distance from the Sun.

Coroll. (1.) Hence we may solve that annual Inequality in the Moon, which respects the middle Motion thereof; that, namely, in which the middle Motion of this Planet doth alternately exceed and fall short of the true middle Motion, by an Excess or Defect of 12′ almost; exceeding it in the Passage of the Earth from the remoter Apsis to the mean Distance; and falling short thereof from the mean Distance to the near-

er

er Apsis; and again, falling short from the nearer Apsis to the mean Distance, and exceeding it from the mean Distance to the remoter Apsis: and so perpetually. And the same thing is to be judged of the Moons about *Saturn* and *Jupiter*, in their Proportion. Albeit this Inequality in these, and the rest likewise, is so very small, that it may very well be neglected in most Cases.

Coroll. (2.) The truly original and primitive periodic Time of every Moon, that is, that Time in which it would revolve about its Primary, if it were without the reach of the Sun's Action, is a little shorter than the middle periodic Time; and the orginal Distance from its Primary, a little less than the present. Namely, because if the Force of the Sun, which debilitates the Force of its Primary, were taken away, it would approach nearer to its Primary; and thus the periodic Time would be the shorter.

Coroll. (3.) Hence also we may infer, with the Famous Dr. *Gregory*, that if any Primary Planet should, through the Accession of new Matter, become greater than it was, and from thence its Attraction become proportionably greater; its Moon would revolve about it at a less Distance, and in a shorter time. As on the contrary, by the Diminution of the Matter of the Primary, the Orbit and periodic Time of its Moon would be enlarged. And the same thing would happen in any Primary, in case the Sun was increas'd or diminish'd.

Coroll. (4.) Since therefore it is manifest from the most ancient Astronomical Observations, as compared with the latter, that the periodic Times of the Primary Planets about the Sun, and of the Moon about the Earth, are the same in this Age, as they were 2000 Years ago; it is certain, that the Quan-

Quantity of Matter both in the Sun and in the Earth, is the same that it was then; and hath had no sensible Addition or Diminution.

Coroll. (5.) But if the Quantity of Matter in the Earth be suppos'd to have been increas'd by *Noah*'s Deluge, or by any other means, the Quantity of the periodic Month of the Moon must necessarily have been diminish'd thereby.

XXXIII. If a Secondary Planet describes an Elliptic Orbit about its Primary, which is placed in the Focus of the Ellipsis; the greater Axis of this Ellipsis, or the Line of Aspes, will, by an angular Motion, go forward and backward by turns; but it will go forward more than it goes back; and in each Revolution of the Secondary, by the Excess of the Progression, it will be carried towards the consequent Signs: That is, In the Conjunction & Opposition with the Sun, it will go forwards; & in the Quadratures it will go backwards.

For the Force wherewith the Secondary Planet P or p is urged towards its Primary about the Quadratures, where the other Force M S or m S is vanish'd away, is compounded of the Force L M or l m, and the Centripetal Force of the Central Body S. The former Force, if the Distance be increas'd or diminish'd, is increas'd or diminish'd almost in the same Proportion directly; so that in the greater Distance from the Primary, the Attraction towards the Center becomes greater, and in a lesser Distance less. But the latter Force arising immediately from the Primary, in a greater Distance becomes less, and in a lesser Distance greater; and is always in the duplicate Proportion of the Distance reciprocally. And consequently the entire Force, or the Sum of the Forces towards the Center of the Primary, doth, upon the Increase of the Distances, increase in a lesser

lesser Proportion than the duplicate Proportion of the Distance is; that is, it is not so much diminish'd in a greater Distance, nor is it so much increas'd in a lesser Distance, as the Motion about the Focus of the unmov'd Ellipsis doth require. But in the Conjunction and Opposition, the Force wherewith the Secondary is urged towards its Primary, is the Difference betwixt the Force wherewith the Secondary is drawn by the Primary, and the Force K L or k l; or in this Case S M or S m. And that Difference, because the Force S M or S m is increas'd nearly in the very Proportion of the Distance directly, decreaseth in less than a duplicate Proportion of the Distance; and consequently is greater in a lesser Distance, and less in a greater, than sufficeth for the describing an unmoved Ellipsis. But if the Centripetal Force decreaseth in more than a duplicate Proportion of the Distance, as it comes to pass about the Conjunction and Opposition; this is a little like the Case of the Decrease of the Centripetal Force in the triplicate Proportion of the Distance; from whence a Motion in a spiral Line, without any Change of the Tangent to the Radius, would follow. The *Satelles* therefore will revolve in some moveable Ellipsis; or a greater angular Motion will be requir'd, that the Tangents oblique to the Radius should become perpendicular to the same, that is, that the *Satelles* should come to its Apsides, than would be required if the Forces were in the duplicate Proportion of the Distances reciprocally; that is, the Line of the Apsides will go forward. And on the contrary, if the Centripetal Force decreaseth in a less than duplicate Proportion of the Distance, as it happens about the Quadratures, the contrary Case follows; and the Motion of the Secondary will arise from a Motion different from that in a Spiral, which keeps the Angle of the Radius and Tangent: So that that Angle should be sooner chang'd, and sooner come unto a right Line, than it would come if the Force were in the very duplicate Proportion of the Distance reciprocally; that is, the Line of Apsides will go back. But in the intermediate Places, betwixt the Conjunction or Opposition, and the Quadratures, the Motion of the Apsis depends upon both Causes conjunctly; so as to make that it should go forwards or backwards, according to the Excess of this or that. From whence, since the Force K L or k l in the Conjunction and Opposition, as we lately noted, is twice as great as the Force L M or l m in the Quadratures; the Excess in every whole Revolution will be on the Side of the greater Force K l or k l; and will transfer the Apsis each Revolution towards the consequent Signs.

Coroll. (1.) Hence we may solve that Inequality, or progressive and regressive Motion of the Lunar Apsis, in which the Apogeum is so mov'd, that in its Conjunction and Opposition it goes forwards more swiftly, and in its Quadratures goes back more slowly; and by the Excess of the progressive Motion above the regressive every Month, it is carried towards the consequent Signs about three Degrees; and thus goes over a whole Circle in the Space of Ten Years, or a little sooner. In the Moons of *Jupiter*, which are mov'd almost in Circles, the Apsides are none at all, or insensible at most, and consequently this Demonstration appertains not to them. In those of *Saturn* it will have place, if at any time some Eccentricity shall be discovered in their Paths; but by reason of the Shortness of their periodic Times, and their vast Distance from the Sun, and consequently the small Force of the same, the

the Change of the Apogeum will be so very small that it cannot fall under our Observation, much less be brought under Computation.

Coroll. (2.) Since therefore the Progress or Regress of the Apsides depends upon the Decrease of the Centripetal Force, which is made in a Proportion greater or less than the duplicate Proportion of the Distance S P or S p in the Transit of the Body from the nearer Apsis to the remoter; as likewise upon the like Increase in the Return to the nearer Apse; and consequently is greatest where the Proportion of the Force in the higher Apsis to the Force in the nearer, is most remote from the duplicate Proportion of the Distances inverted; it is manifest, that the Apsides in their Conjunction and Opposition, by the ablatitious Force K L or S M—L M, or S m—l m, will go forward more swiftly; S P or S p being at that time the least of all; and S M or S m the greatest of all; and S P or S p, or rather the Sum of them on both Sides being the least of all in the Quadratures. From whence, in each Revolution of the *Satelles*, whilst the Apsides are about the Conjunction and Opposition, they will go forwards most swiftly in the Conjunction and Opposition of the *Satelles*, and go back very slowly in the Quadratures thereof; and consequently the Excess of the progressive Motion above the Regressive will be the greatest of all, and the Apsis will be moved very swiftly towards the consequent Signs.

Coroll. (3.) But if the Apsides be about the Quadratures, then contrary Causes will produce contrary Effects; and the Apsides will go forwards more slowly than before, when the *Satelles* is in the Conjunction and Opposition, and go back more swiftly in the Quadratures of it; yea, it may come to pass in the said Position of the

Apsides in some particular Revolution of the *Satelles*, that the Regress of them in the Quadratures of the *Satelles* may surpass the Progress of the same, which is when the *Satellis* is in the Conjunction and Opposition. But because the ablatitious Force S M or S m, that causeth the Progress of the Apsides in the Conjunction and Opposition, is, *cæteris paribus*, about twice as much as the adjectitious Force which brings in the Regress of the Apsides in the Quadratures of the *Satelles*; and because the Apsides do also tarry longer in the Conjunction and Opposition than in the Quadratures; since they move, in the former Place, towards the consequent Signs with the Sun, they go forward, and consequently do accompany him longer; but in the latter Place, moving to the antecedent Signs, they sooner pass the Square of the Sun, which moves in the mean while towards the consequent Signs: From these Reasons it appears, that the Apsides go forward more swiftly and longer in their Conjunction and Opposition, and go back more slowly, but not so long in their Quadratures; and that they by the Excess of the Progress above the Regress in one entire Revolution of them to the Sun, *i. e.* in the Space of about Thirteen Months, are still carried towards the consequent Signs. Thus, in the Orbit of the Moon, the Apogeum thereof is moved so unequally, that it is to be brought under Rule by an Equation amounting to 12 whole Degrees and a Quarter, as is to be seen in the Lunar Tables.

XXXIV. If a *Satellite* be mov'd in an Eccentric Orb about its Primary, the Eccentricity will be changed twice in every Revolution, and will be the greatest, when the Secondary is in the Conjunction and Opposition with the Sun; and the least, when it is in the Quadratures; and consequently

quently will be increas'd continually in the Passage from the Quadratures to the Conjunction and Opposition; and in the contrary Passage continually diminish'd.

For since it appears by what hath already been demonstrated, that the Centripetal Force towards the Primary remov'd at a great Distance, doth sometimes decrease in a greater than the duplicate Proportion of the Distance, sometimes in a less; and since the Motion of the *Satellite* in an immoveable Orbit, and with one certain Eccentricity, depends upon the Decrease of the said Force in the duplicate Proportion of the Distance it self; from the Change of this Proportion the Species of the Orbit must necessarily be changed. Thus, if the Centripetal Forces increase or decrease in more than a reciprocal duplicate Proportion of the Distance; it is manifest, that the *Satellite* in its Descent from the highest Apsis to the lowest, being perpetually impell'd towards the Center by the Accession of that new Force, will incline more to that Center, than it would have done if the Increase of the Centripetal Force had been only in the duplicate Proportion of the Distance diminish'd; and consequently will describe an Elliptic Orb inferiour to the former, and at the lowest Apsis approach nearer to the Center than it did in the highest; and thus the Orb, by the Occasion of this new Force, is made more Eccentric. And now, if in the Return of the *Satellite* from the lowest Apsis to the highest, the said Force should decrease by the same Degrees by which it did before increase, the *Satellite* would return to the former Distance, keeping the Eccentricity lately obtain'd; whereas, if the said Force doth decrease in a greater Proportion than that in which it increas'd before; the Moon being in this Case less attracted, will ascend unto an higher Distance, and so the Eccentricity will be still more increas'd.

In like manner, if the *Satellite* in its Descent from the highest Apsis, be urged with a Force which is increas'd by less than the duplicate Proportion of the Distance diminish'd; is is manifest, that it will describe an Elliptic Orb exterior to the former (than that, I mean, where the Centripetal Force was reciprocally as the Square of the Distance;) and consequently an Orb less excentrical; and that this Eccentricity is still more diminish'd, if in the Ascent the Centripetal Force decreases less or more slowly than it had increased before. If therefore the Proportion of the Increase and Decrease of the Centripetal Force be increas'd in each Revolution, the Eccentricity likewise will be increas'd; and on the contrary it will be diminish'd, where the same Proportion decreaseth. Seeing therefore in every Revolution, that Force decreaseth in the Conjunction and Opposition of the *Satelles* in a greater Proportion than that which is duplicate of the Distance increas'd; and in the Quadratures of the same in a less, as is manifest from what hath been already said, it appears, that about the Conjunction and Opposition of the *Satelles* the Eccentricity of the Orb describ'd is perpetually increas'd, and diminish'd about the Quadratures. And since in many Revolutions compar'd amongst themselves, there is the greatest Proportion of Decrease in the Conjunction and Opposition of the Apsides, and the least in the Quadratures of the same; it is also manifest, that the greatest Eccentricity of the Orbit is when the Apsides are in the Conjunction and Opposition, and the least, when they are in the Quadratures; and consequently that the Eccentricity is diminish'd perpetually in the passing of the Apsides from the Conjunction

junction and Opposition to the Quadratures of the Sun; and are perpetually increas'd in the passing of the same from the Quadratures to the Conjunction and Opposition.

Corollary. Hence we may solve that Eccentricity of the Lunar Orbit which is divers, and daily changing, as being greater in the Moon's Conjunction and Opposition, less in the Quadratures; and likewise continually increasing in the passing of the *Apogeum* from the Conjunction and Opposition to the Quadratures, and in the contrary Case continually decreasing. For in Astronomical Tables we find so great a Diversity assigned to this Eccentricity, that the Distance betwixt the Focus and the Center of the Ellipsis describ'd by this Planet, which we call the Eccentricity of the Orbit, is sometimes of $\frac{66782}{1000000}$ Parts, sometimes of $\frac{43319}{3000000}$ only; such Parts we mean that 1000000 are contain'd in the mean Distance of the Moon. So that the Difference of Eccentricities is found to arise unto above half of the whole least Eccentricity.

June 4. 1705.

LECT. XX.

XXXV. If the *Satellite* be revolv'd about the Primary in an Orb, the Plane whereof is inclin'd to the Plane of the Primary, the Line of the Nodes will be moved with an angular Motion towards the antecedent Signs, but with an unequal Velocity: Most swiftly indeed when the Nodes are in the Quadratures, afterwards by Degrees more slowly, until that be placed in the Conjunction and Opposition, and they wholly rest; and thus being always either Retrograde or Stationary in each Revolution of the *Satellite*, they move back. As likewise in the same Revolution they go back more swiftly, *cæteris paribus*, when the *Satellite* is in the Conjunction and Opposition, than when it is in the Quadratures.

For amongst the disturbing Forces, of which we have spoken so oft, the Force LM or lm which is parallel to SP or Sp, that is always situate in the Plane of the Orbit of the *Satelles*, and can induce no Change of the Plane of the Orbit. The other Force also MS or mS, situate in the Plane of the Elliptic, when the Nodes are in the Conjunction and Opposition, will be also placed in the Plane of the Orbit, as being posited at that time in the common Intersection of both Planes. But when the Nodes are not in the Conjunction and Opposition, this latter and greater Force, which is always in the Plane of the Elliptic, will not be in the Plane of the Orbit; and consequently will affect the Motion of the *Satellite*, as to Latitude, and make the Line of Nodes to go back towards the antecedent Signs. For let the Nodes be supposed to be placed in the Quadratures, this latter Force, which always acts parallel to the Elliptic, will perpetually draw back the *Satelles* whilst it is passing the Nodes on either Side, and about to go forwards in its own Orbit, from the same Plane; so that the Place of the Intersection which is to be next, will be at some Distance from the former Intersection, and towards the antecedent Signs. But when the Nodes are betwixt the Conjunction and Opposition and the Quadratures, this latter Force will sometimes move them towards the consequent Signs, sometimes towards the antecedent; but

will

will always in an entire Revolution of the *Satelles*, by the Excess of the same Force towards the antecedent Signs, carry them back towards that Part: From whence, in the Conjunction and Opposition of the Nodes, they will remain immoveable; in their Quadratures, they will go back most swiftly; and partaking in the intermediate Places of both Conditions, they will go back more slowly; and consequently will always, in a compleat Revolution, be carried back towards the antecedent Signs, notwithstanding their being Retrograde and Stationary in particular Places of the Period. But it is to be noted, that when the Orbit is placed without the Conjunction and Opposition, and Quadratures, whilst the *Satellite* goes forward from the ascending Node to the descending, and, *vice versa*, the Nodes go back more slowly, so long as the Force M S or m S respects that Side of the Plane on which the *Satellite* is placed; and go forward so long as that Force respects the opposite Side. Thus the Line of Nodes being placed in an Octant of the Sun, after its having been placed in the Quadratures, or about R and r, the *Satellite* having pass'd the Plane of the Ecliptic about R, is then towards the Sun: but the disturbing Force from R to the Quadrature C, tends to the contrary Part by an Octant of a Circle; which Force vanishing away in the Quadrature, the Force tending to the Sun takes the Place of it, and continues throughout the three rest of the Octants: So that the Line of Nodes of the moveable Orbit doth first go forward a little, then goes back a little more; and so likewise in the other Semi-circle; until the same Line coming to the Conjunction and Opposition, the Progress and Regress are in a manner equal; but both of them very small, and of very short Continuance, by reason of the near Coincidence of the Situation of the Plane with the Direction of the disturbing Force. But that the

Nodes

Nodes in the ſame Revolution of the *Satellite*, go back more ſwiftly, *cæteris paribus*, when the Satellite is in the Conjunction and Oppoſition, than elſewhere, is manifeſt, by reaſon the diſturbing Force is greater in that Place; and conſequently will obtain a greater Effect.

XXXVI. The ſame things being ſuppoſed, the Inclination or acute Angle of the Plane of the Orb of the Satellite to the Plane of the Ecliptic, is perpetually changed; and is then greateſt, when the Nodes are in the Conjunction and Oppoſition with the Sun; and the leaſt, *cæteris paribus*, when they are in the Quadratures: And is diminiſh'd continually in the Paſſage of the Satellite from the Quadratures to the Conjunction and Oppoſition, and increas'd continually from the Conjunction and Oppoſition to the Quadratures. From whence it comes, that the Satellite being in the Conjunction and Oppoſition, the Inclination of the Planes becomes the leaſt; and returns to the former Magnitude nearly, when the Moon comes to the next Node. And this Inclination of the Planes is diminiſh'd, whilſt the Nodes are carried from the Conjunction and Oppoſition to the Quadratures, and becomes the leaſt of all, *cæteris paribus*, when the Nodes are in the Quadratures; then it increaſeth by the ſame Degrees whereby it had decreas'd before; and the Nodes being again return'd to the Conjunction and Oppoſition, it returns to the former Magnitude. If the former Propoſition be rightly underſtood, this will not ſo much require a particular Explication. For like as, whilſt the Body goes forward by the former Motion from L to F, if an attracting Force, parallel to the Line L M, do ſupervene, which attracts towards M, and is repreſented by the Line L M, the Body will go forwards in the Diagonal L Q, and the Angle of Inclination M L Q will be leſs than M L F the

former

former Angle of Inclination: Or thus, like as whilst the Body goes forward from L to F by its proper Motion, if the like attracting Force parallel to the same L M supervenes, which attracts the contrary way, but nevertheless is represented by an equal Line, the Body will go forward in another Diagonal, and the Angle will be greater than the former Angle: So it must happen in like manner in our present Case, *i. e.* that a divers Inclination of the Plane will follow upon the Motion of the Nodes. For when the Nodes are in the Quadratures, that Motion of them which perpetually draws back the Satellite from the Plane of its Orb, diminisheth the Inclination of the Plane, in the mean while that the Satellite passeth from the Quadratures to the Conjunction and Opposition; and increaseth the same in the contrary Transit; from whence it comes, that the Satellite being placed in the Conjunction and Opposition, the Inclination becomes the least of all, and returns to its former Quantity nearly in the Access of the Moon to the next Node. But if the Nodes be found in the Octants next after their having been in the Quadratures, that is, about P and p; in this Case, according to what hath been said already, the Inclination of the Plane is perpetually diminish'd from either of the Nodes unto the 90th Degree from thence; then it is increased for the Space of 45 Degrees, or in the Transit unto the next Quadratures; and afterwards again is diminish'd for the other 45°, or unto the next Node. So the Inclination is diminish'd more than it is increas'd; and so is always less in the subsequent Node than in the foregoing. And by the like Reasoning, the Inclination is increased more than it is diminish'd, when the Nodes are in the other Octants, or about R and r. Therefore the Inclination is the greatest of all, when the Nodes are

in

in the Conjunction and Opposition. In their Transit from the Conjunction and Opposition to the Quadratures, it is diminish'd in the Access of the Satellite unto them; and becomes the least of all when the Nodes are in the Quadratures, and the Satellite in the Conjunction and Opposition; then it increaseth by the same Degrees by which it had decreas'd before; and the Nodes coming to the next Conjunction and Opposition, it returns to its former Magnitude. *Q. E. D.*

Corollary. From this and the former Proposition, we may solve the most known Phænomena of the Moon; I mean the annual Regress of the Nodes consisting of about 19 $\frac{1}{3}$ Degrees, & that Mutability of the Inclination of the Orbit of this Planet, in which when the Nodes are in the Quadratures, the Angle of Inclination contains only 4°. 59'. 35": But when they are in the *Syrigies*, the same Angle is found to arise to about 5°. 17'. 20".

XXXVIII. All the Inequalities which are in the Motions of the Secondary Planets revolving about their Primaries, are something greater in the Conjunction of the Satellite with the Sun, than they are in the Opposition.

For since Q S bears a greater Proportion to Q A than Q B bears to Q S, by reason that S A, S B, *cæteris paribus*, are equal, and that Q S is greater than Q A; the duplicate Proportion of Q M to Q S, will be greater still than the duplicate Proportion of Q S to Q m. And consequently the Difference M S will be greater than the Difference m S; and L M greater than l m. From whence the Effects derived from this Force, will be greater than those which are derived from the other. *Q. E. D.*

But it is to be noted, that the Distance of the Earth from the Sun is so vastly great, that the Difference of the Forces about the Conjunction

of

of the Moon with the Sun, and about the Opposition of the same, is very small, and hath scarce been distinguished yet by any Observations. From whence it is not to be wonder'd, that Astronomers have taken no notice of this Distinction.

XXXVIII. The absolute Force of the Sun in the disturbing the Secondary Planets, and the Effects thereof in divers Distances from the Sun, is in the triplicate Proportion of those Distances inversly.

For let the Distance of the Satellite from the Sun be altered; let the Radius of the Orbit of the Satellite be in the same Proportion to the other Radius. In this case, the Distance of the Secondary from its Primary, will be in a given Proportion to its Distance from the Sun: From whence, according to this Hypothesis, the absolute disturbing Force will be as the absolute Force of the Sun, or in that duplicate Proportion. Thus the thing would be, if the Radius of the Secondary System had increas'd or decreas'd in the same Proportion, as the Distance of the Sun increas'd or decreas'd; so that they should still keep the same Proportion to one another, as before. But since the Radius doth in no wise decrease by the Access of the Sun, or increase by the Recess thereof, that duplicate Proportion will be to be increas'd again by the other Proportion of the Distance of the Secondary from its Primary. From whence the entire compound Proportion will be triplicate of the former. *Q.E.D.*

As for Example: Let the Sun be suppos'd as near again to the Earth, as it was before; or as 50 to 100. And let A B the Diameter be equal to two Parts, the Quantity of the absolute Force of the Sun at S in the lesser Distance, will be Fourfold of the Quantity of the same Force in the greater Distance; But the Force S M in the lesser Distance,

Distance, will be about Eightfold of the same Force in the greater Distance. For $49 \times 49 = 2401$; and $50 \times 50 = 2500$. From whence $2500 - 2401 = 99$. And $99 \times 99 = 9801$; and $100 \times 100 = 10000$. From whence $10000 - 9801 = 199$. Therefore the Difference of the absolute Force is almost in the double Proportion, or as 199 is to 99. And the mean absolute Forces themselves are in the Quadruple Proportion, or as 4 to 1. Therefore the entire absolute Force compounded of them, is $4 \times 2 = 8$ to $1 \times 1 = 1$, or in the reciprocal triplicate Proportion of the Distance nearly. And since the apparent Diameter of the Sun is almost in the triplicate Proportion of the Distance, and the Force of the Central Body is also nearly the same; the Sun's Force whereby he disturbs the Satellite, and the Effect of it, will be in the direct triplicate Proportion of the Sun's apparent Diameter very nearly.

Scholium (1.) In the same manner wherein the Sun placed without the Orbit of the Secondary Planet disturbs the Motion thereof; the superior Planets will disturb the Motion of the lower, and Comets will disturb the Motion of all the Planets. And the Actions of Planets and Comets upon other Planets, will produce the like Effects; though far less indeed, by reason of the Smalness of their Bodies, if compared with the Sun, and the vast Distances. But some Effects there will be [yea, of the Actions also of the inferior Planets upon the superior] which if they continue, and be for the most part directed the same way, will at length become sensible. As for Example: The Apsis of the Orbit of the Earth will, after many Years, be moved towards the consequent Signs; although this Motion must necessarily be very small, if compar'd with the Motion of the

Apsides

Apsides of the Moon the same way. Thus, indeed, the Eccentricity of the Orbit of the Earth must be subject to some Mutation; which, nevertheless, is so small, that it can scarce be collected from any Phænomenon.

Scholium (2.) And thus the superior Planets will sensibly disturb the Motions of one another, if they be great ones, and tarry long about their mutual Heliocentrick Conjunction, they being then placed at the least Distance from one another. Thus the Action of *Jupiter* upon the Secondaries of *Saturn*, and of *Saturn* upon those of *Jupiter*, the mutual Gravitation of all the Planets one to another, which we have already proved, being supposed, is in no wise to be slighted, at what time they are seen from the Sun in Conjunction. For they are great Bodies, and far exceeding our Earth in Magnitude, and are near enough at that time, to make the Effects of their disturbing Forces become sensible. And that they are indeed sensible to us, will be shewn hereafter from Astronomical Observations.

Scholium (3.) It is easy to estimate the divers Quantities of the Sun's disturbing Force in the System of *Jupiter* and that of *Saturn*, from the known Quantity of the same Force in the Anomalies of our Moon. For from the known Proportions of the Distances of the Earth, and *Jupiter* and *Saturn* from the Sun; and the known Effects of the said Force in the Moon, by a certain Proportion of like Effects on both Sides, observed by Sir *Isaac Newton*; the Effects of that Force, in the Systems of *Jupiter* and *Saturn*, may be determin'd without much Difficulty.

XXXIX. *A Problem.* To find the Proportion betwixt the Force whereby the Motion of a Satellite is disturbed by the Sun, and the Force where whereby a Satellite is retained in its own Orb about its Primary, which is called its Gravitation towards its Primary.

For the whole disturbing Force is compounded of the disturbing Forces L M or l m, and S M or S m: And also by reason of the vast Distance of the Sun, the Line L Q or l Q is almost parallel to the Line M Q; and consequently the Force L M or l m is very near equal to its mean Quantity, or to the Radius of the Satelles S P or S p: And likewise by reason of the Sun's vast Distance S M or S m, or L P and l p, are equal to treble the Line K P or k p. From whence, since in the Triangle S K P or S k p, which is Rectangular at K or k, the Angle K S P or k S p is the Distance of the Satellite from the Quadrature; and the Side K P or k p is the right Sine to the Radius S P; the disturbing Force S M or S m will be to the disturbing Force L M or l m, as is the Radius to the treble of the Right Sine of the Distance of the Satellite from the next Quadrature. From whence, if the Proportion of the disturbing Force S P or S p to the Centripetal Force of the Primary, or to the Force of Gravity, were once known, the disturbing Force S M or S m would easily become known. Which therefore we find out by this Method. The disturbing Force S P or S p is to the Centripetal Force of the Primary towards the Sun, as the Line S p or S p is to the Line S Q; or as the Distance of the Satelles from its Primary, is to the Distance of the Sun from the same Primary. But the Centripetal Force of the Primary towards the Sun, is to the Centripetal Force of the Secondary towards its Primary, as the Squares of the periodic Times drawn into the Radii of the Circles; or as S Q is to S P or S p; and as the Squares of the periodic Times together. From whence, by Equality of Proportion, the Quantity of the disturbing Force will be to the Force of Gravity (the former Proportion of SP or Sp to SQ destroying the other reciprocal Proportion of SQ to SP or Sp,) as the Squares of the periodic Times. *Q. E. D.*

Coroll. Since therefore the periodic Time of the Moon is 39343′; and the periodic Time of the Earth about the Sun is 525969′; the disturbing Force SP will be to the Force of Gravity towards the Earth, which is at the Moon, as 39343 × 39343′ is to 525969′ × 525969′; that is, as 1547871649 is to 276643388961; or as 1 is to $178\frac{11}{13}$. And since the Force SM or Sm in its greatest Quantity, or in the Conjunction and Opposition, is to the former Force as 3 to 1; the Force SM or Sm in the Conjunction and Opposition, will be to the Force of Gravity as 3 is to $178\frac{11}{13}$, or as 1 is to $59\frac{8}{13}$. Therefore that disturbing Force of the Sun SM or Sm, is in the Conjunction and Opposition about a 60th Part of the whole Force of Gravity in the Moon towards the Earth. Or rather the Force SP or Sp being taken away in this Case from the Force SM or Sm, as may very well be done, the whole disturbing Force in the Conjunction and Opposition will be to the Force of Gravity as 1 is to $89\frac{5}{13}$, or a 90th Part of the same nearly. And in other Places, the Force SM or Sm will be to the Force of Gravity, (the whole Sine being put to be equal to Unity) as treble the right Sine of the Distance from the next Quadrature is to $178\frac{11}{13}$.

XL. If many fluid Bodies, either distinct, or gathered together into one Fluid, be moved about a primary Planet; each Part of the Fluid in [illegible] performing its Motion about the Primary after the manner of a Satellite, will come nearer to the Primary, *cæteris paribus*, and be moved more swiftly in the Conjunction and Opposition of the same, and of the Primary, than in the Quadratures. And the Nodes of this Ring, or its Intersections with the Plane of the Ecliptic, will rest in the Conjunction and Opposition. But out of Conjunction and Opposition, they will be carried towards the antecedent Signs; and this most swiftly in the Quadratures, more slowly in other Places. The Inclination of the Ring also will be varied; and the Axis thereof will be moved to and fro in each monthly Revolution; and the Revolution being compleated, it will return to that Position which it had before; so far as it is not carried about by the Precession of the Nodes. All these things do follow of their own accord, from what hath been already demonstrated; and so do not require a peculiar Demonstration.

Corollary. From hence some of the Phænomena of the Ring of *Saturn*, if so be it be a Fluid, may easily be understood. Yea, indeed, if it be solid, the Nodes of the same, its Intersections I mean with the Ecliptic, will rest in their Conjunction and Opposition, when the Sun is found in the Plane of the Ring, as well as in that of the Ecliptic. But out of the Conjunction and Opposition they will go back, and this most swiftly in the Quadratures, and more slowly in other Places. The Inclination of the Ring will also be varied, and the Axis thereof in each Revolution about the Sun will nod, and twice vary its Inclination towards the Ecliptic, and twice return to its former Position, only it will be carried about by the Precession of the Nodes, as is manifest from what has been already said.

XLI. If

XLI. If a Fluid be contain'd in a Channel form'd in the Surface of any Planet, Primary or Secondary, and be uniformly revolv'd together with the Planet with a diurnal periodic Motion; each Part of this Fluid will be accelerated and retarded by turns, in its Conjunction and Opposition; or at Noon-day and Midnight, will be swifter; in the Quadratures, or at the 6th Hour Evening and Morning, it will be slower than the contiguous Surface of the Globe; and thus it will flow in the Channel, and return back by turns perpetually. For the Fluid will be disturbed by the unequal Attraction of the Sun, because the Attraction of the nearer Parts will be greater, and that of Parts more remote less; while the Force L M or l m will draw the Fluid down in the Quadratures, or at the 6th Hour in the Evening and Morning; and make that the Parts of it, which are placed there, should descend unto the Conjunction and Opposition, or unto the Noon and Midnight; and the Force S M and S m will draw the same upwards in the Conjunction and Opposition, or stop the Descent of it, and cause it to ascend unto the Quadratures; and thus perpetually.

Coroll. Hence we learn the Cause of the Flux and Reflux of the Sea. If we allow the disturbing Force of the Moon, as well as of the Sun, and duly apply what hath been already demonstrated to the present Case. But this so well known and stupendious Phænomenon of Nature, will come to be treated of afterwards more largely and distinctly; to which Place therefore we refer our Reader.

Octob. 22, 1705.

LECT. XXI.

XLII. IF a Solid Ring be put about a Globe perfectly ſpherical at the Equator of the ſame, and ſtick to it; there will indeed be no Motion of Flux and Reflux, but the vibrating Motion of Inclination, and the Preceſſion of the Nodes, will remain. Let the Globe have the ſame Axis with the Ring, and compleat its Revolution in the ſame time; and with its Surface touch the Ring inwardly, and cleave to it; by its participating of the Motion thereof, the whole Frame will vibrate to and fro, and the Nodes will go back. For the Globe, as above ſhew'n, is indifferent to receive all Impreſſions. The greateſt Angle of Inclination of the Ring without the Globe, would be where the Nodes are in the Conjunction and Oppoſition. In their Progreſs from thence to the Quadratures, the Ring endeavours to diminiſh its Inclination, and by that Endeavour impreſſeth its Motion upon the whole Globe. The Globe retains the Motion impreſs'd, until that the Ring by a contrary Endeavour takes away this Motion, and impreſſes a new Motion upon the contrary Part. And thus the greateſt Motion of the decreaſing Inclination is in the Quadratures of the Nodes, and the leaſt Angle of Inclination is in the Octants after the Quadratures. Then the greateſt Motion of Inclination is in the Conjunction and Oppoſition, and the greateſt Angle in the next Octants. And the

the Case is the same with a Globe without a Ring, which either is something higher in the Parts about the Equator than about the Poles, or consists of a more dense Matter. For that Excess of Matter in the Parts about the Equator supplies the Place of the Ring.

Coroll. (1.) For the same Reason that the redundant Matter of the Globe causes the Nodes to go back, and consequently by the Increase thereof causeth the Regress to increase, and by the Diminution thereof that the same Regress should be diminished, and by its being taken away that the Regress should cease; it will come to pass, that if more than the redundant Matter be taken away, or, which comes to the same, if the Globe be more depress'd, or of a rarer Substance towards the Equator than towards the Poles, the Motion of the Nodes will be forward, or towards the consequent Signs.

Coroll. (2.) Hence also, from the Motion of Nodes, the Constitution of a Globe may be gathered: To wit, if the Globe constantly keep the same Poles, and the Motion be towards the antecedent Signs, the Matter about the Equator is redundant, but if towards consequent ones, deficient. Let us suppose a Globe uniform, and perfectly spherical, first to rest in a free Space, and then by some Force, whatever it be, impress'd on the Surface, to be driven forwards, and from thence to acquire a Motion partly circular, partly streight forward. Because the Globe is indifferent to all Axes passing through its Center, and is no more determin'd to one Axis, or one Situation of the Axis than to another; it is manifest, that it will never change its Axis, or the Inclination of the same, by any Force of its own. Now, let the Globe be impell'd obliquely in that same

Part of the Surface, as before by some new Impulse; since the Impulse, whether it be sooner or later, makes no Alteration in the Effect; it is manifest, that these two Motions impress'd successively, will produce the same Motion, as if they had been impress'd at the same time; that is, the same as if the Globe had been impell'd at first with a simple Force compounded of both Impulses; and consequently a simple Motion about an Axis of a given Inclination. And the same is the Reason of the Second Impulse made in any other Place of the Equator of the first Motion, as of the first Impulse made in any Place whatever in the Equator of that Motion, which the second Impulse without the first would produce; and consequently of Impulses made upon any Places whatever. These will generate the same circular Motion, as if they had been impress'd at one and the same time upon the Place of the Intersection of the Equators of those Motions, which they had severally generated, if they had been impress'd asunder. The Homogeneous and perfect Globe therefore doth not retain more distinct Motions; but compounds all the impress'd ones, and reduceth them to one; and is in it self perpetually revolv'd, by a simple and uniform Motion, about a single Axis of a given Inclination, as being always invariable. Nor can a Centripetal Force, tending towards any extrinsick Body whatever, change the Inclination of the Axis, or Velocity of the Rotation. If a Globe be understood to be divided into two Hemispheres by any Plane whatever passing through its Center, and through the Center unto which the Force is directed, that Force will always urge both Hemispheres equally, and so the Globe, as to the Motion of Circumrotation, will incline to neither Part. But let new Matter be added

added ſomewhere betwixt the Pole and the Equator, heaped up in the Form of a Mountain: This will both diſturb the Motion of the Globe by the perpetual Endeavour of departing from the Center of its Motion, and will make the Poles to wander over its Surface, and to deſcribe Circles about it ſelf, and its oppoſite Point. Nor will that Irregularity be corrected, but either by placing the ſaid Mountain in one of the Poles, in which Caſe, as was ſaid before, the Nodes will go forwards; or in the Equator, and then the Nodes will go back; or by adding ſome new Matter on the other Part of the Axis to counterpoiſe the Mountain in its Motion. And thus the Nodes will go forwards or backwards, as the Mountain; and the new Matter added on the oppoſite Part, are nearer to the Pole, or to the Equator.

Coroll. (3.) Since therefore it is manifeſt from Aſtronomical Obſervations, that the Nodes of the Equator of the Earth do perpetually go back about 50″ in every Year; which Regreſs is called the Preceſſion of the Equinox; it follows, that the Equatoreal Parts of the Earth are higher than the Polar. And, *vice verſâ*, ſince from the Diurnal Motion the Figure of the Earth is, as will be ſhew'd afterwards, that of an oblate Spheroid, (the Polar Parts being more depreſs'd than thoſe about the Equator;) it is manifeſt from thence, that the Nodes of the Equator muſt go back yearly.

Coroll. (4.) From what hath been ſaid, it is alſo manifeſt, that the Axis of the Earth will vibrate to and fro yearly; and in every annual Revolution be inclin'd twice towards the Equator, and twice return to the former Poſition. It is manifeſt alſo, that the greateſt Motion of the decreaſing Inclination of the Plane of the Equator, and of the Ecliptic, doth happen in the Quadratures of the Nodes; and that the least Angle of Inclination falls in the Octants after the Quadratures, or about the Middle of *Leo* or *Aquarius*. Lastly, that the greatest Motion of Inclination falls in the Conjunction and Opposition of the Nodes, or in the Equinoxes; and the greatest Angle of Inclination in the next Octants, or about the Middle of *Taurus* or *Scorpio*. But by reason of the Smalness of these Motions, these Effects will be altogether insensible, and to be discover'd by no Observations of Astronomers. But it is to be noted, that contrary Effects were to be attributed to our Earth, if so be the Parts about the Equator were more depress'd than the Polar.

Coroll. (5.) And from hence the Evasion devised by the Famous Dr. *Gregory*, to shew that the annual Parallax of the Fixed Stars is built upon a weak Foundation, and that neither the Distance of the Fixed Stars, which are observed, nor the annual Motion of the Earth, can be certainly concluded from thence; this Evasion, I say, falls to the Ground. Let us produce in this place the Words of Dr. *Gregory*, and spare so much time as to debate this Matter with him particularly. Mr. *Flamsteed*'s Method of observing the Parallax of the Fixed Stars, hath been explain'd by us in our Astronomical Lectures, to which I refer you. Now, from this Method rightly understood, it is manifest, that the Polar Star, for Instance, is more distant from the Pole about the Summer, than about the Winter Solstice; and this by a very sensible Difference, as being about 40″ or 45″. From whence Mr. *Flamsteed* concludes, that the Earth must certainly be mov'd about the Sun, and that the Fixed Stars are subject

ject to a Parallax ſenſible enough, and that their Diſtances conſequently may be gathered from thence. Now, what doth Dr. *Gregory* ſay to this? Doth he deny the Obſervation it ſelf? No, in no wiſe. Doth he aſſign for the Cauſe of the ſaid Difference of Diſtance, that very ſmall Nutation of the Axis of the Earth, by which he ſuppoſeth, with Mr. *Flamſteed*, that the Inclination of the Ecliptic to the Equator is leſſen'd about the Solſtices, and increas'd about the Equinoxes? No, not this neither. For Mr. *Flamſteed* had ſhew'd, that that very ſmall Nutation doth rather confirm than weaken his Opinion. What therefore he attributes the ſaid Phænomenon to, as its Cauſe, let us hear his own Words, *Page* 275. " This " Method, ſaith he, ſuppoſeth the Axis of the " Earth to be always moſt exactly parallel to it " ſelf, when it is in the oppoſite Points of its " own Orbit, where the Obſervations are made. [And why ſhould it not ſuppoſe this, or that it is parallel to it ſelf exactly enough for the preſent Purpoſe? But he goes on:] " Although that " ſmall Nutation of the Axis, of which we ſpoke " juſt now, doth in no wiſe hinder Mr. *Flamſteed*'s " Obſervation; yet there is another Nutation of " it, which may produce the Diverſity of the " Diſtance of the Polar Star from the Pole; that " is, if the *Southern* Hemiſphere of the Earth " be of a more denſe Frame than the *Northern* " (whether it be from hence that that hath leſs " Summer than this, and therefore more cold; " or from the Inequality of the Continents about " the Poles, or from ſome other Cauſe unknown " to us,) ſince at the Winter Solſtice, the *Southern* Pole inclines to the Sun, and is withal " nearer to it than the *Northern*, and in the Summer Solſtice this latter inclines to the Sun; the

" Axis

" Axis of the Earth will be more inclin'd to the " Plane of the Ecliptic in Winter Time, than in " the Summer; and the Angle whereby the Polar " Star is distant from the Pole, would be less at " the Winter than the Summer Solstice, altho' " the Pole Star were placed at an infinite Di- " stance, and the Lines drawn from thence to " the *Magnus Orbis* were to be reckoned for pa- " rallel. Since therefore the whole that can be " made of Mr. *Flamsteed's* Observation is this, " that the apparent angular Distance of the Po- " lar Star from the Pole, is less in the Summer " Solstice than in the Winter; and this may " arise from two Causes, either from the Con- " course of right Lines drawn from the Earth to " the Polar Star, in divers Situations of the " Earth to that Star, if the Earth's Axis in one " of the Observations be parallel to it self in " another, which Mr. *Flamsteed* supposeth; or " from the Concourse of right Lines coinciding " with the Axis of the Earth in its divers Situa- " tions, the Polar Star being suppos'd to be infi- " nitely distant; the Parallax of the Fixed Stars " cannot be certainly concluded from that Obser- " vation. Because the whole Observation may " consist, and the right Lines drawn from divers " Places of the Earth in its Orbit to the Pole Star " infinitely distant may remain parallel; tho' the " Parallax of the great Orb, with respect to that " Star, be suppos'd to be none at all. Yea, this " Observation (saith he) doth not so much as " prove immediately the annual Motion of the " Earth. For although the Earth remains in the " middle (making by its Rotation about its Axis, " as in the Semi-Tychonic System, the apparent " Diurnal Motion of the Stars,) the Sun when pla- " ced in the *Southern* Signs may so attract the *Southern*

Hemis-

Plate VII. *fronting Page 234.*

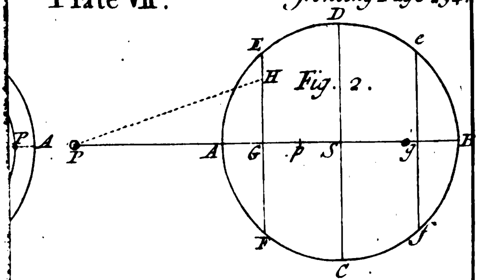


Fig. 2.

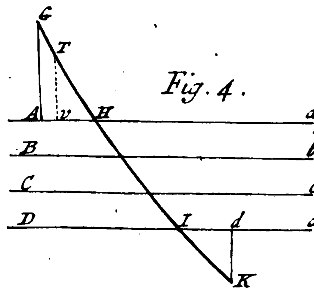


Fig. 4.

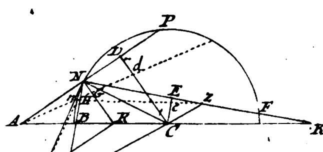
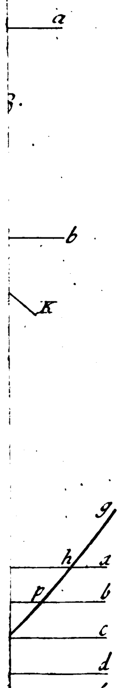


Fig. 6.



I. Senex sculp.t

" Hemisphere of the Earth, which is then nearer, " and is perhaps more dense, that the Distance " of the Polar Star from the Pole at the Time of " the Winter Solstice, should be less than the " same Distance is when the Sun is placed in the " *Northern* Signs, where it is more remote from " the Hemisphere that is then turned to it; from " which and the less Density of the same Hemis- " phere, which is perhaps conjoin'd therewith, " it is no wonder that this Hemisphere should be " less attracted. Thus Dr. *Gregory*, who deviseth the like Evasions for the rest of the Observations of Mr. *Flamsteed* and Dr. *Hook* on this Subject. But I answer,

(1.) That, as to the assigned Causes of the Nutation of the Earth, the less Summer, to wit, of the *Southern* Hemisphere, and greater Cold, or the Inequality of the Continents about the Poles; if this Learned Man would derive that Density of the *Southern* Hemisphere above the *Northern* from these Causes, which may suffice to the moving of the Earth so many Seconds from its former Position; he might as well go about to move Mount *Caucasus* from its Place with a Leaver. I do admire at his Ungeometricalness in this Business, that he would not first estimate in some sort the Force and Quantity of these Causes, before he attributed so huge Effects to them. ***But his Prudence is to be commended***, that he added, or ***from some other Cause unknown to us***: For he knew very well, that an unknown Cause cannot be computed. But in the mean while I will speak freely and openly, that there can be no Cause assign'd of this divers Density of the Hemispheres of the Earth, which he supposes, but what is contrary to the Mechanical Formation of the Planets, and the modern Phænomena of Nature. For,

(2.) If

(2.) If one Hemisphere of the Earth was a little higher or denser than the other, that Nutation of the Earth which he hath devised, would in no wise follow from thence. For in this Case, the Axis of the Globe would nodd indeed, but so that the Angle of the Inclination would twice in a Year return to its greatest, and twice to its least Quantity; and this so that that Angle would be of the same Quantity in both the Solstices, which plainly undermines the Foundations of his Hypothesis.

(3.) From this unequal Altitude or Density of the Hemispheres of the Earth, if so be it exceeds the Altitude or Density of the Equator, the Progress of the Equinoxes would follow: Whereas it is a thing certain, and acknowledg'd by Mr. *Gregory* himself, and every one, that they continually go backwards, and not forwards. But if he assign the Inequality to be such only, as not to infringe the greater Altitude or Density of the Equator; so that so much as the Parts about one Pole do exceed the Equatoreal Parts in Altitude, so much the Parts about the other Pole fall short thereof: Neither will this be any Help to his Cause. For because of the Defect of Force in one Hemisphere, which compensates the Excess in the other; the Forces on both Sides will be in a poize, and there will be no entire Force which should move the Axis, and cause any Nutation. So that neither from that unequal Altitude or Density suppos'd, will his supposed Nutation of the Axis in any wise follow.

(4.) If we should, for Disputation's Sake, suppose that Nutation of the Axis, neither yet would this Learned Man attain his Aim. For he supposeth such a Nutation, as would reduce the Axis in one of the Solstices unto the least Angle of of Inclination, and unto the greatest in the other. Now from the Principles of Sir *Isaac-Newton* before laid down, which are Dr. *Gregory's* Principles likewise, it would follow, that the greatest Angle of Inclination of the Axis will be in the Octants after the Conjunction and Opposition of the Nodes, and the least in the Octants after the Quadratures of the same; so that as we said before, in both the Solstices themselves, which are in the Middle betwixt the greatest and least Angle, no Diversity at all of the Angle of Inclination is to be expected. From whence also, which is to be noted by the way, both Mr. *Flamsteed* himself, and Dr. *Gregory* who follows him, are altogether mistaken, when they suppose that Nutation of the Axis, to which the Precession of the Equinoxes is owing, can have any place here.

(5.) If, lastly, we should be minded to suppose the Nutation of the Axis, to be in the Time, and to the Parts assign'd by Dr. *Gregory*; the Quantity of Inclination would be far less than to produce Mr. *Flamsteed's* Parallax. Let us grant to him, that the Axis of the Earth doth vibrate to and fro every Year; let us grant also, that in one of the Equinoxes this Nutation is to the one Part, and in the other Equinox to the contrary; so that the greatest Difference possible should arise from thence. Yet, how very small will this Difference be: To wit, according to the Calculation we made formerly, (see *Lect. Astron.* Page .) it is manifest, that this huge Nutation which ariseth from the sensible Altitude of about 17 Miles, whereby the Semi-diameter of the Equator exceeds half the Axis, did only amount to a Part of one Second. What therefore is this Minute Difference to the Parallax, which ariseth

to

to three whole Quarters of one Minute? This Cause therefore is in no wise sufficient to that Effect. To conclude, it is most certain that this Evasion of Dr. *Gregory*'s, whereby he would shew that the annual Motion of the Earth doth not follow upon Mr. *Flamsteed*'s Observations, is no small Error of his, and leaves a blemish upon a Work otherwise valuable for Demonstrations strictly geometrical, a Beauty not to be met with often elsewhere in Physical Tracts.

Scholium. But it is to be noted, that the Famous Mr. *Flamsteed* hath not ordered his Reasonings altogether rightly in this Place, which the *French* have lately noted; and hath sometimes deduced the Parallax of the Fixed Stars from Phænomena in no wise proving it. But yet when I looked more narrowly into this Matter, Eleven of Fifteen remarkable Observations, which the *French* allow to be true, and agreeing with their own, do even yet shew the Parallax of the Fixed Stars; and of those Four that seem to disagree with it, there is only one of that Quantity as to give us any Trouble in this Business; which therefore it is reasonable to think to be owing to some Mistake, whether in the observing or in the writing. Especially since the like Parallax seems manifestly to appear from the accurate Observations of Dr. *Hook*. But these Things we leave to the further Diligence and Scrutiny of Astronomers.

Octob. 29. 1705.

LECT.

Lect. XXII.

XLIII. If each particular Body of any System, as A and B severally considered, draws all the rest of the Bodies with accelerative Forces, which are as the Squares of the Distances from the attracting Body reciprocally, the absolute Forces of all those Bodies will be one to another as are the Bodies themselves.

Let the Body A, by its accelerating Force represented by a, draw the Body B; and because of the Distance which is on both Sides the same, let B reciprocally draw A by the Force represented by b. The Quantity of Motion is on both Sides equal, because of the Reaction that is on both Sides equal to the Action: And that Quantity of Motion doth altogether arise from the Velocity drawn into the Quantity of the Matter. Therefore the Rectangle $A \times b$ is equal to the Rectangle $B \times a$. And consequently the accelerating Force of the Body B will be to that of the Body A, at equal Distances, as the Body B is to A. And consequently the absolute Forces of the Bodies will be one to another, as the Bodies themselves: To wit, the Sum of equal Forces tending every where unto equal Parts, at equal Distances. *Q. E. D.*

Scholium. By such like Propositions, we are led unto the Analogy betwixt Centripetal Forces and Central Bodies, to which those Forces are directed. For it is reasonable to think, that the Forces which are directed towards Bodies, ſhould depend upon the Nature and Quantity of thoſe Bodies, as it comes to paſs in Magnetics. And as often as theſe Caſes happen, the Attractions of Bodies are to be eſtimated by aſſigning to each Part of them its proper Force, and ſo gathering the Forces into one total Sum. But as for the Word *Attraction*, we uſe it here generally for any Endeavour whatſoever of coming unto another, which is found in Bodies, whether that Endeavour be from the Action of Bodies, either of themſelves tending to one another, or by mutual Emiſſion of Spirits acting one upon the other; or whether it ariſe from the Action of the Ether, or Air, or any Medium whatever, corporeal or incorporeal, which forces the Bodies floating in it towards one another. In the ſame general Senſe, we uſe the Word *Impulſe*; not conſidering in this place the phyſical Species and Qualities of the Forces, but their Mathematical Quantities and Proportions; as we propos'd above in the Definitions. In which Conſideration of them, the Quantities of the Forces are to be ſearched out and defin'd, and thoſe Proportions which follow upon any Conditions whatever that are ſuppos'd. But when we deſcend unto Phyſics, theſe Proportions are to be compared with the Phænomena, that it may be known what Kind of Force it is which agrees to each Kind of attractive Bodies. And then we may at length, and not till then, ſafely diſpute concerning the Species, Cauſes, and phyſical Reaſons of Forces. Let us ſee therefore by what Forces Spherical Bodies, ſuch as are commonly the greater Bodies of the World, the Sun, Fixed Stars, Planets, and Comets, conſiſting of attractive Particles in the manner juſt now deſign'd,

ought

ought to act one upon another; and what Sort of Motions will follow from thence.

XLIV. If towards each equal Points of a Spherical Physical Surface of equal Thickness every where, but which Thickness is so small that it is not to be regardrd, there be a Tendency of equal Centripetal Forces decreasing in the duplicate Proportion of the Distances from the same Points; any Corpuscle placed any where within this Surface, will not be attracted unto any Part by the said Force; but will either rest, or continue that Motion which is begun without any Disturbance, and in the same manner as if it were acted upon with no Force at all from that Surface.

In *Fig.* 5. *Plate* 6. let HIKL be that spherical Surface, and P a Corpuscle placed within it. Through P let there be drawn to the Surface any two right Lines, intercepting the very small Arches HK, IL. And here because the Triangles HPI, LPK are similar [for the Arches HI and KL are so small, that they are to be taken for right Lines; and the Angles vertically opposite at P are equal; and the Sides containing their equal Angles are (by III. 35. with VI. 14. and VI. 6. of the *Elem.*) on both Sides proportional] therefore those Arches will be proportional to the Distances HP and LP; that is, PH will be to PL, or PI to PK, as IH is to KL. And any little Portions of the spherical Surface at HI and KL, bounded on every Side by innumerable right Lines passing through the Point P, whether they be Polygons or Circular, will be similar Figures, and consequently in the duplicate Proportion of those Arches or Distances from the Corpuscle. And the whole attracting Forces towards the contrary Parts will, by reason of the nearer Situation of the lesser Surface, and the remoter

Situation of the greater, counterpoise and destroy each other. And by the same Argument, all Attractions throughout the whole Surface will be destroy'd by the contrary Attractions. And consequently the Body P will be impell'd to no Part by these Attractions. *Q. E. D.*

Coroll. (1.) Since therefore any Sphere, which hath a Concave, Concentric, Spheric Space within, may rightly be distinguished into innumerable such like spherical Surfaces of an inconsiderable Crassitude; and since from the Force of this Demonstration, no one of these Surfaces can attract a Body placed within it unto any Part: It is manifest, that the whole Sphere can impress no Force upon the Corpuscle within it. But that this Corpuscle, if it was in Rest before, will still rest; or if it was in Motion before, of what sort soever it were, it will still continue that Motion; any Attraction which may be in the exterior Sphere notwithstanding.

Coroll. (2.) And since this thing may with Parity of Reason be demonstrated concerning any Corpuscles whatever, compounding what Body or Mass of Matter soever; it appears, that all Bodies whatever, placed within such a Concave Sphere, are uncapable of receiving any Impression from any attractive Force of that Sphere.

Coroll. (3.) If therefore our Earth, as made of such Spherical Surfaces compos'd of attractive Particles, hath a Spherical Central Cavity, Animals placed there are affected with no Force of Gravity from those Surfaces, and perform their Motions with the same Liberty, as they would do if there was no such thing as Gravity in Nature. And the same is to be said of the Planets and Comets, and of the Sun, and the Fixed Stars.

XLV. The

XLV. The ſame Things ſuppos'd as before, a Corpuſcle placed without the ſpherical Surface will be attracted to the Center of the Sphere by a Force reciprocally proportional to the Diſtance from the ſame Center.

In the double *Fig. 6. Plate 6.* let there be two equal Surfaces, (or rather the ſame Surface put twice) one Mark'd with great Letters, the other with ſmall, A H K B, a h k b deſcrib'd from the Centers S, s with equal Diameters A B, a b; and let P p be two Corpuſcles, (or rather one and the ſame Corpuſcle placed at divers Diſtances from the ſpherical Surface;) placed without in the Continuation of thoſe Diameters. Let the right Lines P H K, P I I: p h k, p i i be drawn from the Corpuſcles, cutting off from the greateſt Circles A T B, a t b equal Arches, H K, h k: and I T I, i t i differing, the latter from the former, as little as may be. And let the Perpendiculars SD, s d be let fall to PK, pk: and S E, s e to P I, p i; and I R, i r to P K, p k. Of which, let S D, s d cut P I, p i in the Points F and F. Let there be let fall alſo to the Diameters the perpendicular Lines I Q, i q; and becauſe of the Equality of the Lines D S, and d s; E S, and e s; and of the moſt ſmall vaniſhing Angles D P E, d p e; the Lines P E, P F, and p e, p f (the Difference F E, f e, and the little Lines D F, d f vaniſhing) may be accounted for equal; as having their laſt Proportion, thoſe Angles D P E, d p e, and D S E, d s e vaniſhing away, the Proportion of Equality. Theſe Things being thus, in the like Triangles P R I, P D F, and p r i, p d f, P I will be to P F, as R I is to D F; and p f will be to p i, as D F or d f is to r i: And both the equal Proportions being compounded into one, the Rectangle P I × p f will be to the

Rectangle P F into p i, as the Rectangle R I × d f is to the Rectangle D F × r i; that is, as R I is to r i: that is, in the last similar Triangles I R H, i r h (because of the right Angle at R and r; and the Angle R H I agreeing to the Angle r h i, if the equal Circles were applied to each other) as the vanishing Arch I H is to the vanishing Arch i h. Again, in the like Triangles P I Q, P S F: p i q, p s f, P I is to P S as I Q is to S E, and p s is to p i as S E or s e is to i q. And both the equal Proportions being compounded, the Rectangle P I × p s will be to the Rectangle PS × p i, as the Rectangle I Q × s e is to the Rectangle S E × i q; that is, as I Q is to i q. And both the principal Proportions being compounded, the Quantity P I × P I × pf × p s will be to the Quantity p i × p i × PF, × PS; that is, PIq × pf × ps will be to p i q × PF × PS, as the Rectangle IH × IQ is to the Rectangle i h × i q; that is, as the Circular Surface or Ring which the smallest Arch I H will describe in the Circumvolution of the Semicircle AHTB about the Diameter A B, is to the Circular Surface or Ring which the smallest Arch i h will describe in the Circumvolution of the Semicircle a h t b about the Diameter a b. And the Forces wherewith these Surfaces do attract the Corpuscles P and p, are, by the Hypothesis, as the Surfaces themselves, so far as the Squares of the Distances do not increase or diminish the the same Forces; and consequently those Forces are as the Surfaces themselves applied to the Squares of their Distances from the Bodies; that is, as $\frac{\text{PI}q \times \text{p f} \times \text{p s}}{\text{PI}q}$ is to $\frac{\text{p i}q \times \text{P F} \times \text{P S}}{\text{p i}q}$:

or as p f × p s is to P F × P S. These entire Forces likewise are to their oblique Parts, which by a Rea Revolution made of the Forces tend to the Centers according to the Lines P S and p s, as P I is to P Q; and as p i to p q; that is, (because of the like Triangles PIQ PSF; and p i q, p s f:) as P S is to P F, and as p s to p f. From whence, by Equality of Proportion, the Attraction of this Corpuſcle P towards the Center S, will be to the Attraction of the Corpuſcle p towards the Center s, as $\frac{PF}{PS} pf \times ps$ is to $\frac{Pf}{Ps}$ $PF \times PS$; or as $PF \times pf \times ps \times ps$ is to $pf \times PF \times PS \times PS$, or alſo as $ps \times ps$ or psq is to $PS \times PS$ or PSq; that is, as the Squares of the Diſtances from their Centers reciprocally.

And by the like Argument, the Forces wherewith the remoter Surfaces deſcribed by the Circumvolution of the remoter Arches H L and h l draw the Corpuſcles, are as the Squares of the Diſtances from their Centers reciprocally. And the Forces of all the like circular or annular Surfaces into which both the ſpherical Surfaces may be diſtinguiſh'd, by taking always equal Arches, as HK, hk and I T I, i t i; or, which is the ſame, by taking the perpendicular S D equal to s d, and S E equal to s e; the Forces of all theſe annular Surfaces, I ſay, are in the ſaid Proportion. And from thence, the Sum of the Forces, or the Force of the whole ſpherical Surfaces, will be exerted upon the Corpuſcles in the ſame Proportion. *Q. E. D.*

Coroll. (1.) Since therefore every entire Sphere may be rightly diſtinguiſh'd into innumerable ſuch like concentrical ſpherical Surfaces; and ſince from the Force of this Demonſtration any one of the Surfaces may ſo attract that Corpuſcle, that the Force of Attraction towards the Center is in the duplicate Proportion of the Diſtance reciprocally; it is manifest, that the whole Sphere also doth so attract that Corpuscle, that the Centripetal Force is in the duplicate Proportion of the Distance from the Center reciprocally.

Corol. (2.) And since the rest of the oblique Forces I Q, i q, estimated from the opposite Hemispheres, are opposite to, and destroy each other; the entire Force exercis'd upon the Corpuscle, will be altogether equal to that Force tending towards the Center.

Corol. (3.) And seeing the Demonstration would proceed in the like manner, if instead of one Corpuscle, any Body compounded of those Corpuscles were supposed (for what agrees to one Particle must, by the same Reason, agree to every one, and consequently to the Sum of them;) it appears, that every Sphere consisting of Particles equally attractive, doth so attract every Body, that the Quantity of the Attraction is in the duplicate Proportion of the Distance from the Center of the Sphere reciprocally.

Corol. (4.) Therefore the Attraction of the Sphere is in the same manner, as if the whole of the Forces tending towards the Center was gathered together in the Center it self, and united and propagated it self on every Side round about from that one Point.

XLVI. If unto each Point of any Spheres which are Homogeneous, or of the same Density, equal Centripetal Forces do tend, decreasing in the duplicate Proportion of the Distances from the Points; and the Proportion of the Diameters of the Spheres to the Distance of the Bodies from the Centers of the same Spheres be given: the Forces wherewith the Bodies are attracted being compar'd amongst themselves, will

be

Plate VI. fronting Page 246

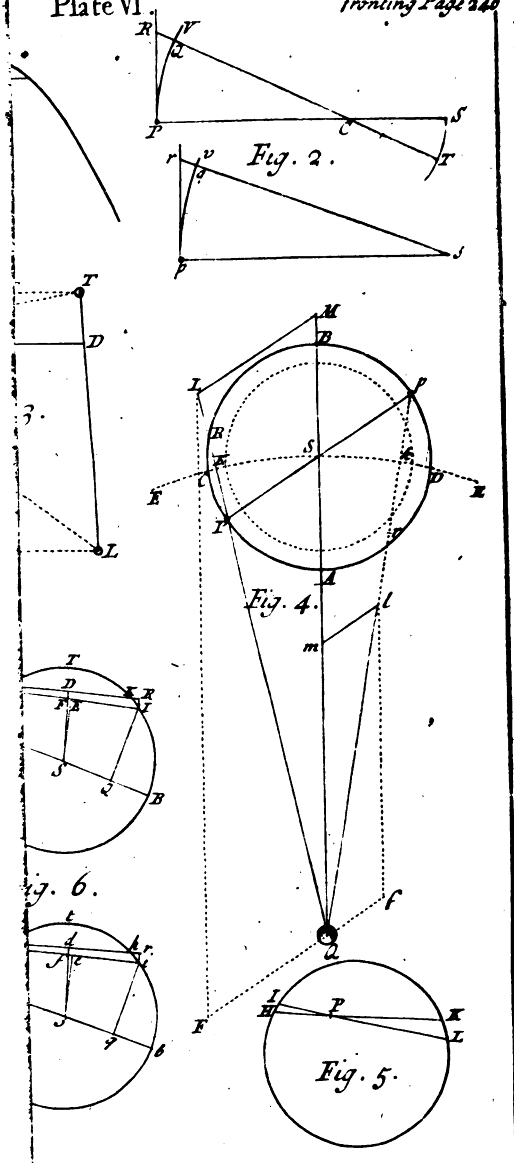


Fig. 2.

3.

Fig. 4.

ig. 6.

Fig. 5.

be found proportional to the Semidiameters of the attracting Spheres.

That is, The Forces of the Spheres are as the attracting Particles themselves, or, as the Spheres themselves; that is, in the triplicate Proportion of the Semi-diameters, to wit, at equal Distances. But when the Distances are suppos'd unequal, and unequal in the very Proportion of the Semi-diameters, the Forces will be diminish'd in Proportion to the Distances; that is, by the Hypothesis in the duplicate Proportion of the Semi-diameters of the Spheres. The remaining Forces therefore, which are to be estimated from the Excess of the triplicate Proportion above the duplicate, will be in the simple Proportion of the Semi-diameters directly. *Q. E. D.*

Corol. (1.) Hence, if any Bodies be revolv'd in Circles about Spheres consisting of Matter equally attractive; and the Distances from the Centers of the Spheres be proportional to the Diameters or Semi-diameters of the same; the periodic Times will be equal. For the Equality of the periodic Times follows from the Forces in the direct Proportion of the Distances; as we have shew'd before.

Corol. (2.) And the Inverse of it is also true; if the periodic Times be equal, the Distances of the revolving Bodies from the Spheres, if so be the same be Homogeneous, or of the same Density, will be proportional to the Semi-diameters of the Spheres.

Corol. (3.) And from the periodic Times given, together with the Distances of the Bodies from the Spheres, the Densities of the Spheres will also be given: To wit, by computing what periodic Times would follow from thence at Distances proportional to the Semi-diameters of the Spheres, and by determining from the Excess or Defect of the periodic Times, the Defect or Excess of Densities reciprocally proportional to the same. Examples of which, in the *Sun*, *Jupiter*, *Saturn*, and the *Earth*, will be produc'd hereafter.

XLVII. If unto each Point of some given Sphere, which is Homogeneous, or of equal Density every where, there be a Tendency of equal Centripetal Forces decreasing in the duplicate Proportion of the Distances from the Points; a Corpuscle placed within the Sphere, is attracted with a Force proportional to its Distance from the Center thereof.

In the Sphere A B C D (of *Fig.* 1. *Plate* 7.) described from the Center S, let the Corpuscle P be placed; and from the Center S with the Interval S P, conceive an inner Sphere to be described, to wit, P E Q F. It is manifest, by *Prop.* 44. That the Concentrick Spherical Surfaces, of which the Difference of the Spheres is composed, the Attractions in one part being every where destroy'd by the contrary Attractions, do not act at all upon the Corpuscle P: There remains only the Attraction of the inner Sphere P E Q F. Therefore the Centripetal Force decreaseth, by reason of the lesser Sphere which attracts in the triplicate Proportion of the diminish'd Distance from the Center; but increaseth in the invers duplicate Proportion of the Distance, because of the Access to the Center. Therefore the remaining Force, to be estimated from the Excess of the triplicate Proportion above the duplicate, will be in the direct Proportion of the Distance from the Center. *Q. E D.*

Corol. (1.) If such a sort of Sphere be bored through the Center, all Bodies let fall from all Distances, whether little or great, will descend unto

unto the Center in an equal Space of Time; in the Space, to wit, of 21′. 9″. in our Earth, as we observed before.

Corol. (2.) And if there be no Medium, which resists the Motion of the descending or ascending Bodies, every Body let fall will, when it hath passed the Center, ascend as far beyond the Center as it before descended to it; and so will, by a perpetual Ascent and Descent, imitate the Motions of pendulous Bodies vibrating in a Cycloid. And these Vibrations, if we may so call them, will be perform'd in equal Times.

Corol. (3.) But if, as many very small Intervals as you will, Concentrical to such a Kind of Sphere, be supposed to be interpos'd betwixt any spherical Surfaces whatever, and any Bodies whatever be understood to be revolv'd in these Intervals about the Center, like so many little Planets; the periodic Times of all these Planets will be equal every where. That is, every Period will be perform'd in the same Space of Time, in which any Body whatever being let down would perform the whole Vibration compounded of Going and Returning: Thus, in our Earth, the said circular Periods would be performed in 1 h. 24′. 36″. As may easily appear from what hath been demonstrated before.

Scholium. It is to be noted, that those Surfaces, of which we suppose solid Bodies to be compos'd, are not purely Mathematical, or void of all Thickness; but such thin Orbs, that their Crassitude is as nothing. In like manner by Points, of which we say Lines are compos'd, and from thence Surfaces and Solids, Particles of equal Magnitude, but which is so small that it is not to be regarded, are to be understood.

Nov. 19. 1705.

LECT.

L E C T. XXIII.

XLVIII. THE ſame Things being ſuppos'd, a Corpuſcle placed without a Sphere is attracted with a Force reciprocally proportional to the Square of its Diſtance from the Sphere. For let the Sphere be diſtinguiſh'd into innumerable Concentrical Spherical Surfaces; the Attractions of the Corpuſcle ariſing from each of the Surfaces, will be reciprocally proportional to the Square of the Diſtance of the Corpuſcle from the Center, by *Prop.* 45. And likewiſe by compounding, the Sum of Attractions, or the Attraction of the whole Sphere, will be in the ſame Proportion. *Q. E. D.*

Corol. (1.) Hence, in equal Diſtances from the Centers of Homogeneous Spheres, the Attractions are as the Spheres themſelves; or as the Cubes of the Diameters are one to another. For, by *Prop.* 46. if the Diſtances be proportional to the Diameters, the Forces of the Spheres will be as the Diameters: Let the greater Diſtance therefore be diminiſh'd in that Proportion; and thus the Diſtance being now made equal, the Attraction will be increas'd in that duplicate Proportion, and conſequently will be to the other Attraction in that triplicate Proportion of the Diameters, that is, in the Proportion of the Spheres themſelves.

Corol. (2.) In any Diſtances whatever, the Attractions will be as the Spheres applied to the Squares of the Diſtances.

Corol.

Corol. (3.) If a Corpuscle placed without an Homogeneous Sphere, be drawn with a Force reciprocally proportional to the Square of its Distance from the Center, and the Sphere in the mean time consists of attractive Particles; the Force of every Particle will decrease in the duplicate Proportion of the Distance from that Particle.

Corol. (4.) Since therefore all the Planets, both Primary and Secondary, are attracted to the Sun; all the Secondaries about *Jupiter* are attracted to the Center of *Jupiter*; all the Satellites of *Saturn* to the Center of *Saturn*; and the Moon to the Center of the Earth: Every one to its own Center in divers Distances, with a Force reciprocally proportional to the Squares of the Distances respectively; the Force of every Particle composing the Body of the *Sun*, *Jupiter*, *Saturn*, and the *Earth*, decreaseth in a duplicate Proportion of the Distance from the same Particle.

XLIX. If unto each Point of a given Homogeneous Sphere, there be a Tendency of equal Centripetal Forces decreasing in the duplicate Proportion of the Distances from the Points; every other similar Sphere will be attracted with a Force reciprocally proportional to the Square of the Distance of the Centers.

For the Attraction of every Particle is reciprocally, as the Square of the Distance thereof from the Center of the attracting Sphere, by *Prop.* 45. and therefore it is the same, as if the whole attracting Force lay in one single Particle situate in the Center of this Sphere. But this Attraction is as great as the Attraction of the same Corpuscle wou'd be, if so be it were attracted by each Particle of the attracted Sphere with the same Force wherewith it attracts them. But this Attraction

of

of the Corpuscle would be by the last *Prop.* reciprocally proportional to the Square of the Distance thereof from the Center of the Sphere; and consequently the Attraction of the Sphere, which is equal to the same, is in the same Proportion. *Q. E. D.*

Corol. (1.) Attractions of Homogeneous Spheres towards other Homogeneous Spheres, are, as it is in those of Points, or the most minute Corpuscles, as the attracting Spheres applied to the Squares of the Distances of their Centers from the Centers of those which attract.

Corol. (2.) The same thing holds, where the attracting Sphere doth also attract it self. For each Point of this will draw each Point of the other with the same Force, whereby it is interchangeably drawn by them. And consequently since in all Attractions, both the attrahent and the attracted Body are urged or acted upon; the Force of the mutual Attraction will be doubled, keeping the Proportions.

Corol. (3.) All those things which have been demonstrated above, concerning the Motion of Bodies, about the *Focus* of Conic Sections, do hold where the attracting Sphere is placed in the Focus, and the Bodies are mov'd without that Sphere.

Corol. (4.) But those things which were demonstrated concerning the Motion of Bodies about the *Center* of Conic Sections, do hold where the Motions are performed within the Sphere; to wit, where a Sphere not perfectly Concave, but full of Concave Parts, is supposed; as we observed before.

L. *Prop.* If Spheres which are dissimilar in the Process from the Center to the Circumference, (as to Density of Matter, and the attractive Force,)

Force,) are neverthelefs altogether fimilar in their Progrefs in a round in every given Diftance from the Center; and the attractive Force of every Point decreafeth in the duplicate Proportion of the Diftance of the attracted Body: the whole Force wherewith one fuch Sphere draws the other, is reciprocally proportional to the Square of the Diftance of the Centers.

For fuch a fort of Sphere may always be divided into fimilar Concentrick Spherical Surfaces. And fince it hath lately been demonftrated, that every Surface feparately confidered, doth fo draw all other Surfaces feparately confidered, that the whole Force wherewith fuch a fpherick Surface draws any other, is reciprocally proportional to the Square of the Diftance from its Center; the Propofition will appear manifeft of entire Spheres compounded of fuch Surfaces. *Q. E. D.*

Corol. (1.) Hence, if many fuch like Spheres being like to one another in all things, do attract each other; the *accelerating Force* of each upon each will be at equal Diftances of the Centers, as the attracting Spheres themfelves; or as the Quantities of Matter contain'd in the fame.

Corol. (2.) And in all unequal Diftances whatever, as the attracting Spheres applied to the Squares of the Diftances betwixt the Centers of the Spheres.

Corol. (3.) But *Moving Attractions*, or the Weights of Spheres in Action upon or towards Spheres at equal Diftances of Centers, are as the attracting and attracted Spheres conjunctly; that is, as the Contents of the Spheres produc'd by Multiplication. For fince the attracting Body, becaufe of Reaction, which every where is equal to Action, tho' tending to the contrary Part, is mov'd towards the Body attracted with the like

Quantity of Motion, that is, with a Celerity reciprocal to the Bodies; and this would be if there were properly no attractive Force of the Body which is attracted: And since to those who inhabit any Sphere, the whole Velocity of Spheres approaching to one another is necessarily referr'd to the other Sphere, and not to the Sphere they dwell upon; because they cannot discern their own Motion; hence it comes to pass, that all the Centripetal Force of the other Sphere, that, to wit, wherewith it approacheth to their own, or that rather wherewith they are both of them carried with the same Tendency towards the mutual Concourse, and which is called the *Weight* of the other; is not only proportional to the attracting Sphere, but to both Spheres taken together. This is that which goes by the Name of the *Weight* of any Body towards the Earth, which makes that that Body and the Earth are carried towards each other with a relative Velocity of Approximation. Thus we shew'd before (*Prop.* 23.) that the Gravity of the Moon towards the Earth, is of that Quantity that it would fall to the Center of the Earth in the Space of 4 h. 20'. almost. Not that all that Velocity is to be referr'd to the Moon it self; but that if all the respective Velocity of approaching, arising from the Motion of both Bodies, were to be referr'd to the Moon alone, as those that inhabit the Earth are wont to refer it; this would make that the Moon would fall to the Center of the Earth in that Space of Time.

Corol. (4.) In unequal Distances *Moving Attractions*, or the Weights of Spheres towards Spheres, will be as those Contents applied to the Squares of the Distances betwixt the Centers.

Corol.

Corol. (5.) The same Things hold also, *a fortiori*, where the whole Attraction ariseth from the attractive Virtue of both Spheres mutually exercised upon the other Sphere. For the Attraction will be doubled, the Proportion being kept.

Corol. (6.) If some such Spheres be revolv'd about others quiescent each about each; and the Distances betwixt the Centers of the revolving ones, and of the quiescent, be proportional to the Diameters of the quiescent; the periodic Times will be equal.

Corol. (7.) And, *vice versa*, if the periodic Times be equal, the Distances will be proportional to the Diameters.

Corol. (8.) All the same Things which we demonstrated above, concerning the Motion of Bodies about the Foci of Conic Sections, do hold where the attracting Sphere of what Form and Condition soever, which hath been already describ'd, is placed in the Focus.

Corol. (9.) As also where the attracting Spheres revolve of what Condition soever, that have been already describ'd, *i. e.* either wholly Homogeneous, or in the same Distances from the Centers so.

LI. If to each equal Point of Homogeneous Spheres, there be a Tendency of Centripetal Forces equal at equal Distances, but at divers Distances directly proportional to the Distances of the Points from the Bodies attracted; the Force compounded of the Forces of all the Parts, wherewith the two Spheres do mutually draw each other, will be as the Distance betwixt the Centers of the Spheres.

Case I. In *Fig.* 2. *Plate* 7. Let A C B D be a Sphere compounded of these attractive Forces: S the Center of the same: P a Corpuscle attracted:

ed: PASB the Axis of the Sphere passing through the Center of the Corpuscle: EF and ef two physical Planes of a Thickness not to be regarded, wherewith the Sphere is cut, which are also perpendicular to the Axis, and on this side and on that equally distant from the Center of the Sphere. The Points G and g, the Intersections of the Planes and the Axis: and H any physical Point in the Plane EF. The Centripetal Force of the Point H upon the Corpuscle P, exercised along the Line PH, is, by the Hypothesis, as the Distance it self PH; which by Resolution of Forces is to be divided into GH, GP. From whence the Force along the Line PS, that is towards the Center S, is as the Length it self PG; [HG, or one Part of the Forces being destroy'd by the Force of the Point which is equally distant from the Axis, on the other Side of the Axis directly opposite in the same Plane.] Therefore the Force of all the Points in the Plane EF; that is, the Force of the whole Plane, whereby the Corpuscle P is drawn towards the Center S, will in like manner be as the Number or Sum of the Points drawn into the Distance PG: that is, as the Content under the Plane EF, and the Distance PG. And in like sort the Force of the Plane ef, whereby the Corpuscle P is drawn towards the Center S, is as the equal Plane drawn into the Distance PG. And the Sum of the Forces of both Planes is as the Plane EF, drawn into the Sum of the Distances PG × Pg; that is, as that Plane drawn into PS the double of the Distance of the Center and the Corpuscle: [Because of the Lines PG, PS, Pg, which are Arithmetically proportional; and from thence the Sum of the Extremes equal to the Double of the Mean.] That is, as the Double of the Plane EF; or the

Sum

Sum of the equal Planes drawn into the Diſtance P S. And by the like Argument, the Forces of all the Planes in the whole Sphere, equidiſtant on this ſide and on that from the Center of the Sphere, are as the Sum of the Planes drawn into the Diſtance P S: that is, as the whole Sphere drawn into the Diſtance of the Center thereof from the Corpuſcle. And becauſe the Sphere is given in every Diſtance, the entire attracting Force will be as P S, the Diſtance of the attracted Corpuſcle from the Center of the Sphere. *Q. E. D.*

Caſe (2.) Now let the Corpuſcle P draw the Sphere, to wit, all Points of it, with a Force directly proportional to the Diſtance of the Points from the Corpuſcle: And by the ſame Argument it will be prov'd, that the Force wherewith that Sphere is drawn will be as the Diſtance P S. *Q. E. D.*

Caſe (3.) Then let another Homogeneous Sphere be compounded of innumerable Particles, as P, attractive in like manner; that is, in the direct Proportion of the Diſtance. And becauſe the Force wherewith every Particle is drawn, is as the Diſtance of the Corpuſcle from the Center of the firſt Sphere drawn into the ſame Sphere; and conſequently is the ſame, as if the whole proceeded from one ſingle Point in the Center of the Sphere: The whole Force wherewith all the Corpuſcles will be drawn in the 2d Sphere, that is, that wherewith that whole Sphere is drawn, will be the ſame as if that Sphere were drawn by a Force proceeding from one ſingle Corpuſcle placed in the Center of the firſt Sphere. And therefore it will be proportional to the Diſtances betwixt the Centers of the Spheres. *Q. E. D.*

Case (4.) Now let the Spheres draw one another; and the doubled Force will still preserve the former Proportion. *Q. E. D.*

Case (5.) Next, let the Corpuscle p be placed within the Sphere A C B D: and here, because the Force of the Plane e f upon the Corpuscle will be as the Content under that Plane, and the Distance p g, or as $ef \times pg$; and the contrary Force of the Plane E F, as the Content under that Plane and the Distance p G, or as $EF \times pG$: or as $ef \times pG$: Therefore the attracting Force will be as the Difference of the Contents, that is, $ef \times pg - pG$; or as the double of e f drawn into half the Difference $pg - pG$, $= 2\,ef \times \frac{1}{2}pg - \frac{1}{2}pG$: That is, because S G, S g are equal, as the Sum of the equal Planes drawn into half the Difference of the Distances, or into p S the Distance of the Corpuscle from the Center of the Sphere. And by the same Argument, the Attraction of all the Planes in the whole Sphere, as E F, e f, equidistant on this side and on that from the Center; that is, the Attraction of the whole Sphere will be as the Sum of all the Planes, or the whole Sphere drawn into p S, the Distance of the Corpuscle from the Center of the Sphere. *Q. E. D.*

Case (6.) And if a new Homogeneous Sphere placed within the former, be compounded of innumerable Particles as p; it will be prov'd, as before, that the Attraction, whether it be simply of one Sphere into another, or mutual and on both sides, will be as the Distance of the Centers p S. *Q. E. D.*

LII. If Spheres in the Progress from the Center to the Circumference (as to Density of Matter, and attractive Force) howsoever dissimilar, are nevertheless in the Progress round about at every

every given Diſtance from the Center every where ſimilar; and the attractive Force of every Point is directly as the Diſtance of the attracted Body: The whole Force wherewith two ſuch Spheres will mutually draw each other, will be proportional to the Diſtance betwixt the Centers of the Spheres.

For ſuch a Sphere may always be divided into equal Circles EF, ef, and in the ſame Diſtances from the Centers G, g, into Homogeneous ones; and ſince from the Force of what hath been already demonſtrated, every circular Perimeter, of which every whole Circle is compounded, doth afford a Force proportional to the Diſtance from the Center of the Sphere; the whole Force will alſo be in the direct Proportion of the Diſtance from the Center.

Corol. What has been above demonſtrated in the Corollaries to the 50th Prop. concerning the Attractions of Spheres where the Law of Attraction was in the duplicate Ratio of the Diſtance inverſly, may be every where applied to this Place, *mutatis rite mutandis.* But eſpecially thoſe which we have formerly demonſtrated, concerning Bodies moving about the Center of Conic Sections, obtain where all the Attractions are made by the Force of ſpheric Bodies of the ſame ſort that was juſt now deſcribed; and the attracted Bodies are Spheres of the ſame Kind.

Scholium. We have now explained the two notable Caſes of Attractions; to wit, where the Centripetal Forces decreaſe in the duplicate, or increaſe in the ſimple Proportion of the Diſtances: Cauſing the Bodies in both Caſes to be revolved in Conic Sections, to wit, by the firſt Law about the Focus, by the ſecond about the Center; (and the firſt Caſe agreeing to Bodies placed without the

the Spheres, but the latter agreeing to the Bodies placed within them:) And compounding the Centripetal Forces of spheric Bodies decreasing by the same Law in their Recess from the Center, or increasing with the Bodies. Which is well worthy to be taken notice of, and is very useful to solve the Phænomena of the Solar System. It wou'd be tedious, and of little Use, strictly to examine the other Cases in this Place, which would exhibit Conclusions less elegant, and more remote from the Constitution of the World. Therefore, since we have already explained the Attractions of spheric Bodies, we may go on to the Laws of Attraction of other Bodies consisting of like attractive Particles: But we don't design to handle them in particular. Therefore we shall only subjoin some more general Propositions of the Forces of Bodies of the like sort, and of the Motions arising from thence, which will be of Use in Physicks.

LECT. XXIV.

LIII. IF two similar Mediums be separated from each other by a Space bounded on both sides with parallel Planes; and a Body in its Passage thro' this Space be attracted or impelled perpendicularly to either Medium, & be neither accelerated nor retarded in its way by any other Force; and if moreover the Attraction be every where the same, at equal Distances, from both Planes taken along the Line of its Motion; the

the Sine of the Angle of Incidence upon the one Plane, will be to the Sine of the Angle of Emergence out of the other Plane, in a given Proportion; *i. e.* whatsoever the Angle of Inclination shall be, the Proportion of those Sines will always be found to be the same.

Case (1.) Let (in *Fig.* 3. *Plate* 7.) A a B b be two parallel Planes. Let a Body fall upon the former along G H, and be in its whole Passage attracted or impell'd towards the Medium of Incidence; and by this Action describe a Curve HI, and then emerge by the Line I K. Let the perpendicular I M be erected upon the Plane of Emergence Bb, meeting both the Line of Incidence G H produc'd in M, and the Plane of Incidence A a in R. And let the Line of Emergence K I produc'd meet H M in L. Thus G M will be the Tangent of this Curve in the Point H; and the Line L K the Tangent of the same in the Point I. Then, from the Center L with the Interval L I, let a Circle be describ'd, which may cut as well H M in P and Q, as M I produc'd in N. And now in the first place, if the Attraction or Impulse be suppos'd to be uniform, that Curve H I will, according to what hath been demonstrated before, be a Portion of a Parabola; (see *Prop.* 8.) and the Line L V, perpendicular to both Planes, will be one of the Diameters thereof; and the right Line H I, bisected by the same L V in the Point C, will be an Ordinate of that Diameter. Now it is a *Property* of this Figure, that the Rectangle contain'd under the *Latus rectum*, belonging to the Vertex H, (which in this Case is always given (by *Corol.* 2. *Prop.* 8.) by reason of the Velocity of the Bodies here suppos'd to be given;) and the Abscis H D or I M, which is equal to it; is equal to the Square of the Semi-ordinate DI or HM, which is equal thereto. The Tangent also of this Parabola HM will be bisected in the Point L: (For in the like Triangles HCL, HIM, as HC is to HI, so will HL be to HM. But HC is half of HI; therefore HL is also half of HM.) From whence, if you let down LO perpendicular to MI, MO and OR will always be equal; and the Equals IO, ON being added, the Wholes MN, IR will likewise be equal. Since therefore IR, the Distance of the Planes, is given in all Inclinations whatever; MN also, which is equal to the same, will be given in all Inclinations. And consequently the Rectangle NM x MI will be to the Rectangle under the *Latus rectum* belonging to the *Vertex* H and MI, as the given NM is to the given *Latus rectum*, or in a given Proportion. Now the Rectangle under HD or MI, and the *Latus rectum*, is equal to DIq or HMq. And therefore the Rectangle NM + MI is to HMq in a given Proportion. But NM + MI is equal to PM + MQ; that is, to the Difference of the Squares of ML and PL, or of ML and LI: And HMq hath a given Proportion to LMq, a fourth Part of it self. Therefore the Proportion of MLq−LIq to LMq is given; and by dividing the Proportion of LIq to LMq, and the subduplicate Proportion of the same Line LI to LM. But in every Triangle, the Sines of the Angles are proportional to the opposite Sides, (*Corol.* 1. *Prop.* 20. *Book* 3. of the *Elem.*) Therefore the Proportion of the Sine of LMR, or of AHG, the Angle of Incidence, to the Sine of the Angle of Emergence MIK or LIR; or of the Angle LIM the Complement of the same unto two right Angles is still given. [For the Sine of the Angle LIR,

Corol. 1. p. 36. Book 3. of the *Elem.*

and

and of L I M, the Complement of the same unto two right Angles, is the same.] *Q. E. D.*

Case (2.) Then let a Body pass successively thro' divers Spaces bounded with parallel Plains, as (in *Fig.* 4. *Plate* 7.) A a b B, B b c C, C c d D, *&c.* and be mov'd with a Force uniform in each Sphere taken a-part, and divers in the divers Spaces: Here, by what hath been demonstrated already, the Sine of Incidence on the Plane A a, will be to the Sine of Emergence from B b in a given Proportion; and this Sine, which is the Sine of Incidence on the Plane B b, will be to that of Emergence from the 3d Plane C c in a given Proportion; and the same is to be said of this last Sine to the Sine of Emergence from D d; and so infinitely. And by Equality of Proportion, the Sine of Incidence on the first Plane will be to the Sine of Emergence from the last, in a given Proportion. Then let the Intervals of the Planes be diminish'd, and the Number increas'd infinitely; to the end that the Action of Attraction or Impulse may be continual, according to any Condition which may be assign'd: And the Proportion of the Sine of Incidence on the first Plane to the Sine of Emergence from the last, will still be given. *Q. E. D.*

LIV. The same Things being suppos'd, the Velocity of a Body before the Incidence will be to the Velocity of the same after the Emergence, as the Sine of Emergence to the Sine of Incidence.

In the former *Figure*, let A H, I d be equal, and let the Perpendiculars A G, d K be erected, meeting the Lines of Incidence and Emergence G H, I K in G and K. In G H let T H be taken equal to I K, and T v be let down perpendicular to the Plane A a. And let the Motion of the

Body be diſtinguiſh'd into two, one perpendicular to the Planes Aa, Bb, Cc, Dd, the other parallel to the ſame. The Force of Attraction or Impulſe, whilſt it acts along the perpendicular Lines, will not at all change the Motion along the Parallels; and therefore the Body by this Motion will, in equal Times, go over thoſe Intervals of the Parallels, which are betwixt the Line A G and the Point H, and betwixt the Point I and the Line D K: that is, will in equal Times deſcribe the Lines G H and I K. And therefore the Velocity foregoing the Incidence will be to that which follows the Emergence, as G H is to I K or T H; as A H or I D is to v H; that is, (in reſpect of the Radius T H or I K) as the Sine of Emergence to that of Incidence. *Q.E.D.*

LV. The ſame Things ſuppos'd; and that the Motion before the Incidence is ſwifter than that afterwards; a Body by the inclining of the Line of Incidence will at length be reflected, and the Angle of Reflection will be equal to that of Incidence.

For (ſee *Fig.* 5. *Plate* 7.) imagine that a Body doth, betwixt the parallel Planes A a, B b, C c, D d, *&c.* deſcribe parabolick Arches, as above; and let HP, PQ, QR, *&c.* be thoſe Arches. And let the Obliquity of the incident Line G H to the 1ſt Plane A a be ſuch, that the Sine of Incidence is to the Sine of the right Angle; that is, to the Radius of the Circle, in that Proportion which the ſame Sine of the firſt Incidence hath to the Sine of the Emergence from the laſt Plane D d, into the Space expreſs'd by D d E e. And here, becauſe of the Sine of Emergence now made equal to the Radius, or Sine of the right Angle, that Angle of Emergence will be a right one; and conſequently the Line of Emergence will

will now coincide with the Plane D d. Let the Body come to this Plane in the Point R: Here, becauſe the Line of Emergence coincides with the ſame Plane, it is manifeſt, that the Body cannot move farther, or towards the remoter Plane expreſs'd by E e. Nor indeed can that go forwards in the Line of Emergence R d, becauſe it is perpetually attracted or impell'd towards the Medium of Incidence: Therefore it will return betwixt C c and D d, deſcribing Q R q the Arch of a Parabola, the principal Vertex whereof will be the Point R; and it will cut the Plane C c in the ſame Angle q, as it did before in Q. Then, in going forwards in the parabolic Arches qp, ph, *&c.* like and equal to the former Q P, P H, it will cut the reſt of the Planes in the ſame Angles in p and h, as before in P and H; and will at length emerge in the ſame Obliquity in h, with which it fell upon H. Conceive now the Intervals of the Planes A a, B b, C c, D d, *&c.* to be diminiſh'd infinitely, and their Number increas'd; ſo that the Action of Attraction or Impulſe may be continual according to any Condition whatſoever; and the Angle of Emergence, which was before always equal to the Angle of Incidence, will remain ſtill equal to the ſame. *Q. E. D.*

Scholium. Not unlike to theſe Attractions, as it ſeems, are the Refractions and Reflections of Light made according to a given Proportion of Secants, as *Snellius* diſcover'd; and by conſequence according to a given Proportion of Sines, as *Des Cartes* has expounded it: [For ſince every Sine is to the Radius, as the Radius is to the Secant of the Complement; and the Angle of Incidence betwixt the Radius and the Plane, called by *Snellius* the refracting Angle, is the Complement of the Angle called by *Cartes*, that of Inci-

dence

dence betwixt the Radius and the Perpendicular; the Proportion of Secants with *Snellius*, will wholly agree and fall-in with the Proportion of Sines us'd by *Cartes*.] For that Light is propagated and returneth from the Sun to the Earth in about 7 or 8 Minutes Space, is what is now manifest from the Phænomena of the *Satellites* of *Jupiter*, confirm'd by the Observations of divers Astronomers; and the Rays of Light which are in the Air, (as *Grimaldus* some while ago discover'd, by letting in Light through a Hole into a dark Chamber; and Sir *Isaac Newton* hath more fully experimented) in their Passage near the Angles of opake or transparent Bodies, are bowed about the Bodies, being, as it were, drawn towards them: And of these Rays, those which in that Passage come nearer to the Bodies, are the more bowed; being, as it were, the more attracted, as Sir *Isaac* himself diligently observed, and hath lately set forth at large in the 3d Book of his Opticks. Now, since there is such an Incurvation of the Rays in the Air without the Knife, the Rays also which fall upon the Knife must have been bended in the Air before they touched the Knife: And there is the like Reason for those which fall upon Glass. The Refraction therefore of the Rays of Light is not made in the Point of Incidence, but by little and little in the continual Incurvation of the Rays, which happens partly in the Air, and before they touch the Glass, and partly as it should seem in the Glass it self after they have enter'd into it. Nor is the thing otherwise in Reflexions, as Sir *Isaac Newton* hath shew'd most accurately in the Book just before cited; whither we refer our Reader, who is studious of these things. Now, because of the Analogy which is betwixt the Propagation of the Rays of Light and the Progress of of Bodies, it ſeem'd proper to premiſe the three foregoing Propoſitions, as preparatory to true Optics. But we muſt note by the way with Sir *Iſaac Newton*, that ſpheric Figures are more accommodate to Optic Uſes than any of the Conic Figures are. And according to his Opinion, if the Objective Glaſſes of Perſpectives were made of two Glaſſes faſhion'd ſpherically, and fitted to contain Water betwixt them; it might come to paſs, that the Errors of Refractions, which happen in the extreme Surfaces of the Glaſſes, would be corrected exactly enough by the Refractions of the Water. And he judgeth, that ſuch Objective Glaſſes are to be preferr'd before Elliptic and Hyperbolic ones; not only becauſe they may be faſhion'd more eaſily and exactly, but alſo becauſe they refract the Pencils of the Rays, ſituate without the Axis of the Glaſs, more accurately. However, the divers Refrangibility of divers Rays will ever hinder Optics from being brought to Perfection by Figures of Glaſſes, either ſpherical or any other whatever. And unleſs the Errors ariſing from the foreſaid Head can be corrected, all our Labour will be employ'd to ſmall purpoſe in correcting the reſt. But as concerning all theſe things, the Famous Author himſelf is to be read in that noble Optic Work mention'd before.

Schol. (2.) But ſince it hath ſeem'd good to that great Man, to propoſe certain Propoſitions in that Book without their Demonſtrations; it will be worth our while to bring in in this place the Demonſtrations of them, which have been either lately found out, or elſewhere delivered by the ſame Author; that ſo there may be nothing in that Famous Treatiſe, which Beginners may

ſtumble

stumble at, as not having it demonstrated before them.

Decem. 10. 1705.

Lect. XXV.

Prop. (1.) LET ACB be a reflecting spherical Surface, the Center whereof is E. Let the Radius EC be bisected in the Point T. And if the Points Q and q be marked in the Line EC, on the same side of the Point T: So that TQ, TE and Tq, be Lines continually proportional; and the Point Q be the Focus of the incident Rays, the Point q will be the Focus of the Reflex Rays. For by the Hypothesis QT : TC : : TC : : Tq. and by compounding QT+TC = QC : QT : : CT + Tq = Cq : CT = ET; that is, QC : QT : : Cq ; ET. And by alternating QC : Cq : : QT : ET. But by V. 19. of the Elements, QT : ET : : QE : Eq. Therefore by Equality QC : Cq : : QE : Eq. From whence, in the Triangle, the Base whereof is Qq, and the Vertex in the spheric Surface ACB, so near to the Point C, that the greater of its Sides should be nearly equal to QC, and the lesser to qC; the Base Qq will be so divided by a Line drawn from the Point of the Sphere to the Center E, that the Parts QE and Eq should be one to another in the Proportion of the Sides QC and qC.

Page 7. Case 2. Book I.

And

And consequently, by VI. 3. of the *Elements*, the Line drawn from the Vertex of the Triangle through the Center E, will bisect the vertical Angle of the Triangle, and make equal Angles on both sides. From whence the Radij passing through Q, because that the Angles of Incidence and Reflection are equal, will be reflected to the Point q; and conversly. *Q. E. D.*

Prop. (2.) Let A Q B be the refracting Surface of a Sphere, the Center whereof is E. In B the Radius E C produc'd on both sides, let the Points T and t be mark'd; so that as well E T as C t (which are equal one to the other) may be to the Radius E C, as the lesser of the Sines of Incidence and Refraction is to the Difference of those Sines. Then let the Points Q and q be marked in the same Line; so that T Q may be to E T or C t, as E t is to t q. But let the Places of the Points be such, that the Line t q may be on that side of the Point which is contrary to that side which the Line T Q is on, with respect to the Point T. Now, if the Focus of the incident Rays be in the Point Q, the Focus of the Refracted ones will be in q. For by the Hypothesis, as T Q is to T C, so is E T to t q. And by compounding, T Q is to T Q + TC = Q C, as is E T = C t to C t + t q = C q; and by alternating, T Q : C t : : Q C : C q. And by compounding and inverting T Q + C t = Q E : : T Q : : Q C + C q = Q q : Q C. Or Q q : Q C : : Q E : : Q T. From whence, together with what *Hugens* hath demonstrated in his *Dioptrics*, *page* 26. &c. the Proposition is manifest.

Lat. page 8. *Case* 3.

Prop. (3.) Let A C B D (*Page* 8. *Case* 4.) be a refracting spherical Glass on both sides Convex or Concave, or at least Plano-convex or Plano-concave,

concave, the Axis whereof (or Line which cuts both Surfaces perpendicularly, and passeth thro' the Center of the Sphere) is C D. In the Axis, let the Points F and f be the Foci of the refracted Rays, found as above: Those, to wit, which would agree to the Radii, parallel on both sides to the Axis, if there were only one refracting Surface. Let the Line F f be bisected in the Point E; and from the Center E, let a Circle be describ'd, with the Radius E F or E f. Now, let any Point Q O be the Focus of the incident Rays. Let Q E be drawn intersecting the former Circle in the Points T and t, and let the Point q be marked in the same Line, so that the Line t q may be to t E, as the same t E or its Equal T E is to T Q. Then let the Line t q lie on that side of the Point t, which is contrary to that which T Q lies on, with respect to the Point T. Thus the Point q will be the Focus of the refracted Rays; of those, to wit, which are near to the Axis, of which alone any Regard is to be had in these Cases. For by the Hypothesis, T Q : T E :: t E : t q. Therefore by compounding TQ : TE + TQ, = Q E :: t E : t E + t q = E q. From whence by V. 12. of the *Elements*, T Q :: Q E :: T Q + t E = Q E : Q E + E q = Q q. From whence, together with what *Hugens* hath demonstrated in Page 67. *&c.* of his *Dioptrics*, the Proposition is manifest.

Prop. (4.) The Mixture of the Rays of the Sun in p t, a refracting Glass (Book 1. Page 46.) is to the Mixture of the Rays of the Sun passing through an empty Hole, as the Breadth of the same Glass is to the Difference of the Breadth and Length of it, or as a g is to g m. For let a h be to a m, as a g is to A G. In this Case the Space a h will be equal to all the Areas of the lesser

ſer Circles, as being, in the duplicate Proportion of the Rays on both ſides: And the Mixture of the Rays would be equal, if ſo be all the leſſer Circles did meet together in that Space. But ſince they are diſpers'd through the Space p t, the Mixture will be as g h to g m. From whence, ſince the Mixture of the Rays in the Glaſs P T, is to the Mixture of them as paſſing through the empty Hole, as A G is to G M, or as a g to g h; and the Mixture in the Glaſs p t, is to the Mixture in the Glaſs P T, as g h is to g m; by Perturbation of Equality, the Mixture in the Glaſs p t will be to the Mixture which agrees to the Rays paſſing without Refraction, as a g is to g m. *Q. E. D.*

Prop. (5.) If a Body in any given Velocity whatever, fall upon a Space (ſee *Book* 1. *Page* 57.) of an inconſiderable Breadth, and terminated on both ſides with parallel Planes, and in its Paſſage towards the remoter Plane, be attracted or impell'd perpendicularly; in ſuch ſort, that the attracting or impelling Force is either every where the ſame, or at leaſt the ſame at equal Diſtances from that Plane; the perpendicular Velocity of the Body, which hath now paſſed that Space, will be equal to the Sum of the Squares of the former Velocity, and the ſquare Root of the Velocity acquir'd in paſſing through. But if the Body be retarded in its paſſing through, the Difference of the ſame Squares is to be taken inſtead of the Sum of them; and thus the *Propoſition will hold good.* It follows from *Newt. Mathem. Principl.* Prop. 39. Probl. 27. Corol. 2.

Prop. (6.) If any Bodies or Rays of Light paſſing through a Space, ſuch as before, and bounded with parallel Planes, be acted upon in their paſſing with a like Force, but which is ſome-

times

times greater, sometimes less; the Motion last acquired will be in the subduplicate Proportion of the Force, which begets the same; in such sort, that the Squares of the Motions will always determine the true Proportions of the same. Let AB be a refracting Surface, or let it represent a Space of a Thickness not to be regarded, bounded with parallel Planes, which is of a refractive Force. And let IC be a Ray of Light falling very obliquely upon the refractive Plane at the Point C, so that ACI the Complement of the Angle of Incidence may be indefinitely small. And let CR be the refracted Ray. From any Point B, let the perpendicular BR be erected, cutting the refracted Ray in R. Where if CR represent the Motion of the refracted Ray, which is resolv'd into two Motions CB and BR; that Part of the Motion CB will be parallel to the refracting Plane, and BR will be perpendicular to the same; and since the Motion along the Plane AB, is in no wise chang'd by the Force perpendicular to the same, CB will be given, by reason of the given Velocity of the Rays, which here suppos'd. And the Line BR will be a Motion produc'd by the Refraction in a given Time; and it will be in the subduplicate Proportion of the Force which produceth it. For, because of the given Latitude of the refractive Space, the Times of the Transit in which the refracting Force would act, will be reciprocally as the Velocities produced, or as the producing Forces reciprocally; and because of the Velocity, the Force being given in the Proportion of the Times, the Line BR would be as the producing Forces reciprocally; and the Time being given, as the same Forces directly. Therefore neither being given,

Book 2. Page 71, 72.

given; the Line B R will be in the subduplicate Proportion of the Forces; for then the Times and Forces being reduc'd unto an Equilibrium, neither will the Force preponderate the Time, nor the Time the Force; which no otherwise could answer each to other. Thus, indeed, if the Forces be put to be in a quadruple Proportion, a double Velocity will be produc'd in half the Time; or the Line B R will be double of the same Line, and thus every-where. *Q. E. D.*

Prop. (7.) In the Solution of the Rainbow, the Arch Q F and the Angle A X R will be the greatest, where N D is to C N, as $\sqrt{II-RR}$ is to $\sqrt{3RR}$. In which Case also, 2 R : I :: NE, ND; and the Angle A Y S, which is made by the Rays A N and H S, will be the least, where N D is to C N, as $\sqrt{II-RR}$ is to $\sqrt{8RR}$. In which Case also, 3 R : I :: N E : N D. Which twofold Proposition we will demonstrate with the Famous Sir *Isaac Newton*, in his Manuscript Optic Lectures.

Problem. If Rays, whether parallel or inclin'd towards some common Point, do fall upon a Sphere to be refracted there; to design the Concourse of the refracted Rays without the Axis, which are next to one another, and lye in the same Plane with the incident Rays. Let A N (in *Fig. 6. Plate 7.*) be an incident Ray, N K the refracted Ray thereof; and N V in the Plane of the Triangle A N K, a right Line touching the Sphere in N. To A N draw N R perpendicular, and meeting the Axis A C in R; and R V parallel, and meeting the Tangent N V in V. Likewise, draw N Q perpendicular to N K, and V Q parallel to the same, meeting it in Q; and draw Q C meeting N K in Z, Z will be the Concourse of the Rays nearest to A N. And let A n be another of the incident Rays, infinitely near to the former AN, and meeting NR in G. Draw nZ meeting NQ in H; and to AN and NK, let down from C the Center of the Sphere, the Perpendiculars CD and CE, meeting An and nZ in d and e. Now, since AN is suppos'd to be infinitely near to An, the Arch Nn, which is infinitely small, may be reckoned for a right Line coinciding with the Tangent NV; and the Triangles NGn, NRV, as also NHn, NQV may be accounted for like. Wherefore it is, DC : : Dd : : (NR : NG : : NV : Nn : : NQ : NH) : : EC : Ee. And conversly, DC : (DC—Dd) dC : : EC : (EC—Ee) eC; and alternately, DC : EC : : dC : eC. But DC is to EC, as the Sine of Incidence is to the Sine of Refraction, because NK is the refracted Ray of AN; and consequently also, dC is to eC as the Sine of Incidence to that of Refraction; and therefore, since the Angles DAd and EZe be infinitely small, and consequently Cd is perpendicular to An, and Ce to nZ, or at least equipollent to perpendicular, nZ will be the refracted Ray of An. *Q. E. D.*

Corol. (1.) ND : NE (or NP : NF) : : NR : NQ. For NC being drawn, because of the Triangle NDC, like to the Triangle NRV; and the Triangle NEC like to the Triangle NQV: it is ND : NR (: : NC : NV) : : NE : NQ: and alternately, ND : NE : : NR : NQ. Hence results a more ready Solution of the Problem; to wit, Erect NR, NQ perpendicular to the Rays AN, NK; of which two Perpendiculars, let NR meet the Axis AC; and let the other NQ be to NR, as NF is to NP. Then draw QC, which will meet with NK in the sought Point Z.

Corol.

Corol. (2.) It is alſo thus, $AN \times DC \times NE : AD \times EC \times ND :: NZ : EZ$. For $AD : AN :: DC : NR$. And from thence $NR = \frac{AN \times DC}{AD}$. Likewiſe, $ND : NE :: NR : NQ$. And from thence, $NQ = \frac{AN \times DC \times NE}{AD \times ND}$. And conſequently, $AN \times DC \times NE : AD \times ND \times EC$ ($:: NQ : EC$) $:: NZ : EZ$.

Corol. (3.) If A the Radiant Point be infinitely diſtant, or *ſend forth parallel Rays*; *it being put thus*, $I : R ::$ the Sine of Incidence: the Sine of Refraction; it will be $I \times NF : R \times NP :: NZ : EZ$. For in this Caſe, AN and AD, ſince they are infinitely long, ought to be reckon'd for equal; and conſequently by *Corol.* 2. of this, $DC \times NE : EC \times ND :: NZ : EZ$. But by the Hypotheſis, $DC : EC :: I : R$; and conſequently $I \times NE : R \times ND$ ($:: NZ : EZ$) $:: NP : NF$. But it is to be noted, that the Reſolution of the Problem is eaſily accommodated to any Caſe whatever, *mutatis mutandis*; whether the incident Rays decline from ſome one Point, or incline to the ſame, or fall parallel.

Problem (2.) From parallel Rays refracted unto a Circle, to determine that Ray; Part whereof being included in the Circle, hath a given Proportion to that Part of the ſame Ray refracted, which is included in the ſame Circle. Let AN be the incident Ray: (ſee *Fig.* 1. *Plate* 8.) NK the refracted: NP and NF, the Parts of them included in the Circle: CD and CE Perpendiculars let down to thoſe Parts from the Center of the Circle; and BC a Semi-diameter drawn parallel to AN. And let it be $CD : CE :: I : R$ and $NP : NF :: p : q$. Theſe Things being put, that the Point N may be found, which determines the Rays A N and N K; erect at B C the Perpendicular B X, and let the Square thereof be to the Square of B C, as $\frac{qq - pp}{pp}$ is to $\frac{II - RR}{II}$. Thus C X being drawn, will cut the Circle in the [illegible] int N. For by the Hypothesis p : q ([illegible] : N F : :) N D : : N E. And I : R : : [illegible] : C E. Wherefore $\frac{q}{p}$ N D = N E; and $\frac{R}{I}$ [illegible] D = C E. Furthermore, since N D q + C D q = N [illegible] q) = N E q + C E q; take from hence N D q + C E q, and there will remain C D q + C E q = N E q — N D q; that is, by substituting the Values of C E and N E, just now found, C D q -- $\frac{RR}{II}$ C D q : = $\frac{qq}{pp}$ N D q ---- N D q; and by making a Reduction $\frac{II - RR}{II}$ C D q, = $\frac{qq -- pp}{pp}$ N D q. Which being resolved into Proportionality, $\frac{qq -- pp}{pp}$: $\frac{II -- RR}{II}$ (: : C D q : N D q) : : B X q : B C q. *Q. E. D.*

April 8. 1706.

LECT.

Lect. XXVI.

Problem (3.) THE Sun enlightning a transparent Sphere, to determine the greatest Inclination to the Axis of the Rays emerging after one Reflection. Let B N K (in *Fig.* 2. *Plate* 8.) be the proposed Sphere: B Q the Diameter, or the Axis parallel to the incident Rays: A N some one of the incident Rays: N F the refracted Ray thereof: F G the reflected: and G R the again refracted; and thus the greatest Angle, which R G can make with the Axis B Q, is to be sought.

To which purpose it is to be observ'd, that in that Case alone, where R G is the most of all inclin'd to B Q, the Rays which are the nearest to A N, can emerge parallel to R G. For in other Cases, of the emerging Rays nearest to it, some are continually more inclin'd to B Q, others less; and consequently are something inclin'd to one another.

It is also to be observ'd, that the Rays which meet at the Point of Reflection, will emerge parallel. For draw a n parallel to A N, and as near to it as may be; and let n f be the refracted Ray thereof, f g the reflected; and g r the second refracted one. And the Points F and f coinciding, when the Angles N F n and G F g are equal; and the Refractions at N n and G g be like, the emerging Rays G R and g r will be parallel, as well as the incident N A and a n.

The Ray A N is therefore to be sought, the refracted Ray whereof concurs with the refracted one of the Ray a n, which is nearest to it in the Point F. And indeed, by *Corol.* 3. *Problem* 1. (if C D and C E be let down from the Center of the Sphere perpendicular to the Rays, and it being put I : R : : C D : C E.) If those Rays meet in any Point Z, it will be I × N F : R×NP (: : N Z : E Z) : : N F : E F, (the Point Z, to wit, falling at F, according to the Hypothesis,) : : 2 : 1. Wherefore I × N F = 2 R × N P; and I : 2 R : : N P : N F. The Proportion therefore of N P to N F is given; and from thence, by the 2d Problem, the Point N will be given. To wit, Let the Tangent B X be drawn at the Top of the Circle, the Square whereof let it be to the Square of the Semi-diameter B C, as 4 R R—II is to II—R R, and let C X be drawn. For this will meet the Circle in N; and from N when found, the other things will easily be determin'd.

Corol. (1.) Hence it comes to be thus, 3 R R : I I--R R : : C N q : N D q. For since it is, 4 R R—I I : I I—R R : : B X q : B C q, by compounding it will be 3 R R : I I—R R (: : C X q : B C q) : C N q : N D q.

Corol. (2.) It is also thus, I : 2 R : : N D : N E. For it was above I : 2 R : : N P : N F. And from these the Resolution of the Problem becomes more expeditious.

Scholium. With the greatest Inclination of the Radius R G, the greatest of the Arches F Q, bounded at the refracted ones N F, is also given. For the Angle F C Q subtended by F Q, is equal to the Angle which C F and A N comprehend, *i. e.* equal to half of the Angle comprehended by R G and A N, or B Q; and consequently that which is defin'd by the Ray A N falling upon the Point which is now found, is the greatest of the Arches

Arches F Q, as well as of the Angles comprehended by R G and B Q.

Problem (4.) The Sun illustrating a pellucid Sphere, to determine the least Inclination to the Axis of the Rays emerging after two Reflections.

Let A N and a n be two incident Rays very near to one another; and let them, after two Reflections in F f and G g, emerge along H S and h s. Now it is manifest, that only in that Case where the acute Angle comprehended by B Q and S H is the least, those Rays H S and h s can be parallel, as was said before of the Rays G R and g r: And where this happens, the Ray F G will be parallel to f g. From whence, double the Arch F f (= to the Arch F f + G g = to the Arch F G — f g = to the Arch N F — n f) is = to the Arch n N — F f; and consequently triple the Arch F f = is equal to the Arch N n. And since N F is divided in Z, in the Proportion of those Arches, as is manifest; N Z will be = to 3 Z F, or 3 E Z. Since therefore by *Corol.* 3. *Problem* 1. I × N F : R × N P : : N Z ; E Z : : 3 : 1. therefore the Proportion of N P to N F is given; and from thence, by *Prob.* 2. the Point N will be given, by drawing B X, which may touch the Circle in the Vertex B; and the Square whereof is to the Square of B C, as 9 R R — I I is to I I — R R: and by drawing C X to meet the Circumference in N. N therefore being found, the other things are easily determined.

Corol. (1.) Hence it is, 8 R R : I I — R R : : C N q : N D q. for 9 R R — I I : I I — R R : : B X q : B C q. And by compounding 8 R R : I I — R R (: : C X q : B C q) : : C N q : N D q.

Corol. (2.) It is also thus: I : 3 R : : N D : N E. Forasmuch as above it was, I : 3 R : : N P : N F.

A *Scholium.* The greatest Inclination of the Ray K T to the Axis, when it emerges after three Reflexions, will be determin'd in the same manner as the greatest of the Arches Q G; to wit, in that Case F G and f g will meet together in G, and the Arch F f (= to the Arch F g — f g = to the Arch N F — n f) will be equal to N n — F f: and from thence doubled, the Arch F f will be = to the Arch N n and N Z will be equal to 2 Z F, and consequently 4 : 1 :: N Z : E Z :: (by *Corol.* 3. *Prob.* 1.) I x N F : R x N P :: or I : 4 R :: N P : N F. And consequently by *Prob.* the 2d, 16 R R — I I : I I — R R :: B X q : BCq. From whence it follows, 15 R R : I I — R R :: C N q : N D q; and I : 4 R :: N D : N E.

And so if the least Inclination of a Ray emerging after four Reflexions be enquir'd, it will be determin'd by making it thus; 25 R R — I I : I I — R R :: B X q : B C q. Or 24 R R : I I — RR :: C N q : N D q. And I : 5 R :: N D : N E. And so on *in infinitum.*

Scholium. From the Determination of the Bounds of the Rainbow by the Famous *Newton,* I will take occasion to solve a certain Phænomenon, or rather the Absence of a certain Phænomenon, which sometime hath seem'd to me very difficult, and almost insolvable. And it is this; Why we do not see a Rainbow about the Sun, at the Distance of about 26 Degrees; since there the Rays come to our Eyes by a double Refraction without any Loss in the Reflexion? For by Computation, there is in that Place a Constipation of the Rays requisite and sufficient for affecting the Sight. And it increaseth the Doubt, that it seems probable at the first that this should be the principal Rainbow of all, and decked with the most lively Colours, as proceeding from a double

double Refraction, without any Loss or Diminution of the reflected Rays. For, as the Primary Rainbow doth far exceed the Secondary, for that it proceeds from a double Refraction, and one single Reflexion, whereas in the Secondary there are two Reflexions; so one would think, that the Rainbow which we spake of, should, in the Splendor of its Colours, as much exceed the Primary one of the two last mention'd, as that doth the Secondary; and should appear about the Sun like a most noble Crown, whensoever the Air affords spherical Drops in that Angle of 26 Degrees for making a Rainbow. And it is to be admir'd, that this Difficulty hath never been touch'd upon by those Philosophers that have treated of the Rainbow. But our Solution of it is this: We do therefore see the Rays that are throng'd about the Limits F and G, rather than the other, because so many of them, as AN an, which enter'd the Drop of Rain parallelwise, return back in the same manner, as RG, rg: SH, sh, and so enter the Eye together; whereas on the other hand, unless they did come forth also in that parallel manner, they would make some Angle, and so could not enter the Eye together, how thick and throng soever they might otherwise be about the Point F and G. From whence, seeing the Rays which come forth about the Point F, do not go forth parallel, but make a certain Angle; it is manifest, that they cannot enter the Eye together, and consequently cannot afford a Rainbow.

LVI. All the Parts of an Homogeneal Mathematical Fluid, [that is, a Body, the Parts whereof yield to any Force whatever impress'd, and in yielding are easily mov'd amongst themselves,] which Fluid is inclos'd in any unmov'd Vessel, and press'd on every Side, are (if you set aside

aside the Consideration of Condensation, of Gravity, and of all Centripetal Force) equally press'd on all Sides; and remain every one of them in its Place, without any Motion arising from that Pressure.

Case (1.) Let a Fluid, contain'd in a spherical Vessel, be uniformly press'd together on all Sides: No Part of the same will be mov'd by that Pressure. For as if some Part as D (see *Fig.* 3. *Plate* 8.) were mov'd thereby, all the like Parts at the same Distance from the Center must be mov'd in like manner; and this because the Pressure of them all is alike and equal, and we have excluded all Motion but what ariseth from the Pressure. But they cannot all come to the Center, but the Parts about the Center must be condens'd, which is contrary to the Hypothesis; nor can they recede farther from it, but there will be a Condensation about the Circumference, which is likewise contrary to the Hypothesis: nor can they be mov'd in a Circle about the Center; for every Force which should determine the Motion of any one Part, or of them all laterally, and to either this Side or that, is here excluded; much less can the same Part be mov'd contrary ways at the same time. Therefore no Part of the Fluid will, in this Case, be mov'd out of its Place. *Q. E. D.*

Case (2.) Each of the spherical Parts of this Fluid are equally press'd on every Side. For let E F be one of those Parts; and if it be not equally press'd on all Sides, let the lesser Pressure be increas'd, until the Pressure be every where uniform and equal; and then the Parts thereof, by the 1st Case (which belongs as well to this little Sphere, as to one contain'd in a solid Vessel) will remain in their Places. But before that Increase, they would remain in their Places by the same first Case;

Caſe; [for we treat here of ſuch a Fluid, the Parts whereof, as we there demonſtrated, do remain in their Places;) and by the Addition of a new Force, they will be mov'd out of their Places by the Definition of a Fluid: Which two Things are contradictory. Therefore it was falſly ſaid, that the Sphere E F was not equally preſs'd on every Side. *Q. E. D.*

Caſe (3.) Beſides, the Preſſure of the divers ſpherical Parts will be equal. For the ſpherical Parts preſs each other equally in the Point of Contact, becauſe that Reaction is always equal and contrary to Action. But alſo by the 2d *Caſe*, all the ſpherical Parts whatever, are equally preſs'd on all Sides. Therefore any two ſpherical Parts, not contiguous, are preſs'd with the ſame Force, becauſe the intermediate ſpherical Part toucheth both. *Q. E. D.*

Caſe (4.) All the Parts of this Fluid are equally preſs'd on all Sides. For any two Parts may be touch'd by ſpherical Parts in any Points whatſoever; and there they do equally preſs thoſe ſpherical Parts by the 3d *Caſe*; and becauſe of Reaction, which is always equal to Action, they are reciprocally equally preſſed by them. *Q. E. D.*

Caſe (5.) Since therefore any Part whatever of this Fluid, as G H I, is incloſed in the reſt of the Fluid, as in a certain Veſſel, and is equally preſs'd on every Side; and the Parts thereof do equally preſs one another, and are at reſt amongſt themſelves; it is manifeſt, that all the Parts of any Fluid whatever, as G H I, do equally preſs one another, and are at reſt amongſt themſelves. *Q. E. D.*

Caſe (6.) Therefore, if that Fluid be incloſ'd in a Veſſel which is not rigid or unyielding, and be not equally preſs'd on every Side, the ſame will

will give way to the stronger Pressure by the Definition of Fluidity.

Case (7.) Therefore, in a rigid Vessel, a Fluid will not sustain a stronger Pressure on one Side than on another; but will give place to the same: and that in a moment of Time, because the stiff Side of the Vessel does not pursue the yielding Liquor. But in yielding, it will press the opposite Side; and so the Pressure will incline to an Equality on every Side. And because the Fluid, as soon as it endeavours to depart from the Part which is more press'd, it is stopp'd by the Resistance of the Vessel on the opposite Part, the Pressure will on every Side be reduc'd to an Equality in a Moment of Time, without local Motion; and immediately the Parts of the Fluid will, by the 5th *Case*, press one another equally, and be at rest amongst themselves. *Q. E. D.*

Corol. Hence the Motions of the Parts of such a Fluid amongst themselves cannot be chang'd by a Pressure, which is upon the external Surface of the same in any Place thereof, unless either the Figure of the Surface be somewhere chang'd, or all the Parts of the Fluid, in pressing one another more vehemently, or more remissly, slide more easily or difficultly amongst themselves.

Corol. (2.) But since the Definition and Affections of this Mathematical Fluid do seem altogether to agree with the Nature and Phænomena of natural Fluids; it is reasonable, that the Demonstrations of these Cases be applied to our natural Fluids, to Water especially, and the like. From whence it will be manifest, that the rest of the interval Parts of a Fluid amongst themselves, doth in no wise contradict the Nature of Fluidity; and that all the Motion of the Parts of Fluids amongst themselves, is to be reckon'd as owing to Fermen-

Fermentation, Heat, or other extrinsick Causes, rather than to the Nature it self of Fluidity. For, if the Parts of a Body be either spherical, or spheroidal, and perfectly polish'd, so that they can never be join'd one to another, but rather do only touch another in physical Points; a Congeries of these Particles will compose a Body, such as is commonly call'd a Fluid, altho' those Particles be altogether at rest amongst themselves: A Fluid therefore seems to consist of Parts very moveable, but not necessarily mov'd.

June 2. 1706.

Lect. XXVII.

LVII. If all the Parts of a spherical homogeneous Fluid, which are at equal Distances from the Center, and lie upon a concentrical, spherical Bottom, do equally gravitate towards the Center of the whole; the Bottom sustains the Weight of a Cylinder, the Base whereof is equal to the Surface of the Bottom; and the Altitude is the same as that of the Fluid, which lies upon the Bottom.

Let (in *Fig.* 4. *Plate* 8.) d h m be the Surface of the Bottom, and a e i the upper Surface of the Fluid. Let the Fluid be distinguished by innumerable spherical Surfaces, into concentrical Orbs of equal Thickness; and imagine the Force of Gravity to act only upon the higher Surface of every Orb, and that the Actions upon equal Parts

Parts of all the Surfaces are equal. The uppermost Surface therefore a e i, is press'd by the simple Force of its own Weight, wherewith as well all the Parts of the supreme Orb, as the 2d Surface b f k (by *Prop.* 56.) are press'd. Besides, the 2d Surface b f k is press'd by the Force of its own Weight, which being added to the former Force makes a double Pressure. By this Pressure, and by the Force of its own Weight withal, that is, by a treble Pressure, the 3d Surface c g l is urg'd: And so the 4th Surface is urg'd with a fourfold Pressure, the 5th with a fivefold, and so on. The Pressure therefore wherewith every Surface is urg'd, is not as the solid Quantity of the Fluid which lies upon it, but as the Number of Orbs unto the Top of the Fluid; and is equal to the Gravity of the lowest Orb, multiplied by the Number of Orbs; that is, to the Gravity of the Solid; the last Proportion whereof unto the before-defin'd Cylinder (if so be the Number of Orbs be increas'd, and their Crassitude diminish'd infinitely; so that the Action of Gravity be render'd continual from the lowest Surface to the uppermost) will become a Proportion of Equality. The lowest Surface therefore sustains the Weight of a Cylinder, the Base whereof is equal to the Surface of the Bottom; and the Altitude the same as that of the Fluid lying upon it. *Q.E.D.*

And by the like reasoning the Proposition is manifest, where the Gravity decreaseth in any Proportion of the Distance from the Center, which may be assign'd; as also where the Fluid upwards is more rare, and more dense beneath. *Q. E. D.*

Corol. (1.) The Fluid therefore is not urg'd by the whole Weight of the incumbent Fluid; but sustains only that Part of the Weight, which is defin'd defin'd in this Propofition ; the reft of the Weight being fuftain'd by the arched Figure of the Fluid.

Corol. (2.) If an entire Sphere confifteth of fuch a Fluid to the very Center, the Center will fuftain no Weight; the whole Weight being upheld by the arched, or rather in this Cafe by the fpherical Figure of the Fluid.

Corol. (3.) But in equal Diftances from the Center, the Quantity of the Preffure is the fame, whether the Surface be prefs'd parallel to the Horizon, or perpendicular, or obliquely ; and whether the Fluid, as continued upwards from the Surface prefs'd, arifeth perpendicularly according to a right Line, or creeps along obliquely by crooked Cavities and Channels, and thofe regular or never fo irregular, wide or narrow. That the Preffure is nothing chang'd by thefe Circumftances, is gathered by applying the Demonftration of this Propofition to every Cafe of Fluids.

Corol. (4.) By the fame Demonftration (and *Prop.* 56. foregoing) it is collected, that the Parts of an heavy Fluid acquire no Motion amongft themfelves from the Preffure of a Body lying upon them ; if fo be the Motion which arifeth from Condenfation be excluded.

Corol. (5.) And therefore, if another Body of the fame fpecifick Gravity, which cannot be condens'd, be immerg'd into this Fluid, it will acquire no Motion from the Weight of the Body lying upon it; it will not afcend nor defcend; nor will it be compell'd to change its Figure: If it be fpherical, it will remain fpherical; if fquare it will remain fo, the Preffure notwithftanding; and this whether it be hard or foft, or even the moft Fluid ; whether it float freely in the Fluid, or fink. For every internal Part of the Fluid hath the Nature of a Body immers'd; and the

fame

ſame is to be ſaid of it, as of all Bodies immerg'd, which are of the ſame Magnitude, Figure, and ſpecifick Gravity. If the Body immerg'd ſhould become liquid, keeping its Weight ſtill, and put on the Form of a Fluid, if it before aſcended or deſcended; or from the Preſſure put on a new Figure, it would do the ſame ſtill; and that becauſe the Gravity thereof, and the other Cauſes of Motion, do remain. But by the 5th *Caſe* of the former Propoſition, it would now reſt, and retain its Figure: Therefore it did ſo before alſo.

Corol. (6.) Therefore a Body, which is ſpecifically more heavy than the Fluid which is contiguous to it, will ſink; and that which is ſpecifically lighter will aſcend, and will obtain ſo much Motion and Change of Figure, as that Exceſs or Defect of Gravity can cauſe. For that Exceſs and Defect hath the Nature of an Impulſe, wherewith the Body, otherwiſe conſtituted in an Æquilibrium with the Parts of a Fluid, is urged; and may be compared with the Exceſs or Defect of Weight in either Part of a Balance.

Corol. (7.) Therefore the Gravity of Bodies placed in Fluids is twofold, the one true and abſolute; the other apparent, vulgar, and comparative. The abſolute Gravity is that whole Force whereby a Body tends downwards, or would deſcend in an empty Place. The relative and vulgar Gravity is the Exceſs of Gravity, whereby a Body doth tend more downwards than the Fluid that encompaſſeth it. The Parts of Fluids, and of all Bodies, gravitate in their proper Places with the former Sort of Gravity; and therefore with their conjoin'd Weights, they compoſe the Weight of the whole. For every whole is heavy, as may be experienced in Veſſels full of Liquors; and the Weight of the Whole is equal to the

Weights

Weights of all the Parts, and therefore is compounded of the ſame; for it can be deriv'd from no where elſe. With the other Sort of Gravity, which may be called apparent, vulgar, and comparative, Bodies do not gravitate in their own Places, or immers'd in their Fluids reſpectively, that is, compar'd amongſt themſelves, are not one heavier than another, but hindering the Endeavours of one another to deſcend, they abide in their own Places, as if they were not heavy: Like as any heavy Bodies whatever, placed within a concave Sphere from the Equality of Gravitation every way, do appear in no wiſe to gravitate, as was obſerv'd above. Thus the things which are in the Air, and do not overweigh, or deſcend at all in the Air, as Clouds and Vapours are judged by the Vulgar not to gravitate at all. What things overweigh the Air, and conſequently deſcend in it, as not being ſuſtain'd by the Weight of the Air, as Hail and Drops of Rain; theſe the Vulgar judge heavy. The vulgar Weights are nothing elſe, but the Exceſs of the true Weights above the Weight of the Air. From whence alſo thoſe things are commonly eſteem'd light, which are leſs heavy; and whilſt they give way to the Weight of the Air, aſcend upwards. And they may be ſaid to be comparatively light, but not abſolutely, for that they would deſcend in a Vacuum. So likewiſe in Water, the Bodies which aſcend or deſcend by reaſon of their leſſer or greater Gravity, are comparatively and apparently light and heavy; and their comparative Levity or Gravity is the Defect or Exceſs wherewith their true Gravity either is exceeded by the Gravity of the Water, or doth exceed it. But what things do neither aſcend nor deſcend, albeit they by their true Weights increase the Weight of the whole; yet comparatively, and in the Sense of the Vulgar (and of Philosophers too of late) they do not gravitate in the Water: For the Demonstration of these Cases is the same.

Corol. (8.) Those things which have been said concerning Gravity, and the Force whereby Bodies descend to the Center of the Earth, in either an absolute, or reciprocal duplicate Proportion of the Distances, are to be understood to hold in all other Centripetal Forces both absolute, and increas'd or diminish'd, according to any Proportion whatsoever of the Diminution or Increase of the Distance.

Corol. (9.) And therefore, if the Medium, wherein any Body is mov'd, be urg'd either with its own Gravity, or any other Centripetal Force; so that the Body, by means of the same, is push'd on more forcibly than otherwise it would be; the Difference of the Forces is to be estimated from that moving Force which, in what goes before, we have consider'd as a Centripetal Force. But, if the Body be urged more lightly, the Difference of the Forces is to be reckon'd for a Force which tends from the Center.

Corol. (10.) Since Fluids, in pressing the included Bodies, do not change their external Figures; it is manifest, by the Corollaries of the foregoing Proposition, that they will not change the Situation of the internal Parts amongst themselves; and consequently will not hurt Animals immers'd in them, neither will they excite any Sensation in them, if Sensation depends upon the Motion of the Parts; only so far forth as these Bodies may be condens'd by a Compression, which is on all Sides of them: And the same is to be said of any System of Bodies whatever, which

which is encompaſs'd with ſome Fluid, and compreſs'd thereby. As to the Parts of the Syſtem, and their Motions, it will be the ſame as if it was in a Vacuum; and they will retain only their comparative Gravity, unleſs the Fluid do reſiſt their Motions, or be neceſſary to the keeping them together by its Compreſſion.

LVIII. Fluids, which are not in a Deſcent, do in every given Baſe preſs themſelves, and other Bodies, as well thoſe which are immers'd, as the containing, in Proportion of the perpendicular Altitude, and not of the Quantity of the Matter; that is, the Preſſure of a Cylinder of Water, of the Heighth of four Feet, where the Area of the Circle of the Cylindrical Column is only of one Inch ſquare, is equal to the Preſſure of a Cylinder of Water of four Feet high, where the Area of the Circle of the Cylindrical Column is of 200 or 1000 ſquare Inches, and thus every where; to wit, if the Baſe of the Water, communicating with the Water contained in the Tube, be in both Caſes equal. This is a moſt known Rule in Hydroſtaticks, often found by Experiments, which I ſhall endeavour to demonſtrate as follows, it not having been demonſtrated hitherto, as I ſuppoſe, phyſically or mathematically. It is well known, that the Quantity of any moving Force, or the Effect anſwering thereto, doth ariſe from the Quantity of the Matter multiplied into the Velocity; and conſequently that whatſoever the Matter which is mov'd be, as to its Quantity, the Preſſure will be the ſame, if the Velocity be reciprocally proportional thereto. Thus the Forces of the Balance, Leaver, and other ſuch mechanical Inſtruments, is deriv'd from the Combination of theſe two things, the Matter and the Velocity; and you may move any Weight whatsoever, by any Force how small soever, if the Distance of the Pressure, or smaller Weight from the Hypomochlion, be reciprocally proportionated to that of Weights. Thus one Poundweight, at the Distance of four Feet from the Hypomochlium, is as much as four Pounds at thr Distance of one Foot; for here the single Pound is mov'd with a Velocity, which is fourfold of that of the other; and consequently is of equal Force in its Motion. Whilst it is moving, I say, but not otherwise, as many seem to reckon. For if at any time the Machine rests, it is manifest, that the Gravity, or Pressure, or Force, is now in Proportion to the Matter, and the four Poundweight is four Pounds, and the one Pound weight no more than one Pound. [If at any time the Machine rests, I say. For indeed, if we speak physically, or at least mathematically, no Body doth wholly rest; but every Body is then said to rest, when the Quantity of the Motion is so small, that it cannot be perceiv'd.] According therefore to what hath been said, the Water contain'd, and the Vessel also which contains it, are always in some sort of Motion, and do never perfectly rest; [which indeed if they did, a Column of Water, as I suppose, that is, an hundred-fold greater, and consequently of an hundred-fold greater absolute Gravity, would press the Vessel an hundred-fold more:] But the absolute Rest of the same not being to be suppos'd, it is to be said, that the Pressure of a Column of Water, the Area of the Base whereof is of one Foot, and that of a Column whose Base is 100 Feet square, is the same; while the Excess, to wit, of the latter Column, in respect of an hundred-fold Magnitude and absolute Gravity, is compensated and counter-balanc'd by the hundred-fold greater

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Velocity of Defcent in the leffer Column: And that the Cafe is the fame here with that of an inverted Syphon of unequal Legs, where the Water keeps an Æquilibrium, by reafon of the Velocities of Afcent or Defcent, which are in both the Channels reciprocally proportional to the Quantity of the Water.

Corol. (1.) Fluids therefore prefs, not in Proportion of the Quantity of the Matter preffing, but of the perpendicular Altitudes.

Corol. (2.) Therefore a Wooden Trencher, thruft down to the Bottom of a Bucket of Water, will rife to the Top, notwithftanding the Quantity of the Water which lies upon it is much greater than that which is under it; for by reafon of the Communication which is, by the fmall Interval that is betwixt the Edge of the Trencher and the Bucket, betwixt that little Cylinder of Water that is under the Trencher, and that Cylinder which is above it; the Weight of the incumbent Water will make that which is under, to lift up the Trencher with a Force equal to the faid incumbent Weight.

Corol. (3.) There is no Occafion therefore for the Hylarchic Principle of the Famous Dr. *More*, for the folving this Effect.

Corol. (4.) Thus it is with Fluids not actually defcending. But if they, with the Veffel containing, do by the common Force of Gravity of them all actually defcend; the Communication of the Preffure, as I fuppofe, paffeth away, and the Effect thereof ceafeth; but this fo, that even ftill, as in the former Cafe, the Preffure is according to the perpendicular Altitude, and is the fame where that Altitude is the fame, whatfoever the Column is otherwife, as to its Magnitude, whether little or great: So that at length we may make the Proposition more universal, and say, without any Restriction, that Fluids do press according to their perpendicular Altitudes, and not according to the Quantity of the Matter.

November 11. 1706.

LECT. XXVIII.

LIX. IF the Density of a Fluid, compos'd of Particles which do flee from each other, be as the Compression; so that if the pressing Force be two, or four, or eightfold, the Density thence arising is so likewise; the Centrifugal Force of the Particles is reciprocally proportional to the Distances from the Center: And, *vice versâ*, where the said Force is reciprocally proportional to the Distances from their Centers, the Particles which flee from each other compose an elastic Fluid, the Density whereof is proportional to the Compression.

Let a Fluid be understood to be inclos'd in the cubic Space A C E, (see *Fig.* 5. *Plate* 8.) and then to be brought by Compression into a less cubic Space a c e. Here the Distances of the Particles, by reason of their keeping the like Situation amongst themselves in both Spaces, according to the Nature of Fluidity, will be as the Sides of the Cubes A B, a b; and the Densities of the Fluid, reciprocally as the cubic Spaces A C E, a c e. Let the Square DP be taken in the Side ABCD of the greater Cube,

Cube, equal to d b the Square of the Side of the lesser Cube. And by the Hypothesis, the Pressure wherewith the Square D P urgeth the inclosed Fluid (or is urged thereby) will be to the Pressure wherewith the Square d b urges its inclosed Fluid, as the Densities of the Medium are to each other; that is, a b cub. to A B cub. But the Pressure wherewith the Square B D urgeth the inclosed Fluid, is to that wherewith the Square D P urgeth its Fluid, as the Square D B is to the Square DP, or as A Bq is to a b q. Therefore, by Equality, the Pressure wherewith the Square D B urgeth its Fluid, is to that wherewith the Square d b urgeth its Fluid, as a b is to A B; or reciprocally, as the Distance of the Particles. For the triplicate Proportion of the Sides a b and A B, being substracted from the duplicate Proportion of the same; the simple Proportion of the Sides, or the Distance of the Particles, remains reciprocally proportional to the Pressure of the same upon the Vessel containing. As for Example: Let the greater Cube be eight-fold of the lesser, or the Side of the greater double of the Side of the lesser. Then indeed, the Density of the Fluid, in the lesser Vessel, will be eight-fold of the Density in the greater, by reason of the same Quantity of Matter contain'd in an eight-fold less Space. And when by the Hypothesis, the Compression into the given Space was made exactly proportional to the Density, the whole Compression, or the compressing Force, adequate to the same in the lesser Cube, will be in the eight-fold Proportion of the Compression, or compressing Force, in the greater: But the entire Surface, wherewith the Compression, or the Surface of every Square in the lesser Cube, is to the Surface of every Homologous Square in the greater, in a sub-quadruple Proportion. The eight-fold Pressure therefore is to be compared with another Pressure of the same Particle, dispers'd into a four-fold greater Space: Therefore, in a Space four-fold less, the same Quantity of Matter, or the same Particles of the Fluid, sustain an eight-fold Pressure; wherefore every single Particle must needs sustain a Pressure two-fold greater; or, that the Centrifugal Forces of the Particles should be reciprocally proportionol to the Distances of the same. *Q. E. D.*

Thus if, by the Planes F G H, f g h drawn through the midst of the Cubes, the Fluid be distinguish'd into two Parts; hese will mutually press each other with the same Force, wherewith they are press'd by the Planes A C, a c; that is, in the Proportion of a b to A B; and consequently the Centrifugal Force, whereby these Pressures are sustain'd, are in the same Proportion. Because of the same Number, and the same Situation of the Particles in both the Cubes, the conjunct Force with all the Particles exercise upon all, according to the Planes F G H, f g h, are as the Force which each exerciseth upon each. Therefore the Force which each exerciseth upon each, according to the Plane FGH in the greater Cube, is to the Force which each exerciseth upon each, according to the Plane f g h in the lesser Cube, as a b is to A B; that is, as we have already demonstrated, reciprocally as the Distances of the Particles from one another. *Q. E. D.* And, *vice versâ*, if the Force of each Particle be reciptocally, as the Distance of the Particles, *i. e.* reciprocally as the Sides of the Cubes A B, a b; the Sums of the Forces will be in the same Proportion, and the Pressure of the Squares D B, d b, as the Sums of the Forces; and the Pressure of the

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Square D P, to the Pressure of the Square D B, a b q, to A B q. And by Equality, the Pressure of the Square D P, to the Pressure of the Square a b, as a b cub. to A B cub. for the Simple Proportion being compounded with the Duplicate, forms a Triplicate one; so that the Force of the Compression in the one, is to the Force of the Compression in the other, as the Density of the Fluid in the former, to the Density of the Fluid in the latter. *Q. E. D.*

Corol. (1.) Since therefore it is manifest by Experiments, that the Density of our Air, compress'd and rarified by Turns, is Proportional every where to the compressing Force, or the Compression it self; it seems that the Air consists of Particles which flee from, or chase away one another in the inverse Proportion of the Distances. For altho' this Centrifugal Force may seem Diametrically opposite to the Universal centripetal Force or Gravity which we spake of, so that it cannot consist of the same; yet it may come to pass, that besides that general Law of Gravity which belongs to Matter as such, and without any Respect had to the Figures, Forms, Circumstances or Motions of the same; there may be other Laws, and natural Forces, whether of attracting, or the contrary, belonging to the special Figures, Forms, Circumstances or Motions of Particles of Matter, and in a peculiar Manner annexed to the same, upon which many of the more difficult Phenomena of Nature may depend. Thus indeed it seems, that the Particles of Air, when they have acquir'd that peculiar Temperament, Figure or Form, from which they are fitted to compose such an Elastic Fluid as we call Air, are immediately subject to the new and special Law or Centrifugal Force belonging

belonging to such Particles, and such only. For our most perspicacious Author doth justly suspect, that the greatest Part of the special Phenomena of Nature depend upon such a Force as hath been mentioned, and are owing to Causes not yet known; whereby Particles of Matter are either driven upon one another, and so cohere in regular Figures; or are driven away, and recede from one another; which Force being unknown, it is no wonder that Philosophers hitherto have in vain attempted to explicate the Works of Nature; and which consequently being now by degrees discover'd, or in the way to be so, it is to be hoped that in Time, at least, we shall come at length unto, tho' not the primary Causes, yet the next to them, and such as will be as well accommodated to Geometrical Calculation, as Humane Uses.

Scholium. But what hath been said concerning the Centrifugal Forces of the Air, and such like Fluids, is to be understood of such Forces only, as are terminated in the next Particles, or diffus'd not much further: Examples of which we have in magnetic Bodies; the attractive Force of which is terminated almost in Bodies of their own Kind which are next to them. The Loadstone's Vertue is contracted by a Plate of Iron which is interpos'd, and almost terminated in the same. For the more remote Bodies are not so much drawn by the Magnet it self, as by the Plate. In like manner, when Particles chase away other Particles of their Kind which are next to them, their Force in the mean while not reaching unto remoter Particles, of them such Fluids will be compos'd, as we have been treating of in this Proposition.

Corol.

Corol. (2.) By the same Reason it seems, that besides the general Force of Gravity, there are other attractive Forces peculiar to the Particles of some Bodies, or to very small Distances, and other Circumstances of particular Bodies, from whence Phenomena, otherwise unaccountable, will naturally proceed. From such an Attraction as this, the Refraction or Inflection of the Rays of Light in Pellucid, or about the Angles of opake Bodies, which, to wit, do attract before the Contact, and the more forcibly at the lesser Distance; as our Author hath observed in his excellent Optic Treatise. And from the like Cause, as he notes in his *Latin* Edition of the same Work, the Spheric Figure of little Drops both of Quick-Silver, and the like Fluids seems to arise. For these Particles, as it seems when at a little Distance from each other, attract strongly; and like as in the great Bodies the Planets, their Spherical Figure results from an equal Gravitation of the Constituant Parts one towards another; so it is reasonable to derive the Spherical Figure of the little Drops which we were speaking of, from an equal Centripetal Force of the Particles that compose them, whilst they approach to one another; especially since we see that these Particles do so quickly, and in a Moment, and so exactly cast themselves into the said Figure; as is manifest in the known Phenomena of the Rainbow, which are wholly owing to an instantaneous and most exact Conformation of the Particles into that Figure. And to the same Cause are some other Phenomena of Fluids, which are otherwise most difficult to be solv'd, reasonably attributed. But this by the Way.

LX. The

LX. The Quantity of Matter in all Bodies, is exactly proportional to their Weight.

For when the Resistance of the Air is taken away, as is done in Mr. *Boyle's Vacuum*; all Bodies, whether they seem most solid and heavy, or most rare and light, descend together with a common and given Velocity, as soon as they are let down from the same Heighth. All Pendulous Bodies also whatever, where the Centers of Vibration are equidistant from the Center of Suspension, do even in the Air Descend and Ascend together, for a great Space of Time, if they begin at the same Time to vibrate, in an equal Arch of the same, or an equal Cycloid, or even an Arch unequal; and where an equal Arch is describ'd, they are mov'd altogether with the same Celerity, whether they be hard or soft, solid or liquid; whether great or small, or of whatsoever Form and Figure. From whence it is manifest, that the moving Force is every-where in the same Proportion with the Matter to be mov'd; or that the same Force of Gravity doth equally affect all Bodies in the same Distance from the Center of the Earth. For that great Bodies do, *cæteris paribus*, descend something more swiftly, and keep their Motions something longer; this is from hence, namely, that the Surface of Bodies, according to which they are expos'd to the Resistance of the Air, or any Medium whatever, is in Similar Bodies, in the duplicate Proportion only of the Diameters, or Homologous Sides; whereas the Solidity of the same, according to which, both the Quantity of Matter, and the Force of Gravity is to be estimated, is in the triplicate Proportion of the said Sides: So that if the Diameter of a Ball of Stone be Three fold of that of another Ball of the same Matter, the Surface

face of the ſame, and conſequently the Velocity being given, the Reſiſtance it meets with in the Air, will be only Nine-fold of that of the other; when, nevertheleſs, its Solidity, and Qantity of Matter, and Gravity which is Proportional thereto, will be Twenty-ſeven-fold of the ſame in the other. From whence it is no Wonder, that the Reſiſtance, which in Proportion of the Gravity, is ſo much leſs in the greater Sphere, ſhould affect and retard the ſame Sphere in a leſſer Proportion than it affects and retards the leſſer. But that there is ſo great a Difference of Velocity of Deſcent in the Air, betwixt Gold and Chaff, ſuppoſe, this depends not only upon this Difference of Surfaces, but eſpecially upon the Difference of Specific Gravity, whereby Gold doth far more exceed the Gravity of the Air, than Chaff doth; but the Exceſs of the Specific Gravity of a Body deſcending in the Air, above the Specific Gravity of the Air it ſelf; this alone is that Gravity, which forces a Body which is placed in the Air to deſcend, as we ſhew'd above. And therefore it is not to be wonder'd, that Gold ſhould fall in the Air much more ſwiftly than Chaff, altho' in a Vacuum they are always obſerv'd to deſcend with equal Velocity.

Scholium. If the Velocity it ſelf of all Bodies in a Vacuum, upon the Surface of the Earth, be requir'd to be given in known Meaſures; we are to know, that as well by the direct Obſervation of Bodies falling Perpendicularly, as by the Vibrations of Pendulums, and Computation made from thence by *Hugens*; it is with the Conſent of Geometricians, determin'd to be of that Quantity, that in one 2d of Time, Bodies deſcend 15 $\frac{1}{2}$ Feet of *Paris*, or 16⌊1 *Engliſh* Feet; or in the Space of one Hour, 208656000 *Engliſh* Feet, *i. e.* almoſt

almost 40000 *English* Miles; as from the Calculation, according to which Bodies descend, in the Proportion of the Time duplicated, it will presently appear.

LXI. In Pendulous Bodies, which are resisted only in the Proportion of the Velocity, when the Vibrations are in a Cycloid, whether the Arches describ'd be greater or lesser, are every where Isochronal.

The Truth of the Proposition, as spoken of in a Vacuum, where there is no Resistance of the Medium, hath been demonstrated above. And if the Resistance be as the Velocity, or as the Arch every where to be describ'd; the rest of the Velocity shall likewise be in the same Proportion; and consequently the Time of Vibrating will be equally retarded on both Sides, and the Vibrations will still remain as before, *i. e.* of equal Time. *Q. E. D.*

Corol. Therefore resisting Mediums, make the Time of each Vibration longer than it would be in a Vacuum. And Experience testifies this of Pendulum Clocks; the Vibrations whereof, have been observ'd to be perform'd something more quickly in a Vacuum, than in the Air. For the Resistance takes off something from the Force of the moving Gravity, and consequently doth something refract or lessen its Effect, and the Velocity of the Motion.

Nov. 25. 1706.

LECT.

LECT. XXIX.

LXXII. BOdies mov'd with an unequal Velocity in a very Subtle Fluid, are resisted by the Fluid in the Duplicate Proportion of the Velocity.

For since the Body which is mov'd the swifter, doth both pass through a greater Part of the Medium, in Proportion of its Velocity, and meets every unequal Part of the Medium with greater Force, in Proportion of the same Velocity, the whole Resistance arising from both Causes conjoin'd, will necessarily be in the Duplicate Proportion of the same Velocity. Which Proportion doth agree well with Experiments. Albeit the Defect, as to Slipperiness of the Parts in the Air, which give Way, arising from Elasticity, and some Cohesion of the Parts of most Fluids, must needs something disturb that Proportion.

Corol. (2.) Since therefore, the Vibrations of Pendulums in a Cycloid, where the Resistance is in the Simple Proportion of the Velocity, would be Isochronal; but the Resistance in the Air, and such like Mediums, is almost in the Duplicate Proportion of the Velocity; it is manifest that the Times of Vibrations in a Cycloid, and much more in a Circle, are, when the Vibration is in the Air, not altogether in divers Arches, but in the greater Arches, something greater, by Reason of the too great Resistance.

Corol. (2.) Hence it follows, that for the most exactly obtaining the Equality of Times in Pendulum Clocks, it is requisite that the Pendulums should always describe the same Arches; otherwise by Reason of the unequal Velocity, where the Arches described are greater, the Motion will be slower; where lesser, it will be swifter than it ought to be. From whence also the Cause may be shew'd, why greater Clocks, placed in a Ship, and tos'd up and down, do not so exactly shew the Hours, as those which are upon Land, and are at Rest. For by Reason of the Concussion, Arches are describ'd sometimes greater, and sometimes lesser; and from thence some Inequality must necessarily follow.

Corol. (3.) Shorter Vibrations, whether in a Cycloid, or in a Circle, are more Isochronal than longer; which is because of the Resistance of the disturbing Medium; and the shortest are perform'd in the same Times nearly as in a Vacuum; where also the Cycloid, and Circle, do just coincide, and the Vibrations in one, scarce differ from those in the other. From whence also, the Pendulum Clocks, which are govern'd by a long Pendulum, do shew the Hours much more exactly, than those which have a shorter Pendulum; forasmuch as far lesser Arches are describ'd by those than these. But the Times of those Vibrations which are made in greater Arches, are something greater, because the Resistance whereby the Time is lengthened, is greater, in Proportion of the Length describ'd in the Descent, (to wit, because of the greater Velocity) than the Resistance in the subsequent Ascent, whereby the Time is Contracted. But the Time of Vibrations, as well short as long, is also something lengthned by the Motion of the Medium.

dium. For when Bodies become slower in their Motion, they are a little less resisted; and when they are accelerated, a little more, than when they are mov'd uniformly; because whilst the Medium goes forward the same way with the Bodies, by that Motion which it hath receiv'd from them, in the former Case it is the more agitated, in the latter less; and consequently doth more or less conspire with the Bodies moved in it. It resists Pendulums therefore in their Descent more, in their Ascent less; and from both Causes the Time is lengthened.

LXIII. All Sounds, whether small or great, do go almost with one given Degree of Velocity; and this given Velocity is so swift that it constantly goes about 1142 Feet in one Second of Time, which is Eight Miles in just 37″, or about 68520 Feet; that is, near 13 Miles in an entire first Minute, or near 780 Miles in an Hour.

For all Sounds, whether great or small, must go with the same Velocity which a Stone, or other descending Body falling from half the Height of the Atmosphere, supposing it uniform, *i. e.* just so high as by its Weight would reduce our Air on the Earth's Surface to its present Degree of Density; I say all Sounds must go with the same Velocity, which a Stone at last would acquire by falling from half that Height; or, which is the same thing, that Sounds must go so far as that entire Height comes to in the same Time that the Stone would descend from the one half of that Height; because the last Velocity acquir'd, if it had been uniformly continued all that while, would have gone twice as far as that Line which had been describ'd by an unequal Velocity, gradually increasing from Rest, as we have above from *Galilæo* demonstrated p.; and therefore in the ſame time would have gone the whole Height of that uniform Atmoſphere: Now this Stone falling from half that Height, deſcends to the Earth in about ſuch a Space of Time as anſwers to the former Obſervations of the Velocity of Sounds. Now, that Sounds muſt needs go with the ſame Swiftneſs that a Stone would arrive at from half that Height beforementioned, is thus demonſtrated. The deſcending Stone is urged downwards only by its own natural Gravity, or infinitely ſmall Degree of Velocity uniformly impreſs'd upon it; and ſo its Velocity in equal Time has an equal Increaſe, and becomes greater exactly in the Proportion of the Time of its Deſcent. The Atmoſphere's Tenſion or Elaſticity, which conveys the Sound with its own natural Degree of Velocity, or attempt to Motion, ariſes alſo in this Caſe from the natural Gravity of each phyſical Part of the upper Surface uniformly augmented by the Addition of equal Parts of the inferior Surfaces quite down to the Bottom; ſo that in both Caſes, the Velocity actually acquir'd in one is the ſame with the *Conatus ad motum*, the Tenſion or beginning Velocity of the other: Juſt like two ſmall or naſcent Quantities originally equal, and which afterward are augmented in the ſame Proportion, whoſe laſt Quantities muſt therefore be equal alſo. This being ſo, and the Tenſion of the Parts of the Air being almoſt the ſame, whether the Motion or Sound be great or ſmall; 'tis plain, that tho' the Quantity of the Sound will be in Proportion to the Quickneſs of the Vibrations of the Sounding Body, and if that Velocity be increas'd by the Concurrence of the Wind, or diminiſh'd by its Oppoſition, the Sound will either be ſtronger and reach farther, or be weaker and ſtop

ſooner;

ſooner; yet will the Velocity of the Sound itſelf be always proportionable to the Tenſion of the Air which conveys it, and that Tenſion being nearly fix'd and certain, this Velocity of all Sounds muſt be nearly fix'd and certain. Now what Time is neceſſary for the Deſcent of a Stone from half the Altitude of ſuch an uniform Atmoſphere, as we have before ſuppos'd, will be thus computed. The ſpecifick Weight of Water to that of Quickſilver, is known by many Trials to be as about 1 to 13 $\frac{3}{4}$; and when the Mercury is 30 Inches in Altitude, the ſpecifick Gravity of Air to that of Water is about that of 1 to near 900, as has been found alſo by many Tryals. Nay indeed, conſidering that moſt of the Elder and Foreign Experiments come nearer to that of 1 to a 1000; and that however the ſpecifick Gravity of thoſe Parts which are properly *elaſtical Air*, if they were freed from Vapors and other Bodies which are not elaſtical, and have nothing to do in the Conveyance of Sounds, would be then not the 1000th Part; I ſhall chuſe 1 to 1100 for the Proportion of Air properly ſpeaking to Water: whence it will follow that true elaſtical Air will be to Quickſilver as 1 to 15125, and 30 Inches of Quickſilver, which is a Balance for an equal Column of Air, will correſpond to 30 times 15125 Inches of Air, or to 453750 Inches thereof; that is, to 37812 *Engliſh* Feet, or about 7 *Engliſh* Miles; which, if the Air were uniform in Denſity, would be its entire Altitude. But falling Bodies are known to deſcend half that Altitude, or 18906 Feet in about 34 Seconds of Time. Whence Sounds ought to propagate themſelves with ſuch a Velocity, as will carry them 37812 Feet; the Altitude of the Air, if it were uniform in Denſity; in the Space of a little above

above 34 Seconds, the Time of a Stone's Descent from half that Altitude; and by consequence will be propagated about 1142 Feet in one Second, about 68520 Feet, or near 13 Miles in one whole Minute, *i. e.* near 780 Miles in an Hour, agreeably to the best Observations. Sir *Isaac Newton*, in his first Edition, calculated the Velocity of Sounds to be somewhat less, by taking only 1 to 850 for the Proportion of Air to Water, which in this Case, as he now owns, ought rather to be taken as only 1 to about 1100: And it plainly appears, that the Observations of the Velocity of Sounds do generally make it greater than his first Numbers did allow. As to his Demonstration of the Conclusion we have here brought this Matter to, tho' it be extremely subtil and ingenious, yet is it too long, too remote, and too intricate to be insisted on in this place; and therefore it was thought proper to make use of this more easy and intelligible Method of Demonstration.

Coroll. (1.) If the Density of Air be increas'd or diminish'd, the Sound it self, or Violence of the Noise, will be increas'd or diminish'd in the same Proportion: Which thing doth well agree with the Experiments of Sounds made in rarified or condens'd Air.

Corol. (2.) If the Wind conspire with the Motion of the Air, the Sound or Noise will be increas'd and carried farther; as being now made up of the Sum of the Motions of the Sound it self, and the Wind. If the Wind be contrary, the Sound will be diminish'd, and sooner stopp'd, as now consisting of the Difference of the said Motions only. Which nevertheless is so to be understood, that the Veloci-

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ty of the Sound it ſelf, which was deſign'd above, alter but very little For Sound depends not on the continual Motion of the Air, but of the Pulſations of the ſame propagated after the manner of Waves by Vibrations, and a continual Viciſſitude of Goings and Returnings, as will be ſhewn afterwards. And of what Sort ſoever the Difference of the Noiſe is, which ariſeth from the different State of the Sonorous Body, or of the Wind; yet the Denſity of the Air, and its Elaſticity, do remain almoſt the ſame; and ſo the Effects of them, or the Velocity of the propagated Sounds, will remain likewiſe almoſt equal.

Corol. (3.) Sounds therefore, of what Kind ſoever, whether they be great or ſmall, are propagated through Air of a given Denſity and Elaſticity almoſt with the ſame Velocity; as the Experiments alſo, which have been made by Philoſophers, do ſhew.

Corol. (4.) The Velocity of Sounds therefore in any Place whatever being given, or that whereby they go about 1142 *Engliſh* Feet in one Second; from the Interval of Time of Sounds given, there is given withal the Interval of Diſtance of the Sonorous Body. Thus, for Inſtance, if we number 10″ of Time betwixt the Fire of a Cannon ſeen, and the Sound heard; it is manifeſt, that the Gun is 11420 Feet diſtant, or ſomewhat above two Miles. As likewiſe, if 5″ paſs betwixt our ſeeing the Flaſh of Lightning, and hearing the Thunder, we may reckon that the Thunder-Cloud is about 5710 Feet, or a little above one Mile diſtant from us.

Decemb. 2. 1706.

Lect. XXX.

LXIV. As the Resistance of Fluids in divers Velocities is in the duplicate Proportion of the Velocity; so in divers Densities the Velocity being given, it is in the direct Proportion of the Density it self; but the Density and Velocity being given in the duplicate Proportion of the Diameters; and consequently the Resistance in general is in a Proportion compounded of the duplicate Proportion of the Velocity, and the same Proportion of Diameters, and the simple Proportion of the Density of the Medium directly.

These Things are easy, and stand in no Need of Demonstration. For if two Spheres do exceed one the other, as to their Diameters, in the double Proportion, or be as 2 to 1, and the greater be mov'd with a Velocity double to that of the other, and in a fluid Medium double to the other in Density; it is plain, that in any given Space of Time the whole Resistance of the greater Sphere, or Motion lost, is to the whole Resistance the lesser Sphere meets with, or its Motion lost, as $2 \times 2 \times 2 \times 2 \times 2$ to $1 \times 1 \times 1 \times 1 \times 1$, or as 32 to 1; and thus every where. But it is to be noted, that Resistance proceeds equally both from Fluids and Solids, *cæteris paribus*; unless so far as in a very fluid Medium, when the Motion is somewhat slow, the Medium it self by a Circulation of Motion, and an Impetus thereby made on on the hinder Part of the Body mov'd in it, doth ſomething promote the Motion of the Body; which reciprocal Impetus of the Medium on the Body ought to be leſs in ſwifter Motions of the Body, and in very ſwift ones none at all; as our Famous Author found the thing to happen in very accurate Experiments, which he made about it.

Corol. (1.) The Mediums therefore in which Projectiles are carried the fartheſt without any ſenſible Diminution of Motion, are not only very fluid, but much rarer than the Bodies moved in them; otherwiſe they would preſently ſtop the Motion of the Projectiles, and bring it to reſt.

Corol. (2.) From whence it follows, that our Air, or all the Matter contain'd in it, is very ſmall, if it be compar'd with the Matter in Bodies, that are carried forward very far and ſwiftly in it; and is ſo far from the *Carteſian Plenum*, that it doth not indeed poſſeſs the 20000th Part of the containing Space.

Corol. (3.) And it follows alſo, that the Ether, or all the Matter contain'd in the Planetary Spaces, thro' which the Planes have revolv'd for ſo many thouſands of Years with ſuch Velocity, and this without almoſt any Loſs of Motion at all, is very ſmall, if compared with the Matter contain'd in the Planets themſelves; ſo that, as will eaſily appear by Calculation, that Space ought rather to be counted a Vacuum than otherwiſe.

Corol. (4.) The whole *Carteſian* Philoſophy therefore falls to the ground, which is entirely built upon a Plenum, and a Celeſtal Matter, which he calls His 1st and 2d Elements. Nor can that ingenious Fiction any longer subsist, when its Basis is thus destroy'd by our Author's Experiments, and what he hath demonstrated: Especially when He has not only taken away that Plenitude of Matter, but shew'd also that there is nothing of the foresaid Matter in the Pores of Bodies. For by the Experiment of a very long Pendulum vibrating in the Air a long while, and by estimating the Loss of Motion, when compar'd with the Resistance of the Air made upon its Surface, he found that either there was none at all, or a plainly insensible Resistance in the internal Parts. From whence it is rightly concluded, that there is either none at all, or a plainly insensible Quantity of any subtle Matter in the Pores of Bodies; whereas, from the *Cartesian* Plenitude, compared with the specifical Gravity of the Pendulum, it ought to be far greater than the gross Substance it self of the Pendulum.

LXV. No Rectilinear Pressure can be propagated through a Fluid, in right Lines only.

For since the Parts of a Fluid are always in Motion every way, or are at least every way easily moveable, and will upon any Occasion be actually mov'd; it cannot be, but that any Pressure whatever, which is first communicated in a right Line, must urge the contiguous oblique Parts more or less; and that these oblique Parts must urge others in like manner that are placed obliquely; and thus *in infinitum*. The Pressure therefore, as soon as it is propagated to Particles which do not lie in the right Line, will begin to divaricate, and be propagated obliquely for ever; and when some Part of the Pressure is intercepted by some

Obstacle,

Obſtacle, the remaining Part now, as well as before, will divaricate into all the Spaces beyond the Obſtacle.

Corol. (1.) Hence the Reaſon appears, why Sounds let into a Chamber, either by the interpos'd Walls, or through the Windows, ſpread themſelves into all Parts of the Chamber, and are heard at all Angles, not only as reflected from the oppoſite Walls, but as propagated thro' the Air on every Side from the Window.

Corol. (2.) The Rays of Light which are propagated through the Ether, or Air, or any other Fluids whatever always in right Lines, are not Impulſes or Modifications of that Fluid, as it is in Sounds, but real Corpuſcles flowing from the Fountain of Light, and propagated by a true Motion through the Medium; as moſt of the other Phænomena of Light do alſo ſhew.

LXVI. Every tremulous Body in an Elaſtic Medium will propagate the Motion of Pulſes on every Side forwards; but in a Medium not Elaſtic, it will excite a circular Motion.

Caſe (1.) For the Parts of the tremulous Body, in their alternate going and returning, will in their going drive forwards, and conſequently preſs and condenſe the Parts of the Medium next thereto; and in their returning will permit the ſaid Parts of the Medium to expand themſelves, and return to their former Situation. Which certain Parts of the Medium going and returning alternately, as doth the tremulous Body it ſelf, will act in the ſame manner upon the Parts of the Medium next to them, as the tremulous Body did

did upon them, and will propagate the same tremulous Motion to those further Parts of the Medium, and these last will propagate it to others more remote than themselves; and thus in *infinitum.* And in every one of the design'd Divisions of the Medium, the Parts will be alternately condens'd and relax'd; in their Going condens'd, and in their Return relax'd, like as it is in the tremulous Body that began the Motion. Not that they all go and return at the same time, but alternately; for the Expansion of the foregoing Division makes the Condensation of the 2d, and is at the same time with it, as the Expansion of the 2d is at the same time that the Condensation of the 3d is. But the Parts which go, and in going are condens'd, because of their progressive Motion wherewith they strike Obstacles, are Pulses; and therefore successive Pulses will be propagated from every tremulous Body through an Elastic Medium; and this at Distances from each other nearly equal, because of the equal Intervals of Time, wherein the Body doth by each Tremor excite each Pulse. *Q. E. D.*

Corol. (1.) Altho' the Parts of a tremulous Body do go and return according to some certain and determinate Direction, or Part; yet the Pulses propagated from thence through the fluid Medium will, by the foregoing Proposition, spread themselves every way on the Sides; and will be propagated every way from the tremulous Body as the Center, according to Surfaces almost spheric and concentric. Of which we have an Example in Waves; which if they be rais'd by a tremulous Finger, will not only go forward, according to the Direction of the Motion of the Finger, but will presently be propagated on all Sides,

Sides, and encompass the Finger in the Form of concentric Circles; for the Gravity of the Water supplies in a sort the Place of Elasticity.

Corol. (2.) Hence we may collect, that the Number of the propagated Pulses is the same with the Number of the Vibrations of the tremulous Body, and is not multiplied in the Progress. For every physical little Line, as soon as by the Expansion it hath return'd to its first Place, would rest there, were it not urged with a new Motion by the Force of the tremulous Body it self, or the Pulses propagated from it. And therefore it will actually rest, when Pulses cease to be propagated from that Body.

Corol. (3.) Hence the Reason appears why Sounds, when the Motion of the sonorous Body ceaseth, do presently cease; and are hear'd at a great Distance no longer than at a lesser: For the Cause ceasing, the Effect must needs cease also.

Corol. (4.) Hence we may understand the Cause of the Increase of Sounds, in the *Stenterophonick* Tubes. For a reciprocal Motion is wont, in each Recourse, to be increas'd by the Cause that produces it: For the Motion in the Tube, which hinders the Dilatation of the Sound, is reverberated more strongly; and therefore is the more increas'd from the new Motion impress'd in each Reflexion. And since all that Force of the sonorous Body, or Voice, which otherwise must in the same time have been propagated through an entire Sphere, which hath the Length of the Tube for its Radius, is now shut up within the Hollow of the Tube, and goes out of the Aperture with a great Strength; it is evident, that the tremu-

tremulous Motion of the Air, or the Violence of its Pulses, is greatly increas'd from thence, and consequently ought to reach unto a much greater Distance; but this so notwithstanding, that the Velocity of the Propagation doth every where remain still the same and unvaried. The Sound therefore, as I suppose, is increas'd in these Tubes in the Proportion of the whole spheric Surface aforesaid, to that Part of it which is contain'd within the Aperture of the Tube. But it would be worth the while that Experiments should be made about this Matter, to determine whether the Increase of Sounds in these Tubes be in that Proportion which hath been defin'd; that we may hereafter pronounce with more Certainty, and may be able to accommodate these Tubes more to the Use of Mankind.

Case (2.) But if the Medium be not Elastic, because the Parts thereof which are pressed by the vibrating Parts of the tremulous Body cannot be condens'd, the Motion will be propagated in an Instant to Parts where the Medium doth more easily give way; that is, to Parts which the Body would otherwise leave empty behind it. The Case is the same here, as with Projectiles in general in any Medium whatever. The Medium, in giving way, doth not go back in *infinitum*, but by a Circulation comes at length to the Spaces which the Body leaves behind it. Thus it is that the Medium gives way to a tremulous Body also, by a circular Retrocession; and as often as the Body returns to its former Place, the Medium is repell'd from thence, and returns to its former Place.

Corol,

Corollary. The *Cartesians* therefore are miſtaken, who ſuppoſe that the Agitation of the Parts of the Sun, or any Flame, ſuffices to a Preſſure, which is to be propagated through the Ambient Medium in right Lines, ſo as to conſtitute the Rays of Light. For ſuch a Preſſure ought to be, not from the Agitation only of the Parts of the Flame, but from the Dilatation of the whole.

Decem. 9. 1706.

Lect. XXXI.

LXVII. IF a ſolid Cylinder, infinitely long, be revolv'd in an uniform and infinite Fluid about its own Axis, the Poſition whereof is given, and the Fluid be mov'd round by the Impulſe of this Cylinder only; and every Part of the Fluid perſeveres uniformly in its Motion; the periodic Times of the Fluid will be as their Diſtances from the Axis of the Cylinder directly; and the Velocities will be every where equal.

For let the Fluid be diſtinguiſh'd into innumerable Cylindrical Orbs concentric to the Cylinder, and of the ſame Thickneſs every where. And becauſe the Fluid is ſuppos'd to be homogeneous,

neous, and the Cylinder, by its circular Motion, endeavours to put all the contiguous Parts of the Fluid, and through them the further Parts in *infinitum*, into its own angular Motion, and consequently into a Velocity of Motion that is in direct Proportion of the Distance, so that each of them shouldbe turn'd about in the same periodic Time with it self; it is plain, that every Orb doth then cease from further Acceleration, and that the Parts of them persevere uniformly in their Motions, where the Resistance or Impression on the Concave Part, is equal to the Resistance or Impression on the Convex Part: (For otherwise the stronger Force prevailing, the Motion will be changed on that Part.) Therefore, where the respective Velocity, according to which Resistance will arise in the given Surface, shall be in the reciprocal Proportion of the Surface, the Impressions on both Parts will be equal; that is, in the present Case, where the angular Velocity is in the reciprocal Proportion of the Distance it self, or where the absolute Velocity is always equal, the periodic Times also will be in the direct Proportion of the Distance. *Q. E. D.*

Corol. (1.) If the Fluid be not infinite, but contain'd in a Cylindrical Vessel; the exterior Cylinder also will be turn'd round, and its Motion will be accelerated until the periodic Times of both Cylinders, and of the inclos'd Fluid, be equal one to another. But if the exterior Cylinder be violently detain'd, it will endeavour to retard the Motion of the Fluid; and unless the interior Cylinders preserve its Motion by some Force continually impress'd, it will make the same to cease by Degrees.

Corol.

Corol. (2.) But since the periodic Times of the Planets are not in the Proportion it self of their Distances from the Sun, but in a Proportion which is sesqui-alteral of the same; and consequently their absolute Velocities are not every where equal, but in a subduplicate Proportion of the Distances, as all Astronomers acknowledge; it appears, that the Constitution of such an Ethereal Fluid doth in no wise agree to the Solar System; nor doth the Supposition of it any ways help the *Cartesian* Vortices.

LXVIII. If a solid Sphere, in an uniform and infinite Fluid, be revolv'd uniformly about its own Axis, the Position whereof is given; and by the Impulse of this alone the Fluid be turned round, and every Part of the Fluid perseveres uniformly in its Motion; the periodic Times of the Parts of the Fluid will be as the Squares of the Distances from the Center of the Sphere.

Let the Fluid be distinguish'd into innumerable Concentric Spherical Orbs of the same Thickness: And, as before, the Fluid will then only persevere in its uniform Motion without any Acceleration or Retardation where the angular Motions of the Parts of the Fluid about the Axis of the Globe be reciprocally as the Spheric Concentric Surfaces themselves, or as the Squares of the Distances from the Center of the Globe reciprocally; or lastly, as the periodic Times of the Parts which are reciprocally proportional to the angular Velocities themselves; where these be as the Squares of the Distances from the Center of the Globe directly.

Corol. (1.) If the Fluid be not infinite, but contain'd in a spheric Vessel; the spheric Vessel also will be turned round, and its Motion will be accelerated until the periodic Times of the Sphere, and

and Veſſel, and incloſ'd Fluid, be equal to one another. But if the ſpheric Veſſel be violently detain'd, it will endeavour to retard the Motion of the Fluid; and unleſs the Sphere preſerve its Motion by ſome Force continually impreſs'd, will make that the ſame, as, in the former Caſe, ſhould by Degrees ceaſe.

Corol. (2.) But ſince the periodic Times of the Planets are not in the duplicate Proportion of their Diſtances from the Sun, as we have ſeen already; it is manifeſt, that the Conſtitution of ſuch an Ethereal Fluid doth in no wiſe agree to the Solar Syſtem; nor are the *Carteſian* Vortices in any wiſe help'd from the Suppoſition of the ſame.

Corol. (3.) Since the Bodies, which being carried in a Vortex, go perpetually the ſame round without conſiderable Acceſs to the Center, or Receſs from it; (as it is in all Planets, both Primary and Secondary;) they muſt needs be of the ſame Denſity with the Vortex, and be carried along together with the contiguous Parts: And ſince this Sort of Vortices muſt be ſo mov'd, that the periodic Times ſhould be in the duplicate Proportion of the Diſtances (contrary to what happens in all the Planets;) it is manifeſt, that the Planets are not carried along in Corporeal Vortices. Which alſo will be made more manifeſt from the following Propoſition.

LXIX. The Velocities of all the Planets, whether Primary or Secondary, about their Central Bodies, by being in the reciprocal ſubduplicate Proportion of the Diſtances from their Centers, do wholly overthrow the *Carteſian* Hypotheſis of Vortices.

For the Planets, as is now known every where, do revolve each of them about the Central Body

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in Ellipses; and this in such sort, that by Rays drawn to their Foci they describe Areas proportional to the Times; and that the Velocities should be in the reciprocal subduplicate Proportion of the Distances. But the Parts of an Ethereal Vortex cannot be revolv'd with such a Motion. For (in *Fig.* 6. *Plate* 8.) let A D, B E, C F be three Primary Orbs describ'd about the Sun S. Of which let the outmost C F be Concentric to the Sun; and let the Aphelia of the two inner be A and B, and their Perihelia D and E. Therefore the Body which is revolv'd in the Orb C F will, by a Ray drawn to the Sun in describing Areas, which are proportional to the Times, be moved with an uniform Motion: But the Body which is revolv'd in B E will, according to the Laws of Astronomy depending both upon Geometrical Demonstrations and Celestial Observations, be mov'd more slowly in the Aphelion B, and more swiftly in the Perihelion C; when yet, according to Mechanic Laws, the Matter of the Vortex must to be mov'd more swiftly in the narrower Space which is betwixt A and C, than in the wider Space which is betwixt D and F; *i. e.* more swiftly in the Aphelion than in the Perihelion. As for Example: In the Beginning of the Sign *Virgo*, where *Mars*'s Aphelion now is, the Distance betwixt the Orbs of *Mars* and *Venus* is, to their Distance in the Beginning of *Pisces*, almost in the sesqui-alteral Proportion, or as 3 to 2: And therefore the Matter of the Vortex betwixt those Orbs in the Beginning of *Pisces*, ought to be carried more swiftly than in the Beginning of *Virgo*, in the same sesqui-alteral Proportion. For by how much the narrower or straiter the Space is through which the same Quantity of Matter passeth in the Time of one Revolution, with so

Y much

much the greater Velocity it must pass thro' it. Therefore if the Earth resting relatively in this heavenly Matter be carried along by the same, and revolv'd about the Sun together with it; the Velolocity thereof, in the Beginning of *Pisces*, ought to be to the Velocity of the same, in the Beginning of *Virgo*, in the sesqui-alteral Proportion, or as 3 to 2. From whence the apparent Motion of the Sun, in one Day's Time in the Beginning of *Virgo*, ought to be greater than 70′, and in the Beginning of *Pisces* less than 48′; when yet (by the Testimony of Experience) that apparent Motion of the Sun is swifter in the Beginning of *Pisces*, than in the Beginning of *Virgo*; and therefore the Vortex is mov'd more swiftly in the Beginning of *Virgo*, than in the Beginning of *Pisces*. The Hypothesis therefore of Vortices doth wholly contradict the Astronomical Phænomena; and serves not so much to explicate, as to disturb the Celestial Motions.

Scholium. Hitherto we have delivered the Principles of Natural Philosophy out of our Famous Author; yet, speaking properly, we have delivered them not Philosophically or Physically, but rather Mathematically. Forasmuch as we have hitherto considered the general Laws and Conditions of Motions and Forces, which chiefly belong to Astronomy and Natural Philosophy, mostly in a Mathematical and Universal Method: Nevertheless, that our Work should not seem altogether dry and barren, we have every where illustrated it with *Scholia*, and *Corollaries* Astronomical, Physical, Optical, and also Mechanical; and so have prepared the Way to true Philosophy and Astronomy, that is, the *Newtonian*. It remains, that we come now at length to the Nature of Things, and to the Philo-

Plate VIII

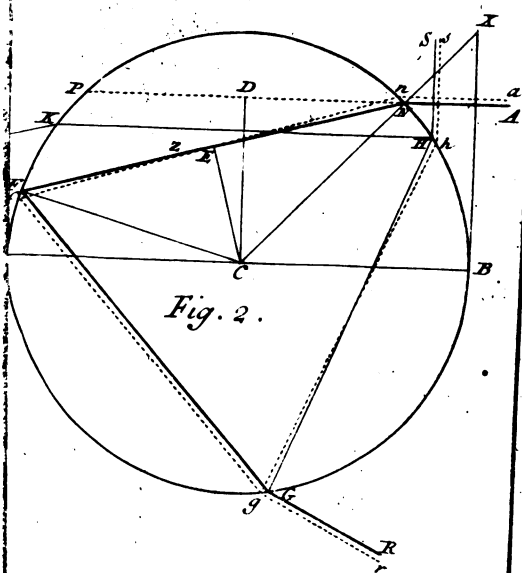
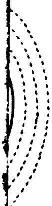


Fig. 2.



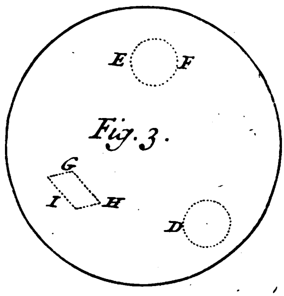
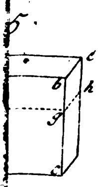


Fig. 3.



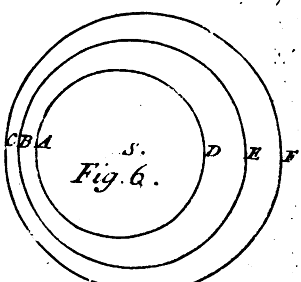


Fig. 6.

I. Senex sculp.

Philosophical Causes of these Phænomena, both Astronomical and Physical, and to the true System of the World; and that we set before you the Frame and Constitution of the same System, so far as it depends upon the Principles already laid down; omitting here, or only lightly touching upon those Things, which we had observ'd in the foregoing *Scholia* or *Corollaries*.

Jan. 29. $170\frac{6}{7}$.

LECT. XXXII.

LXX. THE Six Primary Planets, each with its own Satellites, if they have any, encompass the Sun with their Orbs, and revolve about it.

That *Mercury* and *Venus* revolve about the Sun, is manifestly demonstrated from their Faces exactly imitating those of the Moon; as is every where now known by Telescopic Observations. For sometimes they shine with a full Face about the Conjunctions, but with the least apparent Diameters; they being then situate beyond the Sun, and imitating a Full Moon: and then with an obscure Face about the other Conjunctions, but with the greatest apparent Diameters; they being then situate on this Side the Sun, and imitating a New Moon. And they appear likewise of a gibbous or hollow Face about the Octants, and of an halved and dichotomous one about the Quadratures, like as it it is is in the Moon: Sometimes they paſs thro' the Diſcus of the Sun, and appear as Spots therein, inducing a partial Eclipſe; and ſometimes they paſs beyond the Body of the Sun, being in the mean while inviſible to us. From whence it is certain, that theſe two Planets are revolv'd about the Sun, and not about the Earth. And altho' *Mercury* is ſo rarely ſeen, as appearing to us only about its greateſt Elongations, and when it paſſeth over the Sun, that all the ſaid Faces cannot be actually obſerv'd ſo clearly in this Planet, as in *Venus*; yet notwithſtanding, ſince what Faces of *Mercury* can be ſeen, do exactly anſwer to this Poſition; and ſince thoſe of *Venus*, a Planet of the ſame Condition, lie open to our Obſervation, and do every where fully anſwer the ſaid Poſition; there is no room to doubt concerning the reſt as to *Mercury*. From the full Face of *Mars* alſo, near the Conjunction with the Sun, and the gibbous Face thereof in the Quadratures, it is manifeſt, that it revolves about the Sun. The ſame thing is alſo demonſtrated concerning *Jupiter* and *Saturn*, from their Faces which are always full, as it ought to happen at ſo great a Diſtance. For albeit theſe Planets ought to have their Faces about the Quadratures ſomething diminiſh'd; yet ſince that Diminution of Light is ſo very ſmall that it can ſcarce, or rather not at all be obſerv'd and ſeen by us, their full Face is to be reckoned to agree very well with the ſaid Poſition. But that the Orbit of the Earth encompaſſeth the Sun, is abundantly manifeſt from the annual Parallax, which we have elſewhere explained.

Corollary. From hence with *De Cartes*, and the reſt alſo of the Aſtronomers of the foregoing Age, we gather that the *Ptolemaic* Syſtem of the World, which

which alone was cultivated and celebrated for ſo many Ages foregoing, comes to nothing. And we gather alſo, that the *Tychonic* Syſtem, which was afterwards receiv'd and celebrated by ſo many and great Aſtronomers, doth wholly fall to the Ground; and doth not in any wiſe agree with the Phænomena, which have been obſerv'd of late. And laſtly we gather, that the *Copernican* Syſtem, which hath for ſo long a time been approv'd of and follow'd by moſt of the beſt Aſtronomers, is the true Syſtem of the World, and is that alone which doth preſent to us that Order of all the Planets, which agrees to the Nature of Things, and to Aſtronomical Obſervations. Therefore it may juſtly ſeem ſtrange, that the Famous Dr. *Gregory*, that Excellent Interpreter of the *Newtonian* and *Copernican* Aſtronomy, a Man ſo well skill'd in the true Mundane Syſtem, ſhould beſtow ſo much Time and Pains in delivering and ſetting off thoſe and other falſe and imaginary Hypotheſes. When it is ſo certain, that the *Pythagorean* or *Copernican* Order of the Planets is that alone which is True and Genuine; and that the reſt of the Hypotheſes are only fictitious, To what Purpoſe ſhould we mix the Truth with mere Shadows, and diſturb the Contemplation of the Nature of Things with manifeſt Falſities? Let therefore thoſe, once indeed moſt Noble, moſt Famous Syſtems, be now baniſhed for ever out of the Aſtronomical World; and that only be admitted, cultivated, delivered and taught, which now, at length, we find to be the only one that correſponds to the true Order of Nature, and to real natural Cauſes. But this by the way.

LXXI. The periodic Times of the six Primary Planets, are in the sesqui-alteral Proportion of their mean Distances from the Sun.

This Proportion, which was first found out by *Kepler*, the Parent of the *Newtonian* Philosophy, is now acknowledg'd by all. The Measure of the periodic Times is agreed upon amongst all Astronomers; but as for the Magnitudes of the Orbs, the same *Kepler* and *Bullialdus* have exceeded all others in the Diligence they have us'd for determining the same: And the mean Distances which answer to the periodic Times, do not sensibly differ from the Distances which they have found, and are for the most part in the Middle betwixt them; as may be seen in the following Table.

The mean Distances of the Planets from the Sun.

According to	*Saturn.*	*Jupit.*	*Mars.*	*Earth.*	*Venus.*	*Mercu.*
Kepl.	951000.	519650.	152350.	100000.	72400.	38806.
Bull.	954198.	522520.	152350.	100000.	72398.	38585.
Periods.	953806.	520116.	152399.	100000.	72333.	38710.

And now we will give the true Periods, as also the Distances which come nearest to the Truth, from Mr. *Flamsteed*'s Parallax of the Sun, *viz.* of 10″.

Mer-

		D.	H.	'.
Mercury	revolves about the Sun in the Space of	87	23	16
Venus		224	16	49
Earth & *Moon*		365	6	9
Mars		686	23	27
Jupiter		4332	12	20
Saturn		10759	6	36

Mercury	is distant from the Sun	32,000,000	*English* Miles.
Venus		59,000,000	
Earth		81,000,000	
Mars		123,000,000	
Jupiter		424,000,000	
Saturn		777,000,000	

Now, as to the Methods of finding these Distances, they are thus determin'd.

Of the Distances of *Venus* and *Mercury*, as compar'd with that of the Earth, there is no room to doubt; since these are gathered by plain Trigonometry from their greatest Elongations, known by easy Observation. As for the Superiors, all manner of Dispute concerning their Distances from the Sun, which are deduc'd from the Arch of Retrogradation, is taken away by the Eclipses of the Satellites of *Jupiter* reduc'd to an accurate Calculation, according to the other Distances, and which agree with the Observation, for by those Eclipses, the Position of the Shadow which *Jupiter* casts, is determin'd; and by this means *Jupiter*'s Heliocentric Longitude is had, whilst his Geocentrical is had immediately by Observation. Therefore in the Plane Triangle connecting the Centers of the *Sun*, *Jupiter*, and the *Earth*, all the Angles are given, and consequently the Proportion of the Sides is also given; or the Proportion of the Distances of *Jupiter* and the *Earth* from the Sun.

Corollary. Therefore the Proportion of the Distances from the Sun, is given exactly in all the Planets; so that if the Distance of any one of the Planets was given in some known Measure, as in Miles or Semi-diameters of the Earth, we should withal have the true or absolute Distances of all: But this is what is yet wanting.

LXXII. The six Primary Planets do always, by Rays drawn to the Sun, describe equal Areas in equal Times, and in general Areas proportional to the Times. This Equality of the Areas in equal Times, which is another Foundation of the *Newtonian* Philosophy, is owing likewise to the Observation of the same *Kepler*. Whilst the five other Planets are, in respect of our Earth, sometimes Progressive, sometimes Stationary, and then Retrograde; they do always go forward, in respect of the Sun, and that with an uniform Motion nearly, such that it is something swifter in the Perihelia, and slower in the Aphelia, to preserve the foresaid Proportionality of Areas. This Proposition, which is well known to Astronomers, is demonstrated as *Jupiter* in a peculiar manner; *viz*. by the Calculation of the Eclipses of its Satellites, which is built upon this Hypothesis, and is exactly agreeable to the Observation. For by these Eclipses, as we have said already, *Jupiter*'s Longitude and Distance from the Sun are exactly determin'd.

LXXIII. The Moon, by Rays drawn to the Center of the Earth, describes in equal Times Areas almost equal; and in general, Areas almost proportional to the Times.

This

This appears from the apparent Motion of this Planet, as compar'd with its apparent Diameter, which is in the general nearly reciprocally proportional to the Diſtance. I ſaid in the Propoſition, *almoſt* proportional; becauſe the exact Proportionality is ſomething diſturb'd by the Sun's Force, as we have explained that Matter elſewhere: But taking away that Diſturbance, the Propoſition would be as exact and full, as in the Primary Planets; and that for the ſame Reaſon.

LXXIV. The Satellites of *Jupiter* do, by Rays drawn to the Center of *Jupiter*, deſcribe Areas nearly proportional to the Times: And their periodic Times are in the ſeſqui-alteral Proportion of their Diſtances from their Centers.

Both Parts of the Propoſition are manifeſt from Aſtronomical Obſervations. For their Orbs do not differ ſenſibly from Circles Concentric to *Jupiter*, and their Motions in theſe Circles are found to be almoſt uniform; And as for the Proportion of the periodic Times here meant, it is what all Aſtronomers agree in. Beſides, Mr. *Flamſteed*, who hath ſtated all Things moſt accurately by the Micrometer, and the Eclipſes of theſe Satellites, hath, both by Letters written to Sir *Iſaac Newton*, and by his Numbers themſelves communicated to him, ſignified that that ſeſqui-alteral Proportion doth hold here as exactly as poſſible, ſo far as he can diſcover by Obſervation. Which will be manifeſt from the following Tables.

The Periodic Times.

	D.	H.	′.
1	1	18	27 ½
2	3	13	13 ⅔
3	7	3	42 ⅗
4	16	16	32 ⅙

The

The Distances from the Center of Jupiter.

According to	1	2	3	4	
Cassini.	5	8	13	23	Semi-diam. of *Jupiter.*
Borelli.	$5\frac{2}{3}$	$8\frac{2}{3}$	14	$24\frac{2}{3}$	
Townl. by micr.	5⌊51	8⌊78	13⌊47	24⌊72	
Flamf. by micr.	5⌊31	8⌊85	13⌊98	24⌊23	
Flam. by Ecl. *Satel.*	5⌊578	8⌊876	14⌊159	24⌊903	
Period. Times.	5⌊578	8⌊878	14⌊168	24⌊968	

LXXV. The Satellites of *Saturn* do, by Rays drawn to the Center of *Saturn*, describe Areas proportional to the Times: And their periodic Times are in the sesqui-alteral Proportion of their Distances from the Center of their Primary.

Both Parts likewise of this Proposition are prov'd from Astronomical Observations: For their Orbs scarce differ sensibly from Circles concentric to *Saturn*, and their Motions are found to be almost uniform in these Circles. And as concerning the Proportion of the periodic Times, this will appear to every one that will take the Pains to compute it from the following Table, which we here present the Reader out of Mr. *Hugens*'s *Cosmotheoros*, Page 101, 102.

	The Period. Times. D.	H.	′.	″.	*Distances from the Center of* ♄, *both by Observ. and Period.*	
1	1	21	19		$\frac{39}{40}$	Diameters of the Ring.
2	2	17	41		$1\frac{1}{4}$	
3	4	13	47		$1\frac{3}{4}$	
4	15	22	41		4	
5	79	22	4		12	

Novemb. 17, 1707.

LECT.

Lect. XXXIII.

LXXVI. THE Force whereby the Primary Planets are perpetually drawn back from right Lines, and retain'd in their Orbs, does respect the Sun; and is as the Squares of the Distances from the Center of the Sun reciprocally.

For on account of the foresaid Proportionality of Areas, this Force must tend to the Sun; and because the periodic Times are in the sesqui-alteral Proportion of the Distances, the Quantity of the Force must be every where in the reciprocal duplicate Proportion of the Distances; as we demonstrated before: But this 2d Part is also demonstrated most fully from the Figure of the Orbs. For, if the Planets were mov'd about the Sun in spiral Lines, cutting the Rays in a given Angle, the Centripetal Force would be in the triplicate Proportion of the Distances, or as the Cubes of the Distances, reciprocally. But if they were mov'd about the Sun in Ellipses, which have the Center of the Sun in their Center, the said Force would be in the direct Proportion it self of the Distances: But when the Ellipses, in which they are mov'd, have the Center of the Sun not in their Center, but in one of their Foci, as the Case really is, and all Astronomers do acknowledge; then the said Force must needs be in the duplicate Proportion of the Distances reciprocally.

Which

This is also demonstrated by the Quiescence of the Aphelia. For where the said duplicate Proportion doth hold exactly, there the Aphelia must rest; when the said Proportion approacheth to the simple direct Proportion, then the Aphelia must go back; but when it inclines to the triplicate Proportion, they must go forwards.

LXXVII. The Force wherewith the Satellites of *Jupiter* and *Saturn* are perpetually drawn back from right Lines, and retain'd in their Orbs, respect the Centers of *Jupiter* and *Saturn* respectively; and are as the Squares of the Distances from those Centers reciprocally.

For on account of the aforesaid Proportionality the Areas about the Centers of *Jupiter* and *Saturn*, the said Force must tend to those Centers; and because of the sesqui-alteral Proportion which the periodic Times have to the Distances, the Quantity of that Force must be every where in the reciprocal duplicate Proportion of the Distances, by what was in the foregoing Proposition mention'd to have been demonstrated by us before. But we can fetch no Argument to prove this latter Part of our present Proposition from the Figure of the Orbs; for that those Orbs, of which we speak at present, are Circles, or Ellipses not sensibly different therefrom: Nor consequently from the Quiescence of any Aphelia; for in Circles where there can be no Line of the Apsides, there are no Aphelia.

LXXIX. The Force wherewith the Moon is perpetually drawn back from a Rectilinear Motion, and retain'd in its Orb, respects the Center of the Earth; and is as the Squares of the several Distances from the same Center reciprocally.

For

For on account of the Equality of Areas about theCenter of the Earth in equalTimes,excepting ſo far as the ſame is diſturb'd by the Force of the Sun; the ſaid Force muſt tend unto the Earth: And becauſe of the Elliptic Figure of the Orbit, which hath the Center of the Earth in one of the Foci, the Quantity of the Force muſt be every where in the reciprocal duplicate Proportion of the Diſtances. For altho' the Figure of the Lunar Orbit be not exactly Elliptic, and conſequently the Center of the Earth is not placed exactly in one of the Foci of the ſame Orbit; yet notwithſtanding, ſince all this Variety doth ariſe from the diſturbing Force of the Sun only, the Figure is to be underſtood to be in it ſelf, or primarily an exact Ellipſis, and to have the Earth placed in one of its Foci; and conſequently to have the Centripetal Force in the duplicate Proportion of the Diſtances reciprocally: Yea, whilſt the thing is as it is, the very ſlow Motion of the Moon's Apogeum ſhews, that that Force is in the ſaid duplicate Proportion very nearly, if not exactly. For by our Author's Calculation, it appears from the ſlow Progreſs of the Apogeum, that the Centring Force of the Moon towards the Earth, comes above ſixty times nearer to the duplicate than to the triplicate Proportion. Which ſmall Difference ariſing, as was ſaid, from the Action of the Sun, is to be neglected. It remains therefore, that this 2d Part of our Propoſition holds good, as it was propounded. Which will alſo be more fully manifeſt, by comparing the Centripetal Force of the Moon with the Force of Gravity upon the Surface of the Earth: Which will be done in the next Propoſition.

LXXIX. The Moon gravitates perpetually towards the Earth; and by the Force of Gravity

is

is always drawn back from a Rectilinear Motion; and retain'd in its Orb.

For by Experiments of Pendulums, which have been made as exactly as could be, it appears that the Force of Gravity upon the Surface of the Earth, is of the same Quantity with the Centripetal Force of the Moon; which hath been shew'd to be in the duplicate Proportion of the Distances reciprocally: And consequently from the said Experiments, that Quantity of the Moon's Centripetal Force is more fully demonstrated; and at the same time it is shew'd, that that Centripetal Force of the Moon is no other than that Force which we call Gravity. For if any should say that it is different from it, it must be acknowledg'd however that that Centripetal Force of the Moon, whatever it is, would be felt upon the Face of the Earth; which Force therefore, as join'd with the Force of Gravity, would make Bodies to fall to the Earth as swift again as they do, and in the Space of one Second of Time to describe 32|2 *English* Feet instead of 16|1. [Unless any one should say, that this Force of Feet 16|1, in one 2d of Time, is indeed a Compound Force, compounded of that Force wherewith the Moon tends to the Center of the Earth, and that Tendency thither which Bodies upon the Face of the Earth would have without it.]

LXXX. The Secondaries of *Jupiter* and *Saturn* gravitate towards *Jupiter* and *Saturn* respectively, and the Planets which are mov'd about the Sun, immediately gravitate to the Sun; and by the Force of Gravity are drawn back from Rectilinear Motions, and retain'd in their Orbs.

For the Revolutions of all these Planets about their respective Centers, are Phænomena of the same Kind with the Revolution of the Moon about

about the Earth; and therefore ought to depend upon Causes of the same Kind: Especially when it hath been demonstrated, that the Forces on which these Revolutions depend, respect the Centers of *Jupiter*, *Saturn*, and the *Sun*; and that in departing from *Jupiter*, *Saturn*, and the *Sun*, they decrease in the same Proportion, as the Force of Gravity decreaseth in the Recess from the Earth.

Corol. (1.) Therefore Gravitation is towards all the Planets. For it is certain, that *Venus*, *Mercury*, and the rest of the Planets, are Bodies of the same Kind with *Jupiter* and *Saturn*: But we note also in this place, that by the 5th Law of Motion Gravitation is reciprocal; and that as the Secondaries of *Jupiter* and *Saturn* gravitate towards their Primaries respectively, so their Primaries gravitate respectively towards them; and the Earth towards the Moon; and the Sun towards all the Planets, both Primary and Secondary.

Corol. (2.) The Gravity which respects every Planet, is reciprocally as the Square of the Distance from the Center thereof.

LXXXI. All Bodies gravitate towards each of the Planets; and their Weights towards the same Planets, at equal Distances from the Center of the Planet, are proportional to the Quantity of Matter in each.

The Descent of all heavy Things towards the Earth, if you set aside that unequal Retardation which ariseth from the Resistance of the Air, is in equal Times, as hath been observ'd now for a long time, and we also noted before; whether the descending Bodies be great or small, soft or hard, or of whatsoever Texture of Parts. Which exactly agrees with the Experiments of Pendulums

lums descending in Arches, whether Circular or Cycloidal. For all Bodies being let down at the same Distance of the Center of Oscillation from that of Suspension, and in equal Arches, make their Ascent and Descent in equal Spaces of Time, and vibrate for a long while. Therefore, since the Obliquity of the Curvilinear Motion is, in this Case, every where like and equal; the same Bodies let down together in a Vacuum would, in equal Times, describe equal Spaces in a perpendicular Descent; and consequently are impell'd with a Weight every where exactly proportional to the Quantity of the Matter. For where a double or treble Quantity of Matter is urged with a Force double or treble, and no otherwise; the Velocity of the Motion will always be equal: that is, where any equal Particle of any Body whatever is urged with an equal Force of Gravity, the Sum of all, whether in a great Body or a small, will be urged with a proportional Force of Gravity; and all, neitheir accelerating nor hindring one another's Endeavours, will always descend with equal Velocity, and will in the same degree gravitate towards the Earth. That the Thing is thus in the Experiments of Pendulums, we shew'd before; and our Author try'd the Matter particularly in Gold, Silver, Lead, Glass, Sand, common Salt, Wood, Water, and Wheat. He took two wooden Boxes round and equal, and fill'd one with Wood; and the same Weight of Gold he hanged, as exactly as he could, in the Center of Oscillation of the other. The Boxes hanging by equal Cords, of Eleven Foot each, made Pendulums altogether equal, as to Weight, Figure, and the Resistance of the Air. And being placed just by one another, they were found to vibrate equally, and to go and come together

gether for a long while. And in Bodies of the ſame Weight, the Difference of the Quantity of Matter, which would ſcarce amount to the 1000th Part of the whole, might, by theſe Experiments, be manifeſtly diſcovered. But now that the Nature of Gravity towards the reſt of the Planets, and towards the Sun it ſelf, is the ſame as that towards the Earth, there is no reaſon to doubt. Which is alſo manifeſt from the Spherical Figure of all, which can ſcarce be deduced from any Thing elſe, than an Equilibrium of all the Parts, mutually gravitating towards each other. Furthermore, let Terreſtrial Bodies be ſuppos'd to be lifted up unto the Orb of the Moon, and being together with the Moon, depriv'd of all Motion, to be let down to fall to the Earth. By what hath juſt been demonſtrated it is certain, that in equal Times they would deſcribe Spaces, equal to thoſe which the Moon it ſelf would deſcribe; and conſequently, that they are to the Quantity of Matter in the Moon, as their Weights to its Weight. Beſides, becauſe the Satellites of *Jupiter* and *Saturn* are revolv'd in Times, which are in the Seſquialteral Proportion to their Diſtances from the Centers of *Jupiter* and *Saturn* reſpectively; their accelerating Gravities towards *Jupiter* and *Saturn* will be reciprocally, as the Squares of the Diſtances from thoſe Centers; and therefore in all equal Diſtances from *Jupiter* and *Saturn*, their accelerating Gravities will become equal, and will equally affect all Bodies. And therefore in falling in equal Times, from equal Heights, they would deſcribe equal Spaces, like as it comes to paſs in heavy Bodies on this our Earth. And by the ſame Argument, the Planets about the Sun let down at equal Diſtances from the Sun, would in their Deſcent towards the Sun, in equal Times, describe equal Spaces. Moreover, that the Weights of *Jupiter* and *Saturn*, and their Satellites towards the Sun are Proportional to the Quantity of Matter, is manifest from the Motion of the Satellites, which is most Regular; and their Orbits, which are almost Concentrical with their Primaries. For if some of these were more Attracted to the Sun in the same Quantity of Matter than others are, the Motion of the Satellites would be disturb'd by the Inequality of the Attraction; and so far disturb'd that if at equal Distances from the Sun, the accelerating Gravity of one of *Jupiter*'s Satellites towards the Sun, were greater or lesser than the accelerating Gravity of *Jupiter* it self towards the Sun, though it were but by one 1000th Part of the whole Gravity; then, according to our Author's Computation, the Distance of the Center of the Orb of the Satelles from the Sun, would be greater or lesser than the Distance of *Jupiter* from the Sun, by a 2000th Part of the whole Distance; or in a Sub-duplicate Proportion of the Distance; that is, by a 5th Part of the Distance of the outmost Satelles, from the Center of *Jupiter*; which Occentricity of the Orb would be very sensible. But the Orbs of the Satellites of *Jupiter* are concentrick to *Jupiter*, and therefore the accelerating Gravities of *Jupiter*, and his Satellites towards the Sun, are equal to one another. And by the same Argument, the Weights of *Saturn*, and its Satellites towards the Sun, at equal Distances from the Sun, are as the Quantities of Matter in them. And the Weights of the Moon and Earth towards the Sun, are likewise exactly Proportional to the Mass of Matter contain'd in them. And the Thing is the same, as to the Weights of each Part of every Planet, towards

wards any other whatever; whether they be Internal Parts, or External: For if some Parts did gravitate more, others less, than according to the Quantity of the whole Matter, the whole Planet, or Satelles, would, according to the Kind of Parts with which it most abounded, gravitate more or less than according to the Quantity of the whole Matter; which is contrary to Experience.

Nov. 24. 1707.

Lect. XXXIV.

Coroll. (1.) Hence the Weights of Bodies do in no wise depend upon their Forms and Texture.

For if they were varied with the Forms, they would be greater and less, according to the Variety of the Forms in equal Matter; which is altogether contrary to Experience.

Coroll. (2.) Therefore all Bodies which are about the Earth, whether Wood, or Metals, or Stones, or Water, or Air, or Vapours, gravitate towards the Earth, and according to the Proportion of the Matter, are equally heavy. If Bark, or Wooll, or Air, be of the Weight of one Pound in a Vacuum; and Gold, or Silver, or Brass, be of the same Weight there, the Quantity of Matter will be equal in them all.

Coroll. (3.) Therefore the Weight of all Bodies whatever in a Vacuum, is the most certain Test of the Quantity of the Matter. For in Bodies equal in Bulk, there is wont to be so great Difference as to the Density, that from the apparent Magnitude, the Quantity of the Matter can in no wise be determin'd. But since the Quantity of the same is every where Proportional to the same Weight, it may be determin'd most certainly from the same Weight.

Coroll. (4.) Therefore there must needs be a Vacuum. For if all Spaces were full, the Specifick Gravity of that Fluid, wherewith the Region of the Air, yea, and the Vacuum of Mr. *Boyle* would be filled, by reason of the Density of the Matter, which is the greatest that can be, and most perfect or absolute, or rather infinite, would not fall below, but exceed the Specifick Gravity of Quick-Silver, or Gold, or any other Body, which is counted the densest and heaviest. And therefore Gold it self could not descend in the Air, which is contrary to Experience. To omit here those Arguments which are brought to prove that there could be no Motion in a Plenum, which indeed seem solid enough in themselves to determine us to the same Side of the Question.

Coroll. (5.) Since therefore the Quantity of the Matter is every where known from the Weight, as well as the Resistance; and since it appears from the Weight, that almost all Bodies upon the Face of the Earth contain more void Space than solid Matter in them; since also, from the very little, and almost imperceptible Resistance of Planets and Comets, it appears,

that

that the Heavenly Spaces are void of all Matter; yea, that the Planets and Comets themselves, and also the Sun and fixed Stars, are, as it were, Points in Comparison of the void Space: It is plain, that Nature is so far from abhorring a Vacuum, as some have imagined, the *Cartesians* especially, that it seems to contain little in it besides a Vacuum: So little can Human Wit perform, in tracing out the Works of God, where Mathematical Reasonings, and Experiments, are wanting.

The most sagacious Mind of *Cartes* himself, too much destitute of these Foundations, was never able to find out the true Physical Causes of Things, and those which would agree to the later Discoveries.

Corol. (6.) The Force of Gravity is of a different Kind from the Magnetick Power. For the Magnetick Attraction is not in Proportion to the Matter attracted; since some Bodies are more, others less, others not at all attracted. And the Magnetick Force is far greater, according to the Quantity of the Matter, than the Force of Gravity, since a very small Loadstone may exceed the attracting Force of the whole Earth it self, and lift up an Iron Key from it. Nay, the Magnetick Force may be increas'd or remitted in the same Body; and in the Recess of the Magnet, it decreaseth in more than a duplicate Proportion of the Distance, which yet is the perpetual Proportion of Gravity; because the Force is much stronger in the Contact of the Surfaces, than when the Bodies are in the least separated from one another.

LXXXII. The Force of Gravity hath Place in all Bodies, all those at least, which are in the System of the Sun, and is Proportional to the Quantity of Matter in each.

That all the Planets do gravitate towards each other; and that the Gravitation towards every one separately consider'd, is reciprocally as the Square of the Distance of Places from the Center of the Planet, we have already prov'd. If there should any Doubt arise here, it must certainly be about the Gravity of one primary Planet towards another; for as for the common Gravity of all towards their Central Bodies, the Thing, by what hath been before demonstrated, is plainer, than to be in any wise denied. But as for the other, we have a plain Proof of that also. For when some Years ago, *Saturn* tarried along while near its Conjunction with *Jupiter*; and consequently, by reason of the Magnitude and Nearness of its Body, could not but have some sensible Effect, in disturbing the Satellites of *Jupiter*, if so be *Jupiter*, with its Satellites, did gravitate towards *Saturn*, according to the general Law of mutual Attraction, the Thing was found to be indeed thus: For Mr. *Flamsteed* himself, who at first denied any such Disturbance in the Motions of the Secondary Planets of *Jupiter*, the Thing being better considered, and the Observations being more exactly compared with the Calculations, ingeniously confess'd, that that Universal Law of Gravity holds in this Case also; and that those Motions did indeed appear disturb'd by the Neighbourhood of *Saturn*, and accordingly differ'd from the former Calculations. It follows therefore, by *Prop.* 81. and the Corollaries thereof, that every Planet gravitates towards every Planet, and that this Gravitation is Proportional to the Matter contained in them. Moreover, since all the Parts of every Planet, as of *Mercury* for Instance, do gravitate towards every other Planet, as *Venus* for Instance; and the Gravity of every Particle is to the Gravity of the whole, as the Matter of the Part to the Matter of the whole; and since also all Re-action, by the Sixth Law of Motion, is equal to Action; *Venus* will reciprocally gravitate towards all the Parts of *Mercury*; and the Gravity of *Venus* towards every Part, will be as the Gravity of the same towards the whole, as the Matter of the Part is to the Matter of the whole.

Corollary. Therefore the Gravity towards every whole Planet ariseth from, and is compounded of the Gravity towards each Part; like as it comes to pass in Magnetick and Electrick Attractions, where by how much the greater the Attrahent is, so much the greater, *cæteris paribus*, is the Attraction: For all Attraction towards the whole, arises from the Attractions towards each Part; nor can the Thing be conceiv'd otherwise. This will be more easily understood in Gravity, if we conceive many of the lesser Planets, which attract all Bodies severally, to meet together, and to make one great Planet. For the Force of the whole must be compounded of the Forces of the compounding Parts, and be the adequate Result of the same.

But now, if any one should in the same Place object; That all the Bodies with us, on the Face of the Earth, ought to Gravitate thus towards each other; whereas such a Sort of Gravitation is never perceiv'd: The Answer is ready, namely, That although the Bodies now spoken of, do indeed Gravitate towards each other, yet since the Gravitation of any particular Body towards another, is to the Gravitation of that Body towards the whole Earth, at the same Distance, as the other Body is to the whole Earth; it must needs be far leſs than to fall under the Notice of Senſe.

Corol. (2.) The Gravitation towards each equal Particle of a Body, is reciprocally as the Square of the Diſtances from the Particles.

LXXXIII. If the Matter of Two Globes gravitating each towards the other, on every Side in Places equi-diſtant from the Center, be homogeneous, the Weight of either Globe towards the other, will be reciprocally as the Square of the Diſtance betwixt the Centers.

After that our Author had found that the Gravity towards the whole Planet doth ariſe from, and is compounded of the Gravities towards the Parts, and is towards each Part reciprocally proportional to the Squares of the Diſtances from the Parts; he yet doubted, whether that duplicate reciprocal Proportion would hold exactly in the whole Force compounded of the many Parts, or only very nearly. For it might be that that Proportion, in greater Diſtances, might hold well enough; but near the Surface of the Planet, by Reaſon of the unequal Diſtances of the Particles, and their unlike Situations, it might notably err. But at length, by *Prop.* 44 and 45, and their Corollaries, he underſtood that the ſame Proportion holds exactly in ſuch ſpherical Bodies, as are equally denſe every where at the ſame Diſtance from the Centers.

LXXXIV. A *Prob.* To determine the Weights of Bodies towards the Planets or the Sun, at given Diſtances from the Centers of them.

Caſe (1.) To determine the Weights of Bodies placed without the Surface of the Planets at equal Diſtances. Now ſince the Weights, at equal Diſtances, are as the Quantities of Matter in the Planets towards

towards which the Gravitation is; and ſince that Weight or Quantity of Matter is known only by the Quantity of the Attraction, as the Cauſe by the Effect; and ſince, laſtly, that Quantity of Attraction is directly proportional to the Squares of the Velocities in theſe equal Circles, or reciprocally to the Squares of the periodic Times; the Proportions of the Weights will eaſily be known from the Squares of the Velocities. From the periodic Times therefore of the Planets that have others revolving about them, which Times were declared before; the Proportion of the Weights towards the *Sun*, *Jupiter*, *Saturn*, and the *Earth* reſpectively, will be as follows.

The Weight towards the	*Sun*	——	229600
	Jupiter	——	208⌊72
	Saturn	——	97⌊328
	The *Earth*	——	1
	The *Moon*	——	$0\frac{1}{40}$

Now the ſame Numbers which ſhew the Proportion of the Weight, ſhew likewiſe the Proportion of the Quantity of the Matter. But as for reducing the periodic Times agreeing to the real Diſtances, to periodic Times agreeing to any given Diſtance, it is eaſily done by this Analogy; As the Cube of the real Diſtance is to the Cube of the Diſtance given; ſo is the Square of the real periodic Time, to the Square of the periodic Time ſought. The ſquare Root therefore of this Number will give the periodic Time which is ſought: And by this means the Proportions of the Weights and Matter in the *Sun*, in *Jupiter*, in *Saturn*, and the *Earth*, are obtain'd. And altho' the Moon, which hath no Satelles about it, doth afford no ſuch Argument as this of a Satellit's Weight towards

wards it, or the Quantity of its own Matter; yet notwithstanding, since it presents to us another Argument of the same Thing, to wit, in the Flux and Reflux of the Sea; we thought it not improper to set down in this place, and by way of Anticipation, that Gravitation towards this Planet, which will afterwards be prov'd from that Flux and Reflux.

Case (2.) To determine the Weights of Bodies at the Distances of the Semi-diameters of the Planets, or upon their Surfaces. This is done by the same Method as in the former Case, and by the like Analogy accommodated to these particular Distances. In which Calculation, if we take Mr. *Flamsteed's* Semi-diameters of the Planets for the true ones, they will stand thus:

The *Sun*	is in Diameter	763460	*English* Miles long.
Saturn		67870	
Jupiter		81155	
Mars		4444	
The *Earth*		7935	
Moon		2175	
Venus		7906	
Mercury		4240	

The Weight therefore of equal Bodies upon the Surfaces of those Stars, is as follows:

The Weight towards	The *Sun*	24
	The *Earth*	1
	Jupiter	1|99
	The *Moon*	0|515
	Saturn	1|7

April 26. 1708.

LECT.

Lect. XXXV.

LXXXV. A *Problem.* To determine the Densities of the Planets; Since we have the Quantity of the Matter in five Planets determin'd in the former Case of the last Proposition; and in the latter Case, we have the Diameters of the Planets determin'd according to Mr. *Flamsteed*; it will be no difficult thing, from the given Quantity of Matter contain'd in the given Spheres, to compute the Density of the same Matter; which is done to hand in the following Table.

The Density of	The *Moon*	7,00
	The *Earth*	3,87
	The *Sun*	1,00
	Jupiter	0,76
	Saturn	0,60

LXXXVI. Gravity in proceeding from the Surfaces of the Planets downwards, decreaseth in the simple Proportion of the Distances from the Centers very nearly.

For if the Matter of the Planet were every where the same as to Density, this Proportion would hold exactly by *Prop.* 47. And where it obtains not exactly, the Disagreement is no other than such as the unequal Density ought to produce.

Corollary. Therefore the Gravity of Bodies on the Surfaces of the Planets, is the greatest of all, and

and on both Sides decreaseth; and is upwards in the reciprocal duplicate Proportion of the Distance, and downwards in the simple Proportion direct.

LXXXVII. The Motion of Planets and Comets may be maintain'd for a very long Space of Time in the Heavens.

For since the Resistance of Mediums, which alone can stop or retard these Motions once begun, is diminish'd in Proportion to the Weight or Density of the Matter; so that Water, which is near 14 times lighter than Quicksilver, doth resist less in the same Proportion; and Air, which is almost a thousand times lighter than Water, doth resist less in the same Proportion: If we look beyond our Atmosphere, which doth it self also wax more rare by degrees, as it were infinitely, into the Heavens, where the Weight or Density of the Medium is vastly diminish'd, above what it is in any Part of our Atmosphere; the Resistance will be so very small, that for some thousands of Years it can scarce become any whit sensible; accordingly it is evident that it hath been insensible, because the Celestial Motions have endured from the Infancy of Astronomy unto this Day, without any notable Change or Loss of Motion.

Corollary. But since, in an infinite Duration, that very small Resistance, if there be any, must needs retard and stop all those Motions; it is manifest upon this Hypothesis, that the present State of the Heavens neither was eternal *à parte ante*, nor shall be so *à parte post*. And this will hold good upon another Account also, especially if with Sir *Isaac Newton* we suppose the Force of Gravity to obtain not only in the Solar System, but also thro' the whole Universe. For if the Fixed Stars,

Stars, or Suns with their Planets and Comets, of whatſoever Number they are, ſo that it be not infinite, be ſubject to the Force of Gravity; In an infinite Time it would have come to paſs thouſands of Years ago; that they would have been gathered together into one Heap, and have been reduc'd to reſt in the Center of the Univerſe. Which thing alſo would, ſome time or other, come to paſs in an infinite Time yet to come, without the Interpoſition of the Divine Providence. As therefore the preſent State of Things had a Beginning, which is owing to the good Will, Wiſdom, and Power of Almighty God; ſo at length it may and will have an End upon the foregoing Hypotheſis, that is, according to the Natural or Eſtabliſh'd Order of Things; unleſs it ſhould pleaſe Almighty God, by his extraordinary Interpoſition, to prevent it: Without whoſe continual Interpoſition, on which this wonderful Force of Gravity wholly depends, it cannot laſt the leaſt Space of Time.

LXXXVIII. The common Center of Gravity of the Earth, Sun, and all the Planets, either reſts, or is mov'd uniformly in a right Line. This is manifeſt from what hath been demonſtrated before: But indeed it appears by no certain Token, whether it reſts or is mov'd. This only is to be concluded, That if it be mov'd, and with it the whole Solar Syſtem, the Motion muſt needs be very ſlow [unleſs it be mov'd uniformly and evenly with the Centers of other Syſtems.] For the Fixed Stars, which encompaſs us on every Side, neither appear greater nor leſs to us at this Day, than they did to the Ancient Aſtronomers 2000 Years ago. Which Phænomena ſeems to ſhew the reſt, or at leaſt the very ſlow Motion of the ſaid Center.

Corol. (1.) Hence the common Center of Gravity of the Sun, and all the Planets, is to be reckon'd for the Center of the Solar System, or Planetary World. For since the Sun, and all the Planets, gravitate towards one another, and therefore are in perpetual Agitation, more or less, according to the Force of their Gravity, as it hath been shew'd under the foregoing Laws of Motion; it is plain, that their moveable Centers ought not to be reckon'd for the quiescent Center of the World. If that Body indeed is to be placed in the Center, towards which all other Bodies do most gravitate, and which is next of all to the unmoveable Center, as it is reasonable that we should esteem it; that Privilege certainly is to be allow'd to the Body of the Sun; which therefore, speaking physically, is deservedly accounted *the Center of the Planetary World.* But, if we would speak accurately and mathematically, since the Sun it self is mov'd, and no sensible Body doth rest in the Center; the Center of Gravity of the whole System is to be chosen for the real Center of our World, which Center doth indeed most probably rest, and the Center of the Sun comes very near to it. Upon the whole therefore, Physically the Sun, but Mathematically the Center of Gravity is the Center of our World.

Corol. (2.) There is therefore no perfect Rest of a real Being in Nature. For supposing that the common Center of the System doth rest, that is the *only* thing (if we may so call it) which doth rest; all the Parts of the Systems being in perpetual Motion. I said *real Being*; because this Center of Gravity is not a physical Body, or any thing real, or other than a Mathematical Point, *i.e.* a plain Nothing: from whence, in consequence of our present Argument, it is to be said that nothing

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real doth reſt, or that there is not any real and abſolute Reſt in the whole Solar Syſtem.

LXXXIX. The Body of the Sun doth never reſt, but is in a perpetual Agitation: tho' it never departs far from the common Center of Gravity of all the Planets. For ſince the Quantity of Matter in the Sun is to the Quantity of Matter in *Jupiter*, as 229600 is to 208[72, or as 1100 to 1; and the Diſtance of *Jupiter* from the Sun is to the Semi-diameter of the Sun, as 424,000,000 is to 381,730, or as 1100 to 1; that is, in the ſame Proportion or thereabouts; the common Center of Gravity of the *Sun* and *Jupiter*, which is placed at a Diſtance reciprocally proportional to thoſe Bodies, will fall upon the Surface of the Sun almoſt. By the ſame Argument, ſince the Quantity of Matter in the Sun, is to the Quantity of Matter in *Saturn*, as 229,600 is to 97[328, or as 2360 is to 1; and the Diſtance of *Saturn* from the Sun is to the Semi-diameter of the Sun, as 777,000,000 is to 381,730, or in ſomething leſs Proportion; the common Center of Gravity of *Saturn* and the *Sun* will fall upon a Point ſomething below the Surface of the Sun. From whence the common Center of Gravity of *Jupiter* and *Saturn*, as placed on one Part, and of the Sun as placed on the other, will in no wiſe be diſtant by a whole Diameter of the Sun from its Center. And in purſuance of the ſame Argumentation, if the Earth and all the inferior Planets are, in the Libration, underſtood to be ſet on the ſame Side of the Sun: By reaſon of the Nearneſs and Smalneſs of thoſe Planets; the common Center of Gravity of all will ſcarce be diſtant from the Center of the Sun one entire Diameter thereof. But in other Caſes, which commonly happen, the Diſtance of the Centers

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is less; and where the Planets placed on this Side and on that do counterpoize one another, none at all. Therefore, altho' the Center of Gravity be indeed suppos'd to rest; the Sun, by Reason of the various Situation of the Planets, will be mov'd a little towards all Parts; but will never depart far from that common Center of Gravity.

XC. All the Primary Planets are mov'd in Ellipses, which have a common Focus in the Center of the Sun; and by Rays drawn to that Center, they describe Areas proportional to the Times. This is true also in the Secondaies, as revolving about the Centers of their Primaries. We deduc'd these things above from Astronomical Phænomena; but now the Principles of these Motions being known and established, from these we gather these heavenly Motions *à priori*. For from the Direction of Gravity towards the Centers of the Sun and primary Planets, the foresaid Proportionality of the described Areas doth follow; and from the Law of Gravity towards those Centers, which is in the reciprocal duplicate Proportion of the Distance, that Elliptic Figure of the Orbs about those Centers placed in the Foci is necessarily deriv'd, as we have demonstrated above out of our Author. And these things would be exactly thus, if the Sun and the Primary Planets rested from acting mutually upon one another. For their Orbs would be in Geometrical Srictness Elliptical; and the described Areas would be exactly proportional to the Times. However, those mutual Actions of the Sun and Planets upon one another are so very small, that they ought not to be regarded. And the Motion of the Planets about the Sun as moveable, or any other Planet as such, is less disturb'd than it would be if the same were unmoveable, as we observ'd before:

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From whence, speaking physically, the Proposition is still to be accounted true. The Action indeed of *Jupiter* upon *Saturn*, and its five Satellites; and of *Saturn* upon *Jupiter* and its four Satellits, is not altogether to be neglected: Since these Planets are great ones, and placed at a very great Distance from the Sun. From whence, by their mutual Attractions about their Heliocentric Conjunctions; which, by reason of the Slowness of their Motions, endure for no small Time; some Inequalities will arise on both Sides, as well in the Figures of their Orbits, as in their Motions; but yet scarce to be so much distinguish'd in the unequal Motions of the Primaries themselves, as in those of their Secondaries, of those about *Jupiter* especially.

Scholium. According to our Author's Computation, the disturbing Force or Gravity of *Saturn* towards *Jupiter* is to the Gravity of *Saturn* towards the Sun, about the Conjunction of those Planets, as 1 is to 204, or thereabouts. And the Difference of the Gravities of the Sun towards *Saturn*, and of *Jupiter* towards *Saturn*, is to the Gravity of *Jupiter* towards the Sun, as 1 to 1923. To which Difference the greatest disturbing Force of *Saturn* towards *Jupiter* is proportional. From whence the Disturbance of the Orb of *Jupiter* is far less than is that of *Saturn*: But the Disturbances, which are in the rest of the Orbs, are so very small, that they are not to be regarded.

XCI. The Aphelia and Nodes of the Orbs do rest. Because of the Force of Gravity in the duplicate Proportion of their Distances reciprocally, the Apses and Aphelia ought to rest of themselves; as was noted before. And because the same Force doth always respect a Point almost unmoveable, the Planes of the Orbs ought also to rest; and when the Planes rest, the Nodes or Intersections of the Orbs must rest too. But it is to be noted, that in Succession of Time some Inequalities will arise from the Actions of Planets and Comets upon one another; but that they will be so very small, that by reason thereof they are not to be regarded. It is also to be noted, that we do in this place suppose, with all Astronomers, the Rest of the Center of Gravity of the whole System; altho', as was hinted above, we are not yet able to *demonstrate* that Rest. These Things supposed, we shall deduce the following Corollaries.

Corol. (1.) The Fixed Stars rest, because that they keep their given Positions towards the Aphelia and Nodes which rest. This will seem a new Way of reasoning in Astronomy, to infer the Rest of the Fixed Stars from the Rest of the Systems of the Planets; whereas, on the contrary, we have hitherto been wont to determine the Motions of the Planets from the supposed Rest of the Fixed Stars. And thus it must needs have been, so long as our Famous Author's true Causes of the Celestial Motions were unknown.

Corol. (2.) Since the Parallax of the Fixed Stars, even the Annual, is so very small that it scarce falls under the Observation of the most accurate Observers: The Force of those Stars, by Reason of their immense Distance, can produce no sensible Effects in our System.

Corol. (3.) From whence it follows, that *Judiciary Astrology*, as it is called, (which depends not only upon the Positions and Influences of the Planets, but of the Fixed Stars also,) wants all sure Foundation; since it supposeth the Forces of those Bodies to be exceeding great, which the foregoing Corollary has rightly observ'd,

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are indeed very small, or rather none at all. But we may add this also in the present Case, that the influential Force of all the rest of the Planets, excepting the Sun and Moon, which Astrologers talk so much of, is either by reason of the Immense Distance, or the Smallness of their Bodies, so very little in our Atmosphere, and about the Earth, that it can scarce be by any sure Token *discern'd*; so far is it from being able to produce those great and wonderful Effects which they suppose. Those who, like Idolaters, conceive the Stars to be Gods, or that Gods possess and animate them, have somewhat wherewithal they may maintain their Hypothesis: But as for them who have quitted so gross an Error as that, it is a Wonder how they should come to adhere thus obstinately still, to those Astrological Trifles and Absurdities.

May 17. 1708.

LECT. XXXVI.

XCII. THE Diurnal Motions of the Planets are uniform and equable; and the Librations of the Moon arises from its Diurnal equable Motion, as compar'd with its Menstrual Inequable, and perform'd according to an Axis inclin'd to its Orbit.

These Things are noted elsewhere; and therefore we need not make many Words about them now. But because the Day of the Moon, revolving

Uniformly about its own Axis, is a Month; (I mean here the Periodic Month;) The same Face of this Planet will always nearly respect the *Superior* Focus of the Ellipsis, but not the Earth, which is placed in the *Inferior* Focus; because the Angular Motion also about that Focus is almost equal, but about the Earth plainly unequal. And therefore, according to the Situation of the Superior Focus, it will decline commonly on this Side, and on that from the Earth, and will shew to us sometimes more *Easterly*, sometimes more *Westerly* Parts; which is *the Libration of the Moon as to Longitude.* But *the Libration of the same as to Latitude*, wherein sometimes more *Northerly*, and at other times more *Southerly* Parts are presented to us, must arise from the Inclination of the Moon's Axis to the Plane of the Orbit; as is manifest to him that considers it.

Corollary. We may note also in this place, as we have done elsewhere, how exactly these two Motions of the Moon, which in no wise depend one upon the other, to wit, the Diurnal and Menstrual, do agree together; so that the one hath not been found for above these 2000 Years, to overgo the other in the least. *Which could not be without the Providence of God.*

XCIII. The Axes of the Sun and Planets, which are moved with a Diurnal Motion, are less than those Diameters which are Perpendicular to those Axes. Or the Figure of the Sun and Planets, which are revolv'd each about its own Center, is that of an Oblate Spheriod; that is, that of a Solid produc'd by the Revolution of an Ellipsis about its Lesser Axis.

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The Planets, and all the Celestial Bodies whatever, if all Circular Diurnal Motion were taken away, must needs, by reason of the equal Gravitation of the Parts on all Sides, put on a Spherical Figure. But on the same Diurnal Motion it will come to pass, that the Parts necessarily receding from the Axis of Motion, and thereby detracting from the Gravity about the Equator, must endeavour to ascend, where the Motion is the swiftest. And therefore in that Place the Matter of the Planet, unless it be very Solid, will by its Ascent unto the Equator increase the Diameters of the same; but will diminish the Axis at the Poles, the Gravity of the Parts being nothing diminish'd there. Thus the Diameter of *Jupiter* (the Observations of *Cassini* and Mr. *Flamsteed* agreeing thereto) is found to be shorter about the Poles than from *East* to *West*. And by the same Argument our Earth ought to have its Axis lesser than the Diameters of the Equator. For unless the Thing were so, and that our Earth were something higher at the Equator than about the Poles, the Seas, by reason of the greater Gravity there, would settle downwards about the Poles, and in Ascending about the Equator would overflow all. But by reason of the greater Velocity of the Diurnal Motion, and the lesser Density; *Jupiter* ought to have a much more sensible Difference of its Diameters than any other of the Planets, or than the Sun it self. From whence Astronomical Observers have hitherto been able to discover this Difference in no other Planet but this. But that our Earth is of this Figure, appears not only from the Argument just now produc'd, but also from the most accurate Experiments which have been made by Pendulums. For by how much the nearer Pendulum-Clocks, of the same Length of the Pendulum, are brought to the

the Equator, so much the more Slow their Vibrations are observ'd to be; and by how much the nearer they come to the Poles, their Vibrations are found so much the quicker: because the Center of the Earth, which in the former Case is more remote, and in the latter nearer, doth promote the Acceleration and Retardation of the pendulous Bodies respectively; as it must necessarily come to pass according to the present Proposition.

Scholium. If you would know exactly the Proportion of the Axis of every Planet unto the Diameters of the Equator, you must go through the manifold Intricacies of our Author's Calculation. But if you would have the Benefit of this Calculation without the Trouble of the same, take it thus, By Calculation our Author found that the Centrifugal Force of the Parts of the Earth under the Equator, arising from the Diurnal Motion, is to the Force of Gravity upon the Superficies of the Earth, as 1 is to 289. From whence if (in *Fig.* 1. *Plate* 9.) A P B Q represents the Figure of the Earth, produced from the Revolution of an Ellipsis about the Lesser Axis P Q; and A C Q, a c q be a Canal full of Water, reaching from the Pole Q q to the Center C c, and from thence going forwards towards the Equator A a; the Weight of the Water in the Leg of the Canal A C c a, is to the Weight of the Water in the other Leg Q C c q, as 289 is to 288 almost. Because the Centrifugal Force arising from the Circular Motion, will sustain and take away one Part from the 289 Parts; and the Weight 288 in the other Tube will sustain the rest of the Parts. For the Thing is not only true in the Surface of the Earth, but in all the Parts of both the Tubes, because the Centrifugal Force,

Force, and the Gravity of the inferior Parts, as taken every where at proportional Distances from the Center, are diminish'd in the same Proportion in the Progress to the Center. And then, by continuing the Calculation, the Gravity towards the Earth in the Place Q will be to the Gravity in the Place A, as 501 is to 500; and the Centrifugal Force $\frac{1}{289}$ will make that the Excess of Altitude in the Leg A C c a, should be a $\frac{1}{687} = \frac{1}{229}$th Part of the Altitude in the other Leg Q C c q; or in our Earth, that the Semi-diameter of the Earth at the Equator, should exceed the Semi-axis or Semi-diameter at the Poles by about 17 $\frac{1}{6}$ Miles. These Things, I say, will be thus, in Case that the Earth consists of an uniform Matter. For if the Matter at the Center be more dense, as certainly it ought to be, than at the Surface; the Excess of Altitude at the Equator must be something greater: because that if the redundant Matter at the Center, whereby the Density is made greater, be subducted and considered separately; the Gravity towards the rest of the Earth uniformly dense, will be reciprocally as the Distance of the Weight from the Center; but the Gravitation towards the same redundant Matter, will be reciprocally as the Square of the Distance from that Matter nearly. Therefore the Gravity under the Equator, which is towards that redundant Matter, will be less than the Gravity was towards the Place of that Matter by the foregoing Calculation; and therefore the Earth there, by Reason of the Defect of Gravity, will ascend something higher than was defin'd above. But now the *French* have found by Experiments, that the Length of Pendulums performing their Vibrations in one Second of Time towards the Equator, is less than that in which they perform

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the same towards the Poles in a greater Proportion than the foregoing Calculation requires. And therefore the Earth seems to be much higher at the Equator, than the foregoing Computation makes it to be, and indeed no less than 31 ½ Miles: and accordingly to be denser at the Center than in Mines near the Surface, as Reason altogether requires.

Corol. (1.) If the Excess of Gravity in the Parts about the Poles, above that which is in the Equatoreal Parts, were once more exactly defin'd by more accurate Experiments made to that Purpose, we should at length have an universal Measure determin'd; that, to wit, which would exactly define the due Length of a Pendulum for Seconds, in the several Places which lie betwixt the Equator and the Poles. From whence, as well an Equation of Time, which is now indicated by equal Pendulums in divers Places, as the Proportion of the Semi-diameters of the Earth, and of the Density of the same at the Center, so that the same be suppos'd to increase uniformly, will become known.

Corol. (2.) Since the Proportion is the same in a Canal full of Water, as in one fill'd with any other Fluid; and the same also as in the Earth, which is suppos'd to be fluid within; while in the mean time in a solid Earth the thing is otherwise; since also it is known by Experiments and Observations, that the Earth is indeed higher at the Equator than at the Poles: from thence it is manifest, that either the whole Earth was fluid, when its diurnal Motion first began; or at least that it contain'd a great Fluid within, which, by yielding, might give place to the Elevation at the Equator, and Depression at the Poles.

Corol. (3.) If the diurnal Motion of the Earth should

should be gradually retarded, unless it contain'd within it some great Fluid, which would give way to the Change of its Figure; the Seas would descend towards the Poles, and overflow all there.

Carol. (4.) If the diurnal Motion of a Planet, of a greater or lesser Magnitude, but of a given Density, be accelerated or retarded in any Proportion whatever, the Centrifugal Force will be increas'd or diminish'd from thence in the duplicate Proportion thereof; because of the Increase or Diminution, both of the Curvature and the Velocity in the same Proportion; and therefore the Difference of Semi-diameters will be increas'd or diminish'd in that same duplicate Proportion. But if the Density be increas'd or diminish'd in any Proportion whatsoever, because the Gravity is increas'd or diminish'd in the same Proportion, the Difference of Semi-diameters will be increas'd or diminish'd in that same Proportion also: That is, the Difference of Semi-diameters will be in a Proportion compounded of the duplicate Proportion of the periodic Times, and the simple Proportion of the Density, both reciprocal. From whence, since the Difference of Semi-diameters in the Earth is $\frac{2}{687}$ Parts of the whole Semi-diameter; and the Square of the periodic Time in *Jupiter*, which periodic Time is $9^h. 56'$, is to the Square of $24^h.$ the periodic Time in the Earth, as 5 to 29; and the Density of *Jupiter* is to the Density of the Earth, as 1 is to 5; the Difference of the Semi-diameters in *Jupiter* will be to the Difference of the Semi-diameters in the Earth, as $\frac{29}{5}$ in $\frac{5}{1}$ in $\frac{1}{229}$ is to 1, or as One is to Eight. Therefore the Semi-diameter of the Equator of of the Equator of *Jupiter*, is to the Semi-axis as 9 to 8. From whence, by the way, it is no wonder that so great a Difference should lie open to Astronomical Observation. But it is to be remark'd, that these Things are thus, where the Density of the Planet is uniform. But if the Matter of *Jupiter* be denser at the Center than at the Circumference, as it was before observ'd in general; the Difference of Semi-diameters will be greater still, and more easy to be observ'd. Let the Astronomical Observers therefore take notice how far this Corollary agrees with the Diameters of *Jupiter*, which are measur'd by the Micrometer.

XCIV. The Increase of Weight in going forwards from the Equator to the Poles, is very near as the Square of the right Sine of Latitude; or, which is the same, as the versed Sines of Latitude themselves.

Because the Weights of the unequal Legs of the Canal of Water ACQ qca are equal, and do poize one another; and the Weights of the Parts, like or similar to the whole Legs, and which are alike situated, are to one another as the Weights of the Wholes, and consequently are equal betwixt themselves; the Weights of the Parts, which are equal, and alike situated in the Legs will be reciprocally as the Legs: that is, reciprocally as the Distances of the Bodies from the Center of the Earth. And the thing is the same in all homogeneous and equal Bodies whatsoever, which are alike situated in the Legs of the Canal. Bodies placed in the uppermost Parts of the Canals, or in the Surface of the Earth, will have their Weights in that Proportion to one another reciprocally, as their Distances from the Center are: And the same is to be said of Weights, in any other Regions whatever, through the whole Surface of the Earth. And the Increase of Weight in the Earth, which is a Spheroidal Oblate Body, as the Famous Dr. *Gregory* hath demonstrated (*Astron.* Book III. Prop. 52.) is as the Square of the right Sine of the Latitude of the Place, or, which comes to the same, as the versed Sine of Latitude nearly.

Coroll. Since therefore Dr. *Gregory* hath demonstrated in the same Place, that the Longitudes of Pendulums vibrating in equal Time, are betwixt themselves as the Distances from the Center of the Earth reciprocally; the Difference of the Length of Pendulums will be as the Square of the right Sine of Latitude: And thus every where.

XCV. The unequal Motions of the Satellites of *Jupiter* and *Saturn* are plainly like and analogous to the unequal Motions of the Moon, and arise from like Causes.

I mean the Motion of the Nodes in *Antecedentia*, and of the Apses sometimes in *Antecedentia*, but more slowly, and sometimes in *Consequentia* more swiftly, and by the Excess of this latter Motion their being mov'd in *Consequentia* upon the Whole; the Motion of Variation, and the rest of the like Motions, must be the same in these Secondary Planets as in the Moon, and therefore do not require to be distinctly handled. It is true, that by Reason of the Smallness of these Inequalities, and Slowness of these Motions in the other Secondaries, their Motions appear very regular, when compared with the Motions of the Moon; which hath made some of the later Astronomers to deny all Motion to the Nodes of those other Secondaries. Nevertheless, Mr. *Flamsteed*, in conferring his Observations with

those

those of Mr. *Cassini*, hath found that the Nodes of those about *Jupiter* do indeed go back though more slowly; and it is not to be doubted, but that Time will more certainly and exactly discover and determine the same, and all the other mention'd Inequalities in the Satellites, both of *Jupiter* and *Saturn*.

May 31. 1708.

Lect. XXXVII.

XCVI. THE Flux and Reflux of the Sea arises from the Gravitation of the Water towards the Sun and Moon, or the Attractions of those Luminaries.

That the Sea in the Space of every Day, as well Lunar as Solar, ought to swell twice, and twice to settle and fall-back, is manifest from what hath been demonstrated above. But that the greatest heighth of the Water doth not fall just at the Appulse of the Luminaries to the Meridian, but follows the same by the Space of about three Hours, is what we shall now undertake to explicate. That the thing is indeed so, appears from the Observations of the Tides, as well as in the *Atlantic* Ocean, and the whole *Eastern* Tract of the *Ethiopic* betwixt *France* and the *Cape of Good Hope*, as upon the Coast of the *Pacific* along *Chili* and *Peru*. In all which Shores, the High-Water falls about three Hours after the Time aforesaid; unless

unleſs it be where the Motion is not retarded by its being propagated through Shallows. Now the Reaſon is this: When the Luminary is in the Meridian, the attracting Force is then certainly at the greateſt, but the Effect of that Force is not yet come to its Height. For all impreſs'd Motion perſeveres uniformly until a contrary Motion deſtroys, or at leaſt retards it. The Flux of the Sea, or Ocean rather, which for the Six Morning Hours, if we may ſo call them when we ſpeak of the Moon, is continually increas'd; and by its conſpiring with the diurnal Motion, accelerated; ought, by reaſon of this its greater Celerity, to go forwards ſtill farther, and to accumulate the Waters more and more, until the ſame Force, by tending afterwards contrary to the diurnal Motion, doth by degrees retard the Courſe of that Motion which is going forwards; and by and by to make the ſame Waters to proceed with ſo ſlow a Motion, that a Reflux of the Ocean follows: Which Retardation of the Motion ought to be moſt notable about the Octants, or the third Hour. Examples of ſuch like greateſt Effects, as following ſome ſpace of Time after their greateſt Cauſes, we have yearly in the greateſt Heat of the Summer, and Cold of the Winter; which falls not in the Solſtices themſelves, but about the Octants, if I may ſo ſpeak, about a Month and an half after; and in every Summer-day, in the greateſt Heat of the Day, which happens an Hour or two after Noon, rather than at the Noon it ſelf. So in the preſent Caſe, whilſt after the greateſt Force of all, and that raiſing of the Waters which is thereby, Forces next to the greateſt, and not yet turned to the contrary Part, do ſtill operate; the Forces which are leſs than the greateſt, being ſuper-added to the Motions which

were

were stirr'd up by the greatest, and go forward by their own proper Tendency, must needs obtain a greater Effect, than Forces still increasing, super-added to lesser Motions, could have. Then it is to be noted also, that the attracting Force it self, which lifts the Water directly upwards, doth scarce sensibly fall short of its greatest Quantity for an Hour or two after the Appulse of the Luminary to the Meridian, altho' the Direction of the Attraction which accelerates or retards the Waters, be directly upwards in the Meridian it self, and from thence is chang'd. The Waters therefore will be most accumulated, where the Parts, which have just now pass'd the Meridian with the greatest Velocity, do fall upon other Parts which had before been retarded at the Quadrature; and so by conspiring with the Endeavour of the other, do make the greatest Flood of all: which happens about the third Hour. For in this place we intend not so much the vulgar Hours, as those which are reckon'd from the Appulse of the Sun and Moon to the Meridian of the Place, as well below as above the Horizon; and by the Hours of a Lunar Day, we understand 24 Parts of that Time in which the Moon, by its apparent diurnal Motion, is revolv'd to the Meridian of any Place.

XCVII. The Tides which depend on the Force of the Sun, and on the Force of the Moon severally, do not make a double Tide, but a single one; which is to be estimated from the Conjunction of their Forces.

For like as any Body whatever, which is impress'd by a double Force, cannot go forwards in two Lines, but from the Conjunction of the Forces will proceed in the Diagonal of a Parallelogram, in the same manner as if it had been acted upon by one single Force, according to the Direction of the Diagonal; so those two Motions, which the two Luminaries do excite respectively, will not appear severally, but will make one mix'd Motion. In the Conjunction and Opposition of the Luminaries, their Effects will be conjoin'd; and the greatest Floods of all will be made, as arising from the Sum of the Forces at that Time. In the Quadratures of those Luminaries, the Sun will lift up the Water, whilst the Moon depresseth it; and depress it, where the Moon lifts it up: and there the Flood will be the least of all, as being the Result of the Difference of the Forces only. And because, as it appears by Experience, the Force and Effect of the Moon in the present Case is much greater than that of the Sun, the greatest Heighth of the Water will fall upon the third Lunar Hour. But without the Syzygies and Quadratures, the greatest Flood of all, which by the Lunar Force alone ought to fall in the third Lunar Hour, and by the Solar Force alone in the third Solar Hour, will, by the Composition of the Forces, fall upon some intermediate Time, which will be much nearer to the third Lunar Hour, than to the third Solar one; and consequently, in the passing of the Moon from the Syzygies to the Quadratures, at which Time the third Solar Hour goes before the third Lunar, the greatest Height of the Water will also precede the third Lunar Hour; and this by the greatest Interval of all, a little after the Octants of the Moon. And the greatest Heighth of the Water will follow the third Lunar Hour by the like Intervals, whilst the Moon passeth from the Quadratures to the Syzygies; and this also by the greatest Interval, a little after the Octants of the

Moon.

Moon. Thus will the thing be in the Ocean, or open Sea: For in the Mouths of Rivers, the greater Floods, *cæteris paribus*, require the longer Time, and so come unto their Heighth a little more slowly.

XCVIII. The Tide ought to be different, according to the different Distances of the Luminaries from the Earth, both every Year and every Month; and this in the triplicate reciprocal Proportion of those Distances, or in the triplicate direct Proportion of the apparent Diameters.

This we have demonstrated before: Nor is it to be wondred at, that these Effects should be greater at less Distances, and lesser at greater. Wherefore the Sun in Winter-Time, when it is about the Perigee, will make the Tides after the Syzygies to be something greater, because of the greater Sum of the Forces; and those after the Quadratures to be something less, because of the Difference of the Forces, than they will be in Summer-Time; *cæteris paribus*. And the Moon every Month, when it is about the Perigee, will make greater Tides than 15 Days before and after, when it is in the Apogee. From whence, if the Perigee Situation of the Moon happens about the Conjunction, the Day-Flood will be increas'd, and the Night-Flood diminish'd; but if that Situation happens about the Opposition, the Night-Flood will be increas'd, and the Day-Flood diminish'd. From whence also it comes to pass, that two Tides the greatest of all do not follow one another two Syzygies together. For if the Moon be in one of the Syzygies about the Perigee, and raiseth the greatest Tide at that Time, by the Conjunction of its Force with that of the Sun; in the other of the Syzygies it must needs be about the Apogee, and have less Force.

XCIX. The

XCIX. The Tides likewise ought to be divers, according to the divers Declination of the Luminaries from the Equator.

For if the Luminary were placed in either of the Poles, it would draw the Water constantly without Intention or Remission of the Action; and consequently would make no *Reciprocation* of the Water. Therefore the Luminaries, in departing from the Equator towards either Pole, will by degrees lose their Effects; and therefore will raise lesser Tides after the Solsticial Syzygies, than after the Equinoctial. But after the Solsticial Quadratures, the Tides will become greater than after the Equinoctial; because the Effect of the Moon, which is now placed above the Equator, doth most of all exceed the Effect of the Sun. Therefore the greatest Tides fall after the Equinoctial Syzygies, and the least after the same Quadratures of the Luminaries; and the greatest Flood about the Syzygies is always attended with the least about the Quadratures, as Experience testifies. But by the lesser Distance of the Sun from the Earth in Winter Time than in Summer, it comes to pass, that the greatest Tides and the least do oftner precede the Vernal Equinox, than follow it; and do oftner follow the Autumnal Equinox, than precede it.

C. Some Phænomena of the Tides, and Effects of the Luminaries, are divers, according to the divers Latitude of Places in the Earth; and especially as to the Night and Day-Floods, which follow one another immediately.

In *Fig.* 2. *Plate* 9. let A p E P design the Earth, covered on every side with deep Water. Let C be the Center thereof. p P the Poles. A E the Equator. F any Place without the Equator. F f the Parallel of the Place. D d the correspondent Parallel on the other side of the Equator. H that Place of the Earth which is directly under the Moon's Place, which was Three Hours before, or the middle Point of the elevated Water. h the Place opposite thereto, or the Point of the Water in the other Part of the Earth, where the Water is most elevated. K k, Places distant 90 Degrees from thence. C H, C h the greatest Altitudes of the Sea, measured from the Center of the Earth; and C K, C k the least Elevations. And if from the Axes H h, K k an Ellipsis be describ'd; and then by the Revolution of this Ellipsis about the greater Axis H h, there be describ'd a Spheroid H P K h p k; this will describe the Figure of the Sea nearly: and C F, C f; C D, C d will be the Elevations of the Sea in the Places F f and D d. Moreover, if in the foresaid Revolution of the Ellipsis, any Point whatever, as N describes the Circle N M, which cuts the Parallels F f D d in any Places, as R, T, and the Equator A E in S, C N will be the Heighth of the Sea in all the Places R, S, T situate in this Circle. Hence, in the diurnal Revolution of any Place whatever, as F, the Flood will be the greatest there, three Hours after the Appulse of the Moon to the Meridian above the Horizon; afterwards the Ebb will be the greatest in Q, three Hours after the setting of the Moon; then the Flux will be the greatest in f, three Hours after the Appulse of the Moon to the Meridian below the Horizon; and lastly, the Ebb will be the greatest in Q, three Hours after the rising of the Moon; and the latter Flood in f, will be less than the former Flood in F. For the whole Ocean is distinguish'd into two Hemispherical Floods; one in the Hemisphere K H k C, which looks to the *North*; the other in the opposite Hemisphere

K h k c,

K h k c, which looks to the *South*; which therefore we may call the *Northern* and *Southern* Floods. These Floods, which are opposite each to other, come by turns to the Meridian of each Place, with the Interval of about 12 Lunar Hours betwixt. For since the *Northern* Regions do more partake of the *Northern*, and the *Southern* Regions do more partake of the *Southern* Flood; from thence there proceed Tides alternately greater and less in each Place without the Equator. But the greater Tide, when the Moon declines towards the Vertex of the Place, will fall about three a-Clock after the Appulse of the Moon unto the Meridian above the Horizon; and the Flood, when the Moon changeth its Declination, and recedes from the Vertex, will be changed into a less: And the greatest Difference of Floods will, for this Reason, fall upon the Times of the Solstices, especially if the Moon's ascending Node be in the Beginning of Aries; that so the Moon, when it is nearest to the Vertex, and the remotest from it, may have the same diurnal Revolution. And this is confirm'd from Experience; by which it is found, that the Morning Tides do in Winter-Time exceed the Evening; and in Summer the Evening exceed the Morning Tides: At *Plimouth*, for Instance, by the Heighth of one Foot, and at *Bristol* of fifteen Inches; as appears from the Observations of Mr. *Colepress* and Mr. *Sturmy*. But that these Differences do not seem so great as might be expected in Places so remote from the Equator, may be owing to some other Cause. The Motions describ'd hitherto are something chang'd by that Force of the Reciprocation of the Waters, wherewith the Tide, even though the Actions of the Luminaries should cease, might endure for some Time. This Conservation of the Motion once impress'd, doth diminish the Difference of the alternate Tides; and makes the Tides next after the Syzygies greater, and diminishes those next after the Quadratures. For from hence it comes to pass, that the alternate Tides at *Plimouth* and *Bristol* do not differ much more than by the Heighth of 12 or 15 Inches; and that the greatest Tides of all in the same Ports, are not those which are next after the Syzygies, but the third Tides after them; which agrees exactly with what was said before. For all these Motions are retarded, in their passing thro' Shallows; so that the greatest Tides of all in some Streights, and the Mouths of some Rivers, are the fourth or even the fifth after the Syzygies.

Nov. 8. 1708.

LECT. XXXVIII.

CI. THE Phænomena of the Flux and Reflux of the Ocean in particular Places, as Streights, Ports, Mouths of Rivers, small Seas, and which communicate little or not at all with the Ocean; in those also which are far distant from the Equator, do recede more than a little from the general Laws of the Tide before set down, and are commonly altered by those particular Circumstances.

As for Example; it may come to pass, that the Tide may be propagated from the Ocean thro' divers

vers Streights, and quicker through ſome than others; in which Caſe, the ſame Tide being divided into two or more which come ſucceſſively, may make new Motions of divers Kinds. It may come to paſs alſo through the Length of the Way, or the various Winding of the ſame, or by means of Obſtacles which are in the Way, that the Tide may be diminiſh'd and almoſt ſtopp'd. (From whence it comes, that where there be a great Number of Iſlands, as the *Moluccoes*, the *Philippines*, in the *Mexican-Bay*, the *Antillæ*, there is almoſt no Flux, or far leſs than in the wide and open Ocean.) Again, a Tide which is in a mean State in the Ocean may become very great in Rivers, becauſe of the Narrowneſs of the Paſſages, and the Heighth of the Shores. In ſmall Seas alſo, there is none, or a very ſmall Tide. For ſince the greateſt Tide ought to happen in the deep Ocean only, which is open to the *Eaſt* and *Weſt* for the Space of 90 Degrees; by how much leſs the Sea is, ſo much the leſs the Acceleration and Retardation of the Waters, that is, the Flux and Reflux, muſt needs be; Nor can it be great, unleſs the Sea doth communicate freely with the Ocean. For if it communicateth not, or but little therewith, as it is in the *Mediterranean*, a leſs Tide for that Reaſon is to be expected. In thoſe Seas alſo which are remov'd far from the Equator, where the Tide muſt be propagated in a leſs degree, eſpecially if they have but little Communication with the Ocean; and ſo will be but ſmall, as it comes to paſs in the *Baltic* and the *Northern* Seas. Which happens alſo in the *Euxine* and *Caſpian* Seas, not only by reaſon of their ſomething *Northerly* Situation, and their ſmall Communication with the Ocean, if they have any at all, but by reaſon alſo of the Smalneſs of thoſe Seas. In

Seas which lie open, and extend themselves a great way from the *East* to the *West*, as in the *Pacific* Sea, and the Parts of the *Atlantic* and *Ethiopic* Sea without the Tropics, the Water is wont to be elevated unto the Heighth of 6, 9, 12, or 15 Feet. And in the *Pacific* Sea, which is deeper and wider, the Tides are said to be greater than in the *Atlantic* and *Ethiopic.* In the *Ethiopic* Sea, the Elevation of the Water betwixt the Tropicks is less than that in the Temperate Zones, by reason of the Narrowness of the Sea betwixt *Africa*, and the *Southern* Part of *America.* In the Middle of the Sea the Water cannot ascend, but it must descend at the same time to both Shores, the *Eastern* and *Western*; when yet in our narrow Seas it ought to descend by turns unto the Shores. For this Reason the Flux and Reflux must be very small in Islands, which are very remote from the Continent. In some Ports, as hath been very lately observ'd, where the Water passing thro' shallow Places is forc'd to flow in and out with a great Violence, to fill and empty by turns narrow Bays, the Tides are greater than usual; as at *Plimouth* and *Chepstow-Bridge* in *England*, the Hills of St. *Michael* and the City of *Avranches* in *Normandy*, *Cambaia* and *Pegu* in the *East-Indies.* In these Places the Sea coming and going back with a great Velocity, sometimes overflows the Shoars, and then leaves them dry for many Miles. Nor can the Force of Flowing in and Reflowing be stopp'd, until the Water be elevated or depress'd 30, 40, 50, or sometimes 60 Feet. And the thing is the same, in some measure, in long, shallow, and narrow Streights, as the *Magellanic* and that wherewith *England* is encompass'd. But in Shores which have a steep Descent towards a deep and open Sea, where the Water may be rais'd and

settle

settle freely without any accessional Force of flowing in and returning; the Tide, if we would determine the general mean Quantity, is to be reckon'd to arise to the Heighth of about 12 Feet, *i. e.* if we measure from the Low to the High-Water Mark. But of all Sea-Tides, that is the most to be admir'd, which Dr. *Halley* speaks of from the Observations of Mariners, as being in a Port of the Kingdom of *Tunquin* at *Batsham*, in the *Northern* Latitude of 20°. 50′. There the Water, on the Day following the Passage of the Moon over the Equator, stagnates; then the Moon declining to the *North*, it begins to ebb and flow, not twice as in other Ports, but once only in a Day; and the High-Water falls at the setting of the Moon, and the Low-Water at the rising of the same; and this Tide is increas'd with the Declination of the Moon until the seventh or eighth Day; and for the other seven Days, it decreases by the same Degrees by which it increased before; and the Moon changing its Declination, it ceaseth; and from thence is presently changed into a Reflux. For then the Reflux falls at the setting of the Moon, and the Flux at the rising, until this Planet doth again change its Declination. There is a double Entrance into this Port, and the neighbouring Streights; the one from the *Chinese* Ocean, betwixt the Continent and the *Leuconian* Island; the other from the *Indian* Sea, betwixt the Continentand the Isle of *Borneo*. It seems probable, that two almost equal Tides do come into this Port from the different Tides of this Ocean; the former of which precedes the other by the Space of almost six Hours, and falls 3 Hours after the Appulse of the Moon to the Meridian of the Port. When the Moon in this its Appulse to the Meridian is in the Equator, there will come at each six Hours End equal Fluxes, which falling upon mutual Refluxes will make the ſame equal to the Fluxes; and ſo will cauſe that for that Day the Water will ſeem to be moved with no Tide at all. When the Moon declines from the Equator, the Tides in the Ocean will become by turns greater and leſſer, as we ſhew'd in the laſt Propoſition but one; and from thence two greater Fluxes, and two leſſer ones, will be propagated into this Port by turns. But the two greater Fluxes, by joining their Waters, will make the higheſt Flux in the middle Time betwixt both; the greater Afflux and the leſs will make that the Water ſhould aſcend unto a mean Heighth in the middle Time betwixt them; and betwixt the two leſſer Fluxes, the Water will aſcend unto the leaſt Altitude. Thus, in the Space of 24 Lunar Hours, the Water will come not twice, as it is in other Places, but once only unto its greateſt Altitude, and once unto its leaſt; and the greateſt Altitude, when the Moon declines to the Pole which is above the Horizon of the Place, will fall ſix Hours after the Appulſe of the Moon to the Meridian of the Place; and the Moon changing its Declination, it will be chang'd into a Reflux. Therefore one Tide coming in the Space of 12 Hours from the *Indian* Ocean, and the other in the Space of 6 Hours from the *Chineſe* Ocean through thoſe Streights reſpectively, which were before-mentioned; and ſo falling one at the third, and the other at the ninth Lunar Hour, ſeem to make thoſe anomalous Tides. But theſe and ſuch like particular Phænomena are every where to be left to the Obſervations of the neighbouring Shores and Seas.

Scholium. If we would decline the Intricacy and Tedioufneſs of our Author's Calculation, and deſire only to know the Quantities of the Forces, they

they are thus: The Sum of the Sun's Forces, as well in depressing the Waters in the Places which are 90 Degrees from it, as in elevating them in the Places which are under it, and those opposite thereto, if they be taken conjunctly; or the whole Force of the Sun to move the Sea is to the Force of Gravity with us, as 1 is to 12,868,200. But since the Centrifugal Force of the Parts of the Earth arising from its diurnal Motion, which is to the Force of Gravity as 1 to 289, doth make that the Heighth of the Water under the Equator should exceed its Heighth under the Poles, by the Measure of 85820 Feet of *Paris*: The Solar Force of which we now treat, since it is to the Force of Gravity as 1 to 12,868,200, and consequently to that Centrifugal Force as 289 to 12,868,200, or as 1 to 44,527; it will make, that the Heighth of the Water in the Places under the Sun, and opposite therto, should exceed the Altitude of it in the Places which are 90 Degrees distant from the Sun, by the measure only of one Foot of *Paris*, and a little above 11 Inches; according to this Analogy 44,527 : 1 :: 85,820 : $1\frac{11}{13}$. and $\frac{1}{8}$ of an Inch. Now the Force of the Moon for the moving of the Sea, which is the principal Force, is to be deduced from the Proportion which it bears to that of the Sun, and to be distinguish'd by the Effects or *Sums* of the Motions in the Syzygies, and the *Differences* in the Quadratures: By this Computation, the Force of the Moon is to the Force of the Sun, when Observations are compar'd together, as 4,48 to 1 nearly, or in a round Number almost five-fold.

Coroll. (1.) Since therefore, as we have seen before, the Sun's Force ought to elevate the Water unto the Heighth of almost two Feet, the Moon's Force, which is almost five times as great, ought to elevate the Water unto the Heighth

Heighth of about 9 Feet; and the Lunar Force and the Sun's conjoin'd, as in the Syzygies, will elevate the ſame unto near 11; and when the Sun's Force is ſubſtracted from the Moon's, as in the Quadratures, will elevate it about 7 Feet. Now this Force doth abundantly ſuffice to cauſe all the Motions of the Sea, and doth very well agree with the Quantity of the Motions defin'd above; and by anſwering ſo well to the ſame, doth plainly confirm the Truth of that Cauſe of the Tides which we have aſſign'd.

Corol. (2.) Since the Force of the Moon to move the Sea is to the Force of Gravity, according to what hath been demonſtrated before, only as 1 is to 2871400; it is manifeſt, that that Force is far leſs than to be perceiv'd in any Experiments of Pendulums, or in any Static, or Hydroſtatic Experiments whatſoever. This Force can have a ſenſible Effect in the Sea only.

Corol. (3.) Foraſmuch as the Force of the Moon to move the Sea, is to the Sun's Force upon the ſame as near 5 to 1; and thoſe Forces are as the Denſities of the Bodies, or the Quantities of Matter contain'd in equal Space, and as the Cubes of the Diſtances or Diameters conjunctly; for Bodies equally denſe are as the Cubes of the true Diameters directly, in reſpect of the ſame Diſtance; and the moving Forces in this Caſe are alſo as the Cubes of the Diſtances reciprocally, or as the Cubes of the apparent Diameters directly; and conſequently it is the ſame thing whether the Sun be nearer or more remote, greater or leſs, ſo that the apparent Diameter be certain and determinate: For theſe Reaſons the Denſity of the Moon will be to that of the Sun, as is its Effect, or as 4[48 to 1; and as the Cube of the apparent Diameter of the Moon is to the Cube of the Sun's appa-

apparent Diameter, *i.e.* as 4,48 to 1; and as 720 to 672 conjunctly $= 4,48 \times 720$ to 100×672, or as 67 to 32 almost; but the Density of the Sun to the Density of the Earth, is as 100 to 396. Therefore the Density of the Moon to the Density of the Earth, will be as 21 to 17, nearly, or almost as 5 to 4. Therefore the Body of the Moon is considerably Denser; and if I may use such an Expression, more Terrestrial than the Earth itself, as we observ'd before, by way of Anticipation.

Coroll. (4.) From whence, since the true Diameter of the Moon is to that of the Earth, as 5 to 18, or as 1 to 3,65; the Mass of the Moon will be to that of the Earth, as the Cubes of those Numbers, compounded with the Proportion of Density, or as 1×5 to 49×4; that is, as 1 to 40, very near.

Coroll. (5.) The Accelerating Gravity, or the Weight of equal Bodies on the Surface of the Moon, will be as the Quantity of Matter in the Moon, to the Quantity of Matter in the Earth, with the Reciprocal Duplicate Proportion of the Distance from the Centers compounded; that is, as 1×13 is to 40×1, or near a third Part of the Accelerating Gravity on the Surface of the Earth, as we noted formerly by way of Anticipation.

CII. The Figure of the Body of the Moon (abstracting from the Elevation of the Equatoreal, and the Depression of the Polar Parts, depending upon the Diurnal Motion,) is something Oval, or that of an Oblong Spheroid; the greatest Axis whereof produced, passeth thro' the Center of the Earth; and exceeds the Lesser Axes Perpendicular to the same by the Excess of about 187 Feet. If then the Body of the Moon were Fluid like our Sea, the Force of the Earth to elevate up that Fluid in the hither and opposite Parts, would be to the Force of the Moon upon our Sea, as the attracting Force of the Earth is to that of the Moon; Or as the Quantity of Matter in the Earth is to that in the Moon, by reason of the equal Distances; if the Lesser Diameter of the Moon did not change that Proportion. That whole Force therefore from the Composition of those Proportions, will be in a Proportion compounded of 40 to 1, and 1 to 3|65; or as 40 × 1 is to 1 × 3|65; that is, as 40 is to 3|65. From whence, since by what was before demonstrated, our Sea is lifted up about 9 Feet by the Force of the Moon, the Lunar Fluid ought to be lifted up about 93 Feet by the Force of the Earth. And for this Cause the Moon is of a Spheroidal Figure; the greater Axis whereof being produced, would pass thro' the Center of the Earth, and exceed the Diameters or Perpendicular Axes by about 187 Feet.

Corollary. And from thence perchance it is, that the same Face of the Moon is turn'd more directly to the Earth than otherwise it would be. For the Moon cannot Rest in another Situation, but by Librating to and fro, will always return to this Situation. Nevertheless, the Librations, by reason of the Smallness of the Force in such a small Excess of the Greater Axis above the Lesser ones, will be exceeding Slow; so that the Face which ought always to look to the Earth, may look to the Other Focus of the Lunar Orbit, by reason of the Equability of the Angular Motion about it, as was explain'd before, and not presently be drawn back from thence and turned to the Earth.

CIII. Co-

CIII. Comets are higher than the Moon; and are moved in the Region of the Primary Planets.

CIV. Comets are mov'd in Conic Sections, having their Focus in the Center of the Sun; and by Rays drawn to the Sun, describe equal Area's in equal Times, and in general Area's proportional to the Times.

CV. The Bodies of Comets are Solid, Compact, Fixed, and Durable, like the Bodies of the Planets; and they are commonly encompass'd with huge Atmospheres; and do always acquire Tails from their Neighbourhood to the Sun; but these sometimes longer, and sometimes shorter.

These Propositions contain our famous Author's Cometography, so far as concerns our present Purpose.

Now they are propounded so clearly and fully by our Author himself, that they in no wise need our Explication. Wherefore what follows, we shall take Word for Word out of him.

Novemb. 15. 1708.

Lect. XXXVII.

THAT Comets are higher than the Moon, and shew themselves to us in the Region of the Planets.

As the Want of a Diurnal Parallax hath raised Comets above the Sublunary Regions, so their Descent into the Planetary Regions, is argued from their Annual Parallax. For those Comets which go forwards according to the Order of the Signs, are all of them, about the Time of their Disappearance, either Slower than usual, or Retrograde, if the Earth be betwixt them and the Sun; but swifter than Ordinary, if the Earth tends towards the Opposition. And on the Contrary, they which go contrary to the Order of the Signs, are swifter than usual at the Time of their Disappearance, if the Earth be betwixt them and the Sun; and slower than ordinary, or Retrograde, if the Earth be placed on the Contrary side. This happens especially from the Motion of the Earth in its various Situation; and it is here, as it is in the Planets, which according as the Motion of the Earth conspires with them, or is contrary to them, are sometimes Retrograde, sometimes seem to be mov'd more slowly, sometimes more swiftly. If the Earth goes to the same Part with the Comet, and is carried about the Sun with an Angular Motion more swiftly, the Comet as seen from the Earth, by reason of its slower Motion, appears Retrograde; but if the Earth be carried

more

more slowly, the Motion of the Comet (that of the Earth being substracted) becomes at least slower. And if the Earth be carried to the contrary Part, the Comet from thence appears swifter. Now from this Acceleration, or Retardation, or Retrograde Motion, the Distance of the Comet is thus Collected.

In *Fig.* 3. *Plate* 9. let ♈ Q A, ♈ Q B, ♈ Q C be Three observ'd Longitudes of the Comet about the Time of the Beginning of its Motion; and let ♈ Q F be the Longitude last observ'd, when the Comet begins to disappear. Let the Right Line A B C be drawn, the Parts whereof A B, and B C, which lie betwixt the Right Lines Q A, and Q B, Q B, and Q C, are one to the other, as the Times betwixt the Three first Observations. Let A C be produc'd to G, that A G may be to A B, as the Time betwixt the first Observation and the last, is to the Time betwixt the first Observation and the second; and let Q G be join'd. Now if the Comet were mov'd uniformly in a Right Line, and the Earth either rested or went forward uniformly in a Right Line, the Angle ♈ Q G would be the Longitude of the Comet at the Time of the last Observation; the Angle F Q G therefore, which is the Difference of the Longitudes, arises from the Inequality of the Motions of the Comet and the Earth. But this Angle, if the Earth and the Comet be moved to contrary Parts, is added to the Angle A Q G, and so renders the apparent Motion of the Comet swifter; but if the Comet goes to the same Part with the Earth, it is substracted from the same, and renders the Motion of the Comet either Slower, or perhaps Retrograde, as was said above.

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This Angle therefore arises chiefly from the Motion of the Earth, and is justly to be reckoned for the Parallax of the Comet: some Increase or Decrease of it, to wit, which may arise from the Comet's uneven Motion in its own Orb, being here neglected. But the Distance of the Comet is thus Collected from the Parallax.

In *Fig.* 4. *Plate* 9. let S represent the Sun, *a c* T the *Orbis Magnus*, *a* the Place of the Earth in the first Observation, *c* the Place of the Earth in the second Observation, T the Place thereof in the last Observation, and T ♈ a Right Line drawn towards the Beginning of *Aries*. Let the Angle ♈ T V be taken equal to the Angle ♈ Q F; that is, equal to the Longitude of the Comet when the Earth is in T. Let *a c* be join'd, and drawn out to *g*, that *a g* may be to *a c*, as A G is to A C, and *g* will be the Place which the Earth would reach unto at the Time of the last Observation, its Motion being continued uniformly in the Right Line *a c*; and therefore if *g* ♈ be drawn Parallel to T ♈, and the Angle ♈ *g* V be taken equal to ♈ Q G, this Angle ♈ *g* V will be equal to the Longitude of the Comet seen from the Place *g*, and the Angle T V *g* will be the Parallax which ariseth from the Transferring of the Earth out of the Place *g* into the Place T; and consequently V will be the Place of the Comet in the Plane of the Ecliptic. Now this Place V is wont to be below the Orb of *Jupiter*.

The same Thing is Collected from the Curvature of the Way of Comets. These Bodies go forward almost in great Circles, so long as they are mov'd more swiftly; but in the End of their Course, when that Part of the apparent Motion which ariseth from the Parallax bears a greater Proportion to the whole apparent Motion, they

they are wont to decline from these Circles; and as oft as the Earth is mov'd to one Part, to be carried to the contrary. This Deflexion arises from the Parallax, because that it answers to the Motion of the Earth; and the notable Quantity of it hath by my Computation placed Comets when they disappear far enough below the Orb of *Jupiter*. From whence it follows, that in their Perigees and Perihelia, at which Times they are nearer, they descend oftentimes below the Orbs of *Mars* and the inferior Planets.

The Nearness of Comets is also confirm'd from the Light of their Heads. For the Splendor of a Celestial Body which is illuminated by the Sun, and goes off into far distant Regions, is diminish'd in the Quadruplicate Proportion of the Distance; *i. e.* in one Duplicate Proportion, by reason of the increase of the Distance from the Sun; and also in another Duplicate Proportion, by reason of the Diminution of the apparent Diameter. From whence, if both the Quantity of Light, and the apparent Diameter of the Comet be given, the Distance also will be given, by saying that the Distance is to the Distance of a Planet in the entire Proportion of Diameter to Diameter directly, and in the subduplicate Proportion of Light to Light inversly. Thus the least Diameter of the *Capillitium* of the Comet of the Year 1682, being observ'd by the Famous Mr. *Flamsteed* thro' a Telescope of 16 Feet, and measur'd by a Micrometer, was 2'. 0". But the *Nucleus*, or Star it self, had scarce the 10th Part of this Breadth, as being only 11 or 12" over. But in the Light and Clearness of the Head, it exceeded the Head of the Comet of 1680, and even came near to Stars of the first and second Magnitude. Let us suppose *Saturn* with his Ring to be about 4 Times brighter than that Star; and because the Light of the Ring doth almost equal the Light of the Intermediate Globe, and the apparent Diamerer of the Globe is about 21″; and consequently the Light of the Ring and Globe together doth almost equal the Light of a Globe, the Diameter whereof is 30″; the Distance of the Comet will be to the Distance of *Saturn*, as 1 to $\sqrt{4}$ inversly, and 12″ to 30″ directly; *i. e.* as 24 to 30, or 4 to 5. Again, the Comet of the Year 1665, in the Month of *April*, as *Hevelius* writes, did in Clearness exceed almost all the Fixed Stars; yea, and *Saturn* it self in regard of the Colour, which was far more lively. For the Comet was more lucid than that other which had appear'd in the End of the foregoing Year, and was to be compared with Stars of the first Magnitude. The Breadth of the *Capillitium* was about 6′; but the *Nucleus*, as compared with the Planets, by means of a Telescope, was plainly less than *Jupiter*; and sometimes was judged to be less than the intermediate Body of *Saturn*, sometimes equal thereto. Moreover, seeing the Diameter of the *Capillitium* of Comets doth seldom exceed 8′ or 12″, and the Diameter of the *Nucleus* or Central Star is about a 10th, or perhaps a 15th Part of the Diameter of the *Capillitium*; It is manifest that these Stars are for the most part of the same apparent Magnitude with the Planets. From whence, since their Light may oftentimes be compar'd with that of *Saturn*, and sometimes doth exceed it; It is manifest all the Comets in their Perihelia are placed either beneath *Saturn*, or not far above it. They are widely mistaken therefore, who Remove these Stars into the Region of the Fixed Stars; where certainly they could no more be Illuminated by our Sun, than the Planets which are here, are Illuminated by the Fixed Stars.

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We have, whilst we have been Reasoning hitherto, not considered the Obscuration of Comets by that very copious and gross Fume, wherewith the Head is encompass'd, which always shines as through a Cloud. For the more obscure the Body is rendred by this Fume, so much the nearer it must needs approach to the Sun, that it may equal the Planets in the Plenty of Light reflected from it. From thence it seems probable, that Comets descend far below the Sphere of *Saturn*, as we have prov'd from their Parallax. But the same Thing is especially confirm'd from their Tails. These arise either from the Reflexion of the Fume dispers'd through the Ether, or from the Light of the Head. In the former Case the Distance of the Comets is to be diminish'd, lest the Fume which always ariseth from the Head, should be propagated with an incredible Velocity and Expansion through too large Spaces. In the latter all the Light, as well of the Tail as of the *Capillitium*, is to be referr'd to the *Nucleus* of the Head. Therefore if we imagine all this Light gathered together within the Disque of the *Nucleus*, the *Nucleus* would now certainly, as often as it sends forth a very great and shining Tail, much exceed *Jupiter* it self in Splendor. Since therefore it emits more Light, notwithstanding that it hath a much less Diameter, it must be much more Illuminated by the Sun, and consequently much nearer to the Sun. Moreover, the Heads which lie hid under the Sun, and do at that Time put forth very great and resplendent Tails, like Beams of Fire, as sometimes hath been seen, ought by the same Argument to be placed beneath the Sphere of *Venus*. For all that Light, if it be suppos'd to be gathered together into the Star, would sometimes exceed *Venus* it

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ſelf, that I may not ſay many *Venus's* in Splendor.

Laſtly, The ſame Thing is gathered from the Increaſe of the Light of the Head in the Receſs of Comets from the Earth towards the Sun, and Decreaſe of the ſame in their Receſs from the Sun towards the Earth. For the latter Comet of 1665, (as *Hevelius* obſerv'd) from what Time it begun to be ſeen; remitted always of its Motion, and conſequently had paſs'd the Perigee; but the Splendor of the Head daily increas'd, until the Comet, cover'd with the Solar Rays, ceaſed to appear. The Comet of Year 1683, as the ſame *Hevelius* obſerv'd, in the End of the Month *July*, when it was firſt ſeen, mov'd very ſlowly, making about 40′ or 45′ every Day in its Orb. From that Time its Motion was continually increas'd until *Septemb.* 4. at which Time it aroſe to about 5 Degrees. In this whole Time therefore it approach'd to the Earth. Which is alſo collected from the Diameter of the Head, meaſur'd by a Micrometer: which *Hevelius* found *Aug.* 6. to be only 6′. 5″, the Hair included; whereas *Sept.* 2. it was 9′. 7″. The Head therefore, in the Beginning of its Motion, appear'd far leſs than in the End; but nevertheleſs in the Beginning, in the Neighbourhood of the Sun, it was much brighter than about the End, as *Hevelius* relates. Therefore in all this Time, by reaſon of its Departure from the Sun, it decreas'd as to Light, notwithſtanding its Acceſs towards the Earth.

The Comet of the Year 1618, about the Middle of *December*; and that of 1680, about the End of the ſame Month, mov'd moſt ſwiftly; and conſequently they were then in their Perigees: But the greateſt Splendor of the Heads happened almoſt a Fortnight before, when they had juſt got out

out of the Rays of the Sun; and the greateſt Splendor of the Tails was a little before, when they were nearer to the Sun. The Head of the former Comet, according to the Obſervation of *Cyſatus*, *Decemb.* 1. ſeem'd greater than a Star of the firſt Magnitude; and *Decemb.* 16. (it being now in its Perigee,) it had fallen off a little in its Magnitude, but very much in its Splendor or Clearneſs of Light. *Jan.* 7. *Kepler* being uncertain of the Head, made an End of Obſerving. *December* 12. the Head of the latter Comet was ſeen and obſerv'd by Mr. *Flamſteed*, in the Diſtance of 9 Degrees from the Sun; which a Star of the 3d Magnitude ſcarce could have been. *Decemb.* 15. and 17. the ſame appear'd as a Star of the 3d Magnitude, being at the ſame time rendred leſs conſpicuous by the Brightneſs of the Clouds which were about the Sun-ſetting. *Decemb.* 26. It ing mov'd very ſwiftly, and being then almoſt in its Perigee, it was leſs than the Star call'd *Os Pegaſi*, one of the third Magnitude. *Jan.* 3. It appear'd as a Star of the fourth Magnitude. *Jan.* 9. as one of the 5th. *Jan.* 13. by reaſon of the Splendor of the Moon increaſing, it diſappear'd. *Jan.* 25. It ſcarce equall'd a Star of the ſeventh Magnitude. According to this, if we take equal Times from the Perigee on this Side and on that, the Head, which being placed in Regions very remote, ought, by reaſon of equal Diſtances from the Earth, to have ſhone equally; did on the Part betwixt the Perigee and the Sun, ſhine moſt of all, on the other Part vaniſh'd out of ſight. Therefore from the great Difference of Light in theſe two Situations, the great Vicinity of the Comet to the Sun, in the former Situation, is rightly concluded. For the Light of Comets is wont to be regular, and to appear the greateſt of all, when the Heads are moved most swiftly, and consequently are in their Perigees; only so far as it becomes greater in the Neighbourhood of the Sun.

Coroll. (1.) Comets therefore shine by the Reflexion of the Light of the Sun.

Coroll. (2.) We may gather from what hath been said, why Comets do so much frequent the Region of the Sun. If they were seen in Regions far beyond *Saturn*, they ought always to appear in the opposite Part to the Sun. For those which were in this Part would be nearer to the Earth, and the Sun by its Interposition would obscure the rest. But in running over the Histories of Comets, I have found that four or five Times more have been discover'd in the Hemisphere that is towards the Sun, than in the Opposite, besides others, no doubt not a few, which the Light of the Sun hath wholly hidden. The Thing is this; In their Descent to our Regions, they neither send forth Tails, nor are so illustrated by the Sun as to be seen by the naked Eye, until they have descended beneath *Jupiter*. Now the far greater Part of the Space described in so small an Interval about the Sun, is on that side of the Earth which looks to the Sun; and in that greater Part these Stars, as being then, for the most Part, nearer to the Sun, are wont to be enlightned.

Coroll. (3.) From hence it is manifest that the Heavens are destitute of Resistance. For Comets taking oblique Ways, and sometimes contrary to the Course of the Planets, are mov'd every Way most freely, and hold their Motions for a long time, contrary to the Course of the Planets. I am mistaken, if they be not a kind of Planets; and which being in perpetual Motion, return in a Round. For whereas some Writers will have them

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to be Meteors, taking their Argument from the perpetual Changes of the Heads of them, This seems to want all Foundation. Their Heads are encompass'd with huge Atmospheres, and the Atmospheres ought to be more dense beneath. From whence it comes to pass, that it is not the Bodies themselves of the Comets, but Clouds about them, which those Mutations are seen in. Thus, if the Earth were seen from the Planets, it would shine without doubt by the Light of its Clouds, and the firm Body would almost lie hid under those Clouds. Thus the Girdles of *Jupiter*, which are form'd in the Clouds of that Planet, change their Situation amongst themselves; so that the firm Body of *Jupiter* is difficultly discern'd thro' those Clouds. And much more ought the Bodies of Comets to be hid under their Atmospheres, which are both more deep and more crasse.

Now to him that Revolves in his Mind the Orb of the Comet of 1680, and $168\frac{0}{1}$, and the rest of the Phænomena it will be easily manifest, that the Bodies of Comets are Solid, Compact, Firm and Durable, like the Bodies of the Planets. For if they were nothing else than Vapours or Exhalations of the Sun, Earth and Planets, this Comet ought in its Transit through the Neighbourhood of the Sun to have been immediately dissipated. For the Heat of the Sun is as the Density of the Rays; that is, reciprocally, as the Square of the Distance from the Sun. And therefore, since the Distance of the Comet from the Sun, *Decemb.* 8. at which Time it was in its Perihelion, was to the Distance of the Earth from the Sun, as 6 to 1000, or thereabouts; the Heat of the Sun upon the Comet at that time was to the Heat of the Summer-Sun with us, as 1,000,000 is to 36: or as 28,000 is to 1. But the Heat of boiling

Water is about three times greater than that Heat which the dry Earth conceives from the Summer-Sun, as I have try'd my ſelf; and the Heat of Red-hot Iron (if I gueſs right) is about three or four times greater than the Heat of boiling Water; and conſequently the Heat which the dry Earth in the Comet contracted from the Rays of the Sun when it was in its Perihelion, might be about 2000 times greater than that of Red-hot Iron. Now by ſo great an Heat as this, Vapours and Exhalations, and all Volatile Matter muſt have been preſently conſum'd and diſſipated.

The Comet therefore, in its Perihelion, contracted an immenſe Heat from the Sun, and would hold that Heat for a very long time. For a Globe of Red-hot Iron of one Inch Diameter, would ſcarce loſe all its Heat in one Hour's Space, if it were expos'd to the open Air. But a greater Globe would keep its Heat longer, and this in the proportion of its Diameter; becauſe the Surface (according to which it is cool'd by the Contact of the Ambient Air) is in that Proportion leſs, if compar'd with the quantity of the hot Matter included. And therefore a Globe of Red-hot Iron, of the Bigneſs of this Earth, *i. e.* of 40,000,000 Feet Diameter, would ſcarce be wholly cool'd in ſo many Days, or 50,000 Years. I ſuſpect nevertheleſs, that the Continuance of Heat, by reaſon of latent Cauſes, is increas'd in leſs Proportion than that of the Diameter; and wiſh that the true Proportion were ſearched out by Experiments.

Furthermore, it is to be noted, that the Comet in the Month of *December*, when it was ſo heated by the Sun, ſent forth a Tail far greater, and more reſplendent than it had done before in *November*, when it had not yet reached to its

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Perihelion. And in general, all the Tails which exceed in Magnitude and Brightneſs are then ſeen, when the Star hath lately paſs'd through the Region of the Sun. The Heating therefore of the Comet conduceth to the Magnitude of the Tail. And from thence I am apt to conclude, that the Tail is nothing elſe but a moſt thin Vapour, which the Head or *Nucleus* of the Comet emits through its Heat.

Now there is a threefold Opinion concerning the Tails of Comets; that they are either the Beams of the Sun propagated thro' the Tranſlucid Bodies of thoſe Stars; or ariſe from the Refraction of the Light in the Progreſs thereof, from the Head of the Comet to the Earth; or, laſtly, are a Cloud or Vapour ariſing continually from the Head of the Comet, and which is turn'd off to the Part oppoſite to the Sun. The firſt is the Opinion of thoſe who are not yet inſtructed in the Knowledge of Optics. For the Beams of the Sun, let into a dark Chamber, are not ſeen there, any further than the Light is reflected from the Particles of Duſt and Fumes floating in the Air; and conſequently are much more bright in the Air when ſtuffed with groſs Fumes, and ſtrike the Senſe more forcibly; in a thinner Air are leſs perceiv'd, and in the Heavens, where there is no reflecting Matter, are not to be perceiv'd at all. Light is not ſeen as it is in the Beam, but as it is from thence reflected to our Eyes. For Sight is only by Rays which fall upon the Eyes. Some reflecting Matter therefore is requir'd in that Part of Heaven which the Tail takes up; Otherwiſe the whole Heaven it ſelf, which is illuminated by the Sun, would ſhine uniformly. The Second Opinion is urged with many Difficulties. The Tails are never varied with Colours, which yet are inſeparable

separable Concomitants of Refractions. The Light of the Fixed Stars and Planets, which is distinctly transmitted to us, shews that the Celestial Medium is without any refractive Force. For as for what is said, that the Fixed Stars have sometimes been seen with bright Streams by the *Egyptians*; this, because it is a thing which happens very seldom, is to be ascribed to the accidental Refraction of the Clouds. The Radiation also, and twinkling of the Fixed Stars, is to be referr'd to Refractions both in the Eyes and the tremulous Air; because, when they are seen through a Telescope, it vanisheth away.

By the Tremor of the Air, and ascending Vapours it comes to pass, that the Rays are easily turn'd off by Turns from the narrow Space of the Pupil of the Eye; but from the wider Aperture of the Object Glass never. From hence it is, that in the former Case a twinkling is produc'd, but in the latter none at all; and the Absence of it in the latter Case demonstrates the regular Transmission of Light through the Heavens, without any sensible Refraction. And if any one should say in this Place, that Tails are not wont to be seen in the Fixed Stars, only because their Light is weak and feeble; so that their Secondary Rays have not Force enough to move the Eyes, that Tails should appear about them; He may take notice, that the Light of Fixed Stars may be increas'd by Telescopes above an Hundred Times, and yet no Tails are seen. The Light of Planets also is greater than that of Comets, but yet they have no Tails; yea, and Comets have the longest Tails, when the Light of their Heads is weak and very obtuse. For the Comet of 1680, in the Month of *December*, at what time its Head did scarce equal a Star of the Second Magnitude, sent forth a Tail of notable Splendor unto 40, 50, 60 Degrees

Degrees of Length, and more. Afterwards, *Jan.* 27, and 28, the Head appear'd as a Star of the Seventh Magnitude only; but the Tail, which was of a dim Light indeed, but sensible enough, was seven or eight Degrees long, and with a very obscure Light, which could scarce be perceiv'd, it was stretch'd forth 12 Degrees, and more, as was said above. And *Febr.* 9, and 10, when the Head could no longer be seen with the naked Eye, I saw a Tail two Degrees long through a Telescope. Furthermore, if the Tail proceeded from the Refraction of Celestial Matter, and turn'd aside from the Opposition to the Sun, according to the Figure of the Heavens; that Deflection ought in the same Regions of Heaven always to be to one Part. But the Comet of 1680, *Decemb.* the 28th. at 8½ a Clock in the Evening at *London*, was in ♓ 8 deg. 41′, with a *Northern* Latitude 28 deg. 6′; the Sun being in ♑ 18 deg. 26′. And the Comet of the Year 1557, *Decemb.* 29, was in ♓ 8 deg. 41′. with *Northern* Latitude 28 deg. 40′; the Sun also being in ♑ about 18°. 26. In both Cases, the Earth was in the same Place, and the Comet appear'd in the same Part of Heaven; yet notwithstanding in the former Case (by mine own and others Observations) the Tail of the Comet declin'd with an Angle of 4½ Degrees from Opposition to the Sun towards the *North*; whereas in the latter (according to *Tycho*'s Observation) the Declination was 21 Degrees towards the *South*. The Refraction therefore of the Heavens being taken away, it remains that the Phænomena of the Tails are deriv'd from some reflecting Matter.

Now that the Tails do proceed from the Heads, and do ascend towards the Region opposite to the Sun, is confirm'd from the Laws which they observe. As that lying in the Plains of the Orbs of of the Comets passing through the Sun, they decline from Opposition to the Sun, always unto that Part which the Heads going forward in those Orbs do leave behind them. That they appear to the Spectator when placed in these Plains, turn'd away directly from the Sun; but the Spectator going aside from these Plains, the Deviation is perceiv'd by degrees, and grows every Day greater. That the Deviation, *cæteris paribus*, is less when the Tail is more oblique to the Orb of the Comet, as also when the Head of the Comet approacheth nearer to the Sun; especially if the Angle of Deviation be taken at the Head of the Comet. Besides, that the Tails which deviate not, are straight; but those which do, are bowed. That the Curvature is greater, where the Deviation is greater, and more sensible when the Tail, *cæteris paribus*, is longer; for in short Ones the Curvature is hardly perceiv'd. That the Angle of Deviation is less at the Head of the Comet, greater at the other End of the Tail; and consequently, that the Tail on its Convex-side looks to that Part from which the Declination is, and which is in a Right Line drawn from the Sun through the Head of the Comet *in infinitum*. And that the Tails which are longer and broader, and shine with a more lively Light, are upon their Convex-sides something more splendid, and bounded on the Concave-side with a Limit not very distinct.

The Phænomena therefore of the Tail depend upon the Motion of the Head, and not upon that Region of Heaven in which the Head is seen; and therefore are not made by a Refraction of the Heavens, but arise from the Head, which affords Reflecting Matter. For as in our Air the Smoke of a Body set on Fire ascends upwards, and that either Perpendicularly if the Body rest-

eth;

eth; or Obliquely, if it be mov'd to one Side: So in the Heavens, where Bodies gravitate to the Sun, the Fumes and Vapours ought to ascend from the Sun (as hath been already said) and this in a Right Line if the Body rests, but Obliquely, if the Body be mov'd, and in going forwards always forsakes the Places from which the upper Parts of the Vapour had ascended. And that Obliquity will be less, where the Ascent of the Vapour is more quick; as in the Neighbourhood of the Sun, and near to the Surface of the smoking Body. Now from the Diversity of the Obliquity, the Column of the Vapour will be bowed. And because the Vapour on that side of the Column which goes before is something more fresh, for this Reason it will be something more Dense in the same Place, and will therefore reflect Light more copiously, and be bounded with more distinct Limit. As to the sudden and uncertain Agitations of their Tails, and of the irregular Figures of the same, which some describe, I add nothing here; because they either arise from the Mutations that are in our Air, and the Motions of the Clouds which do in some part obscure the Tail; or perchance from Parts of the *Via Lactea*, which may seem united with the Tails that pass by them, and taken to be Parts of them.

Now that Vapours which suffice to fill such vast Spaces may arise from the Atmospheres of Comets, will be understood from the Rarity of our Air. For the Air, near to the Surface of the Air, possesses 850 Times more Space than a Quantity of Water of the same Weight; and therefore a Cylindrical Column of Air 850 Feet high, is of the same Weight with a Column of Water of the same Breadth, which is only one Foot high. But

a Column

a Column of Air arising to the Top of the Atmosphere, doth equal in its Weight a Column of Water, which is about 33 Foot high; and therefore if the inferior Part of the whole airy Column be taken away, which is of the Heighth of 850 Feet, the upper remaining Part will be equal in its Weight to a Column of Water of 32 Feet high. But from thence (according to the Hypothesis which hath been confirm'd by many Experiments, namely, that the Compression of the Air is as the Weight of the Atmosphere which lies upon it, and that Gravity is reciprocally, as the Square of the Distance from the Center of the Earth) by making Computation according to Coroll. Prop. XXII. Book II. I found that Air, at the Heighth of a Semi-diameter of the Earth, from the Surface of the same, is rarer than it is with us in a Proportion far greater than is that of all the Space beneath the Orb of *Saturn* to a Globe of one Inch Diameter. And therefore a Globe of our Air of one Inch Diameter in that Rarity, which it would have in the Heighth of a Semi-diameter of the Earth, would fill all the Regions of the Planets unto the Sphere of *Saturn*, and much farther. Therefore since Air, which is yet higher, grows more rare infinitely; and the Atmosphere of a Comet, in ascending from its Center, is about ten times higher than the Surface of the *Nucleus*, and when the Tail doth ascend yet higher, the Tail ought to be rare in the highest degree. And altho' by reason of the more gross Atmosphere of Comets, and the great Gravitation of Bodies towards the Sun; and the Gravitation of the Particles of the Air and Vapours towards one another; it may come to pass, that the Air doth not grow Rare in so great a Degree in the Heavenly Spaces and in the Tails of Comets; yet is it manifest from this Computation, that a very small Quantity of Air and Vapours sufficeth abundantly to all those Phænomena of the Tails. For the great Rarity of the Tails is also collected from the Stars which are visible through them. The Atmosphere of the Earth shining with the Light of the Sun, doth by its Crassitude, which is but for a few Miles, [frequently] both obscure, and even hide all the Stars and the Moon it self; but the least Stars are observ'd to be visible without any detriment of their Clearness through the Tails of Comets, which are of an exceeding great Depth, and are at the same time illuminated with the Light of the Sun. Nor is the Splendor of many of their Tails wont to be greater, than is that of our Air, when in a dark Chamber it reflects the Light of the Sun in the Form of a Beam, for the Breadth of an Inch or two.

In what Time the Vapour ascends from the Head to the End of the Tail, may almost be known by drawing a Right Line from the End of the Tail to the Sun, and noting the Place where that Line cuts the Trajectory. For a Vapour in the End of the Tail, if it ascends straight from the Sun, begun to ascend from the Head, at what Time the Head was in the Place of Intersection. And tho' it doth not ascend straight from the Sun, yet by retaining that Motion which the Comet had before its Ascension, and Compounding the same with the Motion of its Ascension, it ascends Obliquely. From whence the truer Solution of the Problem will be, that the Right Line which cuts the Orb, be Parallel to the Length of the Tail; or rather (by reason of the Curvi-linear Motion of the Comet) that the same deflect a little from the Line of the Tail.

By

By this means I found, that the Vapour which was in the End of the Tail, *Jan.* 25. began to aſcend from the Head, *Decemb.* the 11th, and conſequently had ſpent above 45 Days in its Aſcenſion. But all that Tail which appear'd *Dec.* 10. had aſcended in the Space of thoſe two Days which had paſs'd from the Time of the Perihelion. Therefore the Vapour aſcended moſt ſwiftly in the Neighbourhood of the Sun, and afterwards went on aſcending with a Motion ſtill retarded by its own Gravity; and by its aſcending increas'd the Length of the Tail; but the Tail, ſo long as it appear'd, conſiſted almoſt all of the Vapour which had aſcended from the Time of the Perihelion; and the Vapour which aſcended firſt, and compoſed the End of the Tail, vaniſhed not out of ſight before that it ceaſed to appear, by reaſon of its great Diſtance both from the Sun which illuſtrated it, and from our Eyes. From whence alſo the Tails of other Comets which are ſhort, do not aſcend with a ſwift and perpetual Motion from the Heads, and ſo preſently vaniſh away, but are permanent Columns of Vapours, propagated from the Heads with a very ſlow Motion of many Days; which by partaking of that Motion of the Heads which they had at the Beginning, go on to be mov'd through the Heavens, together with the Heads. And from hence it is again collected, that the Heavenly Spaces are deſtitute of all Force of Reſiſting; ſince not only the Solid Bodies of Planets and Comets, but alſo the exceeding thin and rare Vapours of the Tails of Comets do perform their Motions in them moſt freely and ſwiftly; and hold the ſame for a very long Time.

Plate IX

fronting Page 4

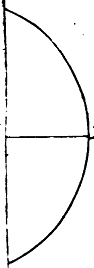
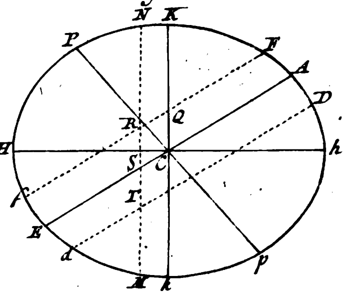


Fig. 2.



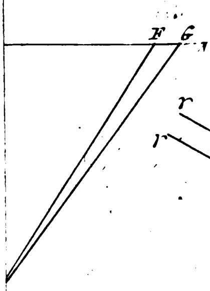


Fig. 4.

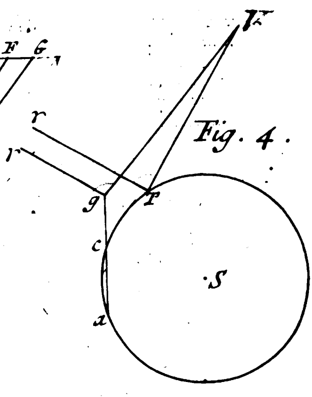
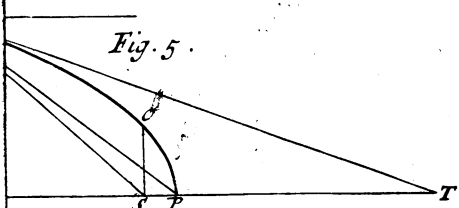


Fig. 5.



I. Senex sculpt

Kepler aſcribes the Aſcent of the Tails from the Atmoſpheres of the Head, and their Progreſs to the Part oppoſite to the Sun, to the Action of the Rays of Light, which carries away with them that Matter of which the Tail conſiſts. And it is not altogether unreaſonable to think, that a very thin Air may give way to the Rays of Light in free Spaces; notwithſtanding that groſs Subſtances cannot in our Regions be ſenſibly impell'd or mov'd by the Solar Rays. Another is of Opinion, that there may be as well Light as heavy Particles of Matter; and that the Matter of the Tails are light, and by their Lightneſs aſcend from the Sun. But ſince the Gravity in Terreſtrial Bodies is as the Matter in thoſe Bodies, and conſequently cannot be increas'd or diminiſh'd, the ſame Quantity of Matter remaining; I am prone to think, that that Aſcent doth rather ariſe from the Rarefaction of the Matter of the Tails. The Smoke in a Chimney aſcends by the Impulſe of the Air in which it floats. That Air being rarified by the Heat, aſcends by reaſon of the Diminution of its Specific Gravity, and carries away the Smoke entangled in it, together with it. What ſhould then hinder, but that the Tail of a Comet ſhould aſcend in the ſame manner from the Sun? For the Rays of the Sun do not agitate the Mediums through which they paſs, but in their Reflection and Refraction. The Reflecting Particles being heated by that Action, will heat the Ethereal Air which is about them.

This, by the Heat communicated to it, will grow rare; and by reaſon of the Diminution of its Specific Gravity, by that Rareneſs will aſcend, and carry away the Reflecting Particles, of which the Tail consists. It conduces also to the Ascent of the Vapours, that they are turn'd about the Sun, and by that Action endeavour to Recede from the Sun; whilst the Atmosphere of the Sun, and the Matter of the Heavens, do either wholly rest, or are turn'd round slowly, only with a Motion, which they have receiv'd from the Rotation of the Sun. These are the Causes of the Ascent of the Tails in the Neighbourhood of the Sun where the Orbs are more Curve, and the Comets are within the more Dense, and by that means the more heavy Atmosphere of the Sun. For the Tails which arise then, will, by keeping their Motion, and Gravitating in the mean while towards the Sun, be mov'd about the Sun in Ellipses after the manner of the Heads, and by that Motion always accompany the Heads, and stick to them. For the Gravity of Vapours towards the Sun, will no more cause that the Tails should fall from the Heads afterwards towards the Sun, than the Gravity of the Heads can make, that they should fall from the Tails. Therefore by their common Gravity they will either fall together towards the Sun, or be retarded together in their Ascent; and consequently that Gravity hinders not, but that the Tails and Heads should most easily receive, and afterwards most freely keep any Position to one another, whatsoever it is, which they may receive from the Causes which have been mention'd, or any other whatever.

The Tails therefore which arise in the Perihelia of Comets, will go away with their Heads into far distant Regions; and after a long Series of Years, return with the same to us; or rather being

being there rarified, will by little and little vanish away. For afterwards, in the Descent of the Heads to the Sun, new little short Tails ought to be propagated from their Heads by a slow Motion, and afterwards to be immensely increas'd in the Perihelia of those Comets which descend into the Atmosphere of the Sun. For the Vapour in those most free Spaces is perpetually rarified and dilated. By which means it comes to pass, that the Tail is broader at the upper End, than near the Head of the Comet. Now it seems reasonable to think, that the Vapour being by that Rarefaction perpetually dilated, is dispersed at length through the whole Heaven, and is by little and little drawn unto the Planets, and mingled with their Atmospheres. For like as the Seas are altogether requir'd unto the Constitution of this our Earth; and this, that Vapours may be rais'd sufficiently out of them by the Heat of the Sun; which being gather'd together into Clouds, may fall down in Rains, and water and nourish all the Earth, for the Procreation of Vegetables; or being Condens'd in the cold Tops of Mountains, (as some do reasonably enough suppose) runs down unto the Heads of Springs and Rivers: So to the Preservation of Seas and Rivers in Planets, Comets seem necessary; from the Condens'd Vapours and Exhalations whereof, what Liquor is spent by Vegetation and Corruption, and turn'd into dry Earth, is continually supplied and renew'd. For all Vegetables grow from Liquors or Juices; and then in great Part they pass by Putrefaction into dry Earth, and Mud ariseth perpetually from the putrefied Liquors. Hence the Bulk of dry Earth is continually increas'd; and what is Humid, unless it be increas'd from elsewhere, ought perpetually to decreaſe, and at length to fail. Yea, I ſuſpect that that Spirit which is the leaſt indeed, but the moſt ſubtile and the beſt Portion of our Air, and which is requir'd to the Life of all things, doth come from Comets eſpecially.

The Atmoſpheres of Comets are in their Deſcent to the Sun diminiſh'd by running out into Tails, and (on that Part certainly which looks to the Sun) become more narrow: And on the other hand in their Receſs from the Sun, at which time they run leſs out into Tails, they are enlarg'd; if ſo be *Hevelius* hath rightly noted the Phænomenons thereof. But they appear the leaſt, when their Heads having been juſt now heated at the Sun, iſſue forth in great and refulgent Tails; and their *Nucleus*'s are perhaps ſurrounded with a more craſſe and black Fume in the loweſt Parts of their Atmoſphere. For all Smoke which is ſtirred up by a very great Heat, is wont to be ſo much the more craſſe and black. Thus the Head of the Comet of which we have been treating, did at equal Diſtances from the Sun and the Earth, appear more obſcure after the Perihelion than before. For in the Month of *December*, it was compar'd with Stars of the third Magnitude; whereas in *November* it equall'd Stars of the firſt and ſecond. And they who had ſeen both, deſcribe the former Comet as the greater. For *November* the 19th, this Comet, how obtuſe ſoever the Light was wherewith it ſhined, did then equal *Spica Virginis*, as it appear'd to a Young-man of *Cambridge*, and ſhin'd more clearly than it did afterwards. And Mr. *Storer*, in Letters which fell into my Hands, wrote, that the Head of it in *December*, at which time it ſent forth the greateſt and moſt refulgent

fulgent Tail of all, was small, and in its visible Magnitude, did fall far short of what it appear'd before the Rising of the Sun in *November*. The Reason whereof he guessed to be this, That the Matter of the Comet at the beginning was more copious, and was wasted by degrees.

We may here also fitly Note, that the Heads of other Comets, which have sent forth most great, and refulgent Tails, are describ'd as being somewhat dim and small. For in the Year 1668, *March* 5. *New-Style*, at 9 a-Clock in the Evening, the Reverend Father *Valentine Estance*, being then in *Brasile*, saw a Comet near to the Horizon at the *South West*, of a very little Head, and scarce to be seen, but with a Tail exceedingly Refulgent; so that those who stood on the Shore might easily see the Image of it reflected from the Sea. It had the Appearance of a shining Beam, of the Length of 23 Degrees, inclining from the *West* to the *South*, and almost Parallel to the Horizon. But this great Splendor endur'd only for two Days, and from that time notably decreas'd; and in the mean while that the Splendor decreas'd, the Tail did increase in Magnitude. From whence also it is said to have possess'd in *Portugal* almost a 4th Part of Heaven, (or 45 Degrees) stretching from the *West* to the *East*, with a very great Splendor; nor did it all appear notwithstanding, the Head still in these Regions lying hid below the Horizon. From the Increase of the Tail, and Decrease of the Splendor it is manifest, that the Head was receding from the Sun; and was next to it about the beginning, as it was in the Comet of 1680. And we read of a like Comet in the Years 1101, or 1106; "The Star whereof was small "and obscure (as was that of 1680,) but the "Splendor which went forth from thence was "very clear; and as it were a great Beam, reaching to the *East* and *North*, as *Hevelius* hath it, "out of *Simeon*, the Monk of *Durham*. It appear'd "in the Beginning of *February* about Evening, at "the *South-West*." But from thence, and the Situation of the Tail, it is gathered, that the Head was near to the Sun. "It was distant, saith "*Matthew* of *Paris*, about one Cubit from the "Sun, from the third, (more truly the sixth "Hour) until the ninth, sending forth a long "Ray from it self." Such also was that fiery Comet describ'd by *Aristotle*, Book I. Meteor 6. "The Head whereof was not seen the "first Day, because that it had Set before the "Sun, or at least under the Rays of the Sun; "but the following Day it was seen as far as its "Situation allow'd. For it had left the Sun but "with a Distance as small as might be; and then "Set By reason of the very great Ardor [of the "Tail,] the dispers'd Fire of the Head did not "yet appear; but afterwards, when the Tail did "burn less, the Head was restor'd to its former Ap"pearance. And it extended its Splendor unto "a third Part of Heav'n, (*i. e.* unto 60 Degrees.) "Now its Appearance was in Winter-time, and "Ascending unto the Girdle of *Orion*, it there "Vanish'd."

That Comet of the Year 1618, which came with a very long Tail out of the Rays of the Sun, seem'd to equal, if not exceed a Star of the first Magnitude; but there have not a few greater Comets appear'd, which have had shorter Tails. Some of them are related to have been equal to *Jupiter* Others to *Venus*, or even to the Moon.

I said

I ſaid before, that Comets are a kind of Planets, Revolving in very eccentric Orbits about the Sun. And like as amongſt the Planets, which have no Tails, thoſe are wont to be leſs which are revolv'd in leſſer Orbits, and nearer to the Sun; ſo likewiſe is it reaſonable to think, that the Comets, which in their Perihelia approach nearer to the Sun, are for the moſt Part leſſer, and are Revolv'd in leſſer Orbits. But as for the Tranſverſe Diameters of their Orbits, and their Periodical Times, we leave theſe Things to be determin'd from the Compariſon of Comets, returning in the ſame Orbits after long Intervals of Time.

LECT. XXXIX.

HAving finish'd the Explication of Sir *Isaac Newton*'s Philosophy, we will now endeavour to Explain Dr. *Halley*'s Cometography, which is built upon that Philosophy. And tho' this Work of Dr. *Halley*'s be an excellent Piece, yet is it something too succinct and obscure, as being only preparatory to a greater Work intended; and indeed is no-where else represented plain enough for Beginners: My Purpose therefore is to give you the whole in the Author's own Words; but so, that I intend to improve it all along, and illustrate it with a Commentary. The Historical Preface, which is prefix'd to the same, needs no Explication: However, I shall not think much to transcribe it, that no Part of this excellent Work may be wanting in this Place.

A SY-

A
SYNOPSIS
OF THE
Aſtronomy of COMETS.

By EDMUND HALLEY, L.L.D. *Savilian* Profeſſor of Geometry at *Oxford*.

THE Ancient *Egyptians* and *Chaldeans* (if we may credit *Diodorus Siculus*) by a long Courſe of Obſervations, were ſaid to be able to predict the Apparitions of Comets. But ſince they are alſo ſaid, by the help of the ſame Arts, to have prognoſticated Earthquakes and Tempeſts; 'tis paſt all doubt, that their Knowledge in theſe Matters, was the Reſult rather of meer Aſtrological Calculation, than of any Aſtronomical Theories of the Celeſtial Motions. And the *Greeks*, who were the Conquerors of both thoſe People, ſcarce found any other Sort of Learning amongſt them, than

than this. So that 'tis to the *Greeks* themselves as the Inventors (and especially the great *Hipparchus*) that we owe the Astronomy we have, and which is now improv'd to such a heighth. But yet, amongst the *Greeks*, the Opinion of *Aristotle* (who wou'd have Comets to be nothing else, but Sublunary Vapours, or Airy Meteors) prevail'd so far, that this most difficult part of the Astronomical Science lay altogether neglected; for no Body thought it worth while to take notice of, or write about, the wandring uncertain Motions of what they esteemed Vapours floating in the Æther; whence it came to pass, that nothing certain, concerning the Motion of Comets, can be found transmitted from them to us.

But *Seneca* the Philosopher, having consider'd the Phænomena of two remarkable Comets of his Time, made no scruple to place them amongst the Celestial Bodies; believing them to be Stars of equal Duration with the World, tho' he owns their Motions to be govern'd by Laws not as then known or found out. And at last (which was no untrue or vain Prediction) he foretells, that there should be Ages sometime hereafter, to whom Time and Diligence shou'd unfold all these Mysteries, and who shou'd wonder how 'twas possible the Ancients cou'd be ignorant of them, after some lucky Interpreter of Nature had shewn, *in what parts of the Heavens the Comets wander'd, what sort of Beings, and how great they were.* Yet almost all the Astronomers differ'd from this Opinion of *Seneca*; neither did *Seneca* himself think fit to set down those Phænomena of the Motion, by which he was enabled to maintain his Opinion; nor the Times of those Appearances, which might be

of

of use to Posterity, in order to the determining these things. And indeed, upon the turning over very many Histories of Comets, I find nothing at all that can be of service in this affair, before *A. D.* 1337. at which time *Nicephorus Gregoras*, a *Constantinopolitan* Historian and Astronomer, did pretty accurately describe the Path of a Comet amongst the Fix'd Stars, but was too lax as to the Account of the Time; so that this most doubtful and uncertain Comet, only deserves to be inserted in our Catalogue, for the sake of its appearing near Four-hundred Years ago.

The next of our Comets was in the Year 1472; which being the swiftest of all, and nearest to the Earth, was observ'd by *Regiomontanus*. This Comet (so fearful upon the account both of the Magnitude of its Body, and the Tail) mov'd forty Degrees of a great Circle in the Heavens, in the Space of one Day; and was the first, of which any proper Observations are come down to us. But all those that consider'd Comets, until the Time of *Tycho Brahe* (that great Restorer of Astronomy) believ'd them to be below the Moon, and so took but little Notice of them, reckoning them no other than Vapours.

But in the Year 1577, (*Tycho* seriously pursuing the Study of the Stars, and having gotten large Instruments for the performing Celestial Mensurations, with far greater Care and Certainty than the Ancients cou'd ever hope for) there appear'd a very remarkable Comet; to the Observation of which *Tycho* vigorously applied himself; and found by many just and faithful Trials, that it had no Diurnal Parallax that was perceptible: And consequently was not only no Aerial Vapour, but also much higher than the

the Moon; nay, might be plac'd amongst the Orbs of the Planets, for any thing that appear'd to the contrary; the cavilling Opposition made by some of the School-men in the mean time, being to no purpose.

Tycho was succeeded by the most Sagacious *Kepler.* He having the Advantage of *Tycho's* Labours and Observations, found out the true Physical System of the World, and vastly improv'd the Science of Astronomy.

For he demonstrated that all the Planets perform their Revolutions in *Elliptic Orbits, whose Plains pass thro' the Center of the Sun*, observing this Law, that *the Area's of the Elliptic Sectors, taken at the Center of the Sun (which he proved to be in the common Focus of these Ellipses) are always proportional to the Times in which the correspondent Elliptical Arcs are describ'd.* He discover'd also that *the Distances of the Planets from the Sun are in the Sesquialtera ratio of the Periodical Times*, or (which is all one) that *the Cubes of the Distances are as the Squares of the Times.* This great Astronomer had the opportunity of observing two Comets, one of which was a very remarkable one. And from the Observations of these (which afforded sufficient Indications of an Annual Parallax) he concluded, That *the Comets mov'd freely thro' the Planetary Orbs, with a Motion not much different from a Rectilinear one; but of what kind he cou'd not precisely determine.* Next, *Hevelius* (a noble Emulator of *Tycho Brahe*) following in *Kepler's* Steps, embraced the same Hypothesis of the Rectilinear Motion of the Comets, himself accurately observing many of them. Yet he complain'd that his Calculations did not perfectly agree to what he observed in the Heavens: And was aware, that *the Path of a Comet*

was

was bent into a Curve Line concave towards the Sun. At length came that prodigious Comet of the Year 1680; which descending (as it were from an infinite Distance) perpendicularly towards the Sun, arose from him again with as great a Velocity.

This Comet, (which was seen for four Months continually) by the very remarkable and peculiar Curvity of its Orb (above all others) gave the fittest Occasion for investigating the Theory of its Motion. And the Royal Observatories at *Paris* and *Greenwich* having been for some time founded, and committed to the Care of most excellent Astronomers, the apparent Motion of this Comet was most accurately (perhaps as far as humane Skill cou'd go) observ'd by Mrs. *Cassini* and *Flamsteed.*

Not long after, that great Geometrician the Illustrious *Newton*, writing his *Mathematical Principles of Natural Philosophy*, demonstrated not only that what *Kepler* had found, did necessarily obtain in the Planetary System; but also, that all the Phænomena of Comets wou'd naturally follow from the same Principles; which he abundantly illustrated by the Example of the aforesaid Comet of the Year 1680; shewing at the same time, a method of delineating the Orbits of Comets Geometrically; therein solving (not without meriting the highest Admiration of all Men) a Problem, whose Intricacy render'd it scarce Accessible to any but himself. This Comet he prov'd to move round the Sun in a Parabolical Orb, and to describe Area's (taken at the Center of the Sun) proportional to the Times.

Where-

Wherefore (following the Steps of so Great a Man) I have attempted to bring the same Method to Arithmetical Calculation; and that with all the Success I cou'd wish. For, having collected all the Observations of Comets I cou'd, I have fram'd this following Table, the result of a prodigious deal of Calculation; which, tho' but small in Bulk, will be no unacceptable Present to Astronomers. For these Numbers are capable of representing all that has been yet observ'd about the Motion of Comets, by the help only of the annex'd General Table; in the making of which I spar'd no Labour, that it might come forth perfect, as a Thing consecrated to Posterity, and to last as long as Astronomy it self.

The

The Astronomical Elements of the Motions in a Parabolic Orb of all the Comets that have been hitherto duely observ'd.

Com. An.	*Nodus Ascend.* gr. ′ ″	*Inclin. Orbitæ.* gr. ′ ″	*Perihelion.* gr. ′ ″	*Distan. Periheli. à Sole.*	*Log. Dist. Perihelia. à Sole.*	*Temp. æquat. Perihelii.* d. h. ′	*Perihelion à Nodo.* gr. ′ ″	
1337	♊24.21. 0	32.11. 0	♉ 7.59. 0	40666	9.609236	*June* 2. 6.25	46.22. 0	Retrog.
1472	♑11.46.20	5.20. 0	♉ 5.33.30	54273	9.734584	*Feb.* 28.22.23	123.47.10	Retrog.
1531	♉19.25. 0	17.56. 0	♒ 1.39. 0	56700	9.753583	*Aug.* 24.21.18½	107.46.10	Retrog.
1532	♊20.27. 0	32.36. 0	♋21. 7. 0	50910	9.706803	*Oct.* 19.22.12	30.40. 0	Direct
1556	♍25.42. 0	32. 6.30	♑ 8.50. 0	66390	9.666424	*Apr.* 21.20. 3	103. 8. 0	Direct
1577	♈25.52. 0	74.32.45	♌ 9.22. 0	18342	9.263447	*Oct.* 26.18.45	103.30. 0	Retrog.
1580	♈18.57.20	64.40. 0	♋19. 5.50	59628	9.775450	*Nov.* 28.15.00	90. 8.30	Direct
1585	♉ 7.42.30	6. 4. 0	♈ 8.51. 0	109358	0.038850	*Sept.* 27.19.20	28.51.30	Direct
1590	♍15.30.40	29.40.40	♏ 6.54.30	57661	9.760882	*Jan.* 29. 3.45	51.23.50	Retrog.
1596	♒12.12.30	55.12. 0	♏18.16. 0	51293	9.710058	*July* 31.19.55	83.56.30	Retrog.
1607	♉20.21. 0	17. 2. 0	♒ 2.16. 0	58680	9.768490	*Oct.* 16. 3.50	108.05. 0	Retrog.
1618	♊16. 1. 0	37.34. 0	♈ 2.14. 0	37975	9.579498	*Oct.* 29.12.23	73.47. 0	Direct
1652	♊28.10. 0	79.28. 0	♈28.18.40	84750	9.928140	*Nov.* 2.15.40	59.51.20	Direct
1661	♊22.30.30	32.35.50	♋25.58.40	44851	9.651772	*Jan.* 16.23.41	33.28.10	Direct
1664	♊21.14. 0	21.18.30	♌10.41.25	102575½	0.011044	*Nov.* 24.11.52	49.27.25	Retrog.
1665	♏18.02. 0	76.05. 0	♊11.54.30	10649	9.027309	*Apr.* 14 5.15½	156. 7.30	Retrog.
1672	♑27.30.30	83.22.10	♉16.59.30	69739	9.843476	*Feb.* 20. 8.37	109.29. 0	Direct
1677	♏26.49.10	79.03.15	♌17.37. 5	28059	9.448 0	*Apr.* 26.00.37½	99 12. 5	Retrog.
1680	♑ 2. 2. 0	60.56. 0	♐22.39.30	00612½	7.787106	*Dec.* 8.00. 6	9.22.30	Direct
1682	♉21.16.30	17.56. 0	♒ 2.52.45	58328	9.765877	*Sept.* 4.07.39	108.23.45	Retrog.
1683	♍23.23. 0	83.11. 0	♊25.29.30	56020	9.748343	*July* 3. 2.50	87.53.30	Retrog.
1684	♐28.15. 0	65.48.40	♏28.52. 0	96015	9.982339	*May* 29.10.16	29.23.00	Direct
1685	♓20.34.40	31.21.40	♊17.00.30	32500	9.511883	*Sept.* 6.14.33	86.25.50	Direct
1698	♐27.44.15	11.46. 0	♑00.51.15	69129	9.839660	*Oct.* 8.16.57	3. 7. 0	Retrog.

This Table needs little Explication, since 'tis plain enough from the Titles, what the Numbers mean. Only it may be observ'd, that the Perihelium Distances, are estimated in such Parts, as the Middle Distance of the Earth from the Sun, contains 100000.

A

A General Table for Calculating the Motions of Comets in a Parabolical Orbit.

Med. mot.	Angul. à perihelio.	Logar. pro dist. à Sole.	Med. mot.	Ang. à perihelio	Logar. pro dist. à Sole.
o	gr. ′ ″		o	gr. ′ ″	
1	1.31.40	0.000077	41	53.29.44	0.098300
2	3. 3.15	0.000309	42	54.27.32	0.102019
3	4.34.43	0.000694	43	55.24.21	0.105752
4	6. 6. 0	0.001231	44	56.20.12	0.109490
5	7.37. 1	0.001921	45	57.15. 6	0.113240
6	9. 7.43	0.002759	46	58. 9. 3	0.116995
7	10.38. 2	0.003745	47	59. 2. 4	0.120756
8	12. 7.54	0.004876	48	59.54.11	0.124518
9	13.37.17	0.006151	49	60.45.25	0.128278
10	15. 6. 7	0.007564	50	61.35.45	0.132035
11	16.34.20	0.009115	51	62.25.14	0.135792
12	18. 1.54	0.010798	52	63.13.52	0.139544
13	19.28.47	0.012609	53	64. 1.40	0.143291
14	20.54.54	0.014550	54	64.48.38	0.147029
15	22.20.14	0.016607	55	65.34.50	0.150762
16	23.44.44	0.018783	56	66.20.13	0.154482
17	25. 8.22	0.021072	57	67.04.50	0.158192
18	26.31. 8	0.023470	58	67.48 42	0.161890
19	27.52.55	0.025969	59	68.31.50	0.165578
20	29.13.47	0.028570	60	69.14.16	0.169254
21	30.33.40	0.031263	61	69.55.58	0.172914
22	31.52.32	0.034045	62	70.36.56	0.176557
23	33.10.23	0.036916	63	71.17.16	0.180188
24	34.27.12	0.039864	64	71.56 56	0.183803
25	35.42.59	0.042892	65	72.35.57	0.187404
26	36.57.41	0.045989	66	73.14.15	0.190978
27	38.11.20	0.049154	67	73.51.59	0.194540
28	39.23.54	0.052382	68	74.29. 6	0.198085
29	40.35.23	0.055668	69	75.05.38	0.201614
30	41.45.47	0.059009	70	75.41.35	0.205122
31	42.55.06	0.062400	71	76.16.56	0.200612
32	44. 3.20	0.065838	72	76.51.43	0.212080
33	45.10.29	0.069319	73	77.25.57	0.215529
34	46.16.35	0.072839	74	77.59.41	0.218963
35	47.21.36	0.076396	75	78.32.54	0.222378
36	48.25.33	0.079984	76	7[illegible]. 5.35	0.225769
37	49.28.27	0.083600	77	7[illegible].37.45	0.229142
38	50.30.19	0.087244	78	80. 9.23	0.232488
39	51.31. 8	0.0909[illegible]0	79	80.40.34	0.235809
40	52.30.56	0.094596	80	81.11.16	0.239127

Med. mot. °	*Angul. à Perihelio.* gr. ′ ″	*Logar. pro dist. à Sole.*	*Med. mot.* °	*Angul. à Perihelio.* gr. ′ ″	*Logar. pro dist. à Sole.*
81	81.41.31	0.242416	142	102.32.41	0.407380
82	82.11.19	0.245684	144	103.00.31	0.411784
83	82.40.40	0.248933	146	103.27.47	0.416132
84	83. 9.34	0.252159	148	103.54.31	0.420430
85	83.38. 4	0.255366	150	104.20.43	0.424676
86	84. 6. 8	0.258552	152	104.46.22	0.428866
87	84.33.49	0.261720	154	105.11.33	0.433012
88	85. 1. 5	0.264865	156	105.36.16	0.437110
89	85.27.58	0.267989	158	106.00.32	0.441164
90	85 54.27	0.271092	160	106.24.23	0.445178
91	86.20.34	0.274176	162	106.47.47	0 449144
92	86.46.20	0.277239	164	107.10.44	0.453060
93	87.11.43	0.280284	166	107.33.17	0.456936
94	87.36.45	0.283306	168	107.55.27	0.460772
95	88.01.27	0.286308	170	108.17.14	0.464208
96	88.25.49	0.289293	172	108.38.37	0.468318
97	88.49.48	0.292252	174	108.59.39	0.472030
98	89.13.32	0.295201	176	109.20.20	0.475705
99	89.36.54	0.298122	178	109.40.40	0.479340
100	90.00.00	0.301030	180	110.00.40	0.482937
102	90.45.14	0.306782	182	110.20.20	0.486498
104	91.29.18	0.312469	184	110.39.41	0.490022
106	92.12.14	0.318060	186	110.58.44	0.493512
108	92.54. 4	0.323587	188	111.17.28	0.496965
110	93.34.52	0.329042	190	111.35.55	0.500384
112	94.14.40	0.334424	192	111.54.05	0.503769
114	94.53.30	0.339736	194	112.11.58	0.507121
116	95.31.22	0.344979	196	112.29.34	0.510441
118	96. 8.22	0.350153	198	112.46.55	0.513729
120	96.44.30	0.355262	200	113. 4.00	0.516984
122	97.19.48	0.360306	204	113.37.25	0.523406
124	97.54.17	0.365284	208	114. 9.52	0.529705
126	98.28.00	0.370200	212	114.41.23	0.535886
128	99.00.57	0.375052	216	115.12.02	0.531958
130	99.33.11	0.379842	220	115.41.51	0.537922
132	100. 4.43	0.384576	224	116.10.52	0.553782
134	100.35.45	0.389252	228	116.39. 7	0.559538
136	101. 5.48	0.393868	232	117.06.38	0.565199
138	101.35.22	0.398428	236	117.33.27	0.570762
140	102. 4.19	0.402930	240	117.50.25	0.5[illegible]22

Med. mot. o	Angul. à perihelio. gr. ′ ″	Logar. pro dist. Sole.	Med. mot. o	Ang. à perihelio gr. ′ ″	Logar. pro dist. à Sole.
244	118.25. 5	0.581616	620	137.33.12	0.882575
248	118.49.57	0.586912	640	138. 3.58	0.892649
252	119.14.14	0.592122	660	138.33.21	0.902401
256	119.37.56	0.597252	680	139. 1.29	0.911866
260	120. 1. 6	0.602301	700	139.28.25	0.921012
264	120.23.44	0.607274	720	139.54.16	0.929907
268	120.45.52	0.612174	740	140.19. 5	0.938549
272	121. 7.30	0.616998	760	140.42.56	0.946951
276	121.28.39	0.621750	780	141. 5.55	0.955124
280	121.49.22	0.626438	800	141.28. 3	0.963082
284	122. 9.38	0.631056	820	141.49.24	0.970836
288	122.29.28	0.635608	840	142.10.00	0.978397
292	122.48.54	0.640098	860	142.29.56	0.985771
296	123. 7.57	0.644525	880	142.49.10	0.992970
300	123.26.36	0.648893	900	143. 7.48	0.100000
310	124.11.40	0.659559	920	143.25.51	1.006871
320	124.54.36	0.669880	940	143.43.21	1.013586
330	125.35.34	0.679876	960	144.00.18	1.020155
340	126.14.44	0.689568	980	144.16.46	1.026583
350	126.52.12	0.698970	1000	144.32.46	1.032876
360	127.28. 6	0.708104	1500	149.26. 8	1.158188
370	128. 2.33	0.716976	2000	152.26.15	1.246058
380	128.35.38	0.725606	2500	154.32.20	1.313703
390	129. 7.27	0.734006	3000	156.7. 27	1.368678
400	129.38. 4	0.742186	3500	157.22.49	1.414974
410	130. 7.34	0.750160	4000	158.24.36	1.454950
420	130.36. 2	0.757930	4500	159.16.36	1.490125
430	131. 3.30	0.765516	5000	160. 1.12	1.521521
440	131.30. 2	0.772918	5500	160.40. 5	1.549874
450	131.55.41	0.780148	6000	161.14.24	1.575718
460	132.20.30	0.787216	6500	161.45.00	1.599460
470	132.44.32	0.794122	7000	162.12.34	1.621417
480	133. 7.50	0.800882	7500	162.37.34	1,641838
490	133.30.25	0.807494	8000	163.00.23	1.660922
500	133.52.20	0.813969	8500	163.21.20	1.678834
520	134.34.18	0.826522	9000	163.40.42	1.695708
540	135.14. 0	0.838600	5900	163.58.38	1.711662
560	135.51.28	0.850187	10000	164.15.20	1.726784
580	136.27. 6	0.861369	50000	170.52. 0	2.197960
600	137.00.57	0.872155	100000	172.45.44	2.399655

The

The Construction and Use of the General Table.

"AS the Planets move in Elliptic Orbs, so
" do the Comets in Parabolic ones, having
" the Sun in their common Focus, and describe
" equal Areas in equal Times. Now since all
" Parabola's are similar to one another, there-
" fore if any determinate Part of the Area of a
" given Parabola, be divided into any number of
" Parts at liberty, there will be a like division
" made in all Parabola's under the same Angles,
" and the Distances will be proportional: Con-
" sequently this one Table of ours will serve for
" all Comets." Thus far Dr. *Halley*.

But it is to be noted, that our famous Author doth not assert in this Place the Trajectories of Comets to be compleatly Parabolical; but only means, that whereas they are indeed Elliptical, they are withal so Eccentrical, that that Part of the Orbits of Planets which respects the Planetary World, and which we the Inhabitants of this Earth can see, doth so little differ from the curvest Part of a Parabolic Line, that it may safely, and without any sensible Error, be assum'd to be a Parabola. For it was before noted, that there may be Ellipses of all Species, and that the Concentrical do at length degenerate into Circles, the infinitely Eccentrical into Parabola's. Nor is it therefore to be wonder'd at, if instead of an Ellipsis, a Figure of more difficult Contemplation, and generally of an unknown Species, we chuse to use a Parabola, a Figure more easy to be Contemplated, and of one Species only; in that Place, espe-

 cially

cially where the Phænomena of Comets mark out to us Trajectories ſcarce other than Parabolical. We have before ſhew'd that the Proportionality of the Area to the Time is common to Comets as well as Planets; and ſhall not go over the ſame Thing again. It is alſo manifeſt, that like Figures, as Circles and Parabola's do admit and require, that the like Diviſions of them, or their Proportional or Correſpondent Parts ſhould be expreſs'd by the ſame Numbers. But let our Author proceed.

" Now the Manner of the Calculation of this " Table is thus: In *Fig.* 5. *Plate* 9. let S be " the Sun, POC the Orbit of a Comet, P the " Perihelion, O the Place where the Comet is " 90 gr. diſtant from the Perihelion, C any o- " ther Place. Draw the Right Lines C P, C S, " and make S T, S R, equal to C S; and having " drawn the Right Lines C R, C T, (whereof the " one is a Tangent, and the other a Perpendicu- " lar to the Curve) let fall C Q Perpendicular " to the Axis P S R.

It is here as it is in the Planetary Aſtronomy, (where we firſt enquire the Place of the Planet, or the Angular Diſtance from the Axis of the Ellipſis, which we call the *True Anomaly* of the ſame, together with the abſolute *Diſtance* from the Sun); Even ſo here, we muſt in the firſt Place find out the like Angle and Diſtance. But it is to be noted, that according to the Nature of all Parabola's, the Line S O is half the *Latus Rectum.* S P is a 4th Part of the ſame *Latus Rectum*, or half of S O; and that a Tangent C T being drawn unto any Point C, and there being erected Perpendicular to the ſame the Line C R, cutting the Axis; and there being let down from the ſame Point,

Point C, to the Axis, the Perpendicular CQ cutting the Axis in Q; SC, SR and ST are equal amongst themselves; and the Line QR is equal to SO, or half the *Latus Rectum*. All which Things are well known from the Conics. But our Author proceeds.

"Now any Area, as COPS being giv'n, it "is requir'd to find the Angle CSP, and the Di-"stance CS. From the Nature of the Parabola "RQ is ever $= \frac{1}{2}$ the Parameter or *Latus Rectum* "of the Axis, and consequently if the Parame-"ter be put $= 2$, then $RQ = 1$. Let $CQ = z$; "PQ shall $= \frac{1}{2}zz$, and the Parabolic Segment "$COP = \frac{1}{12}zzz$; But the Triangle CSP will "$= \frac{1}{4}z$, and so the Mixtilineal Area $COPS =$ "$\frac{1}{12}z^3 \times \frac{1}{4}z = a$, whence $z^3 \times 3z = 12a$. Where-"fore resolving this Cubical Equation, z, or the "Ordinate CQ will be known.

Thus far our Author. But it is to be well observ'd, that we have here the Analytical way for finding the Coequate Anomaly in a Parabola from the Mean Anomaly given, that is, from the Area describ'd, which is every-where Proportional to the Time of the Description. Nor can the Angle CST, or the Coequate Anomaly be found directly from the Given Area or Mean Anomaly, without Analysis. But that upon the Hypothesis, that the Line CQ, which is first to be sought, (for when that is found, the Angle CST will easily be found, as will be manifest presently) be called z, the Line PQ will be equal to $\frac{1}{2}zz$, is easy to be demonstrated: For as $RQ = 1$, is to $CQ = z$; so the same $CQ = z$ is to QT, or zz, the half whereof consequently QP will be equal to $\frac{1}{2}zz$. But that the Parabolic Segment COP, according to the same Hypothesis, will be

 rightly

rightly expreſs'd by $\frac{1}{12}zzz$, eaſily follows from the Conics. For the Area COPSQ is to the Triangle CPQ, or CPT (equal to the ſame) as 4 to 3; and conſequently the Parabolic Area COP is to CPQ as 1 to 3; and ſince the Triangle CPQ made of the Perpendicular CQ or z, drawn into half the Baſe $\frac{1}{4}zz$, becomes $\frac{1}{4}zzz$, the 3d Part of it will be $\frac{1}{12}zzz$, equal to the Parabolic Area COP. The Triangle CSP alſo is made of the Perpendicular z, drawn into half the Baſe $\frac{1}{2}$, equal to $\frac{1}{4}z$; and conſequently the Sum of the Area's COP and CSP, or the whole Area COPS, Proportional to the Time, will be equal to the Sum of theſe Quantities, which is called a: or there will ariſe this Equation $\frac{1}{12}z^3 * \frac{1}{4}z * = a$; and by multiplying on both Sides by 12, $z^3 + 3z = 12a$; which is a Cubic Equation, the 2d and 4th Terms whereof are wanting. The Root therefore of this Equation being found, or the Value of z being found in Numbers by Dr. *Halley*'s Method, or otherwiſe, the Length of the Line CQ will be known. *Q. E. I.* And now let us hear our Author himſelf.

Dr. *Halley*. "Now let the Area OPS be pro-"poſ'd to be divided into One Hundred Parts; "this Area is $\frac{1}{12}$ of the Square of the Parameter, and "conſequently 12 a is equal to that Square $= 4$. "If therefore the Roots of theſe Equations $z^3 + 3z = 0{,}04 : 0{,}08 : 0{,}12 : 0{,}16$, *&c.* be ſuc-"ceſſively extracted, there will be obtain'd ſo "many z or Ordinates CQ reſpectively, and "the Area SOP will be divided into One Hun-"dred equal Parts. And in like manner is the Cal-"culation to be continued beyond the Place O. "Now the Root of this Equation (ſince RQ is "$= 1$) is the Tabular Tangent of the Angle "CRQ, or of $\frac{1}{2}$ the Angle CSP, wherefore the

"Angle

" Angle C S P is given. And R C, the Secant " of the same Angle C R Q, is a mean Propor- " tional between R Q, or Unity and R T, which " is the double of S C, as is plain from the Co- " nics. But if S P be put = 1, and so the *Latus* " *Rectum* = 4 (as in our Table) then R T will " be the Distance sought, *viz.* the double of S C " in the former Parabola. After this manner " therefore, I compos'd the foregoing Table, " which serves to represent the Motions of all " our Comets; of which hitherto there has been " none observed, but those that come within " the Laws of the Parabola.

Now that the Area O P S is a twelfth Part of the Square of the *Latus Rectum*, it is manifest: Because, according to the Conics, the Area O P S is $\frac{2}{3}$ of the Rectangle of O S, multiplied by S P; that is, of the Rectangle of half the *Latus Rectum*, multiplied by a 4th Part of the same. For $\frac{2}{3} \times \frac{1}{8} = \frac{1}{12}$. But any Numbers, as 4. 8. 12. 16. If they be put in the 2d Place of Decimals, as here is done, will rightly express 100th Parts. And we are therefore content with a Right Angle as the principal Guide of Computation, because we want an entire Period in Parabola's. But because of the equal Angles S C, S R, the external Angle of the Isosceles Triangle C R S will be equal to the double Angle C R S. And there being giv'n consequently by the Tables of Tangents the Angle C R Q, the Double thereof, or the Angle C S T; that is, the Coequate Anomaly of the Comet is found. In like manner, the Angle C S T being now giv'n, if you make by the Golden Rule: As R Q = 1 is to the Secant of that Angle to be taken out of the same Tables; so that Secant is to the 3d Proportional R T; The half hereof R S is equal to S C, or to the Distance of the Comet from the Sun. *Q. E. I.*

Nov. 29. 1708.

LECT. XL.

Dr. *Halley*. "It now remains, that we "give the Rules for the Calculation, "to shew the way of determining the "Visible Place of a Comet, by these "Numbers. *The Velocity of a Comet* "*moving in a* Parabola, *is every-where to the Velo-* "*city of a Planet describing a Circle about the Sun, at* "*the same Distance from the Sun*, *as* $\sqrt{2}$ to 1. as "appears from *Cor.* 7. *Prop.* 16. *Lib.* 1. of the "*Princip. Phil. Nat. Math.* If therefore a Comet "in its Perihelion were suppos'd to be as far di- "stant from the Sun as the Earth is, then the "Diurnal Area which the Comet wou'd describe, "woud be to the Diurnal Area of the Earth, as "$\sqrt{2}$ to 1. And consequently, the Time of the "Annual Revolution, is to the Time in which "such a Comet wou'd describe the Quadrant of "its Orbit from the Perihelium, as 3.14159, *&c.* "(that is the Area of the Circle) to $\sqrt{\frac{8}{9}}$.

That the Velocity in a Parabola is to the Velocity at the same Distance in a Circle, as $\sqrt{2}$ is to 1, or as 10 to 7 almost, was demonstrated in Prop. XXII. foregoing; or rather deduced

deduced as it were a Corollary, from the Nature of Circular and Parabolic Curvity, and the Proportion of the Subtenses of the Angle of Contact. But the Annual Time in an Elliptic Circle, or the Time of an Entire Revolution, represented by the whole Area of a Circle, which is to be estimated from the Multiplication of half the Circumference by the Radius; will be to the Time of the Description of a Quadrantal Arch in a Parabola, which is to be *represented by a Quadrantal Area of the Parabola, to be estimated from the Multiplication of* $\frac{2}{3}$ *of half the* Latus Rectum, by a quarter of the same *Latus* or *Radius*; as the Area's themselves; or as the Heighths of the Rectangles to the Common Base; only so far as the Velocity of Description in a Parabola doth disturb and diminish that Proportion of the Times, in the Proportion of 1 to $\sqrt{2}$; and therefore instead of $\frac{2}{3}$, let $\sqrt{\frac{4}{9}}$ be taken: and let the Numerator be doubled, because of the Square Number two, the double of Unity; that is, for the Circle, let the Area of it 3,141,59, be taken, for the Parabola $\sqrt{\frac{8}{9}}$. And thus the Truth of our Author's Reasoning will easily be understood.

Dr. *Halley*. "Therefore the Comet wou'd describe "that Quadrant in 109 Days, 14 Hours, 46 Mi-"nutes; and so the Parabolic Area (analogous "to the Area POS) being divided into One "Hundred Parts, to each Day there wou'd be al-"lotted 0.912.280 of those Parts, the Log. of "which, *viz.* 9.960128, is to be kept for con-"tinual use. But then the Times in which Co-"mets, at a greater or less Distance, wou'd de-"scribe similar Quadrants, are as the Times of "the Revolutions in Circles, that is, in the Ses-"quiplicate Ratio of the Distances: Whence the

"Diur-

" Diurnal Areas estimated in Centesimal Parts of
" the Quadrant (which Parts we put for Mea-
" sures of the mean Motion, like Degrees) are
" in each, in the Subsesquialtera proportion of
" the Distance from the Sun in the Perihe-
" lion.

Mr. *Whiston.* The Mean Diurnal Motion, to wit, 0,912,280, to be express'd by a Negative Logarithm after the Old Manner— 0,039,872, is in this Place express'd in a New Way by a Positive One 9,960,128, to avoid the Difficulty about the Negative Characteristic; but is presently made equivalent to the wonted Form, by casting away Ten in the Addition when occasion shall require. But our Author observes here rightly, that in divers Parabola's, a Quadrant is always reckon'd of the same Number of Parts, I mean an Hundred; in such Sort nevertheless, that those Parts be indeed unequal, and according to the Magnitude of the Parabola greater or less, but not in that Proportion greater or less, in which the Distances increase or decrease from the Sun, but in the Subsesquialteral Proportion of the same: So that the Squares of the Distances be betwixt themselves, as the Cubes of these Parts reciprocally.

Dr. *Halley.* " These necessary Things premis'd,
" let it be propos'd to compute the apparent Place
" of any one of the forementioned Comets for a-
" ny given Time. Therefore,

[" *1. Let the Sun's Place be had, and the Log. of its Distance from the Earth.*

2. Let the difference between the Time of the Perihelion and the Time given be gotten, in Days and Decimal Parts of Days. To the Log. of this Number, let there

there be added the constant Log. 9.960.128, *and the Complement Arithmetical of three halves of the Log. of the Perihelion Distance of the Comet from the Sun: The Sum will be the Log. of the mean Motion, to be sought in the first Column of the general Table.*

3. *With the mean Motion let there be taken the correspondent Angle from the Perihelion in the Table, and the Log. for the Distance from the Sun: Then in Comets that are Direct, add, and in Retrograde ones subtract; if the Time be after the Perihelion, the Angle thus found, to or from the Perihelion: or in Direct Comets, substract; and in Retrograde ones add; if the Time be before the Perihelion, the foresaid Angle to or from the Place of the Perihelion; and so we shall have the Place of the Comet in its Orbit. And to the Log. for the Distance found, let there be added the Log. of the Distance at the Perihelion, and the Sum will be the Log. of the true Distance of the Comet from the Sun.*

4. *The Place of the Node, together with the Place of the Comet in its Orbit, being given, let the Distance of the Comet from the Node be found; then the Inclination of the Plane being given, there will be given also (from the common Rules of Trigonometry) the Comet's Place reduced to the Ecliptic, the Inclination or Heliocentric Latitude, and the Log. of the Curt Distance.*

5. *From these things given (by the very same Rules that we find the Planet's Places, from the Sun's Place and Distance given) we may obtain the Apparent or Geocentric Place of the Comet, together with the Apparent Latitude. And this it may be worth while to illustrate by an Example or two."*]

As

As to the Place of the Sun, and the Distance thereof from the Earth, we have elsewhere taught how to find both by Astronomical Calculation. But the Logarithms of the Distances, we through some neglect, omitted in that Place; and therefore shall add them in the End of this Work. But the Logarithm of Days is therefore added to the Given Logarithm of one Day, that the Motion of one Day may be understood to be multiplied by the Number of Days: For it is known, that the Addition of Logarithms doth infer the Multiplication of Numbers corresponding to those Logarithms.

And these Things may suffice, if so be the Comet be suppos'd to pass in its Perihelion at a Distance equal to a Radius of the great Orb. But if, which commonly is the Case, the Comet doth not pass at that Distance, but at a greater, as it is sometimes; or at a less, as oftner happens; that Area, proportional to the Time, is to be increas'd or diminish'd; and this in the Sub-sesquialteral Proportion of that least Distance from the Sun; so that at length that Area may rightly represent the Mean Anomaly. From whence the Logarithm of that Sesqui-plicate Distance is to be added to the former Sum of the Logarithms, and the Radius to be substracted according to the Exigence of the Golden Rule, to be practis'd in Logarithms; or which is the same, the Arithmetical Complement only of that Sesqui-alteral Logarithm is to be added. Neither ought it to seem strange, that in lesser Distances we, by adding the Logarithm, obtain the true Proportion increas'd, and the same in greater Distances diminish'd: For Multiplication by a Fraction, or Decimal

cimal Parts, doth no less diminish the Sum, than Multiplication by whole Numbers doth increase it. And the Thing is the same in Logarithmetical Addition. But we are to observe, that the Logarithms marked in the 3d Column of the General Table, are not the Logarithms of the Numbers of the Distances from the Sun, to be added over and above the Radius to the Mean Distance; but of Numbers, by the Multiplication of which, that true Distance were to be obtained. From whence the Logarithms of the same being superadded to one another, will easily give us the Logarithm of that whole Distance from the Sun. These Things being well understood, we shall be able to undertake and perform the Calculation.

EXAM-

EXAMPLE I.

Let it be requir'd to find the Place of the Comet of the Year 1664, March 1^d, 7^h, 00', *P. M.* London. *That is*, 96^d, 19^h, 8', *after the Perihelion, which happen'd* Novemb. 24°, 11^h, 52'.

Log. Dift. Perihel.	0. 011044
Log. Sesquialt.	0. 016566
Comp. Arith.	9. 983434
	9. 960128
Log. Temp.	1. 985862
Log. Med. Mot.	1. 929424
Medius Motus	85.001
Perihel. ♌	10. 41. 25
Ang. Correfp.	83. 38. 05 —
Comet in Orb. ♉	17. 3. 20
Afcend. Nod. ♊	21. 14. 00
Com. à Nodo	34. 10. 40
Red. ad Eclip.	32. 19. 05
Com. Helioc. ♉	18. 54. 55
Incl. Bor.	11. 46. 50
Log. pro dift.	0. 255369
Log. Perihel.	0. 011044
Co-fin. Incl.	9. 990754
Log. dift. Cur	0. 257167
Log. dift. ☉	9. 997918
☉ ♓	21. 44. 45
Com. Vifus ♈	29. 18. 30
Lat. Vifa (Bor.)	8. 36. 15

EXAM-

EXAMPLE II.

Let it be requir'd to find the Place of the Comet of the Year 1683, July 23°, 13^h, 35', *P. M.* London; *Or*, 13^h, 40' *Equat. Time. That is*, 21^d, 10^h, 50' *after the Perihelion.*

Log. Dist. Perihel.	9. 748343
Log. Sesquialt.	9. 622514
Comp. Arith.	0. 377486
	9. 960128
Log. Temp.	1. 310723
Log. Med. Mot.	1. 648337
Medius Motus	44. 498
Perihel. ♊	25. 29. 30
Ang. Corresp.	56. 47. 20
Comet. in Orb. ♈	28. 42. 10
Nod. Descend. ♓	23. 23. 00
Com. à Nodo	35. 19. 10
Red. ad Eclip.	4. 48. 30
Com. Helioc. ♓	28. 11. 30
Incl. Bor.	35. 2. 00
Log. pro dist.	0. 111336
Log. Perihel.	9. 748343
Co-sin. Incl.	9. 913187
Log. dist. Curt.	9. 772866
Log. dist. ☉	0. 006104
☉ Locus ♌	10. 41. 25
Com. Visus ♋	5. 11. 50
Lat. Bor.	28. 52. 00

But

But now, that we may rightly perform this Calculation; It is to be noted,

(1.) That the Logarithm of the least, or perihelion Distance, is only set down here, that we may obtain the other Logarithm, which is Sesquialteral of the same, or is thereto as 3 to 2.

(2.) That the Arithmetical Complement of this last Logarithm being added to the Constant Logarithm of one Day, doth make the Logarithm of the whole Time before or after the Perihelion. For working by Logarithms, the Numbers in the former of the Examples will be thus. The Logarithm of one Day is 9,960,128; and the Logarithm of Days is 1,985,862. These alone being added together, would make the Logarithm of the Mean Motion, if the Perihelion Distance were equal to Unity, or the Radius of the great Orb: But when the Area of that Mean Motion is to be increas'd in the Proportion of that Sesquialteral Perihelion Distance to the Radius of the Annual Orbit, that Sesquialteral Logarithm 0,016,566, is to be added to the former Logarithm; and the Logarithm of the Number 10 is to be substracted; or, which comes to the same, the Arithmetical Complement of the Sesqui-alteral Logarithm is only to be added: which is done in this Place. Now the Mean Motion will easily be known, when the Logarithm of the same is given.

(3.) The Mean Motion, or Mean Anomaly, being now given, the Angle Corresponding thereto in the General Table, is 83°. 38′. 5″. (the intermediate proportional Parts being every-where found, where there is Occasion, by the Golden Rule.) Which being deducted from the Place of the Perihelion in *Leo* 10°. 41′. 25″, because of the Retrograde Motion of the Comet giveth

us

us the Place of the Comet in its own Orb, 17°. 3′. 20″, in *Taurus*.

(4.) Substract this Place from the Place of the descending Node in *Gemini*; the Remainder will be the Distance of the Comet from the Node, 34°. 10′. 40″.

(5.) And now that we may reduce the Place of the Comet in its own Orb to the Ecliptic, we must resolve a Rectangular Spherical Triangle; and from the Given Angle and the Hypotenuse, must find the other Sides. And for Reduction to the Ecliptic, for the Heliocentrical Longitude, the following Analogy will suffice.

As Radius		10,000,000
is to Cosin.of the Ang.	21°. 18′. 30″.	9.969.248
So is Tangent,	34. 10. 40.	9.831.890
To Tangent,		9.801.138=
		=32°. 19′. 5″

Then for the Inclination, or Heliocentrical Latitude,

As Radius,		10.000.000
To Sin.	34°. 10′. 40″.	9.749.553
So is Sine of the giv'n Ang.	21. 18. 30	9.560.369
To Sine of Ang. sought		9.309.922=
		11°. 46′. 44.

(6.) For obtaining the Logarithm of the true Distance of the Comet from the Sun, we must add the Logarithm for the Distance from the Sun, which in the General Table belongs to the Mean Motion, to the Logarithm of the least or perihelium Distance; that is, 0.255.369, to 0.011.044, which make 0.266,413. And then say,

As Radius	10.000.000
To Dist. from Sun	0.266.413

So is Co-sin of Inclin. —— 9. 990. 754
To Curt. Dist. —— —— 0. 257. 167

Or, which comes to the same; the three Logarithms are to be added, and the Logarithm of the Radius to be cast away; as is done in our Examples.

(7.) For obtaining the Geo-centrical Longitude of the Comet, or the visible Place in the Ecliptic, do thus. Substract the Helio-centrical Longitude 1s. 18°. 54′. 55″ out of the true Place of the Sun in the Ecliptic 11s. 21°. 44′. 45″; there will Remain the Angle of Commutation 10s. 2°. 49′. 50″; the Complement whereof unto a Circle is 1s. 27°. 10′. 10″, or 57°. 10′. 10″. The half hereof is 28°. 35′. 5″. From whence say,

As dist. of the Earth —— 9.997.918
To Curt. Dist. of the Com. 10.257.167
So is Radius —— —— 10.000.000
To Tangent —— —— 10.259.249 = 61°.10′. 3″

Now 45 Deg. being cast away, there rests 16.10.3. Therefore,

As Radius —— —— 10.000.000
To Tang. 16°. 10′. 3″. —— 9.462.265
So is Tang. of Semi-Sum.-- 9.736.294 = 28°. 35′. 5″.
To Tang. of Semi-differ. — 9.198.559 = 8.58.36.

Which Half-difference being taken away out of the Half-Sum, there remains 19°. 36′. 29″; that is, the Parallax of the Orb. But the Parallax being in this Case substracted from the Heliocentrical Place of the Comet, the Geocentrical Place of the same is ♈. 29. 18°. 26. Something more exactly, as I suppose, than our Author's Calculation hath it.

But if the Curtated Distance of the Comet from the Sun be less than the Distance of the Earth from the Sun, as it is in the other Example, we must work

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in the Calculation, as is done for the Inferior Planets, (like as we have Calculated here, as we do for the Superior.) And the Half-difference of the Angles, which in that Case will represent the Elongation from the Sun, is to be added to the Longitude of the Sun in the Ecliptic, or substracted from the same, for obtaining the Geocentrical Place of the Comet.

(8.) For determining the Geocentrical Latitude of the Comet, we are to Work thus; (the Angle of Elongation being made up of the Aggregate of the Half-Sums.)

	° ′ ″	
As Sin. of Ang. of Commut.	57.10.10.	9.924.423
Is to Sin. of Ang. of Elong.	37.33.41.	9.785.053
So is Tang. of Inclination—	11.46.44.	9.319.161
To Tang. of Latitude ——	(8.36.09.)	9.179.791

"At the Instant of Time specified in the first "Example, 'twas observ'd (at *London*) that the "Comet applied to the second Star of *Aries*; so "that it was found to be 9′ more Northerly, "and 3′ to the East, according to Dr. *Hook*'s Ob-"servation. But at that of the second Example, "I my self (near *London*, with the same Instru-"ments whereby I formerly observ'd the Southern "Constellations) found the Place of the Comet "to be ♋, 5°, 11′½, and 28°, 52′ North Latitude, "which agreed exactly with the Observation "made at *Greenwich*, almost at the very same Mo-"ment.

"As for the Comet of the Year 1680, which "came almost to the very Sun it self (being in its "Perihelion, not above one third of the Semidia-"meter of the Sun distant from the Surface of it,) "since the *Latus Rectum* of its Orb is so very small, "it could hardly be contained within the Limits

" of the General Table, because of the excessive
" Velocity of the Mean Motion. Wherefore in
" this Comet, the best way will be (after the
" Mean Motion is found) to get from thence (by
" the help of the foregoing Equation $z^3 + 3z = \frac{4}{100}$
" of the Mean Motion) the Tangent of half the
" Angle from the Perihelion, together with the
" Log. for the Distance from the Sun. Which be-
" ing found, we are to proceed by the same Rules,
" as in the rest.

" After this manner therefore, the Astronomi-
" cal Reader may examine these Numbers, which
" I have calculated with all imaginable Care, from
" the Observations I could meet with. And I
" have not thought fit to make them publick be-
" fore they have been by my self duly examin'd,
" and made as accurate as 'twas possible, not with-
" out the Labour of many Years. I have publish'd
" this Specimen of Cometical Astronomy, as a
" *Prodromus* of a future Work I have in design,
" lest, happening to be prevented by Death,
" these Papers might chance to be lost, which e-
" very Man would not be capable to retrieve, by
" reason of the great Difficulty of the Calculation.

" Now it may not be amiss to put the Reader
" in mind, that our five first Comets, (the third
" and fourth observ'd by *Peter Apian*, the fifth by
" *Paulus Fabricius*) as also the tenth, seen by *Mæst-*
" *lin*, if I mistake not, in the Year 1596, are not
" so certain as the rest; for the Observations
" were made neither with sufficient Instru-
" ments, nor due Care, and upon that account are
" disagreeing with themselves, and can by no
" means be reconcil'd with a regular Computus.
" The Comet which appear'd in the Year 1684,
" was only taken notice of by *Blanchinus*, who
" observed it at *Rome*: And the last, which ap-

" pear'd

" pear'd in the Year 1698, was seen only by the " *Parisian* Observers, who determin'd its Course " after a very uncommon manner. This Comet " was a very obscure one, and altho' it mov'd " swift, and came near enough to our Earth; yet " we, who are wont not to be incurious in these " Matters, saw nothing of it. For want of Ob-" servations, I have also left out of the foregoing " Catalogue, those two remarkable Comets which " have appear'd in this our Age, one in *Novem-* " *ber* in the Year 1689, the other in *February* in " the Year 1702. For they directing their Cour-" ses towards the Southern Parts of the World, " and being scarce conspicuous any where in " *Europe*, met with no Observers proper for the " purpose. But if any one shall bring from *India*, " or the Southern Parts, an accurate Series of Ob-" servations, I will willingly fall to work again, " and undergo the Fatigue of representing their " Orbits in Numbers, as I have done the rest.

" By comparing together the Elements of the " Motions of these Comets, 'tis apparent, their " Orbits are dispos'd in no manner of Order; nor " can they, as the Planets are, be comprehended " within a Zodiac; moving indifferently every " way, as well retrograde as direct; from whence " it is clear, they are not carry'd about or mov'd " in a Vortical System. Moreover, the Distances " in their Perihelia are sometimes greater, some-" times less; which makes me suspect, there may " be a far greater Number of them, which may " move in Regions more remote from the Sun, " and being therefore very obscure; and wanting " Tails, may pass by us unseen.

" Hitherto I have consider'd the Orbits of Co-" mets as exactly Parabolic; upon which Suppo-" sition it wou'd follow, that Comets being im-" pell'd towards the Sun by a Centripetal Force,

" would descend as from Spaces infinitely distant, " and by their so falling acquire such a Velocity, " as that they may again fly off into the remotest " Parts of the Universe, moving upwards with a " perpetual Tendency, so as never to return again " to the Sun. But since they appear frequently " enough, and since none of them can be found " to move with an Hyperbolic Motion, or a Mo- " tion swifter than what a Comet might acquire " by its Gravity to the Sun, 'tis highly probable " they rather move in very Eccentric Elliptic Or- " bits, and make their Returns after long Periods " of Time: For so their Number will be determi- " nate, and, perhaps, not so very great. Besides, " the Space between the Sun and the Fix'd Stars is " so immense, that there is room enough for a " Comet to revolve, tho' the Period of its Revo- " lution be vastly long. Now, the *Latus Rectum* of " an Ellipsis, is to the *Latus Rectum* of a Parabola, " which has the same Distance in its Perihelium; " as the Distance in the Aphelium in the Ellipsis, " is to the whole Axis of the Ellipsis. And the Ve- " locities are in a Subduplicate Ratio of the same: " Wherefore in very Excentric Orbits the Ratio " comes very near to a Ratio of Equality; and the " very small difference which happens on account " of the greater Velocity in the Parabola, is easily " compensated in determining the Situation of the " Orbit. The principal Use therefore of this Table " of the Elements of their Motions, and that which " indeed induced me to construct it, is, that when- " ever a new Comet shall appear, we may be a- " ble to know, by comparing together the Ele- " ments, whether it be any of those which has " appear'd before, and consequently to deter- " mine its Period, and the Axis of its Orbit, " and to foretel its Return. And, indeed there " are many things which make me believe that

" the

“ the Comet which *Apian* obſerv'd in the Year 1531, was the ſame with that which *Kepler* and *Longomontanus* more accurately deſcrib'd in the Year 1607; and which I my ſelf have ſeen return, and obſerv'd in the Year 1682. All the Elements agree, and nothing ſeems to contradict this my Opinion, beſides the Inequality of the Periodic Revolutions. Which Inequality is not ſo great neither, as that it may not be owing to Phyſical Cauſes. For the Motion of *Saturn* is ſo diſturbed by the reſt of the Planets, eſpecially *Jupiter*, that the Periodic Time of that Planet is uncertain for ſome whole Days together. How much more therefore will a Comet be ſubject to ſuch like Errors, which riſes almoſt four times higher than *Saturn*, and whoſe Velocity, tho' increaſed but a very little, would be ſufficient to change its Orbit, from an Elliptical to a Parabolical one. And I am the more confirmed in my Opinion of its being the ſame; for that in the Year 1456, in the Summer-time, a Comet was ſeen paſſing Retrograde between the Earth and the Sun, much after the ſame manner: Which tho' nobody made Obſervations upon it, yet from its Period and the manner of its Tranſit, I cannot think different from thoſe I have juſt now mention'd. And ſince looking over the Hiſtories of Comets I find, at an equal Interval of Time, a Comet to have been ſeen about *Eaſter* in the Year 1305, which is another double Period of 151 Years before the former: Hence I think I may venture to foretel, that it will return again in the Year 1758. And, if it ſhould then ſo return, we ſhall have no reaſon to doubt but the reſt may return alſo: Therefore Aſtronomers have a large Field wherein to exerciſe themſelves for many Ages, before they will be able to know the Number of theſe many and great Bodies revolving about the common Center of the Sun, and to reduce their Motions to certain Rules. I thought indeed that the Comet which appear'd in the Year 1532, might be the same with that observ'd by *Hevelius* in the Year 1661. But *Apian*'s Observations, which are the only ones we have concerning the first of these Comets, are too rude and inaccurate for any thing of certainty to be drawn from them, in so nice a matter. But as far as probability from the equality of Periods, and similar appearance of Comets, may be urged as an argument, the late wondrous Comet of $168\frac{0}{1}$, seems to have been the same, which was seen in the Time of our King *Henry* I. *Anno* 1106, which began to appear in the *West* about the middle of *February*, and continued for many Days after, with such a Tail as was seen in that of $168\frac{0}{1}$. And again in the Consulate of *Lampadius* and *Orestes*, about the Year of Christ 531, such another Comet appeared in the *West*, of which *Malela*, perhaps an Eye-witness, relates that it was μέγας ϗ φοβερὸς, *a great and fearful Star*; that it appeared in the *West*, and emitted upwards from it a long white Beam; and was seen for 20 Days. It were to be wish'd the Historian had told us what Time of the Year it was seen; but 'tis however plain, that the Interval between this and that of 1106, is nearly equal to that between 1106 and $168\frac{0}{1}$, *viz.* about 575 Years. And if we reckon backward such another Period, we shall come to the 44th Year before *Christ*, in which *Julius Cæsar* was murder'd, and in which there appear'd a very remarkable Comet, mentioned by almost all the Historians of those Times, and by *Pliny* in his Natural History, *lib.* 11. *c.* 24. who recites the Words of *Augustus Cæsar* on this Occasion, which lead us to the very Time of its Appear-

"ance,

ance, and its Situation in the Heavens. These Words being very much to our purpose, it may not be amiss to recite them. *In ipsis Ludorum meorum diebus, sydus crinitum per septem dies, in regione Cœli quæ sub Septentrionibus, est conspectum. Id oriebatur circa undecimam horam diei, clarumq; & omnibus terris conspicuum fuit.* Now these *Ludi* were dedicated *Veneri genetrici*, (for from *Venus* the *Cæsars* would be thought to be descended,) and began with the Birth-day of *Augustus*, viz. *Sept.* 23. (as may be collected from a Fragment of an Old *Roman* Calendar extant in *Gruter*, pag. 135.) and continued for 7 Days, during which the Comet appeared. Nor are we to suppose that it was seen only those 7 Days, but possibly both before and after. Nor are we to interpret the Words *sub Septentrionibus*, as if the Comet had appear'd in the *North*, but that it was seen under the *Septem triones*, or brighter Stars of *Ursa major*. And as to its rising *Hora undecima diei*, it can no ways be understood, unless the word *diei* be left out, as it is by *Suetonius*; for it must have been very far from the Sun, either to rise at Five in the Afternoon, or at Eleven at Night; in which Cases it must have appeared for a long time, and its Tail have been so little remarkable, that it could by no means be call'd, *Clarum & omnibus Terris conspicuum Sydus.* But supposing this Comet to have traced the same Path with that of the Year 1680, the ascending part of the Orb will exactly represent, all that *Augustus* hath said concerning it; and is yet an additional Argument to that drawn from the Equality of the Period. Thus 'tis not improbable but this Comet may have four times visited us at Intervals of about 575 Years: Whence the Transverse Diameter of its Elliptic Orb will be found $\sqrt[3]{575\times575}$ times greater than the annual Orb;

“ or

" or 138 times greater than the mean Diſtance
" of the Sun; which Diſtance, tho' immenſely
" great, bears no proportion to that of the Fix'd
" Stars.

" I have lately found out a ready Method to
" compute the Motion of Comets in theſe Ellip-
" tic Orbs, of which perhaps ſhortly we may ex-
" hibit a Specimen, giving this Comet for an Ex-
" ample. In the mean time, thoſe that deſire to
" know how to conſtruct Geometrically the Orb
" of a Comet, by three accurate Obſervations gi-
" ven, may find it at the End of the 3d Book of
" Sir *Iſaac Newton*'s Principles of Natural Philoſo-
" phy, entituled *De Syſtemate Mundi*, in the Words
" of its renowned Inventor. Which have ſince been
" more fully explain'd by my very worthy Col-
" legue Dr. *Gregory*, in his learned Work of *Aſtro-*
" *nomia Phyſica & Geometrica.*

" One thing more perhaps it may not be im-
" proper or unpleaſant to advertiſe the Aſtrono-
" mical Reader; That ſome of theſe Comets have
" their Nodes ſo very near the Annual Orb of the
" Earth, that if it ſhall ſo happen, that the Earth
" be found in the Parts of her Orb next the Node
" of ſuch a Comet, whilſt the Comet paſſes by;
" as the apparent Motion of the Comet will be
" incredibly ſwift, ſo its Parallax will become
" very ſenſible; and the proportion thereof to
" that of the Sun will be given. Wherefore ſuch
" Tranſits of Comets do afford us the very beſt
" means, tho' they ſeldom happen, to determine
" the Diſtance of the Sun and Earth: Which hi-
" therto has only been attempted by *Mars* in his
" Oppoſition to the Sun; or elſe *Venus* in Peri-
" gæo, whoſe Parallaxes, tho' triple to that of the
" Sun, are ſcarce any ways to be perceived by
" our Inſtruments; whence we are ſtill in great
" Uncertainty in that Affair. This Uſe of Comets
" was

“was the ingenious Thought of that excellent Geometrician Mr. *Nicolas Fatio.* Now the Comet of 1472, had a Parallax above twenty Times greater than the Sun's. And if the Comet of 1618, had come down, about the middle of *March*, to his descending Node; or if that of 1684, had arriv'd a little sooner at its ascending Node, they would have been yet much nearer the Earth, and consequently have had more notable Parallaxes. But hitherto none has threaten'd the Earth with a nearer Appulse, than that of 1680. For by Calculation I find, that *Novemb.* 11°, 1h, 6', *P. M.* that Comet was not above the Semidiameter of the Sun to the Northwards of the Way of the Earth. At which time, had the Earth been there, the Comet would have had a Parallax equal to that of the Moon, as I take it. This is spoken to Astronomers: But what might be the Consequences of so near an Appulse; or of a Contact, or, lastly, of a Shock of the Celestial Bodies, (which is by no means impossible to come to pass,) I leave to be discuss'd by the Studious of Physical Matters.

FINIS.

ERRATA.

PAge 4. Line 15. dele , p. 5. l. ult. dele *made less by the Distance H I.* p. 10. l. 5. dele X. l. 28. read *m I.* p. 11. l. 22. r. *Plate* 1. *Fig.* 3. p. 12. l. 12. dele *or R Z.* l. 13. r. *P l and K T.* l. 17. del. *T.* p 13. l. 26. r. *since.* l. 29. r. *Semi-ordinates.* p. 15. l. 21. r. *as long again as.* p. 17. l. 8. r. *L L.* l. 25. r. *of the.* p. 18. l. 4. 5. del. *whether of the following Sections, or of the former.* l. 32. r. *K H. D H.* l. 34. r. *i b, h b. i a. h a.* p. 19. l. 8, 9. r. *double.* p. 21. l. 9. r. *Fig.* 6. p. 27. l. 20. del. *should be laid down.* l. 32. del. *so.* p. 28. l. 31. del. *therefore.* p. 29. l. 5. del. *not.* l. 6. del. *to.* p. 31. l. penult. r. 21. l. ult. r. *therefore where the ambient Bodies are moved, those that relatively rest in those ambient Bodies are really moved.* p. 38. l. 6.

ERRATA.

l. 6. r. *a certain*. p. 43. l. 17. del. *the* l. 33. r. *of*. p. 44. l. 11. r. *the*. p. 46. l. 13. r. *it*. p. 56. l. 18. r. *coming with*. p. 61. l. 32. r. $\frac{8}{5}$ or $1\frac{3}{5}$. l. 37. r. $\frac{12}{5}$ or $2\frac{2}{5}$. p. 70. l. 29. r. *this*. p. 74. l. 9. r. *Axels, Ropes, Strings*. p. 77. l. 27. r. *CE*. p. 78. l. 7. r. *Corollary after*. p. 88. l. 14. r. *CHB*. p. 94. l. ult. r. *if in*. p. 97. l. 11. r. *Plate 3. Fig. 5*. p. 98. l. 19. r. *db, DB*. p. 99. l. 31. r. *Eb*. p. 105. l. 21. r. *ax*. l. 26. r. *ed*. l. 31. r. *gq*. p. 106. l. 25. r. *a b1*. l. 29. r. *1b*. l. penult. r. *Lines*. p. 108 l. 20. del. *bg*. p. 113. l. 1. r. *are*. l. 9. r. *EH*. l. 29. r. *were the Descent*. p. 116. l. 8. r. *are*. l. 21. r. *EA*. p. 120. l. penult. r. *Te*. p. 121. l. 9. r. *I Te*. l. 30. r. *Fl*. l. 35, 36. r. *l* and *m*. p. 122. l. 3. r. *Tb*. p. 123. l. 18. r. *cr*. p. 124. l. 5. del. *D*. l. 15. r. *sF, sg will be lesser*. p. 126. l. 23. r. △ *i*. l. penult. r. △ *i squared, so is the Quadruple of the*. p. 127. l. 30. del. *of*. p. 128. l. 2. r. *to double*. l. 28, 29. r. *sg, the longest horizontal Range is the half*. p. 129. penult. r. *or* 90 + 49. p. 131. l. 31. r. *mP*. l. penult. r. *in n*. p. 134. l. 16. r. *Fig. 6*. p. 140. l. 13. r. 1. l. 20, 21. r. *the Dimidiate or Subduplicate of* 81 *to* 9. i. e. *that of* 81 *to* 27. p. 144. l. 21. r. *along Bc*. p. 150. l. 8. r. *nearer*. p. 152. l. 30. r. *a*. p. 153. l. 13. r. *Point D*. p. 161. l. 21. r. *Pf*. p. 166. l. 3. r. *Fig. 3*. l. 12. r. *be taken as the Distance, as*. p. 168. l. 10. r. $\frac{pq}{xi}$ PF. l. 20. r. *as also do pq and xi*. p. 169. l. 11, 12. r. *at the same Distance*. l. 15. r. *the*. p. 171. l. 9 r. *will*. p. 175. l. 6. r. *a*. l. penult. r. *ABd*. p. 176. l. 14. r. +. l. 17. r. *AD*. l. 24 r. 18. r. 50. l. 26. r. 1130. *or*. p. 177. l. penult. r. 1 × 10. p. 178. l. 5. r. *BD*. p. 180. l. 2. r. *ARPB*. p. 181. l. 16. r. *ARPV*. p. 185. l. 17. r. 2⌊19. l. 25. r. *Fh*. l. antepen. r. *sq*. p. 186. l. 3. r. ⌊57. p. 187. l. 1. r. *h*. l. 21 r. 4672⌊85. l. 22, 25. r. *Fh*. l. 32 del. T. p. 188. l. 5. r. *the Number* 2. p. 192. l. 29. r. *Coroll*. 1. p. 193. l. 2. del. *and*. p. 196. l. 34. del. *which*. p. 197. l. 25. r. *a*. p. 200, 204, &c. r. *Syzygies*. p. 210. l. 28, 29. r. *nine*. p. 219. l. 12. r. *Attraction*. p. 237. l. 32. r. *pag*. l. 34. r. 31. p. 241. l. 19. r. *HI, KL*. p. 256. l. 31. r. ⊥ l. 32. del. *PS*. p. 260. l. 21. r. *Decemb*. 3. 1705. l. ult. r. *on the same Side of the Plane*. p. 262. sæpe for + r. ×. l. 29. r. *that of the*. p. 265. l. penult. r. *Angle of Incidence*. p. 269 l. 11. del. *B*. l. 13. Marg. del. *Lat*. p. 270. l. 11. del. *O*. l. 31. & p. 271. l. 7, 10, 11, 12. instead of Glass r. *Spectrum*. p. 273. l. ult. r. *For*. p. 279. l. 22. add *and* I × *NF will be equal to* 3 *R* × *NP. or I* : 3 *R* : : *NP* : *NF*. p. 280. l. 4. r. *as well as*. l. 8. r. *the doubled arc Ff*. p. 284. l. 34. r. *internal*. p. 286. l. 18. r. *a*. p. 303. l. antepenult. r. *the same in*. p. 305. l. ult. r. *p.* 106. p. 321. l. 22. del. *to*. p. 345. r. 227500. 220. 94. p. 347. r. 123. 100. $25\frac{1}{2}$. 19. 15. p. 365. l. 1. del. *not*. l. 31. r. *two or three Hours*. p. 372. l. 4. r. *Now*. p. 375. l. 4. r. *scarce to*. p. 383. l. 24. r. *QC*.